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HEAT TRANSFER TO A BOILING LIQUID;
MECHANISM AND CORRELATIONS

Progress Report No. 7

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Heat Transfer to a Boiling Liquid;

Mechanism and Correlations

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Abstract

Various heat transfer mechanisms which have been previously proposed are analyzed in the light of recent experiments. Evidence is presented in favor of a liquid-vapor exchange mechanism and against the widely accepted "micro-convection" mechanism. The vapor-liquid exchange mechanism is shown to explain the insensitivity of boiling heat flux to the level of sub-cooling. A "Reynolds Analogy" for nucleate boiling is presented in some detail. A procedure is given for calculating the superheat at which the liquid bulk velocity ceases to contribute to the heat flux. An expression for the growth of a vapor bubble in a highly superheated liquid is deduced. A method is presented which allows the deduction of correlations for nucleate boiling which give the dependence of heat flux on superheat and system pressure. Two such correlations are presented and results are compared with experiment. It is shown that one correlation yields the heat flux for different liquids varying from water to mercury, without necessitating any change in constant or exponent of the correlation.

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NOMENCLATURE

$\bar{q}$	= heat flux, Btu/hr. sq. ft.
T	= temperature, ° F.
p	= pressure, lb./ft. ²
$T_s = T_{sat}$	= saturation temperature, ° F.
$T_w = T_{wall}$	= temperature of heating surface, ° F.
$T_w - T_s = \Delta T_1$	= superheat, ° F.
$T_w - T_L = \Delta T_2$	= subcooling, ° F.
$T_w - T_L = \Delta T_1 + \Delta T_2$	= over-all temperature difference, ° F.
Δp	= pressure difference corresponding to the superheat ΔT_1 , lb./ft. ²
R	= bubble radius
$R_o = \frac{2\sigma}{\Delta p}$	= radius of the critical bubble
W	= velocity of liquid bulk
V	= liquid velocity induced by bubble growth and collapse.
L	= latent heat of vaporization, Btu/lb.
ρ	= density, lb./cu.ft.
c	= specific heat at constant pressure, Btu/lb. ° F.
k	= thermal conductivity, Btu/hr. sq. ft. ° F per ft.
σ	= surface tension, lb./ft.
μ	= viscosity, lb./hr. ft.
α	= thermal diffusivity, ft. ² /hr.
C_F	= friction factor, dimensionless, in Fanning equation.
C_H	= heat-convection number, dimensionless, defined by eq. (6).
A_o	= effective cross sectional area of pipe.

Subscripts

w	= wall = heating surface
L	= liquid
v	= vapor
s	= sat = saturation

INTRODUCTION

Until now the knowledge of nucleate boiling heat transfer has not been sufficiently detailed to permit the calculation of heat flux, even approximately, for various liquids, for various system pressures, superheats, levels of subcooling and convective velocities.

Such calculations can, of course, only be successful if they are based on the correct physical mechanism of boiling heat transfer. In the last 20 years a good deal of research has been performed in this area and several different mechanisms were proposed. They have in common that they all give adequate account, qualitatively, of the high heat flux encountered in nucleate boiling without subcooling and convection (pool boiling). It is the purpose of this paper to show that if the experimental facts on subcooling and forced convection are considered, sufficient information is available to decide in favor of one of these proposed mechanisms and against the others and further, that based on these considerations a method has been worked out which permits the prediction of heat flux in boiling liquids with forced convection and subcooling.

Experiments in boiling heat transfer show considerable scatter. Our aim is to relate the heat flux to the fundamental properties of the boiling liquid such as temperature, pressure, superheat, convective velocity, heat of vaporization, thermal conductivity, while we do not consider such influences as surface condition, aging, tube diameter, etc. We are aware of the influence of the latter parameters, but we also find that the essential trend of the heat flux curve is determined by the properties of the fluid while surface conditions shift but do not much alter the basic relationships and the basic mechanism of heat transfer, and it is the latter which we are trying to find and uncover.

PHYSICAL CONSIDERATIONS CONCERNING THE HEAT TRANSFER MECHANISM

To begin, we state three important experimental findings on boiling heat transfer with subcooling and forced convection:

- (1) For every convective velocity W and temperature difference $(T_{\text{wall}} - T_{\text{liquid}})$ there is a certain point P_1 on the heat flux curve (see Figure 1) such that for heat fluxes higher than P_1 the relation between heat flux and superheat is essentially the same as for pool boiling.*
- (2) Beyond the aforementioned point P_1 the heat flux is highly insensitive to subcooling. In detail: beyond point P_1 the heat flux for a given temperature difference $(T_{\text{wall}} - T_{\text{sat}})$ remains essentially unchanged while subcooling may be changed by a large factor (by as much as 100 or 150° F).

The experimental facts (1) and (2) are also illustrated in Figure 2 which is reproduced from Maidment's book (Reference 1); the heat flux values shown are for levels of subcooling which vary from 20° F to 150° F. It is seen that the heat flux is quite independent of the level of subcooling.

- (3) For forced convection boiling heat transfer there is definitely a relation between the friction coefficient C_F and the heat transfer coefficient C_H ; this was demonstrated by the experiment of Saborisky and Mulligan (2), which yielded the approximate relation $C_F = 2 C_H$. Saborisky pointed out that this is the relation that one would expect on the basis of Reynolds' analogy for turbulent flow either without a laminar boundary layer or with the laminar boundary layer when the Prandtl number is unity.

*In this report we refer, of course, to heat flux below the "burnout point."

Figure 1. Typical heat transfer curve for forced convection, with and without nucleate boiling.

Figure 2. Surface boiling for various velocities and levels of sub-cooling. (From McAdams, W. H., Heat Transmission, 3rd. ed., 1954, p. 392.)

Let us now consider and discuss the various mechanisms of boiling heat transfer in the light of these experimental findings. (A more complete discussion of the various mechanisms may be found in Reference 3, from which the following is in part condensed).

Mechanism I: Micro-convection in the sublayer.

This is the mechanism most widely accepted at the present time. The heat path leads from the heating surface to liquid between the bubbles. Thermal conduction through a laminar sublayer cannot adequately account for the observed large heat fluxes unless large convective velocities are assumed. "Some form of random micro-convection excited by bubble activity in the normally laminar sublayer" (cf. Reference 4) is therefore postulated. Bubble growth velocities are sufficiently large to explain the order of magnitude of the observed heat flux when tested with the Sieder-Tate convective heat transfer equation. We mention though that the flow conditions and therefore the temperature distribution caused by the random micro-convection is quite different from that caused by turbulent steady flow over a surface for which the Sieder-Tate equation is derived.

A schematic view of this mechanism is shown in Figure 3. According to this viewpoint, the liquid velocities V (induced by the bubbles growing with radial velocities of the order of 10 or 20 feet/sec) are so large that they (and not the convective velocity W) determine the temperature distribution in the sublayer near the heating plate.

It is important to observe though that the heat flux is from the heating surface through the sublayer to the liquid. Therefore, according to this mechanism, the heat flux should strongly depend on the temperature difference ($T_{wall} - T_{liquid}$) which is the driving potential for the heat flux; in fact, according to the Sieder-Tate convective heat transfer

SCHEMATIC DIAGRAM SHOWING LIQUID VELOCITY, BUBBLE INDUCED VELOCITY AND HEAT FLUX FOR "MICRO-CONVECTION" MECHANISM

FIGURE 3

equation the heat flux should be directly proportional to this temperature difference. But, according to experimental finding (2) above, this is not at all the case. Experiment shows that for the same superheat ($T_{\text{wall}} - T_{\text{sat}}$) the heat flux in nucleate boiling remains essentially unaffected while the subcooling may increase by a factor of ten and the temperature ($T_{\text{wall}} - T_{\text{liquid}}$) may increase by a factor of three. Mechanism I is thus incompatible with experimental findings and must be abandoned.

Mechanism II: The bubbles act in the manner of surface roughness.

Led by their experiments reported in Reference 2, Saborsky and Mulligan explored a very interesting alternative mechanism for heat transfer in nucleate boiling: It had apparently been suggested by H. S. Tsien that for boiling heat transfer with forced convection the bubbles on the heating surface, acting much in the same way as roughnesses, thereby increase the turbulent exchange of liquid between the heating surface and the bulk liquid moving at velocity W . By this mechanism the bubbles would affect heat transfer and pressure drop similarly as in flow in a very rough pipe, that is, in accordance with Reynolds' analogy.

Saborsky's experimental findings are most valuable in furthering our understanding of the nucleate boiling process, but we do not find Tsien's hypothesis tenable: firstly, just as for Mechanism I, the heat flux would again have to depend directly on the temperature difference ($T_{\text{wall}} - T_{\text{liquid}}$) which is not borne out by experiment; secondly, if Tsien's hypothesis were correct the heat flux would have to depend strongly on the ratio of (mean) bubble diameter to pipe diameter because the friction coefficient is dependent on the ratio of mean size of roughness to pipe diameter and, in accordance with Reynolds' analogy, the heat transfer coefficient must then be similarly affected. But it is found experimentally that, for

instance, doubling the pipe diameter from, say, two to four inches leaves the heat flux practically unchanged.

Mechanism III: Latent heat transport by bubbles.

By latent^{heat} transport is meant: While a bubble grows it absorbs latent heat of vaporization which is then returned to the liquid bulk when the bubble collapses.

It has already been shown by Jacob and others that the latent heat transport of the bubbles fails to explain the high heat transfer rates observed in boiling. We reproduce here a demonstration of this fact by Gunther and Kreith (Reference 4) because we shall make use of some of their data later on:

observed heat flux $\bar{q}$	$= 2.0 \text{ Btu/sq. in sec.}$
liquid temperature T_L	$= 98^\circ \text{ F}$
wall temperature T_w	$= 270^\circ \text{ F}$
average maximum radius of bubble	$= .015 \text{ in.}$
frequency with which the bubble pattern cycles ν	$= 1000 \text{ per second}$
number of bubbles visible per square inch	$N = 280$

The amount of latent heat delivered to the bulk liquid per bubble is then (V_B is the bubble volume)

$$L \rho_v V_B = (970 \text{ Btu/lb}) (2 \times 10^{-5} \text{ lb/in}^3) (7 \times 10^{-6} \text{ in}^3) = 1.4 \times 10^{-7} \text{ Btu/bubble}$$

Multiplication with the number of bubbles and cycling frequency yields the latent heat transport

$$(\bar{q})_{\text{latent}} = 1.4 \times 10^{-7} \times 280 \times 1000 = .04 \text{ Btu/in}^2 \text{ sec.}$$

When this value is compared with the observed heat flux of 2 Btu/in² sec it is seen that the latent heat transport accounts for only 2% of the total heat flux.

Snyder (cf. Reference 3) pointed out that this heat flux carried by latent heat, though insufficient by itself, could be increased by assuming

additional heat flux through the bubbles by mass transfer; in detail: heat flows into the individual bubble near the superheated base at the heating plate, is absorbed as heat of vaporization at the vapor-liquid boundary, is carried as steam to the top of the bubble where the vapor condenses while giving off the latent heat to the subcooled liquid bulk. Experiments are being performed by Snyder and Edwards to verify this hypothesis on the path of heat flow and further analysis of this mechanism will depend upon the outcome of the experiments.

Mechanism IV: Vapor-liquid exchange action.

The most satisfactory of the nucleate boiling mechanisms so far proposed is Mechanism I, but we have also seen that its adoption leads to contradictions with experiment insofar as the dependence of heat flux on subcooling is concerned. We are therefore looking for an alternative and we shall presently show that the assumption of strong microconvection in the sublayer is an unnecessary assumption and that the heat flux observed in nucleate boiling can be accounted for without it.

Referring to Mechanism III, it was observed that the latent heat contained in the bubbles accounts for only a few percent of the total heat absorbed from the heating surface. But, in addition to the latent heat transport, the bubbles also transfer heat during their growth by pushing a quantity of hot liquid from the heating surface into the stream. As illustrated in Figure 4, whether a bubble grows and then detaches (Figure 4, (a)) or grows and then collapses (Figure 4, (b)) the total effect is that (at least) the bubble volume of hot liquid is transferred from the heating surface.

The ratio of heat transferred by this vapor-liquid exchange to the heat transferred by latent heat transport is evidently the dimensionless

A VOLUME V_B OF HOT LIQUID IS PUSHED IN-TO THE BULK.

THE BUBBLE IS DETACHED AND A VOLUME OF COLD LIQUID REPLACES THE BUBBLE AT THE HEATING SURFACE.

THE BUBBLE HAS COLLAPSED AND A VOLUME OF COLD LIQUID IS BROUGHT IN CONTACT WITH THE HEATING SURFACE.

SCHEMATIC DIAGRAM FOR VAPOR-LIQUID EXCHANGE MECHANISM
(a) FOR DETACHING BUBBLES, (b) FOR GROWTH-COLLAPSE CYCLE

FIGURE 4

ratio M of the heat content of the bubble volume filled with hot liquid to the latent heat content of the vapor in the same bubble:

$$(1) \quad M = \frac{c \rho_L}{L \rho_V} \Delta T_{\text{mean}} .$$

A mean temperature difference ΔT_{mean} between the wall temperature T_W and the liquid temperature T_L must be taken to account for the temperature distribution near the heating surface. One may at first think that this ratio will be small because L , the heat of vaporization is large (of the order of 10^3 in British units) but the fraction (ρ_L/ρ_V) is also large and, e.g. for water at atmospheric pressure

$$(2) \quad M = \frac{(60 \text{ lbm/ft}^3) (1 \text{ BTU/lbm deg. F})}{(970 \text{ BTU/lbm}) (3.6 \times 10^{-2} \text{ lbm/ft}^3)} (\Delta T_{\text{mean deg. F}}) = 1.7 \Delta T_{\text{mean}}$$

Now, the temperature difference $(T_W - T_L)$ is of the order of 10^2 and ranges (for water at 1 atm.) from, say, 30° F. to over 150° F. whereby the ratio M ranges from 50 to over 250.

The foregoing shows that about 100 times more heat is transferred by the process of liquid-vapor exchange at the heating surface than by the latent heat transport. But it is well known that the latent heat transport accounts for a few per cent of the total heat flux. It follows that the simple mechanism of the exchange of liquid and vapor for each bubble volume is sufficient to account for the observed heat flux in nucleate boiling.

It is fortunate that Gunthor and Kreith measured in one experiment both the bubble radii and the temperature distribution near the heating surface. Their measurements are reproduced in Figure 5 (which is Figure 12 of Reference 4.). This permits us to test our considerations more exactly in this case. From the size of the maximum bubble and the temper-

Figure 5. Temperature distributions in water related to size of hemispherical bubbles; (From Gunther, F. C., and Kreith, F., "Photographic Study of Bubble Formation in Heat Transfer to Subcooled Water", Prog. Rept. 4-120, Jet Propulsion Lab., California Institute of Technology, March 1950, p. 16, Figure 12).

ature distribution as given in Figure 5 the heat content of the volume of liquid (later) occupied by the vapor bubble can be easily computed and it is found to correspond to a mean temperature difference of $\Delta T_{\text{mean}} = 95^{\circ} \text{F}$. This makes the ratio $M = 1.7 \times 95 = 160$, or in words: the heat content of the bubble volumes in the form of hot liquid is 160 times their heat content in the form of latent heat. In this example the latent heat transport was shown to account for $.04 \text{ BTU/in}^2 \text{ sec}$ while the observed total heat flux was $2.0 \text{ BTU/in}^2 \text{ sec}$. Since $.04 \times 160 = 6.4 \text{ BTU/in}^2 \text{ sec}$, the following is evident: if all of the liquid displaced by the growing bubbles were pushed into the main stream of temperature T_L this liquid-vapor exchange alone could account for 3 times the observed heat flux. Of course, for each growth and collapse cycle only part of the displaced liquid mixes with liquid of temperature T_L and this accounts for the fact that the heat flux is not as high as the maximum theoretically possible by this mechanism.

We summarize what we have established:

The assumption that the action of the bubbles is to increase the heat flux by causing strong micro-convection in the highly superheated sub-layer at the heating surface (Mechanism I) is incompatible with the experimental finding that the heat flux is quite insensitive to the level of subcooling.

The amount of heat transferred by the liquid-vapor exchange taking place every time a bubble grows and then collapses on, or detaches from, the heating surface is by itself sufficient to account for the heat flux in nucleate boiling (Mechanism IV).

THE EFFECT OF SUBCOOLING

According to Mechanism IV, the pulsating bubble volumes draw liquid from the bulk (at temperature T_L) to the heating surface of temperature T_w and then push the heated liquid back into the bulk. At first, it would appear that this mechanism too (like Mechanism I) cannot explain the insensitivity of heat flux to the level of subcooling. However, detailed considerations do yield the following explanation.

When a bubble grows to some maximum size $R_{\max}$ and then collapses, it causes the exchange of a volume of liquid proportional to $R_{\max}^3$; the amount of heat withdrawn from the heating surface by each bubble growth-collapse cycle is then proportional to $(\Delta T_1 + \Delta T_2) \cdot R_{\max}^3$ where ΔT_1 and ΔT_2 are the superheat and subcooling, respectively. Let the period of a growth-collapse cycle be τ ; then the amount of heat transferred by each cycling bubble per second is proportional to

$$(2) \quad q \sim \rho_L R_{\max}^3 (T_w - T_L) \frac{1}{\tau}$$

since the sum of superheat and subcooling is $(T_w - T_L)$.

The insensitivity of heat flux to subcooling is a result of the combined effect of the amount of subcooling on the maximum radius $R_{\max}$ and on the bubble lifetime τ ; That is, as the liquid temperature T_L drops (subcooling increases) the maximum radius decreases so strongly that the decrease of the factor $R_{\max}^3$ in equation (2) more than compensates for the increase in the factor $(T_w - T_L)$. This may be seen from Table I which is compiled from Ellison's data (5) on the effect of liquid temperature on bubble dynamics.

T_L	ΔT_2	$(T_w - T_L)(^{\circ}F)$	$R_{max}(in)$	$R_{max}^3(T_w - T_L) \times 10^3$	$\frac{1}{\tau} \frac{(cycles)}{(sec)}$	$R_{max}^3(T_w - T_L) \frac{1}{\tau}$
177	35	80	.022	.85	903	.77
140	72	117	.019	.80	1100	.88
100	112	157	.016	.64	1430	.92
62	150	195	.013	.43	2000	.86

TABLE I: The influence of subcooling on the heat transferred by one bubble.

In columns 1 and 2 the liquid temperature and subcooling is listed; the superheat ΔT_1 in this test was 45° which gives a total temperature difference as shown in column 3. As the subcooling (listed in column 2) increases the maximum radius attained by the bubbles (shown in column 4) decreases. It is seen from column 5 that the product of bubble volume and temperature difference not only does not increase with increasing subcooling but, in fact, decreases. Thus, despite the fact that larger subcooling increases the total temperature difference between heating surface and bulk liquid, the heat transferred according to Mechanism IV for each bubble cycle decreases.

Subcooling has a further effect: The period τ of a bubble cycle decreases with an increase in subcooling. Ellison (5) reports that τ decreased from 1.1 milliseconds to .5 milliseconds when the liquid temperature was lowered from $177^{\circ} F$ to $62^{\circ} F$. This is shown in column 6 where the reciprocal of the period is listed.

In accordance with equation (2) it is the product of columns 5 and 6 which determines the heat flux according to Mechanism IV. This product is shown in column 7. By comparing column 7 with column 2 it is seen that while subcooling was changed by more than 400% the product appearing in

equation (2) changed only by about 15%; an insignificant variation in view of experimental accuracy.

From the foregoing we conclude that the combined effects of subcooling largely cancel each other and this is the explanation for the apparent insensitivity of heat flux to subcooling.

Before closing this section we can use the data in Table I to show again that the liquid-vapor exchange taking place at the heating surface accounts fully for the observed heat flux. Let us assume that the mean temperature of the bubble volume of liquid at the heating surface is about half of $(T_w - T_L)$; this yields (cf. col. 7, Table I) a mean value of about .44 for the quantity $R_{\max}^3 (T_w - T_L)_{\text{mean}} / \gamma$. The heat transferred per bubble per second is then (for hemispherical bubbles)

$$q \approx \frac{2\pi}{3} \rho_L .44 = \frac{.44 \times 1 \times 60}{1728} \frac{2\pi}{3} = .032 \frac{\text{BTU}}{\text{sec}}$$

for each bubble. Since it is well known that for the superheat considered in this example there are about 100 bubbles on the square inch the heat flux becomes $3.2 \text{ BTU}/(\text{in}^2) \text{ sec}$; the observed heat flux was $1.6 \text{ BTU}/(\text{in}^2) \text{ sec}$. This example shows a simple way of calculating the heat flux. It is seen that the feasibility of calculating the heat flux directly (that is, not by dimensionless correlations) depends largely upon the possibility of calculating the maximum radius; a new method for the calculation of such problems in bubble dynamics is reported in Ref. 6.

"Reynolds Analogy" in nucleate boiling.

Reynolds analogy is based on the identity of differential equations describing the diffusion of vorticity and the diffusion of heat in a fluid. For a fluid with Prandtl number equal to 1 the relation between shear

stress and heat transfer is the same for laminar and for turbulent flow (i.e. in the laminar sublayer and in the turbulent core) and the resulting relation between friction coefficient C_F and heat transfer coefficient C_H is then

$$(3) \quad C_F = 2 C_H .$$

If there is no laminar sublayer equation (3) is not restricted to fluids with Prandtl number equal to 1.

Sabersky and Mulligan (2) showed by their experiments that the relation given by equation (3) is also satisfied for boiling heat transfer with forced convection. Since we propose in this paper that the vapor-liquid exchange mechanism is the correct one, we wish to show that equation (3) can be deduced as a consequence of this mechanism.

One can see the basis of the analogy, qualitatively, by reference to Figure 4. Suppose a bubble grows and then detaches itself from the heating strip. When it reaches the main stream it has caused two things to occur: firstly, the volume of liquid which the bubble displaces is being heated at the heating surface whereby the fluid gains heat; secondly, the volume of liquid which the bubble displaces had previously a certain momentum which was lost by the liquid when the liquid replaced the bubble and became essentially stationary at the heating surface. This alternating gain of heat and loss of momentum of the liquid is the essence of the analogy. If the bubble is not detached but collapses on the heating strip the heat and momentum exchange is similar.

To make these considerations quantitative we start with the heat flux $\bar{q}$ per second per unit area; then, in accordance with the vapor-liquid exchange mechanism, the volume of liquid exchanged per second per unit area is $\bar{q}/\rho_L \cdot \Delta T$. Consider a section of pipe of length dx and

radius r as illustrated in Figure 6 with flow velocity W ; the mass of liquid exchanged at the surface is then obtained by multiplying the above expression by the area $2 r \pi dx$ and density ρ_L ; the change of momentum per second due to vapor-liquid exchange becomes:

$$(4) \quad \text{momentum change} = \frac{2 \bar{q} r \pi W dx}{c \Delta T}$$

Let A_0 be the projected area in the crosssection occupied by liquid (cf. Figure 6; part of the crosssection is occupied by vapor as shown); then the momentum change for a length dx is also equal to $(-A_0 \frac{dp}{dx} dx)$ and therefore from equation (4)

$$(5) \quad -A_0 \frac{dp}{dx} = \frac{2 \bar{q} r \pi W}{c \Delta T}$$

The heat transfer coefficient C_H is defined by the equation

$$(6) \quad \bar{q} = C_H \rho_L c W \Delta T$$

which is inserted in equation (5) to give

$$(7) \quad -A_0 \frac{dp}{dx} = 2 C_H \rho_L r \pi W^2$$

The pressure change is related to the coefficient of friction C_F by the Fanning equation (see, e.g. Ref. (1) P. 145):

$$(8) \quad \frac{-dp}{dx} = C_F \frac{W^2 \rho_L}{2 r_h}$$

In our case the hydraulic radius $r_h = A_0 / 2 r \pi$; thus, equation (8) inserted in equation (7) yields

$$(9) \quad A_0 C_F \frac{\rho_L W^2}{2 A_0} 2 r \pi = 2 C_H \rho_L r \pi W^2$$

Figure 6. Schematic diagram showing effective cross section of pipe for "Reynolds Analogy" in nucleate boiling.

or, after cancellations, $C_F = 2 C_H$. This relation is the one reported by Sabersky and Mulligan.

We mention that in our deduction we have tacitly assumed that the velocity and temperature profiles are similar in a region roughly corresponding to one bubble diameter above the heating strip. The agreement with Sabersky's experiment seems to bear this out for temperatures and pressures as reported. If conditions prevail for which the aforementioned profiles differ greatly, the factor of 2 between C_F and C_H will change accordingly.

THE EFFECT OF FORCED CONVECTION

It may be seen from Figures (1) and (7) that the slope of heat flux versus temperature changes very strongly and quite abruptly when boiling starts: for non-boiling the heat flux depends strongly on the convective velocity W of the liquid; the slope is small and the heat flux can be calculated from non-boiling correlations for forced convection like the well-known equations of Dittus and Boelter (7). Once boiling is well established the slope is seen to increase strongly and the heat flux is seen to become independent of convective velocity W .

Here again the micro-convection mechanism fails to yield an explanation: why should the bubble induced convective velocities completely nullify the effect of the convective velocity W of the liquid while the latter velocity is frequently much larger than the former? Consider, for instance, the situation illustrated in Fig. (7) where convective velocities up to 20 or 40 feet per second prevail in the boundary layer which is, just like the bubbles, in the immediate vicinity of the heating surface. The mean velocity with which the bubble surfaces move is about 10 ft./sec. and the velocity thereby induced in the liquid is only a few

Figure 7. Forced convection heat transfer with and without boiling for aniline. (From Kreith, F., and Summerfield, M., "Pressure Drop and Convective Heat Transfer with Surface Boiling at High Heat Flux; Data for Aniline and n-Butyl Alcohol," Trans. ASME, vol. 72, 1950, p. 869.)

feet per second because the bubbles are so small and occupy at most, about 25% of the heating surface (5). Moreover, the fact that these micro-convective velocities change their direction at random does not permit the development of the thin boundary layer (which would normally develop for unidirectional turbulent flow) without which a high heat transfer rate cannot be produced. In short, the effect of liquid velocities W of 30 or 40 feet per second in the boundary layer cannot be nullified by the small additional increase caused by the micro-convective velocities. The micro-convective velocities parallel to the heating plate are certainly present (cf. Ref. (4), Fig. 13) but their contribution cannot explain the experimental fact that the heat flux in a boiling liquid with convection is quite the same as that for a boiling liquid at rest.

In contrast, the vapor-liquid exchange mechanism shows immediately why the heat flux in boiling is so much larger than in convection and why it is independent of liquid velocity. In convection, whether we call it micro- or macro-convection, the heat absorbed at the heating surface can reach the bulk only by diffusion (laminar and/or turbulent). When surface boiling occurs, the bubbles, in growth and collapse, act as highly efficient piston pumps working at about 1000 cycles per second which pump mechanically the hot liquid from the heating strip to the bulk and the cold liquid from the bulk to the strip. Since the efficiency of these "pumps" in transporting hot liquid to the bulk is entirely independent of liquid velocity W (while diffusion by eddies is not!) it is clear that convection loses its importance for heat transfer once surface boiling is well established. The fact that this pumping action is more than sufficient to account for the observed heat flux was already shown in section 3 of this paper.

We shall deduce in the next section a number of correlations giving the heat flux for various liquids at rest. But, we have just discussed

the fact that the influence of convection on heat flux disappears, once vigorous boiling has started. It only remains to find the point at which this takes place, or, to find the following relation: given a liquid at velocity W , find that heat flux and superheat beyond which the liquid velocity ceases to be of influence. This point for the respective velocity is designated as P_1 in our Figures (1) and (7).

We make use of the foregoing considerations to find a means for computing the location of P_1 on the heat flux curve. At this point the influence of convection is nullified by the bubble action; we therefore expect that the heat flux $\bar{q}_1$ at point P_1 is reached when the heat flux due to bubble action becomes equal to and somewhat surpasses the heat flux due to eddy diffusion (convection). Let $\bar{q}_0$ signify the flux for the point P_0 (of. Fig. 1) where the curve for purely convective heat transfer intersects the curve for heat transfer due only to bubble activity (pool boiling). Then, we expect from our previous considerations that $\bar{q}_1$ will be between one or two times $\bar{q}_0$. A test of the usefulness and correctness of our considerations will come with the question whether the ratio $(\bar{q}_1/\bar{q}_0)$ is in this range for different liquids and various liquid velocities and subcooling. This ratio is shown in Fig. (8) for various conditions and for different liquids. It is seen to be in the vicinity of 1.4 for most data and between 1 and 2 for all data:

$$(10) \quad \bar{q}_1 \approx 1.4 \bar{q}_0, \quad \bar{q}_1 < 2 \bar{q}_0.$$

Anticipating that we shall produce a correlation for pool boiling in the next section it is possible to find $\bar{q}_0$ analytically as the intersection of the correlation for pool boiling and the Sieder-Tate equation for forced convection heat transfer. Then, eq. (10), together with the Sieder-Tate equation and the correlation for pool boiling will permit the prediction of heat flux with and without subcooling, boiling and convection.

Figure 8. Ratio $\bar{q}_1/\bar{q}_0$ for different liquids at various velocities, bulk temperatures, superheats and system pressures.

CORRELATIONS FOR THE HEAT FLUX IN NUCLEATE BOILING

Until analytical expressions for the maximum radius $R_{\max}$ will be available the direct calculation of heat flux along the lines of section 3 is not possible and correlations must be derived from dimensional considerations. We must first decide from physical considerations what the important parameters and dimensionless groups are. One of these is certainly the Prandtl group

$$(11) \quad Pr = \frac{\mu c}{k} \quad .$$

Another parameter should be representative of cycle frequencies and of the dynamics of bubble growth in general. Parameters important in nucleation, e.g. the critical radius R_0 (which depends on surface tension) will be included in a third group.* We shall include viscosity in the

*Properties of the heating surface will not be considered here. We are aware that by special treatment of surfaces, like oiling, scratching, pitting, aging, etc. the heat flux can be changed considerably, but a quick survey shows that most experiments of importance to the designer are performed with clean surfaces and yield a heat flux which depends on the second to third power of superheat. For practical purposes we are content to aim our analysis at this body of experimental evidence.

we shall include
second group and designate it π_1 ; the heat flux $\bar{q}$ in the third group, π_2 . The two groups then play in our correlations the role of Reynolds modulus and Nusselt modulus, respectively, but since both contain mechanical as well as thermal properties we shall refer to them simply as dimensionless groups.

The problem of the growth of a vapor bubble in a superheated liquid was analyzed previously to the first order in superheat $\Delta T_1(8)(9)(10)$.

For large superheats which occur in boiling and in bubble chambers an analysis to the second power in superheat would be desirable, but the mathematical difficulties are forbidding. An advance in this direction can be made, however, in the following manner.

Rayleigh's equation which is to be solved

$$(12) \quad \rho_L (R\ddot{R} + \frac{3}{2}\dot{R}^2) = \Delta p - \frac{2\sigma}{R}$$

contains the excess pressure Δp ; the associated heat conduction-convection problem (10) yields ΔT_1 as a function of the radius R and time t and consequently Δp becomes a function of R and t , to be inserted into eq. (12). If the exact expression for Δp were inserted, eq. (12) would become too complex for further analysis. Therefore, only the linear term in ΔT_1 entered the original calculation and the radius as a function of time became proportional to the first power in superheat (Ref. 9, eq. (14)). Since eq. (12) depends explicitly on excess pressure Δp we can develop its solution $R = G(\Delta p, t)$ (for a specific instant)

$$(13) \quad R(\Delta p) = R_0 + A_1 \Delta p + A_2 \Delta p^2 + \dots$$

If the exact solution to eq. (12) in terms of ΔT_1 were known, say $R = F(\Delta T_1, t)$, it could similarly be developed into the power series

$$(14) \quad R(\Delta T_1) = R_0 + B_1 \Delta T_1 + B_2 \Delta T_1^2 + \dots$$

Now, the development of excess pressure in terms of superheat is

$$(15) \quad \Delta p = \frac{dp}{dT} \Delta T_1 + \frac{1}{2} \frac{d^2 p}{dT^2} \Delta T_1^2 + \dots$$

where the derivatives are to be evaluated at saturation temperature T_s . When the excess pressure Δp from eq. (15) is inserted into eq. (13),

eq. (14) must result; this permits us to calculate the coefficients A_1 , A_2 , ... if we know B_1 , B_2 , ...:

$$(16) \quad A_1 \frac{dp}{dT} = B_1, \quad \frac{1}{2} A_1 \frac{d^2 p}{dT^2} + \left(\frac{dp}{dT} \right)^2 A_2 = B_2 \quad \text{etc.}$$

But B_1 was previously calculated as

$$(17) \quad B_1 = \frac{c \rho_L}{L \rho_v} \sqrt{\pi a}$$

and we obtain from eq. (17) the coefficient for the growth of a vapor bubble in a highly superheated liquid, to the first order in excess pressure

$$(18) \quad A = A_1 \Delta p = \frac{c \rho_L \sqrt{\pi a} T_s}{(L \rho_v)^2 J} \Delta p$$

where Clapeyron's equation was used for dp/dT ; J is the mechanical heat equivalent. Inasmuch as Δp increases exponentially with superheat, the coefficient A describes bubble growth correctly to an order higher than the first in superheat.

We make use of eq. (18) in forming the dimensionless group

$$(19) \quad \pi_1 = \frac{\rho_L}{\mu} A^2.$$

In many problems the heat flux has been found to depend on the $1/3$ power of the Prandtl number; we shall also assume such dependence in our problem. We are thus led to consider correlations of the form

$$(20) \quad \pi_2 = C \left(\frac{\rho_L}{\mu} A^2 \right)^n \left(\frac{\mu c}{k} \right)^{1/3}$$

For every judicious choice of a dimensionless group π_2 eq. (20) yields a correlation. For the determination of the exponent n we propose the following method. Bonilla and co-workers (11) (12) have found

that a simple relation exists between superheat and system pressure at constant heat flux and for low pressure

$$(21) \quad \left[\frac{d(\log \Delta T_1)}{d(\log p)} \right]_{\bar{q}=\text{const.}} = -\frac{1}{4} \quad (\text{Bonilla et al.})$$

Professor Bonilla reports that eq. (21) holds for many liquids which are quite different in nature (e.g. for water, iso-propanol and mercury) from which one may conclude that eq. (21) expresses a rather fundamental relation in nucleate boiling. We shall therefore use this equation for the determination of the exponent n in eq. (20). We illustrate the procedure by deriving the following two correlations.

Correlation I. As the simplest possible choice for Π_2 we form a Russelt modulus $(\bar{q}l/k\Delta T_1)$ using the critical radius $R_0 = 2\sigma/\Delta p$ as the characteristic length l

$$(22) \quad \Pi_2 = \frac{\bar{q} \frac{2\sigma}{\Delta p}}{k \Delta T_1}$$

Equation (20) then takes the form

$$(23) \quad \frac{2\bar{q}\sigma}{k \Delta T_1 \Delta p} = C_I \left(\frac{\rho_L A^2}{\mu} \right)^n \left(\frac{\mu c}{k} \right)^{1/3}$$

The system pressure p is contained in eq. (23) implicitly via the quantities T_{sat} and ρ_v ; other quantities (e.g. ρ_L, σ, c) are, by an order of magnitude, less pressure dependent. To carry out the differentiation indicated in eq. (21) for low superheat and low pressure we express Δp in terms of ΔT by Clapeyron's equation and we use the ideal gas law for ρ_v ; from eq. (23) the heat flux then depends on boiling temperature T_b and system pressure in the following manner

$$(24) \quad \bar{q} \sim \Delta T_1 \frac{p \Delta T_1}{T_s^2} \left(\frac{T_s \Delta T_1}{p} \right)^{2n}$$

where the second and third term on the right side of eq. (24) stems from Δp and A^2 in eq. (23) respectively. Making use of

$$\frac{dy(x)}{d \log x} = x \frac{dy}{dx}$$

we differentiate eq. (24) while keeping $\bar{q}$ constant and obtain

$$(25) \quad \left[\frac{d \log \Delta T_1}{d \log p} \right]_{\bar{q}=\text{const}} = - \left[\frac{1-2n}{2+2n} + \frac{2n-2}{2n+2} \frac{RT_s}{L} \right]$$

For low pressure (RT_s/L) is small and eqs. (25) and (2) yield an equation for the exponent n

$$(26) \quad \frac{1-2n}{2+2n} = \frac{1}{4}, \quad n = \frac{1}{5}$$

Correlation I according to eq. (23) is then

$$(27) \quad \bar{q} = C_1 \frac{k}{2\sigma} \Delta p \Delta T_1 \left(\frac{\rho}{\mu} A^2 \right)^{1/5} \left(\frac{\mu c}{k} \right)^{1/3}$$

Since A is proportional to Δp the heat flux is seen to be proportional to $(\Delta T_1 \cdot \Delta p^{1.4})$. Thus, $\bar{q}$ depends on superheat to the 2.4 power for low ΔT_1 and follows a moderate exponential increase (since Δp depends exponentially on superheat) as the superheat increases. Experimentally, a dependence of heat flux on superheat raised to a power between 2 and 4 is generally reported; Correlation I is seen to be in good agreement. We may point out that this exponent in superheat was not inserted in the course of the derivation, since n was obtained from the dependence on system pressure.

Comparison of Correlation I with experimental data for water is shown in Fig. 9 with $C_1 = 0.7 \times 10^{-2}$ in this case. The correlation has practical value because, after C_1 is determined by one measurement of heat flux at atmospheric pressure, the heat flux, say, at 50 atmospheres is accurately

Figure 9. Comparison of correlation I, equation (27), with experimental data of heat flux in water at 1 and 50 atmospheres.

predicted by equation (27), as illustrated in the Figure. Equation (27) was also tested for other liquids including liquid mercury; again, the equation predicted the heat flux correctly for high and low pressure once the coefficient C_I was determined from a measurement at atmospheric pressure. The coefficient C_I varied (from water to mercury) by a factor of 2.

Correlation II. While Correlation I does predict the heat flux in its dependence on pressure and superheat the coefficient C_I varies from one liquid to another. Although a variation by a factor of 2 (for liquids as different physically as water and mercury) is small, we wish to present another correlation which seems to have quite general validity since we have found it to predict, without any change whatever, the heat flux in nucleate boiling of all liquids which we have tested so far.

We arrive at this correlation by choosing for Π_2 in eq. (20) the dimensionless group

$$(28) \quad \Pi_2 = \frac{\bar{q}}{L \rho_v} \left(\frac{R_o}{a} \right)^{1/2} \left(\frac{\rho_L}{\Delta p} \right)^{1/4}$$

obtained from dimensional analysis. From eq. (20) we then obtain, analogous to eq. (23),

$$(29) \quad \frac{\bar{q}}{L \rho_v} \left(\frac{2\sigma}{a \Delta p} \right)^{1/2} \left(\frac{\rho_L}{\Delta p} \right)^{1/4} = C_{II} \left(\frac{\rho_L}{\mu} A^2 \right)^n \left(\frac{\mu c}{k} \right)^{1/3}$$

Exactly as before we use the ideal gas law for ρ_v and Clapeyron's equation for Δp at low superheat and differentiate eq. (29) for constant heat flux $\bar{q}$; equation (21) then gives an equation for n . After some arithmetic eq. (21) yields $n = 5/8$. The coefficient C_{II} was then obtained from eq. (29) by comparison with experiment for one liquid; the correlation, for any consistent system of units, becomes

$$(30) \quad \bar{q} = 1.2 \times 10^{-3} \frac{a c \rho_L T_s}{J L \rho_v \sqrt{\sigma}} \left(\frac{c T_s \sqrt{a}}{J (L \rho_v)^2} \right)^{1/4} \left(\frac{\rho_L}{\mu} \right)^{5/8} \left(\frac{\mu c}{k} \right)^{1/3} \Delta p^2$$

For the heat flux $\bar{q}$ in (Btu/hr. ft.²) and the other quantities in units as given in the nomenclature

$$(30a) \quad \bar{q} = 4.3 \times 10^{-5} \frac{a c \rho_L T_s}{\sigma^{1/2} (L \rho_v)^{1/2}} \left(\frac{c T_s \sqrt{a}}{J (L \rho_v)^2} \right)^{1/4} \left(\frac{\rho_L}{\mu} \right)^{5/8} \left(\frac{\mu c}{k} \right)^{1/3} \Delta p^2$$

We have tested equation (30) by comparison with experimental data for the following liquids and physical conditions:

water: for pressures from 1 to 50 atmospheres (cf. Fig. 10);

n-butyl alcohol: for 50 psia;

aniline: for 35 psia (cf. Fig. 7);

mercury: for 1 and 3 atmospheres (cf. Figures 11 and 12).

The extent to which eq. (30) represents the experimental data can be judged from the graphs. While this correlation may not, for each liquid, be as accurate as Correlation I it must be remembered that eq. (30) is as accurate as it is, without any variation in the coefficient C_{II} and thus permits the prediction of heat flux without resort to experiment.

We have also compared eq. (30) with whatever data we could gather for boiling heat transfer in liquid sodium (13) and we found fair agreement. For liquid Cesium eq. (30) yields reasonable values but no comparison with experimental data could be made because the sparse data reported in the literature are obviously for conditions of film boiling.

Figure 10. Comparison of correlation II, equation (30), with experimental data of heat flux in water at 1 and 50 atmospheres.

It is interesting to comment on the slope of the heat flux curve. The exponent of superheat has variously been reported in the range from 2 to 4; measuring the slope in Fig. 10 with a simple ruler reveals that for atmospheric pressure and 15° F. superheat the slope is 2.1 while for 80° F superheat a slope of 3.5 is found. Equation (30) brings about this increase in slope of heat flux versus superheat by being explicitly dependent on excess pressure Δp which, in turn, is not a linear but an exponential function of superheat.

In closing, we wish to outline how the results presented in this paper may be used to calculate the heat flux in a subcooled liquid with forced convection. Let it be required, for example, to calculate the heat flux in aniline (cf. Fig. 7) for liquid temperature T_L , superheat ΔT_1 and velocity W .

First, the heat flux $\bar{q}_0$ is calculated (cf. section 5 of this paper) as the intersection of eq. (30) (for nucleate boiling) with the equation of Sieder-Tate (14) for convective heat transfer (without boiling) at velocity W and temperature difference $(T_W - T_L)$. With $\bar{q}_0$ known, eq. (10) gives $\bar{q}_1 \approx 1.4 \bar{q}_0$; the corresponding superheat and temperature of the heating surface T_1 is then directly found from eq. (30) or from the graph which was plotted to find $\bar{q}_0$.

With T_1 known, there are three regimes to consider: for wall temperatures higher than T_1 eq. (30) (or eq. (27)) is applicable; for a wall temperature at the boiling point or below, the Sieder-Tate equation is applicable. For the narrow range between incipient boiling and T_1 the heat flux can be easily computed by assuming a linear increase between the point of incipient boiling and T_1 . Hence, one is now able to predict the heat flux in a moving, subcooled liquid for the entire temperature range below the "peak heat flux."

Figure 11. Comparison of correlation equation (30), with experimental data of heat flux in mercury at 1 atmosphere.

Figure 12. Comparison of correlation II, equation (30), with experimental data of heat flux in mercury at 3 atmospheres.

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