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RADIAL PLASMA BUILDUP CODE FOR NEUTRAL BEAM INJECTION INTO A MIRROR MACHINE

B. W. Stallard

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RADIAL PLASMA BUILDUP CODE FOR NEUTRAL BEAM INJECTION INTO A MIRROR MACHINE

Abstract

A computer code has been developed for calculating the time-dependent radial profile of plasmas in a mirror machine built up and maintained by injection of high current, energetic neutral beams into a target plasma. The code models plasma expansion caused by ions trapped on the boundary and the competing effect of plasma erosion by charge exchange of the plasma on thermal gas at the boundary. For equilibrium the maximum allowed ratio of thermal gas current to injected beam current is determined by the geometry, ion energy and mass, and magnetic field.

The computer code calculates the self-consistent thermal gas in the plasma boundary taking into account gas from the neutral beam sources, wall reflux of charge exchange neutrals, and pumping by the plasma, gettered walls, and conducting apertures in the plasma chamber.

Examples are presented for beam injection into the 2XIIB mirror experiment.

Introduction

A computer code has been developed for calculating the time-dependent radial profile of plasmas in a mirror machine. The plasma is built up and maintained by injecting high-current, energetic neutral beams into a target plasma. The code described here is an improved and expanded version of a code written by Ralph Moir for use in the Fusion Engineering Research Facility (FERF) studies. An important effect is plasma expansion caused by ions trapped on the boundary contributing density outside the boundary because of the non-zero Larmor radius. Competing with radial expansion is plasma erosion by charge exchange of the plasma on thermal gas at the boundary. This is a particularly severe problem for mirrors, since there is little radial excursion of plasma ions and charge exchange losses on the boundary must be replaced by beam trapping in a depth of the order of a Larmor radius.¹ For equilibrium, the maximum allowed ratio of thermal gas current to injected beam current is determined by the geometry, ion energy and mass, and magnetic field. If the ratio is less than the maximum, the plasma size will increase or decrease from its initial value until the equilibrium is reached. If the thermal gas is zero, the plasma will expand until limited by

the plasma chamber walls. The computer calculation described in this paper is one-dimensional and assumes symmetry about the mirror machine axis. Finite length and fanning effects of the field lines of a minimum-B magnetic well are taken into account by an appropriate volume of the flux tube. Although finite beta effects are not included, a beta limit is imposed on the density.

Two plasma ion groups are treated. One group, with energy at the injected beam energy WZERO, represents the ions added by trapping of the injected neutral beam. Ions of energy WTAR represent a plasma target which exists at the start of the computation and is allowed to decay away by coulomb collisions or, if desired, at an exponential rate. The electron energy is taken to be a fraction WEDIVW of the average ion energy. No energy exchange between ions and electrons is presently included in the calculation. Ion loss occurs by charge exchange and coulomb scattering into the loss cone.

Seven particle beams are treated. These are:

- The injected neutral beam of energy WZERO which sustains the plasma,
- Two beams of energy WZERO and WTAR which result from charge exchange of the trapped and target plasmas on the injected beam,
- A directed beam of streaming molecular gas incident on the plasma from the neutralizing cell of the neutral beam source,
- A thermal molecular beam equivalent to the random molecular current incident on the plasma, resulting from gas diffusing into the chamber from the sources and return of fast charge exchange neutrals from the walls as molecules,
- Franck-Condon neutrals from dissociation of the molecular gas on the plasma boundary,
- A thermal atomic beam resulting from reflux of fast neutrals off the walls.

Pumping of thermal gas occurs by ionization and charge exchange with the plasma. The cold ions created are swept out of the plasma chamber by the ambipolar potential. Pumping also occurs through apertures in the plasma chamber and by rettered walls. The calculation has been used to study neutral beam injection into the 2XIIIB and Baseball mirror experiments.

Finite Larmor Radius Effects and the Guiding Center Density

To accurately calculate plasma expansion and the effects of thermal gas on the plasma boundary, the effects of finite larmor radius must be properly treated. This is conveniently done by solving the rate equations for the plasma guiding center densities and then calculating the resulting plasma particle densities using an appropriate radial weighting function.

For calculating changes in the guiding center density, it is convenient to treat the ions trapped at the beam energy by ionization and charge exchange separately from the

plasma ions lost by charge exchange. We thus divide beam plasma interactions into two classes: beam trapping and charge exchange loss.

For beam trapping (Class A reactions), all interactions of the fast neutral beams with the plasma, i.e., charge exchange and ionization, result in ions trapped at the beam energy. These interactions increase the guiding center density and the guiding center is allocated to the radial zone in which the trapping event takes place.

For charge exchange loss (Class B reactions), all charge exchange events result in loss of plasma density. A charge exchange event in a given zone affects the guiding center density in adjacent zones to a distance of one larmor radius. In turn, since each ion contributes to the density over one larmor radius, the charge exchange events in a given zone result in a change of plasma density in adjacent zones to a distance of two larmor radii. By considering these two classes of interaction, a charge exchange event of a fast neutral with the plasma results in trapping of a hot ion of the neutral beam energy and the loss of an ion at the plasma energy.

Calculating charge exchange in the manner described automatically takes into account radial ion diffusion by charge exchange. For example, charge exchange with a density gradient results in loss of a guiding center at a point of greater density than the position at which the charge exchange event occurs. Since a guiding center is added at the position of charge exchange, there is an effective diffusion of guiding centers from regions of greater density to regions of lower density.

The relation between guiding center density and plasma particle density in cylindrical geometry is given by,²

$$n(r) = \frac{2}{\pi} \int \frac{R \mathcal{N}(R) [U(R - |r - a_i|) - U(R - r - a_i)] dR}{\sqrt{[r^2 - (R - a_i)^2] [(R + a_i)^2 - r^2]}}, \quad (1)$$

where $n(r)$ [$\mathcal{N}(r)$] are the plasma [guiding center] densities, a_i is the ion larmor radius, and $U(x)$ is the unit step defined by $U(x) = 1$ for $x \geq 0$, $= 0$ for $x < 0$.

For calculation Eq. 1 is used to define coefficients $f(r, r')$ which give the contribution to plasma density in a radial zone at radius r from unity guiding center density in a zone at radius r' . Hence,

$$n(r) = \sum_{r'} f(r, r') \mathcal{N}(r'), \quad (2)$$

where r' sums over all zones to within one larmor radius of r . The equation for f is

$$f(r, r') = \frac{1}{\alpha(r)} \int_{r-\Delta R/2}^{r+\Delta R/2} \left(\frac{2}{\pi} \int_{r'-\Delta R/2}^{r'+\Delta R/2} \frac{R [U(R - |x - a_i|) - U(R - x - a_i)] dR}{[x^2 - (R - a_i)^2] [(R + a_i)^2 - x^2]^{1/2}} \right) 2\pi x dx, \quad (3)$$

where $\alpha(r)$ is the cross-sectional area of the radial zone of thickness ΔR centered at radius r .

For class A interactions, let $\frac{d}{dt} N_A(r')$ be the rate of change of guiding center density at r' . Thus the rate of change of plasma density at r is

$$\frac{d}{dt} n_A(r) = \sum_{r'} f(r, r') \frac{d}{dt} N_A(r'). \quad (4)$$

Now consider charge exchange losses (class B). The loss in guiding center density at r by charge exchange at r' is proportional to the contribution of density at r' by guiding centers at r . Thus

$$\alpha(r) \frac{d}{dt} N_B(r) = \alpha(r) \left[\frac{f(r, r') N(r)}{n(r')} \right] \frac{d}{dt} n_B(r'). \quad (5)$$

The quantity in brackets weights the charge exchange loss $\frac{d}{dt} n_B(r')$ at radius r' . Multiplying by α conserves particles.

Using Eq. (5) and summing over adjacent radial zones, the total rate of change in guiding center density is

$$\frac{d}{dt} N(r) = \frac{d}{dt} N_A(r) + \sum_{r'} \frac{\alpha(r')}{\alpha(r)} f(r, r') N(r) \frac{d}{dt} \frac{n_B(r')}{n(r')}. \quad (6)$$

Applying Eq. (2), we obtain the rate of change of plasma density

$$\frac{d}{dt} n(r) = \sum_{r'} f(r, r') \left[\frac{d}{dt} N_A(r') + \sum_{r''} \frac{\alpha(r'')}{\alpha(r')} f(r', r'') N(r') \frac{d}{dt} \frac{n_B(r'')}{n(r'')} \right] \quad (7)$$

Beam Equations

The neutral beam sustaining the plasma is assumed injected perpendicular to the plasma at the midplane. Figure 1 shows several radial zones of the geometry at the midplane. Three energetic neutral beams are treated. These are the injected beam and two charge exchange beams, one at the target energy and the other at the trapped or hot ion energy which is assumed equal to the injected beam energy. The beams are assumed to have height D and to propagate from left to right as shown in the figure.

The angular scattering of charge exchange neutrals is approximated by assuming a fraction F of the neutrals are headed in the original beam direction and $1-F$ are headed backward. For the calculation it is equivalent to place the "back-scattered" component $1-F$ on the opposite side of the plasma as shown in the figure. In general, F is a

Fig. 1. Geometry of plasma at the midplane. For convenience, all beams are assumed to propagate from left to right in the calculation.

function of energy (Appendix A) and depends on the interacting species. Thus F_{ij} refers to charge exchange of neutral beam i on a plasma ion of species j .

In addition to the energetic beams there are two thermal molecular beams, a "warm" Franck-Condon atomic beam, and a thermal atomic beam. Molecular gas in the plasma chamber arises from several sources. The first is leakage of gas from the charge exchange cell of the injected neutral beam source. A fraction of this current I_s streams directly incident onto the plasma and is counted as one of the molecular beams. The remainder of the source gas I_d is assumed to uniformly fill the volume outside the plasma. Depending on wall conditions, fast charge exchange neutrals from the plasma and the injected beam which penetrates the plasma may reflux off the walls as neutral molecules or as atoms. The second molecular beam is set equal to the randomly directed current at the plasma surface arising from the source gas I_d and molecular wall reflux.

Franck-Condon dissociation of the molecular beams on the plasma boundary creates the warm atomic beam. Only one-half of the atoms are assumed to be directed inward to interact with the plasma.

If wall reflux is atomic rather than molecular, an atomic gas density is created outside the plasma. The random current at the plasma surface is the thermal atomic beam in the calculation. Although reflux atoms can be energetic at a substantial fraction of the incident energy, they are assumed to rapidly thermalize by further interactions with the walls before striking the plasma.

All thermal beams and the Franck-Condon atomic beam are assumed to propagate radially inward. We assume the coordinate z as shown in Fig. 1 and arrive at the following beam rate equations.

Injected neutral beam:

$$\frac{dI_b(z)}{dz} = - \left[\frac{n_t \langle \sigma v \rangle_T^{h-t} + n_h \langle \sigma v \rangle_T^{h-h}}{v_b} \right] I_b(z). \quad (8)$$

Charge exchange beam at hot ion energy:

$$\begin{aligned} \frac{d}{dt} I_{cxh}(z) = & \frac{n_h \langle \sigma v \rangle_{cx}^{h-h}}{v_b} [F_{hh} I_b(z) + (1 - F_{hh}) I_b(2R_w - z)] \\ & - \left[\frac{n_t \langle \sigma v \rangle_T^{h-t} + n_h \langle \sigma v \rangle_i^{h-h} + (1 - F_{hh}) n_h \langle \sigma v \rangle_{cx}^{h-h}}{v_h} \right] I_{cxh}(z) \\ & + \frac{(1 - F_{hh}) n_h \langle \sigma v \rangle_{cx}^{h-h}}{v_h} I_{cxh}(2R_w - z) + \frac{1}{2} \frac{n_h \langle \sigma v \rangle_{FC}^{h-h}}{v_{FC}} [I_{FC}(z) + I_{FC}(2R_w - z)] \\ & + \frac{n_h \langle \sigma v \rangle_{cx}^{t-h}}{v_t} [F_{th} I_{cxt}(z) + (1 - F_{th}) I_{cxt}(2R_w - z)]. \quad (9) \end{aligned}$$

Charge exchange beam at target energy:

$$\begin{aligned} \frac{d}{dz} I_{cxt}(z) = & \frac{n_t \langle \sigma v \rangle_{cx}^{h-t}}{v_b} [F_{ht} I_b(z) + (1 - F_{ht}) I_b(2R_w - z)] \\ & + \frac{n_t \langle \sigma v \rangle_{cx}^{h-t}}{v_h} [F_{ht} I_{cxh}(z) + (1 - F_{ht}) I_{cxh}(2R_w - z)] \\ & + \frac{1}{2} \frac{n_t \langle \sigma v \rangle_{FC}^{h-t}}{v_{FC}} [I_{FC}(z) + I_{FC}(2R_w - z)] \\ & - \left[\frac{n_t \langle \sigma v \rangle_i^{t-t} + n_h \langle \sigma v \rangle_T^{t-h} + (1 - F_{tt}) n_t \langle \sigma v \rangle_{cx}^{t-t}}{v_t} \right] I_{cxt}(z) \\ & + \frac{(1 - F_{tt}) n_t \langle \sigma v \rangle_{cx}^{t-t}}{v_t} I_{cxt}(2R_w - z). \quad (10) \end{aligned}$$

Franck-Condon atomic beam:

$$\frac{dI_{FC}(z)}{dz} = - \left(\frac{n_t \langle \sigma v \rangle_T^{FC-t} + n_h \langle \sigma v \rangle_T^{FC-h}}{v_{FC}} \right) I_{FC}(z) + \frac{1}{2} \frac{n_e \langle \sigma v \rangle_{DISS}^{m-e}}{v_m} [I_s(z) + I_{rm}(z)]. \quad (11)$$

Streaming thermal molecular beam:

$$\frac{dI_s(z)}{dz} = - \left(\frac{n_t \langle \sigma v \rangle_T^{m-t} + n_h \langle \sigma v \rangle_T^{m-h}}{v_m} \right) I_s(z). \quad (12)$$

Random thermal molecular beam:

$$\frac{dI_{rm}(z)}{dz} = - \left(\frac{n_t \langle \sigma v \rangle_T^{m-t} + n_h \langle \sigma v \rangle_T^{m-h}}{v_m} \right) I_{rm}(z). \quad (13)$$

Random thermal atomic beam:

$$\frac{dI_{ra}(z)}{dz} = - \left(\frac{n_t \langle \sigma v \rangle_T^{a-t} + n_h \langle \sigma v \rangle_T^{a-h}}{v_a} \right) I_{ra}(z). \quad (14)$$

The quantities n , v , I , $\langle \sigma v \rangle$, and F are respectively the plasma particle density, beam velocity, beam current, average over velocity of the cross-section σ times the relative velocity v , and the forward scattered fraction F of the charge exchange neutrals. The subscripts b, t, h, ext (cxh), FC, e, and rm (ra) refer respectively to the beam, target, hot or trapped, charge exchange at target (hot) energy, Franck-Condon, electron, and random molecular (atomic) gas. For the cross-section averages, the superscripts refer to the two species over which the average is performed, and the subscripts i, cx and T refer to ionization (ion and electron impact), charge exchange, and total cross-section.

The large angular brackets $\langle \rangle$ in Eq. (11) denote the average molecular current in the Franck-Condon source term. Although attenuation of the energetic beams is small in a zone thickness, it is usually not true for the thermal and Franck-Condon neutral beams. Thus the average current taking into account the exponential decrease from zone to zone is appropriate. The average for a thermal beam of velocity v is

$$\begin{aligned} \langle I \rangle &= \frac{1}{\Delta R} \int_{z-\Delta R/2}^{z+\Delta R/2} I(z) \exp[-(z'-z)/\lambda] dz' \\ &= I(z) \frac{2\lambda(z)}{\Delta R} \sinh \left(\frac{\Delta R}{2\lambda} \right) \end{aligned} \quad (15)$$

where $\lambda(z)$ is the mean free path for attenuation. The factor $1/2$ in Eqs. (9) and (10) for the Franck-Condon term is based on the assumption that the forward scattering fraction equals $1/2$. The factor $1/2$ in Eq. (11) assumes that $1/2$ of the Franck-Condon neutrals head radially inward.

Rate Equation for Guiding Center Densities

We first consider the source term for guiding centers (class A). A generalized current source term Δ_A in a cylindrical shell of radius r and thickness ΔR is obtained by integrating beam attenuation throughout the shell. The source term is*

$$\Delta_A = \frac{1}{D} \int_{-D/2}^{D/2} dy \int_{r-\Delta R/2}^{r+\Delta R/2} \left[\frac{dI_A}{dz}(y, z) + \frac{dI_A}{dz}(y, 2r+z) \right] dz \quad (16)$$

where the y, z axes are shown in Fig. 1 and $z = R_w - r$ for $z \leq R_w$; $R_w + r$ for $z > R_w$. There are two terms since each beam passes through a cylindrical shell twice.

Converting Eq. (16) to cylindrical coordinates r, θ , we obtain

$$\Delta_A = \frac{2}{D} \int_0^{\sin^{-1} \frac{D}{2r}} d\theta \int_{r-\Delta R/2}^{r+\Delta R/2} \left[\frac{dI_A}{dz}(y, z) + \frac{dI_A}{dz}(y, 2r+z) \right] r dr. \quad (17)$$

If the difference between beam attenuation at the beam edge and at the beam center is neglected, then evaluating $\frac{dI}{dz}$ along the beam axis,

$$\Delta_A = \frac{2r\Delta R}{D} \left[\frac{dI_A}{dz}(z) + \frac{dI_A}{dz}(2r+z) \right] \begin{cases} \sin^{-1}(D/2r), & r \geq D/2 \\ \pi/2, & r < D/2 \end{cases}. \quad (18)$$

The arcsine term takes into account the increased path length in a zone, since all beam particles are not directed along a radius. However, evaluating dI/dz only along the beam axis overestimates total beam attenuation.

The integral of the rate of change of guiding center density over a flux tube of volume ΔV is

$$\frac{d}{dt} \int_{\Delta V} \mathcal{N}_A dv = \frac{\Delta_A}{e}.$$

*Equation (16) is strictly correct only for planar geometry, since the guiding center of the trapped ion is displaced one larmor radius from the point of trapping. Thus, in cylindrical geometry a fraction of the trapping events in a zone should be allocated to adjacent zones. Neglecting this effect will produce little error for radii greater than several larmor radii. At smaller radii, the plasma radial profile will be altered to some degree.

To account for a finite plasma length and the fanning of magnetic field lines of a minimum-B magnetic well, the following integral relation is used:

$$\int_{\Delta V} \mathcal{N}_A^n dv = \mathcal{N}_{A0}^n 2\pi r \Delta R L C_n, \quad (19)$$

where L is the length between mirrors, and $\mathcal{N}_{A0}$ is the density at the mid-plane. Thus for $n = 0$, the volume of a flux tube is

$$\Delta V = C_0 2\pi r \Delta R L.$$

The average density is $\bar{n} = n_0 C_1 / C_0$ and the mean square density is $\overline{n^2} = n_0^2 (C_2 / C_0)$.

Equation (19) has been evaluated as a function of mirror ratio by Moir and Taylor³ for normal mode collisional distributions, and Table 1 shows the value of C_n for several mirror ratios. These coefficients are "filling factors" to determine the mid-plane density from the trapping rate in a flux tube.

Table 1. Density filling factors.

R_m	C_0	C_1	C_2
2	0.7059	0.3734	0.2932
3	0.6075	0.2983	0.2214
5	0.4968	0.2794	0.2141
10	0.3710	0.2210	0.1783

Thus in terms of the mid-plane density,

$$\frac{d\mathcal{N}_{A0}}{dt}(r) = \frac{1}{2\pi r \Delta R L C_1} \left(\frac{\Delta \mathcal{I}_A}{e} \right) \quad (20)$$

Defining

$$A(r) = \begin{cases} \frac{1}{e\pi L D C_1} \sin^{-1} \frac{D}{2r}, & r \geq D/2 \\ \frac{1}{e2L D C_1}, & r < D/2 \end{cases} \quad (21)$$

and dropping the 0 subscript, Eq. (20) becomes

$$\frac{d\mathcal{N}_A(r)}{dt} = A(r) \left[\frac{d\mathcal{I}_A}{dz}(z) + \frac{d\mathcal{I}_A}{dz}(2r+z) \right]. \quad (22)$$

Now $d\mathcal{I}_A/dt$ is proportional to plasma particle density and Eq. (22) has the form:

$$\frac{d\mathcal{N}_A(r)}{dt} = A(r) \frac{n(r) \langle \sigma v \rangle}{v_A} T \left[\mathcal{I}_A(z) + \mathcal{I}_A(2r+z) \right]. \quad (23)$$

The cross-section average $\langle \sigma v \rangle_T$ is the total cross-section for ionization and charge exchange and v_A is the beam velocity. For thermal beams assumed to propagate radially, $A(r)$ is replaced by $A_{th}(r) = \frac{1}{e2\pi LC_1} \left(\frac{1}{r} \right)$.

The guiding center charge exchange loss terms (class B) are derived in a similar manner.* First, analogous to Eq. (23), the loss rate for the particle density in a zone is:

$$\frac{dn_B(r)}{dt} = -A(r) \frac{n(r) \langle \sigma v \rangle_{cx}}{v_B} [I_B(z) + I_B(2r + z)], \quad (24)$$

where the appropriate cross-section is charge exchange.

Applying the results of Eq. (5), we obtain the guiding center loss rate

$$\frac{d}{dt} \mathcal{N}_B(r) = -\mathcal{N}_B(r) \sum_{r'} \frac{\alpha(r')}{\alpha(r)} f(r, r') A(r') \frac{\langle \sigma v \rangle_{cx}}{v_B} [I_B(z') + I_B(2R_w - z')]. \quad (25)$$

An additional loss term is coulomb scattering into the loss cone. Let $(t/\tau)_{r, r'}$ be the fraction of an orbit time that an ion with guiding center in a zone at radius r spends in the zone at radius r' . Counting all particles and using Eq. (2), we have

$$\alpha(r) \mathcal{N}(r) \left(\frac{t}{\tau} \right)_{r, r'} = \alpha(r') f(r, r') \mathcal{N}(r) \quad (26)$$

If $\tau_{ii}(r)^\dagger$ is the lifetime for ion-ion scattering into the loss zone at radius r , then the loss time for the guiding center located at r must be averaged over nearby zones. The average is

$$\frac{1}{\langle \tau_{ii}(r) \rangle} = \sum_{r'} \frac{\left(\frac{t}{\tau} \right)_{r, r'}}{\tau_{ii}(r')} = \sum_{r'} \frac{n(r') \left(\frac{t}{\tau} \right)_{r, r'}}{n \tau_{ii}(r')} \quad (27)$$

Using Eq. (26) and averaging over the flux tube, we obtain the loss rate for guiding center

$$\frac{d}{dt} \mathcal{N}(r) = \frac{\mathcal{N}(r)}{\langle \tau_{ii}(r) \rangle} = \mathcal{N}(r) \frac{C_2}{C_1} \sum_{r'} \left[\frac{\alpha(r')}{\alpha(r)} f(r, r') \frac{n(r')}{n \tau_{ii}(r')} \right].$$

Using Eqs. (22), (25), and (27), and defining $\tilde{f}(r, r') = [\alpha(r')/\alpha(r)] f(r, r')$, we obtain the guiding center rate equations for hot and target ion species as follows:

* In contrast to ion trapping, Eq. (16) is correct in cylindrical geometry for charge exchange losses since the zone for loss of guiding centers is properly handled by the formulation of Eq. (5).

† The ion-ion scattering term includes a term for scattering between ions species as well as self-scattering.

Guiding center density for hot ions:

$$\begin{aligned}
 \frac{d}{dt} \mathcal{N}_h(r) = & A(r) \left(\frac{n_i \langle \sigma v \rangle_T^{h-t} + n_h \langle \sigma v \rangle_T^{h-h}}{v_b} \right) \left[I_b(R_w - r) + I_b(R_w + r) \right. \\
 & \left. + I_{cxh}(R_w - r) + I_{cxh}(R_w + r) \right] \\
 & - \mathcal{N}_h(r) \sum_{r'=r-a_h}^{r+a_h} \tilde{f}_h(r, r') \left(A(r') \left\{ \frac{\langle \sigma v \rangle_{cx}^{h-h}}{v_b} \left[I_b(R_w - r') + I_b(R_w + r') \right] \right. \right. \\
 & + \frac{\langle \sigma v \rangle_{cx}^{h-h}}{v_h} \left[I_{cxh}(R_w - r') + I_{cxh}(R_w + r') \right] \\
 & \left. \left. + \frac{\langle \sigma v \rangle_{cx}^{t-h}}{v_t} \left[I_{cxt}(R_w - r') + I_{cxt}(R_w + r') \right] \right\} \right. \\
 & + A_{th}(r') \left\{ \frac{\langle \sigma v \rangle_{cx}^{FC-t}}{v_{FC}} \left[\langle I_{FC}(R_w - r') \rangle + \langle I_{FC}(R_w + r') \rangle \right] \right. \\
 & + \frac{\langle \sigma v \rangle_{cx}^{m-h}}{v_m} \langle 1 + FMOL \rangle \left[\langle I_s(R_w - r') \rangle + \langle I_s(R_w + r') \rangle \right. \\
 & \left. \left. + \langle I_{rm}(R_w - r') \rangle + \langle I_{rm}(R_w + r') \rangle \right] \right. \\
 & \left. + \frac{\langle \sigma v \rangle_{cx}^{a-h}}{v_a} \left[\langle I_a(R_w - r') \rangle + \langle I_a(R_w + r') \rangle \right] \right\} + \left(\frac{C_2}{C_1} \right) \frac{n_h(r')}{(\bar{n}_{ii})_h(r')} \quad (28)
 \end{aligned}$$

Guiding center density of target ions:

$$\begin{aligned}
\frac{d}{dt} \mathcal{N}_t(r) = & A(r) \left(\frac{n_t \langle \sigma v \rangle_T^{t-t} + n_h \langle \sigma v \rangle_T^{t-h}}{v_t} \right) [I_{\text{cxt}}(R_w - r') + I_{\text{cxt}}(R_w + r')] \\
& - \mathcal{N}_t(r) \sum_{r'=r-a_t}^{r+a_t} \tilde{f}_t(r, r') \left(A(r') \left\{ \frac{\langle \sigma v \rangle_{\text{cx}}^{h-t}}{v_b} [I_b(R_w - r') + I_b(R_w + r')] \right. \right. \\
& + \frac{\langle \sigma v \rangle_{\text{cx}}^{h-t}}{v_h} [I_{\text{cxh}}(R_w - r') + I_{\text{cxh}}(R_w + r')] \\
& + \left. \frac{\langle \sigma v \rangle_{\text{cx}}^{t-t}}{v_h} [I_{\text{cxt}}(R_w - r') + I_{\text{cxt}}(R_w + r')] \right\} \\
& + A_{\text{th}}(r') \left\{ \frac{\langle \sigma v \rangle_{\text{cx}}^{\text{FC}-t}}{v_{\text{FC}}} [\langle I_{\text{FC}}(R_w - r') \rangle + \langle I_{\text{F}}(R_w + r') \rangle] \right. \\
& + \frac{\langle \sigma v \rangle_{\text{cx}}^{m-t}}{v_m} (1 + \text{FMOL}) [\langle I_{\text{S}}(R_w - r') \rangle + \langle I_{\text{S}}(R_w + r') \rangle \\
& + \langle I_{\text{rm}}(R_w - r') \rangle + \langle I_{\text{rm}}(R_w + r') \rangle] \\
& + \left. \frac{\langle \sigma v \rangle_{\text{cx}}^{a-t}}{v_a} [\langle I_a(R_w - r') \rangle + \langle I_a(R_w + r') \rangle] \right\} + \left(\frac{C_2}{C_1} \right) \frac{n_t(r')}{\langle n \tau_{ii} \rangle_t(r')} \Bigg) \quad (29)
\end{aligned}$$

The quantity FMOL in the molecular charge exchange term takes into account charge exchange on molecular ions. These effects are discussed in Appendix B.

Using Eq. (2) the rate equations for particle density are,

$$\begin{aligned} \text{TARGET} \quad \frac{dn_t(r)}{dt} &= \sum_{r'=r-a_t}^{r+a_t} f_t(r, r') \frac{d}{dt} \mathcal{N}_t(r'), \\ \text{HOT} \quad \frac{dn_h(r)}{dt} &= \sum_{r'=r-a_h}^{r+a_h} f_h(r, r') \frac{d}{dt} \mathcal{N}_h(r'). \end{aligned} \quad (30)$$

Thermal Gas and Wall Reflex

The source terms for molecular and thermal gas have already been discussed.

Pumping of gas occurs by ionization and charge exchange on the plasma, by wall gettering, and by conduction through apertures in the mirrors. Thermal gas produced by wall reflux of charge exchange neutrals may be either atomic or molecular. The rate equation for molecular gas is

$$\frac{d}{dt} (n_m V_c) = \frac{I_d(t)}{e} + \Gamma_m (\beta I_p + I_{cxh} + I_{surh}) - 1/4 n_m v_m (A_p + A_w S_{wm} + 2A_{end}). \quad (31)$$

The equation for atomic gas is

$$\frac{d}{dt} (n_a V_c) = \Gamma_h (\beta I_p + I_{cxh} + I_{surh}) + \Gamma_t (I_{tcx} + I_{surt}) - 1/4 n_a v_a (A_p + A_w S_{wa} + 2A_{end}). \quad (32)$$

The symbols are defined as follows:

- $n_m (n_a)$ = molecular (atomic) neutral density
- $v_m (v_a)$ = molecular (atomic) thermal velocity
- $S_{wm} (S_{wa})$ = molecular (atomic) wall sticking fraction
- V_c = chamber volume outside plasma
- A_w = wall area
- A_p = plasma surface area = $2.6 \times 2\pi R_p L$
- A_{end} = opening into end tanks
- Γ_m = reflux coefficient for fast neutrals at the target (hot) energy returning as atoms
- $\Gamma_t (\Gamma_h)$ = reflux coefficient for fast neutrals at the target (hot) energy returning as atoms
- I_p = penetrating beam

Fig. 2. Simplified flow diagram of wall reflux and calculation of random atomic and molecular beams on plasma surface.

$I_{\text{cxt}}(I_{\text{cxh}})$ = charge exchange beam from the bulk of the plasma at the target (hot) energy

$I_{\text{surt}}(I_{\text{surrh}})$ = charge exchange current from the plasma surface at the target (hot) energy

I_d = molecular gas current from sources which fills the volume outside the plasma

The coefficient β is set equal to zero for a beam dump. In these equations, pumping by the plasma and through the end apertures is assumed 100% efficient. The factor 2.6 in the definition of A_p takes into account the surface area of the fan-shaped plasma of a minimum-B well.

Equations (31) and (32) are solved for the gas densities. Figure 2 shows a flow diagram for solution of the gas equations. The random currents $(en_a v_a A_p)/4$ and $(en_m v_m A_p)/4$ and the direct streaming current I_s are the appropriate currents for charge exchange on the plasma surface.

Solutions of Equations

At each time step, the beam current amplitudes in each zone are calculated. For the energetic beams [Eqs. (8)-(10)], the solution is

$$I(z + \Delta z) = I(z) + \frac{dI}{dz}(z) \Delta z,$$

where $\Delta z = \Delta R$. The initial conditions at $z = 0$ are $I_b(0) =$ injected beam current, $I_{\text{cxh}}(0) = I_{\text{cxt}}(0) = 0$. The solutions for the Franck-Condon and thermal beams are [Eqs. (11)-(14)]

$$I(z + \Delta z) = I(z) \exp \left[-\frac{\Delta z}{2} \left(\frac{1}{\lambda(z)} + \frac{1}{\lambda(z + \Delta z)} \right) \right].$$

The initial conditions are:

$$I_{\text{FC}}(0) = 0, \quad I_s(0) = I_{s0}, \quad I_{\text{rm}} = en_m v_m A_p / 4 \quad \text{and} \quad I_{\text{ra}} = en_a v_a A_p / 4.$$

With the current amplitudes determined in each radial zone, the solution for guiding center density [Eqs. (28)-(29)] for the next time step is

$$\mathcal{N}(r, t + \Delta t) = \mathcal{N}(r, t) + \frac{d}{dt} \mathcal{N}(r, t) \Delta t,$$

and Eq. (2) is used to find the particle density $n(r, t + \Delta t)$.

Equations (31) and (32) for the molecular and atomic gas densities are also solved in similar fashion using the values of I_b , I_{cxt} , and I_{cxh} evaluated at $z = 2R_w$. The surface charge exchange loss currents I_{cxt} and I_{cxh} are obtained by summing the charge exchange losses in each radial zone resulting from the thermal beams. The plasma surface area varies with time in direct proportion to the plasma radius.

This completes the cycle. The newly determined plasma density is used to calculate the beam attenuation for the next time step, etc.

Numerical Examples for 2XIIB

The buildup code has been used to study beam injection into a target plasma in the 2XIIB mirror experiment. The target plasma is formed by trapping and adiabatic compression of an injected deuterium plasma. Typical parameters of the decaying trapped plasma are initial density $>5 \times 10^{13} \text{ cm}^{-3}$, mean ion energy of a few keV and $n_T \geq 1 \times 10^{10} \text{ cm}^{-3} \text{ sec}$. To sustain the plasma there are 12 LBL pulsed 20 kV, 50A neutral beam modules.

Two injected beam energy distributions have been investigated. These are mono-energetic injection at 600A and 20 keV energy and a spread beam distribution of 275A and mean energy of 14 keV. The latter distribution, obtained by extracting beams at different energies from the 12 independently controlled neutral beam modules, results in a broad plasma ion distribution throughout the transition from target plasma to a hot ion plasma. The energy program for the module was obtained by Fokker-Planck calculations. Such distributions should be stable against double-hump instabilities.⁴

As already mentioned, the buildup code does not take into account energy exchange between species. In particular, slowing down of ions or electrons is neglected. Although this is an important effect, the cases investigated have sufficiently large currents of injected beam that the charge exchange lifetime of a plasma ion on the injected neutrals is less than the lifetime resulting from cooling of ions on electrons. Thus, neglect of energy exchange probably does not result in large errors.

The effect of thermal gas on the rate of plasma expansion is shown in a series of runs where the ratio of molecular gas current incident on the boundary to the injected beam current is varied from 0 to 8% in factors of 2. The injected beam is the spread beam distribution with a beam focus of 10 cm at the plasma.

Figure 3 shows the result of the calculation. The rate of expansion of the plasma is greatly reduced as the gas current is increased. The maximum gas current for equilibrium is about 10%.

To calculate realistic cases of buildup for the experiment, several runs were made including the effects of gas from the injected beam modules, reflux, as thermal atomic neutrals, of fast charge exchange neutrals from the plasma chamber walls, and gas pumping by titanium gettered walls. The calculation was carried out for the mono-energetic beam and the spread beam distribution of injected currents. Ion scattering is assumed classical, and no beam dump is present in the experiment. The mirror ratio is 2.

The molecular gas currents from the sources, I_g and I_d , were assumed constant and estimated from streaming of gas through the baffling apertures between the beam module vacuum tanks and the plasma vacuum chamber. For these cases, the initial target plasma density and ion energy are $5 \times 10^{13} \text{ cm}^{-3}$ and 2 keV respectively. The focused beam height is 6 cm, appropriate for curved beam extraction grids. A beta limit of 0.5 was imposed on the plasma density of trapped ions.

Fig. 3. Computer calculations showing reduced radial expansion with molecular gas on the plasma boundary.

Fig. 4. Radial plasma density profiles and time history of density on axis for monoenergetic injection, including effects of wall reflux, source gas, and wall gettering.

Fig. 5. Molecular and atomic currents at plasma surface for monoenergetic beam.

Fig. 6. Radial plasma density profiles and time history of density on axis for spread beam injection, including effects of wall reflux, source gas, and wall gettering.

Fig. 7. Molecular and atomic currents at plasma surface for spread beam injection.

The monoenergetic beam case is shown in Figs. 4 and 5. Figure 4 shows the plasma density profiles at the midplane at several times and the time history of target and trapped density on the axis. The density is seen to increase rapidly to the beta limit in about 300 μ sec. The plasma radius rapidly increases in time, doubling the initial radius, measured at one-half peak density, in about 6 msec. Minimum injected beam attenuation is 50%.

Figure 5 shows the self-consistent molecular and atomic thermal currents incident on the plasma surface. The coefficients for reflux of energetic neutrals from walls are dependent on wall conditions, and little experimental data is available. The coefficients assumed for the calculations are based on theoretical estimates with an increase of a factor 3 at high energies to bring into agreement the theory and experimental data.⁵ The total gas load on the plasma is about 3%, approximately a factor of 3 below the maximum allowed for equilibrium.

The results for the spread beam case are shown in Figs. 6 and 7. With the reduced current the plasma density increases more slowly and the plasma size expands much less rapidly. In both examples, the density remains high throughout the transition from target to hot plasma and the quantity $\omega_{pe}^2/\omega_{ce}^2$ remains greater than 6.

Discussion

The results of the calculation have shown that charge exchange of thermal gas on the plasma boundary is a severe problem for mirror machines. For plasma equilibrium, the thermal gas must be small compared to the injected sustaining beam current. On the other hand, if the thermal gas is very small, the plasma can rapidly expand as a result of ion trapping on the boundary.

The calculation has not taken into account radial transport by ion-electron collisions, nor has energy exchange between species been included. For plasmas of a few tens of keV energy, presently of interest in experiments such as 2XIIIB, neglecting these effects probably does not introduce appreciable error. At these energies, charge exchange is the dominant process for diffusion. As already pointed out, the hot ion lifetime during buildup tends to be limited by charge exchange on the injected beam rather than slowing down on the electrons. As the ion energy increases, however, the charge exchange cross-section falls, and eventually ion-electron collisions become important. Their effects should then be included for a more accurate calculation.

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Appendix A: Forward Scattering of Charge Exchange Neutrals

Consider charge exchange between a beam of velocity v_i and ions of velocity v_j . The rate of charge exchange for neutrals scattered into the angle interval $d\phi$ at an angle ϕ measured with respect to the beam direction is proportional to $\langle \sigma v(\phi) \rangle_{cx} = \iint f(E_j, \theta_j) \sigma_{cx}(v_r) v_r (1/2) \times \sin \phi d\phi dE_j d\theta_j$. The function f is the energy distribution function for ions of energy E_j and pitch angle θ_j , and $v_r = (v_i^2 + v_j^2 - 2v_i v_j \cos \phi)^{1/2}$ is the relative velocity of ions and beam neutrals. The forward scattered fraction is defined by

$$F_{ij} = \frac{\int_0^{\pi/2} \langle \sigma v(\phi) \rangle_{cx} d\phi}{\int_0^{\pi} \langle \sigma v(\pm) \rangle_{cx} d\phi} \quad (33)$$

Neglecting the angle dependence θ_j , Eq. (34) has been evaluated as a function of beam energy using the beam-sustained energy distribution given by Kuo-Petravic, et al.⁶ Figure 8 shows the forward scattered fraction for hydrogen and deuterium plasmas for a mirror ratio of 3.

Fig. 8. Forward scattered fraction of charge exchange neutrals versus beam energy.

Appendix B: Charge Exchange Losses on the Plasma Boundary

A thermal molecule incident on the boundary can undergo ionization, charge exchange, and dissociation. Plasma erosion by charge exchange is most severe for reactions whose mean free path is less than a Larmor radius. Energetic Franck-Condon neutrals of a few eV energy or so are more penetrating. Thus, their effect on erosion is neglected.

The following list of interactions have been considered. Reactions on the neutral molecule are termed primary and on the molecular ions, secondary. The reactions listed are for hydrogen. The subscript c refers to a "cold" (or thermal) incident gas molecule.

Primary Reactions

1. $H_{2c} + e \rightarrow H_{2c}^+ + 2e$	ionization by electrons
2. $H_{2c} + e \rightarrow H + H^+ + 2e$	dissociative excitation
3. $H_{2c} + e \rightarrow 2H + e$	Franck-Condon by dissociation
4. $H_{2c} + \underline{H}^+ \rightarrow H_{2c}^+ + \underline{H} + e$	ionization by ions
5. $H_{2c} + \underline{H}^+ \rightarrow H + H^+ + \underline{H}^+$	dissociative excitation
6. $H_{2c} + \underline{H}^+ \rightarrow H_{2c}^+ + \underline{H}$	charge exchange

Secondary Reactions

	~3 eV	
7. $H_{2c}^+ + e \rightarrow H + H^+ + e$		dissociative excitation
8. $H_{2c}^+ + e \rightarrow 2H^+ + 2e$		dissociative ionization
9. $H_{2c}^+ + \underline{H}^+ \rightarrow H + H^+ + \underline{H}^+$		dissociative excitation
10. $H_{2c}^+ + \underline{H}^+ \rightarrow 2H^+ + H^+ + e$		dissociative ionization
11. $H_{2c}^+ + \underline{H}^+ \rightarrow 2H^+ + \underline{H}$		charge exchange

The cross-sections for reactions 2 and 5 are small relative to the remaining reactions. Thus, they are neglected for the calculation of FMOL.

Let I_c be the equivalent current of molecules incident on the boundary. The immediate charge exchange loss on the molecule is

$$I_{x1} = I_c \frac{\langle \sigma v \rangle_6}{\langle \sigma v \rangle_{1-6}}, \quad (34)$$

where the numbered subscript refers to the reactions listed, and $\langle \sigma v \rangle_{i-j}$ implies a summation over the cross sections i to j.

Now, if a large enough electric field exists on the plasma boundary, e.g., the result of the mirror ambipolar potential, the molecular ions will be swept along the field lines through the mirror before appreciable losses via the secondary processes can occur. The total charge exchange loss is then I_{x1} . Figure 9 shows a plot versus energy of I_{x1} for hydrogen and deuterium.

On the other hand, if the field on the boundary is weak, perhaps as a result of surface cooling on the cold gas, then charge exchange losses on the molecular ions can occur. The maximum loss is I_{x2} given by

$$I_{x2} = I_c \left[\frac{\langle \sigma v \rangle_1 + \langle \sigma v \rangle_4 + \langle \sigma v \rangle_6}{\langle \sigma v \rangle_{1-6}} \right] \frac{\langle \sigma v \rangle_{11}}{\langle \sigma v \rangle_{7-11}}. \quad (35)$$

Estimates of I_{x2} have been made based on some published measurements and some theoretical calculations of dissociation and charge exchange by Gerjuoy.⁷

If FMOL is defined by $FMOL = I_{x2}/I_{x1}$, then over the energy range 20 to 100 kV, $FMOL \approx 0.5$ for hydrogen and deuterium. Thus an upper bound on charge exchange losses is $I_{x1} + I_{x2}$. A lower bound is I_{x1} .

Fig. 9. Fractional charge exchange loss on neutral molecules at plasma surface versus plasma ion energy.

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