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CONTROL OF XENON INSTABILITIES

IN LARGE PWR'S

QUARTERLY PROGRESS REPORT

FOR THE PERIOD ENDING

JUNE 30, 1967

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Project Engineer

Prepared for the New York Operations Office
U. S. Atomic Energy Commission
Under AEC Contract AT(30-1)-3680

July 1967

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PROGRAM DESCRIPTION

This program investigates the characteristics and control of spatial instabilities in large pressurized-water reactors (large PWR's), with particular emphasis on azimuthal xenon instabilities (x-y plane). The program consists of the following technical tasks:

Task EUXE-200 - Effect of Core Parameters on Spatial Oscillations

The aim of this task is to analyze the effect of variations in core design and operating parameters on the propensity for spatial oscillations, with emphasis on those resulting from xenon redistribution. Parameters to be analyzed include, but are not limited to, core dimensions, fuel and moderator temperature feedbacks, and power distributions.

Task EUXE-300 - Remedial Control Procedures

Under this task, two-dimensional x-y calculations will be performed to establish control methods and detector locations which will prevent divergent oscillations. Calculated results will be used to develop criteria for the application of remedial measures to large pressurized-water reactors.

Task EUXE-400 - Three-Dimensional Analysis

Under this task, either direct or synthesized three-dimensional calculations will be performed to further study spatial instabilities in large PWR cores with conditions conducive to oscillations. If a three-dimensional oscillation develops after perturbations have been introduced, selected remedial methods (as developed under Task EUXE-300) will be applied and evaluated. The unique characteristics of three-dimensional oscillations(if observed) will be identified and the criteria for the application of remedial measures will be developed.

PROGRESS SUMMARY

Major progress is summarized below for the three-month period ending June 30, 1967:

A simple correlation was developed to correct xenon-oscillation results for the errors caused by using finite time-steps in diffusion theory calculations. The use of a finite time-step introduces errors because the flux is assumed constant during the time-step. The calculations performed verified the intuitive deductions that for unstable cores, somewhat removed from the stability threshold, a non-zero time-step will result in calculated oscillations which are relatively more stable than in the limit of a zero time-step.

A useful index of the influence of various core parameters on xenon-induced spatial instabilities was developed within the framework of linearized modal-expansion theory. This index, which is the real part of the roots of the characteristics equation, was determined as a function of selected core parameters. These modal analysis results were found to be in relatively good agreement with diffusion theory results when the results from the latter were corrected for the error caused by using finite time-steps.

One-dimensional diffusion theory calculation were performed for a core known to be unstable, where perturbations of zero magnitude were introduced into the core purposely to see if oscillations could be excited by very small perturbations. Results, which are very sensitive to the convergence criteria and the time-step lengths used in the calculations, suggest that very small perturbations, of the order of the flux criterion, can excite diverging oscillations in an unstable core.

Xenon-induced oscillations in the x-y plane were studied in the 11-ft core with fuel temperature feedback for a dished, steady-state power distribution. While the calculations with a four-hour time-step predicted convergent oscillations, extrapolation to zero time-step showed the core to be unstable in the first harmonic. Further calculations indicate that it is important to include the variation in the fast removal cross section with temperature along with the fast absorption cross section in stability calculations.

DETAILED PROGRESS REPORTS

EUXE-100

PROGRAM MANAGEMENT

M. J. O'Boyle, Project Engineer

Development Projects
PWR Plant Division

L. W. Weidman

This is the fourth in a series of technical progress reports on the U.S.-Euratom program titled "Control of Xenon Instabilities in Large PWR's". The preceding reports in this series are:

EURAEC-1721 "Control of Xenon Instabilities in Large PWR's",
WCAP-3680-1 Technical Progress Report for the Period Ending
September 30, 1966

EURAEC-1781 "Control of Xenon Instabilities in Large PWR's",
WCAP-3680-2 Quarterly Progress Report for the Period Ending
December 31, 1966

EURAEC-1830 "Control of Xenon Instabilities in Large PWR's",
WCAP-3680-3 Quarterly Progress Report for the Period Ending
March 31, 1967

Program management is not discussed in this technical progress report, since separate reports emphasizing the administrative aspects of the program are published monthly for limited distribution.

EFFECT OF CORE PARAMETERS ON SPATIAL OSCILLATIONS

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A. Introduction and Summary

Previous quarterlies /1,2,3 have described how two-dimensional calculations were used to evaluate both the influence of various core parameters on xenon-induced oscillations in the x-y plane and the spatial characteristics of these oscillations. The parameters studied were core size, power density, fuel temperature feedback, xenon absorption cross section, and Xe-135 and I-135 fission yields.

During the current reporting period the following investigations were carried out to complete essentially all the work under this task:

(1) Modal Expansion Calculations and Sensitivity Analyses

The use of modal stability analysis in large PWR's, based on λ -mode expansions, was evaluated. Relatively good agreement was found between modal results and numerical calculations with diffusion theory programs, when the latter results were corrected for the use of a non-zero time-step length. A useful stability index can be defined as the real part of the roots of the characteristic equation of the first overtone mode.

-
- /1 M. J. O'Boyle, et al, "Control of Xenon Instabilities in Large PWR's Technical Progress Report for the Period Ending September 30, 1966", WCAP-3680-1 (1966)
 - /2 M. J. O'Boyle, et al, "Control of Xenon Instabilities in Large PWR's Quarterly Progress Report for the Period Ending December 31, 1966", WCAP-3680-2 (1966)
 - /3 M. J. O'Boyle et al, "Control of Xenon Instabilities in Large PWR's Quarterly Progress Report for the Period Ending March 31, 1967", WCAP-3680-3 (1967)

Analytical expressions were derived which express the dependence of the stability index on variations in core parameters. Extensive data, in the form of curves, have been generated to provide a quick guide to the effect of variations in core parameters on spatial stability.

(2) Variations in Average Core Enrichment

The average U-235 enrichment was varied in two-dimensional calculations to evaluate the dependence of the spatial stability characteristics on this parameter. The average power density was held constant, while an approximately flat power distribution was maintained in all cases. The results of the calculations were inconclusive. The results, however, provided a good example of the importance of and the necessity of correcting for the use of a non-zero time-step length in diffusion theory digital calculations.

(3) Fuel Temperature Feedback

A two-dimensional calculation was performed for the 11-ft core with fuel temperature feedback, for a dished steady-state distribution. Although the calculations with a four-hour time-step predicted convergent oscillations, extrapolation to a zero time-step showed the core to be unstable in the first harmonic. The sensitivity of the stability characteristics to the magnitude of the fuel temperature feedback was studied with the one-dimensional program ZEST. Calculations which neglected the variation of the fast removal cross section with fuel temperature showed the importance of accounting for the effect of fuel temperature on both the fast absorption and removal cross sections in diffusion theory calculations.

(4) Time Step Length Analysis

One- and two-dimensional digital calculations have shown that both the damping factor and the oscillation period are sensitive to the length of the time-step used in the calculations. A theoretical analysis, based on Davison's ideas^{/1} has been performed. Very good

^{/1} B. Davison, "Correction for Non-Zero Time Step in the Numerical Simulation of Spatial Oscillations of Xenon in a Nuclear Reactor", AECL-1292 (1961)

agreement was found between the theoretical prediction and numerical calculations of the effect of a non-zero time-step on stability characteristics. Simple correlations were derived which permit one to extrapolate digital results to essentially zero time-step lengths.

(5) Small Perturbations

Calculations with program ZEST have shown that very small perturbations will excite divergent oscillations in an unstable core. The growth rate and period of oscillation are, however, sensitive functions of the flux convergence criterion and the time-step length used in the calculations, both of which act as sources of pseudo-fluctuations.

(6) Program ZEST

A one-dimensional, two-group diffusion theory program was written, which solves the coupled neutron flux diffusion equations and the equations describing the time dependence of I-135 and Xe-135, on a discrete time mesh basis. Fuel temperature and moderator density feedback effects are accounted for. The program is geared to xenon oscillation studies and provides useful output edits for that purpose.

B. Sensitivity and Parametric Analyses

In this and previous progress reports^{/1,2}, the sensitivity of xenon-induced spatial instabilities in the x-y plane to variations in core parameters was studied numerically with one- and two-dimensional multigroup diffusion theory codes. Analytical sensitivity and parametric evaluations are useful both as a guide in evaluating the numerical results, and as a means of providing a continuous and complete mapping of the parameters through the ranges of

/1 M. J. O'Boyle, et al, "Control of Xenon Instabilities in Large PWR's Technical Progress Report for the Period Ending December 31, 1966" WCAP-3680-2 (1966)

/2 M. J. O'Boyle, et al, "Control of Xenon Instabilities in Large PWR's Technical Progress Report for the Period Ending March 31, 1967", WCAP-3680-3 (1967)

interest. This section describes such an analysis based on a modal expansion of the linearized equation. The modal expansion method of Randall and St. John^{/1}, in terms of the eigenvector of the steady-state diffusion equation, or in terms of the geometric eigenvector, is used primarily because it leads to simple analytical expressions for the spatial stability characteristics. The expansion in terms of complex natural modes, originally proposed by Kaplan^{/2}, and used extensively at the Westinghouse Bettis Laboratories^{/3}, possesses the property of finality (no modal interaction), and is superior to the above. However, the analysis is more complex, and must be implemented numerically. In the following section the applicability of a simple modal expansion method to large pressurized water reactors is briefly evaluated.

(1) Evaluation of a Simple Modal Expansion Method

Within the framework of linear theory, the main disadvantages of the Randall-St. John modal analysis is the neglect of modal interactions, and the difficulty in representing the spatial flux distribution in the analysis. However, one can define a spatial distribution index, which gives an indication of the amount of flux flattening in the core. Moreover, as shown on the following Figure 200.7, the dependence of the stability characteristics on the steady-state flux distribution is very much reduced when the core is operated with a negative net temperature coefficient.

As shown by Pearce^{/4}, the space shape of the excess xenon distribution changes continuously during a cycle, as a result of the local xenon

-
- /1 D. Randall and D. S. St. John, "Xenon Spatial Oscillations", Nucleonics, 16, 3, 82-86 (1958)
- /2 S. Kaplan, "The Property of Finality and the Analysis of Problems in Reactor Space-Time Kinetics by Various Modal Expansions", Nuclear Science and Engineering, 9, 357-361 (1961)
- /3 T. R. England, G. L. Hartfield, and R. K. Deremer, "Xenon Spatial Stability in Large Seed-Blanket Reactors", WAPD-TM-606 (1967)
- /4 R. M. Pearce, "Method of Studying Xenon Spatial Instability with an Analogue Computer", Nuclear Science and Engineering, 11, 328-337 (1961)

loss rate. In the analysis of the stability of the first overtone, however, it is assumed that the excess xenon and flux distributions have the same spatial shape. Figures 200.1 and 200.2 show the spatial distributions of the excess thermal flux and xenon concentrations, respectively, in a one-dimensional reflected slab with uniform fuel loading, as calculated with program ZEST (see section G). This core is unstable with a damping factor of 0.022 hrs^{-1} . The distortion in the excess xenon distribution is apparent in Figure 200.2. As pointed out by Pearce, the peak of the excess xenon distribution occurs at different times in different regions of the core. From Figures 200.1 and 200.2, one can see that in a large PWR the effect of the reflector is small, and that the distortion of the excess xenon distribution is not very great. The amount of modal interaction, as originally discussed by Gyorey^{/1}, appears to be small for the cases of interest with PWR cores. Gyorey has also shown that flux flattening decreases the amount of modal interaction^{/2}. Because of economic considerations, PWR cores are normally designed to operate with a flattened steady-state power distribution, providing some justification for neglecting modal interaction.

Figure 200.3 shows a comparison of the stability characteristics of a reflected 10-ft slab, with a uniform fuel loading (2.6 a/o U-235), as calculated by modal analysis and with the diffusion theory program ZEST, with a correction for the use of a finite time-step. The stability index plotted in the figure corresponds to the damping factor of the first overtone and is defined in the next section. For a sinusoidal steady-state distribution, the flux distribution index Λ_1 is 3.0, and for a flat distribution, Λ_1 is 1.0. With no temperature

^{/1} A. L. Gyorey, "The Effect of Modal Interaction in the Xenon Instability Problem, Nuclear Science and Engineering, 13, 338-344 (1962)

^{/2} A. L. Gyorey, "The Effect of Modal Interaction on the Xenon Instability Problem", Trans. Am. Nuc. Soc. 4, 1-83 (1961)

Figure 200.1 Spatial Distributions of Thermal Flux Variations

Figure 200.2 Spatial Distribution of Excess Xenon Concentration

Figure 200.3 Comparison of Numerical Diffusion Theory Calculations and Modal Expansion Analysis

feedback, the stability index is very sensitive to the flux distribution index Λ_1 , as shown on Figure 200.3. For the core studied here, the steady-state distribution is not a true sinusoidal because of the flattening effect of the spatial equilibrium xenon distribution and a small reflector effect. The value of Λ_1 has been estimated to be between 2.8 and 2.9. The agreement between the modal analysis and digital calculations exhibited on Figure 200.3 is deemed to be satisfactory. Note, however, that in making such a comparison, it is essential to correct the digital calculations for the effect of a non-zero time-step. This effect is discussed at length in section E. For instance, for the core operating at 106 w/cc, a digital calculation with a four-hour time-step actually predicts a stable condition. Thus, it appears that digital calculations with time-steps greater than about one hour are probably less accurate than the simple modal calculations. However, in section E, a simple correction for the effect of a finite time-step length is introduced, which allows extrapolation of the results to effectively zero time-step length.

2. Parametric Analyses by a Modal Expansion Method

The behavior of the first overtone of the excess flux distribution is described by the function

$$\delta \phi(\underline{r}, t) = A_1 \psi_1(\underline{r}) e^{b_1 t} \cos(\omega_1 - \theta)t \quad (200.1)$$

where $\psi_1(\underline{r})$ is the spatial distribution of the first overtone, and A_1 and θ are constants which depend on the perturbation. The damping factor b is a useful stability index, since it simply defines the regions of stability and instability:

According to simple modal analysis^{/1}, the stability index b is given by

$$b = - \frac{(\lambda_1 + \lambda_x + \sigma_x \bar{\phi}_1)}{2} - \frac{a_x \sigma_x \bar{\phi}_1 (\bar{X}_1 - \gamma_x)}{2(\mu_1^2 - a_T \bar{\phi}_1)} \quad (200.2)$$

where

$$\bar{\phi}_1 = \int d\underline{r} \phi_o(\underline{r}) \psi_1^2(\underline{r})$$

$$\bar{X}_1 = \int d\underline{r} X_o(\underline{r}) \psi_1^2(\underline{r}) \quad (200.4)$$

$$\gamma_x = \frac{y_x}{y_1 + y_x} = \frac{y_x}{y_T} \quad (200.5)$$

In equations (200.3) and (200.4), $\phi_o(\underline{r})$ and $X_o(\underline{r})$ are the steady-state flux and xenon distributions, respectively. The xenon concentration is in units relative to the saturation value. In equation (200.2), μ_1^2 is the eigenvalue corresponding to the eigenvector ψ_1 , and a_x and a_T are coefficients which represent the dependence of B^2 on the variation in xenon concentration and flux level, respectively. In the x - y plane, a_T represents the effects of fuel and moderator temperature feedback. The remaining notation is standard.

Equation (200.2) has been used to calculate the stability index for diametral oscillations in the x - y plane as a function of various core parameters. Variations in core enrichment, core diameter, power density, power distribution, power coefficient, iodine yield, xenon yield, and xenon cross section were studied. Simple functional relationships between the stability index and the various parameters

^{/1} D. Randall and D. S. St. John, "Xenon Spatial Oscillations", Nucleonics, 16, 3, 82-86 (1958)

listed above can be derived if the following simplifying approximations are made. The volume-averaged value of ϕ_0 is used in lieu of $\bar{\phi}_1$ in equation (200.2). For a flat flux distribution this is exact, while for a sinusoidal distribution, $\bar{\phi}_1/\bar{\phi}_0 = 1.0667$ ^{/1}. We further assume

$$\bar{\phi}_0 = \frac{p}{\Sigma_f \kappa} \quad (200.6)$$

where p is the average core power density, κ is a conversion factor, converting fissions/sec to watts, and Σ_f is an effective fission cross section for the core-averaged fuel enrichment:

$$\Sigma_f = \Sigma_{f_{th}} + \Sigma_{f_f} \frac{\phi_f}{\phi_{th}} \quad (200.7)$$

where subscripts th and f refer to the thermal and fast groups, respectively, in a two-group scheme. The average equilibrium xenon concentration (relative to the saturation value) is calculated from

$$\bar{X}_0 = \frac{\bar{\phi}_0 \sigma_x}{\lambda_x + \bar{\phi}_0 \sigma_x} \quad (200.8)$$

The error introduced by the use of (200.8) has been shown by Randall and St. John^{/1} to be small. In reference 1, a flux distribution index Λ_1 is introduced, such that the eigenvalue μ_1^2 is defined as

$$\mu_1^2 = \Lambda_1 B_0^2 \quad (200.9)$$

^{/1} D. Randall and D. S. St. John, "Xenon Spatial Oscillations", Nucleonics, 16, 3, 82-86 (1958)

where B_0^2 is the fundamental geometric buckling. Here a reduced index Γ is introduced, which is defined as

$$\Gamma = \frac{B_1^2 - B_0^2}{\Lambda_1 B_0^2} \quad (200.10)$$

where B_1^2 is the eigenvalue of the first overtone of the geometric modes. If L is a characteristic extrapolated dimension (core height or diameter), μ_1^2 can be written as

$$\mu_1^2 = -\frac{\xi}{\Gamma L^2} \quad (200.11)$$

where $\xi = 0.031871$ for axial oscillations, $\xi = 0.038319$ for azimuthal oscillations, and L is in units of feet. For a slab, Γ is related to Λ_1 by

$$\Gamma = \frac{3}{\Lambda_1} \quad (200.12)$$

For a sinusoidal distribution, $\Lambda_1 = 3$ ($\Gamma = 1$) and for a flat distribution, $\Lambda_1 = 1$ ($\Gamma = 3$). If the flux is flattened in all directions, the values of Γ for diametral oscillations can be approximated by those for axial oscillations.

The coefficient a_x is calculated from

$$a_x = \frac{\rho_{x,\infty}}{M^2} \quad (200.13)$$

where $\rho_{x,\infty}$ is the reactivity effect of saturated xenon and M^2 is the migration area. To a good approximation,

$$\rho_{x,\infty} = -\frac{k_{th}}{\Sigma_{a_{th}}} \Sigma_{fyT} = -c \Sigma_{fyT} \quad (200.14)$$

where k_{th} is the contribution of the thermal group to k_{eff} of the core, and Σ_f is defined in equation (200.7).

The coefficient a_T can be written in terms of a net power coefficient $(\partial\rho/\partial P)$:

$$a_T = \frac{(\partial\rho/\partial P) V \Sigma_f \kappa (10^{-6})}{M^2} \quad (200.15)$$

where V is the core volume in cm^3 and $(\partial\rho/\partial P)$ is in units of reactivity change per MWT. Note that with fuel temperature and moderator density feedback in the x-y plane, $\partial\rho/\partial P$ is defined as

$$\partial\rho/\partial P = \frac{\partial\rho}{\partial T_f} \frac{\partial T_f}{\partial P} + \frac{\partial\rho}{\partial T_m} \frac{\partial T_m}{\partial P} \quad (200.16)$$

where T_f and T_m are the fuel and moderator temperatures, respectively.

When the average core enrichment is varied, we introduce the following approximation

$$\Sigma_f = \Sigma_f^N \frac{\epsilon}{\epsilon^N} \quad (200.17)$$

where ϵ is the U-235 enrichment, and superscript N refers to a nominal value.

We also assume that $\rho_{x,\infty}$ is independent of enrichment.

With the above approximations, the dependence of the stability index b on the various core parameters, each taken individually, can be represented generally as

$$b = \frac{a_0 x^3 + a_1 x^2 + a_2 x + a_3}{a_4 x^2 + a_5 x + a_6} \quad (200.18)$$

where x represents the core parameter of interest. The constants a_i in equation (200.18) are defined as follows:

1. $x = \text{U-235 enrichment, } \epsilon$:

$$a_1 = (\lambda_i + \lambda_x) \lambda_x$$

$$a_2 = (\lambda_i + 2\lambda_x) A - D$$

$$a_3 = A^2 + C$$

$$a_4 = \lambda_x$$

$$a_5 = A$$

$$a_0 = a_6 = 0$$

$$A = \frac{\sigma_x p \epsilon^N}{\Sigma_f^N \kappa}$$

$$D = B \gamma_i \lambda_x$$

$$C = B A (1 - \gamma_x)$$

$$B = - \frac{c \sigma_x p t^N y_T}{\kappa M^2 \left[\mu_1^2 - (\partial \rho / \partial P) v_p (10^{-6}) \right]}$$

2. $x = \text{core size, } L$:

$$a_1 = \frac{A D \Gamma}{\xi} - C$$

$$a_3 = A$$

$$a_4 = \frac{DF}{\xi}$$

$$a_6 = 1$$

$$a_0 = a_2 = a_5 = 0$$

$$A = \lambda_i + \lambda_x + \sigma_x \bar{\phi}$$

$$D = -\frac{(\partial \rho / \partial P) v_p (10^{-6})}{M^2}$$

$$C = \frac{c \sigma_x y_T p \Gamma}{\kappa M^2 \xi} \left[\frac{\bar{\phi} \sigma_x}{\lambda_x + \bar{\phi} \sigma_x} - \gamma_x \right]$$

3. x = flux level, $\bar{\phi}$:

$$a_0 = -E \sigma_x^2$$

$$a_1 = -(\lambda_i + \lambda_x) E \sigma_x + \sigma_x (\mu_1^2 \sigma_x - E \lambda_x) + B \sigma_x (1 - \gamma_x)$$

$$a_2 = (\lambda_i + \lambda_x) (\mu_1^2 \sigma_x - E \lambda_x) + \mu_1^2 \lambda_x \sigma_x - B \gamma_x \lambda_x$$

$$a_3 = (\lambda_i + \lambda_x) \lambda_x \mu_1^2$$

$$a_4 = -E \sigma_x$$

$$a_5 = \mu_1^2 \sigma_x - E \lambda_x$$

$$a_6 = \mu_1^2 \lambda_x$$

$$E = \frac{(\partial \rho / \partial P) V \Sigma_f \kappa (10^{-6})}{M^2}$$

$$B = - \frac{c \sigma_x \Sigma_f y_T}{M^2}$$

Note that equation (200.6) can be used with the above to find the dependence on power density.

4. x = power distribution, Γ :

$$a_2 = \frac{A D L^2}{\xi} - C$$

$$a_3 = A$$

$$a_5 = \frac{D L^2}{\xi}$$

$$a_6 = 1$$

$$a_0 = a_1 = a_4 = 0$$

$$A = \lambda_1 + \lambda_x + \sigma_x \bar{\phi}$$

$$D = - \frac{(\partial \rho / \partial P) V_p (10^{-6})}{M^2}$$

$$C = \frac{c \sigma_x y_T p L^2}{\kappa M^2 \xi} \left[\frac{\bar{\phi} \sigma_x}{\lambda_x + \bar{\phi} \sigma_x} - \gamma_x \right]$$

5. x = power coefficient, $\partial \rho / \partial P$:

$$a_2 = - A C$$

$$a_3 = A \mu_1^2 - B$$

$$a_5 = -C$$

$$a_6 = \mu_1^2$$

$$a_0 = a_1 = a_4 = 0$$

$$A = \lambda_1 + \lambda_x + \sigma_x \bar{\phi}$$

$$C = \frac{V_p (10^{-6})}{M^2}$$

$$B = \frac{c \sigma_x y_T p}{\kappa M^2} \left[\frac{\bar{\phi} \sigma_x}{\lambda_x + \bar{\phi} \sigma_x} - \gamma_x \right]$$

6. x = total xenon yield, y_T : (γ_x constant)

$$a_2 = -B$$

$$a_3 = A$$

$$a_6 = 1$$

$$a_0 = a_1 = a_4 = a_5 = 0$$

$$A = \lambda_1 + \lambda_x + \sigma_x \bar{\phi}$$

$$B = \frac{\frac{c \sigma_x p}{\kappa M^2} \left[\frac{\bar{\phi} \sigma_x}{\lambda_x + \bar{\phi} \sigma_x} - \gamma_x \right]}{\mu_1^2 - \frac{(\partial \rho / \partial P) p V (10^{-6})}{M^2}}$$

7. x = xenon cross section, σ_x :

$$a_1 = \bar{\phi} \left[\bar{\phi} + B (1 - \gamma_x) \right]$$

$$a_2 = \bar{\phi} (A - \lambda_x) - B \gamma_x \lambda_x$$

$$a_3 = A \lambda_x$$

$$a_5 = \bar{\phi}$$

$$a_6 = \lambda_x$$

$$a_0 = a_4 = 0$$

$$A = \lambda_1 + \lambda_x$$

$$B = \frac{c y_T p}{M^2 \left[\mu_1^2 - \frac{(\partial \rho / \partial P) V_p (10^{-6})}{M^2} \right]}$$

8. x = fractional xenon yield, γ_x : (y_T constant).

$$a_2 = B$$

$$a_3 = A - B C$$

$$a_6 = 1$$

$$a_0 = a_1 = a_4 = a_5 = 0$$

$$A = \lambda_1 + \lambda_x + \sigma_x \bar{\phi}$$

$$B = \frac{c \sigma_x y_T P}{\kappa M^2 \left[\mu^2 - \frac{(\partial \rho / \partial P) V_P (10^{-6})}{M^2} \right]}$$

$$C = \frac{\bar{\phi} \sigma_x}{\lambda_x + \bar{\phi} \sigma_x}$$

Figures 200.4 through 200.11 show plots of the stability index for diametral oscillations as a function of core parameters. The standard parameters used as a basis for the calculations are those of the 11-ft. diameter core (12-ft axial length), with an average U-235 enrichment of 2.6 a/o, average power density 84.5 w/cc, and a flat flux distribution. Other parameters used in the calculations are:

$$\lambda_1 = 2.88 \times 10^{-5} \text{ sec}^{-1}$$

$$\lambda_x = 2.11 \times 10^{-5} \text{ sec}^{-1}$$

$$\sigma_x = 1.622 \times 10^{-18} \text{ cm}^2$$

$$y_1 = 0.062$$

$$y_x = 0.002$$

The nominal power coefficient due to fuel temperature changes is -3.10×10^{-6} /MWT. The total power coefficient, for different values of moderator temperature coefficient can be obtained from equation (200.16). For example, taking $\partial T_m / \partial p = 0.3^\circ\text{F/w/cc}$, $\partial \rho / \partial P$ takes on values of -4.5×10^{-6} /MWT, and -1.7×10^{-6} /MWT for moderator temperature coefficients of $-1.5 \times 10^{-4}/^\circ\text{F}$ and $+1.5 \times 10^{-4}/^\circ\text{F}$, respectively.

An important observation from Figures 200.4 through 200.11, is that the more negative the power coefficient, the less sensitive is the stability index to variations in core parameters.

Figure 200.4 Variation of Stability Index with U-235 Enrichment - Modal Analysis

Figure 200.5 Variation of Stability with
Core Diameter - Modal Analysis

Figure 200.6 Variation of Stability Index with Average Power Density - Modal Analysis

Figure 200.7 Variation of Stability Index with Power Distribution - Modal Analysis

Figure 200.8 Variation of Stability Index with
Net Power Coefficient-Modal Analysis

Figure 200.9 Variation of Stability Index with
I-135 Yield - Modal Analysis

Figure 200.10 Variation of Stability Index with Xenon Absorption Cross Section-Modal Analysis

Figure 200.11 Variation of Stability Index with Direct Xe-135 Yield - Modal Analysis

Figures 200.4 through 200.11 are useful as a quick guide to the effect of variations in core parameters on spatial stability. Because of the approximate nature of the calculations, they should be used only as a guide, with more exact multi-dimensional calculations required for accurate assessment of core stability.

C. The Effect of Change in Core Enrichment on Spatial Stability

A change in core enrichment at the same average power density affects the neutron flux level and, therefore, the spatial stability characteristics of the core. The effect is not similar to a change in power level, since a change in enrichment at constant power level only affects the xenon loss through absorption, while the xenon production remains practically unaffected.

Two-dimensional x-y TURBO^{/1} calculations were performed for the 11-ft core, with no temperature feedback. The average core enrichment was set at 2.2 a/o, 2.6 a/o, and 3.0 a/o, while maintaining an approximately flat steady-state power distribution in all cases. A description of the core and the method of perturbing it can be found in reference 2.

Figure 200.12 shows the resulting oscillations across a diameter. It would appear from the figure that the core with an average enrichment of 2.2 a/o is relatively more stable than that with the nominal enrichment of 2.6 a/o. However, the modal expansion results shown on Figure 200.4 do not predict such a behavior. Further analysis tends to lead to the following interpretation of the results of Figure 200.12:

1. Although both calculations had an approximately flat steady-state power distribution, the distribution for the 2.2 a/o case was slightly peaked, while that for the 2.6 a/o case was slightly dished. As can be observed from Figure 200.7, the stability index is particularly sensitive to the steady state distribution for flattened distributions.

/1 S. M. Hendley and R. A. Mangan, "TUREO* - A Two-Dimensional Few-Group Depletion Code for the IBM-7090", WCAP-6059 (1964)

/2 M. J. O'Boyle, et al, "Control of Xenon Instabilities in Large PWR's Quarterly Progress Report for the Period Ending March 31, 1967", WCAP-3680-3 (1967)

Figure 200.12 Effect of Average Core Enrichment on Xenon-Induced Spatial Oscillations

2. From the analysis summarized in section E, it appears that the error introduced by the use of a finite time step can be defined by a function which is particularly sensitive to the average neutron flux level. It is shown in section E that the use of a finite time-step length can lead to apparent trends in stability characteristics, which are opposite to the correct trend. The results of Figure 200.12 are further discussed in section E.

D. The Effect of Fuel Temperature Feedback on Core Stability

In the previous progress report^{/1}, results of a study of oscillation characteristics in the x-y plane for the standard 11-ft core with a flat power distribution and nominal fuel temperature feedback were reported. The same core was studied with TURBO* for a significantly dished distribution, with nominal fuel temperature feedback. The steady-state power distribution is that corresponding to case D on Figure 200.11 in reference 2. Figure 200.13 shows the TURBO* results. The power oscillations when no temperature feedback is included are also shown for comparison. It is observed that the initial transient following the perturbation is significantly reduced in magnitude. Figure 200.14, which represents the oscillations when fuel temperature feedback is included shows that these are convergent.

In interpreting the above results, the following important considerations must be borne in mind:

1. The TURBO* calculations were performed with a four-hour time-step. As is shown in section E, a correction for the finite time-step results in the core actually being unstable. The same comment applies to the results for the case with a flattened power distribution. This situation brings out a major difficulty in the use of multidimensional diffusion-theory codes for the analysis and control studies of xenon-induced oscillations. Although TURBO* is not representative of present-day capability in multi-dimensional calculations, it has been until now the only code which allowed the explicit representation of fuel temperature feedback in the x-y

/1 M. J. O'Boyle, et al, "Control of Xenon Instabilities in Large PWR's Quarterly Progress Report for the Period Ending March 31, 1967", WCAP-3680-3 (1967)

/2 M. J. O'Boyle, et al, "Control of Xenon Instabilities in Large PWR's Quarterly Progress Report for the Period Ending December 31, 1966" WCAP-3680-2 (1966)

Figure 200.13 Xenon-Induced Oscillations with and without Fuel Temperature Feedback

Figure 200.14 Xenon-Induced Oscillations with Fuel Temperature Feedback

plane. Running time with the temperature feedback included is, however, excessive, being of the order of 15-30 minutes per time step (IBM-7090), thus precluding the use of shorter time-steps.

2. The cases studied here included only fuel temperature feedback, and thus correspond to a situation where the moderator temperature coefficient is zero. In the x-y plane, the effects of fuel temperature and moderator density feedback are, to a first approximation, additive (as opposed to the axial plane, where the moderator density variations are dependent on an integral of the flux distribution). Therefore, any negative value of the moderator temperature coefficient will tend to stabilize the core against azimuthal or diametral oscillations. An estimate of the magnitude of this effect can be had from Figures 200.4 - 200.8 and equation (200.16).

The sensitivity of the stability characteristics to the fuel temperature feedback was studied with the one-dimensional program ZEST (see section G). The standard 11-ft core with a flattened power distribution was used as a basis for the calculations. Oscillations across a diameter in the x-y plane were simulated in one-dimensional slab geometry. Two-hour time-steps were used in the calculations which, for the nominal fuel temperature feedback, resulted in slightly divergent oscillations.

ZEST includes a fairly complete treatment of fuel temperature feedback, in that explicit correlations between fuel temperature and power density, fast absorption cross section and fuel temperature, and fast removal cross section and fuel temperature, are used by the program. The sensitivity to the magnitude of fuel temperature feedback was studied by modifying the correlation between fuel temperature and power density. Calculations were performed where the slope $\partial T_f / \partial p$ was varied uniformly by $\pm 20\%$ from its nominal value. The resulting oscillations are shown in Figure 200.15, compared to the nominal case. Note that the effect of a -20% change in the magnitude of the power coefficient is relatively greater than a $+20\%$ change in magnitude. This nonlinear behavior is evident in the modal expansion results of Figure 200.8.

Figure 200.15 Effect of Variation of Fuel Temperature Feedback on Diametral Xenon-Induced Oscillations

A ZEST calculation with nominal fuel temperature feedback was repeated with the approximation that only the fast absorption cross section varies with fuel temperature. That is, the removal cross section was assumed to be constant. The results of the calculations are shown in Figure 200.16. It is observed that assuming invariance of the removal cross section leads to a significant overestimate of the instability of the core. This sensitivity to the variation of the removal cross section with fuel temperature is to be expected, since the rate of absorption in Xe-135 relative to its production, by decay of I-135, is proportional to the removal cross section. These results point to the importance of accounting for variations in both fast absorption and removal cross sections with fuel temperature in two-group simulations of xenon-induced oscillations.

E. The Effect of a Finite Time Step Length on Calculated Stability Characteristics

In digital simulations of xenon-induced oscillations with diffusion theory codes, the time dependence of the spatial flux distribution is discretized by using a time mesh, or a number of time steps of non-zero lengths. In the majority of diffusion theory codes, the assumption is made that the spatial flux distribution is constant in time during the length of the time step. It can be readily deduced that for unstable cores, somewhat removed from the stability threshold, a non-zero time-step will result in observed, or calculated, oscillations which are relatively more stable than in the limit of a zero time-step length. Similarly, for stable cores, somewhat removed from the stability threshold, the calculated oscillation will be relatively less stable than in the limit of a zero time-step length. For example, when the flux of a point is increasing, the assumption of a constant flux during the duration of the time-step results in an underestimate in the xenon concentration change.

Since diffusion theory calculations, and in particular two- and three-dimensional calculations, are time consuming, economic considerations preclude the use of very short time-steps. Therefore it is essential to ascertain the error introduced by the use of a finite time-step length.

Figure 200.16 Effect on Assuming Σ_R Constant in Xenon Oscillation Calculation with Fuel Temperature Feedback

In the following subsections numerical results are first displayed where the same core was studied with different time-step lengths. A theoretical analysis of the effect is next introduced, which leads to an estimate of the error introduced by a non-zero time-step length. It is shown, at least for uncontrolled calculations, and within the limitations of linear theory, that a simple correction can be made to digital calculations to account for the use of a non-zero time-step length.

1. Numerical Evaluation

Xenon-induced diametral oscillations in the standard 11-ft core, with nominal fuel temperature feedback, were simulated in the ZEST program, with time-steps of one, two, and four hours duration. The resulting thermal flux oscillations are shown in Figure 200.17. Note that whereas the use of a four-hour time-step predicts spatial stability, the use of a shorter time-step length actually shows the core to be spatially unstable. It is observed that both the damping factor and the oscillation period are sensitive to the time-step length.

The oscillations in Figure 200.17 are of relatively small magnitude. Figure 200.18 shows a similar analysis for relatively large oscillations. The core is a one-dimensional slab, 12 ft in length, with uniform fuel (2.6 a/o U-235) and no temperature feedback. The sensitivity to the time-step length for both damping factor and oscillation period is again evident.

Figure 200.19 shows results for two-dimensional TURBO* calculations with two-hour and four-hour time-steps. The core is the standard 11-ft core with a flattened power distribution and no temperature feedback. The reactivity effect of the perturbation for the core was 1×10^{-4} . From Figures 200.17-200.19, it is evident that the sensitivity to the time-step length is greater for relatively more unstable cores.

Figure 200.17 Variation of Xenon-Induced Spatial Stability Characteristics with Time Step Length in Digital Calculations

Figure 200.18 Variation of Xenon-Induced Spatial Stability Characteristics with Time Step Length in Digital Calculations

Figure 200.19 Variation of Xenon-Induced Spatial Stability Characteristics with Time Step Length in Digital Calculations

Although the numerical results displayed here are in qualitative agreement with similar calculations published by Harris and Lacy^{/1} for a seed-blanket reactor, they are in contradiction with conclusions reached by Strawbridge^{/2} for slightly enriched PWR's. It is shown in the following section, however, that the particular cases studied by Strawbridge fell in a portion of the stability plane where the sensitivity to the time-step length is a minimum.

2. Analytical Evaluation

Davison has published a theoretical investigation of the error introduced in digital calculations by the use of a non-zero time-step length.^{/3} In this section an analytical evaluation of the time-step length problem is presented which, although it is based on Davison's ideas, uses a different treatment of the spatial distributions than was assumed by Davison. In addition, simple correlations are derived for slightly enriched PWR's which can be used to estimate the error introduced by a non-zero time-step length.

In one-group theory, the describing equations for the time-dependent xenon problem are written in the form

$$\begin{aligned} \nabla \cdot D(\underline{r}) \nabla \phi(\underline{r}, t) - \Sigma_a(\underline{r}, t) \phi(\underline{r}, t) - N_x(\underline{r}, t) \sigma_x \phi(\underline{r}, t) \\ + \nu \Sigma_f(\underline{r}) \phi(\underline{r}, t) = 0 \end{aligned} \quad (200.19)$$

$$\frac{\partial N_i(\underline{r}, t)}{\partial t} = \gamma_i \Sigma_f(\underline{r}) \phi(\underline{r}, t) - \lambda_i N_i(\underline{r}, t) \quad (200.20)$$

$$\begin{aligned} \frac{\partial N_x(\underline{r}, t)}{\partial t} = \lambda_i N_i(\underline{r}, t) + \gamma_x \Sigma_f(\underline{r}) \phi(\underline{r}, t) \\ - N_x(\underline{r}, t) \left[\sigma_x \phi(\underline{r}, t) + \lambda_x \right] \end{aligned} \quad (200.21)$$

^{/1} D. R. Harris and P. S. Lacy, "A Simple Approximate Test for Spatial Xenon Stability", Trans. Am. Nucl. Soc. 3 (2), 437 (1960)

^{/2} L. E. Strawbridge, E. C. Allard, and C. P. Bhalla, "Xenon-Induced Core Instabilities", WCAP-3269-48 (1965)

^{/3} B. Davison, "Correction for Non-Zero Time Step in the Numerical Simulation of Spatial Oscillations of Xenon in a Nuclear Reactor", AECL-1292 (1961)

The notation in equations (200.19) - (200.21) is standard. Temperature feedback is accounted for in $\Sigma_a(\underline{r}, t)$. In the usual way, small variations of the dependent variables about their steady-state values are now introduced:

$$\phi(\underline{r}, t) = \phi_o(\underline{r}) + \delta\phi(\underline{r}, t)$$

$$N_i(\underline{r}, t) = N_{i_o}(\underline{r}) + \delta N_i(\underline{r}, t) \quad (200.22)$$

$$N_x(\underline{r}, t) = N_{x_o}(\underline{r}) + \delta N_x(\underline{r}, t)$$

$$\Sigma_a(\underline{r}, t) = \Sigma_{a_o}(\underline{r}) + \delta\Sigma_a(\underline{r}, t) = \Sigma_{a_c}(\underline{r}) + K_a(\underline{r}) \delta\phi(\underline{r}, t)$$

The last of equations (200.22) is a representation of linear feedback. Substitution of equations (200.22) into equations (200.19) - (200.21), linearization of the resulting equations, and elimination of the steady-state equations lead to the following set of perturbed equations:

$$\begin{aligned} \nabla \cdot D \nabla \delta\phi - \Sigma_{a_o} \delta\phi - N_{x_o} \sigma_x \delta\phi + \nu \Sigma_f \delta\phi \\ - K_a \phi_o \delta\phi - \phi_o \sigma_x \delta N_x = 0 \end{aligned} \quad (200.23)$$

$$\frac{\partial \delta N_i}{\partial t} = y_i \Sigma_f \delta\phi - \lambda_i \delta N_i \quad (200.24)$$

$$\frac{\partial \delta N_x}{\partial t} = \lambda_i \delta N_i + y_x \Sigma_f \delta\phi - N_{x_o} \sigma_x \delta\phi \quad (200.25)$$

$$- \delta N_x (\sigma_x \phi_o + \lambda_x)$$

where the space and time dependence were left out for clarity.

The steady state equation for the flux can be written

$$L_o \phi_o = 0 \quad (200.26)$$

where L_o is the unperturbed, steady-state operator corresponding to equation (200.19). Equation (200.23) can be written in the form

$$(L_o - L_1) \delta \phi = 0 \quad (200.27)$$

where

$$L_1 = (K_a + \sigma_x) \phi_o \quad (200.28)$$

A modal expansion of the flux perturbation is now assumed, of the form

$$\delta \phi(\underline{r}, t) = \sum_n a_n e^{\omega_n t} \psi_n(\underline{r}) \quad (200.29)$$

where ψ_n are the eigenvectors corresponding to the eigenvalues μ_n^2 of the steady-state operator L_o :

$$L_o \psi_n = \mu_n^2 \psi_n \quad (200.30)$$

Note that $\mu_o = 0$, and $\psi_o = A \phi_o$ where A is a constant.

The eigenvectors ψ_n are orthonormal:

$$\int d\underline{r} \psi_n \psi_m^+ = \delta_{n,m} \quad (200.31)$$

The modal expansion (200.29) is similar to that used by Randall and St. John^{/1,2}.

/1 D. Randall and D. S. St. John, "Xenon Spatial Oscillations", Nucleonics, 16, 3, 82-86 (1958)

/2 D. Randall and D. S. St. John, "Xenon Spatial Oscillations", Nuclear Science and Engineering 13, 60-62 (1962)

One mode is now concentrated on -- the first overtone ψ_1 assuming the oscillations to be primarily in this mode. Noting that in one-group theory the eigenfunctions ψ_n are self-adjoint, equations (200.24) and (200.25) are multiplied by ψ_1 and integrated over the core volume. The resulting equations are written in the form:

$$\frac{\partial \langle I \rangle}{\partial t} = \langle \beta_i \rangle \langle \delta \phi \rangle - \lambda_i \langle I \rangle \quad (200.32)$$

$$\begin{aligned} \frac{\partial \langle X \rangle}{\partial t} = & \lambda_i \langle I \rangle + \langle \beta_x \rangle \langle \delta \phi \rangle - \langle \gamma \rangle \langle \delta \phi \rangle - \langle A \rangle \langle X \rangle \\ & - \langle A \rangle \langle X \rangle \end{aligned} \quad (200.33)$$

where

$$\langle I \rangle = \int d\underline{r} \delta N_i(\underline{r}, t) \psi_1(\underline{r}) \quad (200.34)$$

$$\langle X \rangle = \int d\underline{r} \delta N_x(\underline{r}, t) \psi_1(\underline{r}) \quad (200.35)$$

$$\langle \delta \phi \rangle = \int d\underline{r} \delta \phi(\underline{r}, t) \psi_1(\underline{r}) \quad (200.36)$$

$$\langle \beta_i \rangle = \frac{\int d\underline{r} y_i \Sigma_f(\underline{r}) \delta \phi(\underline{r}, t) \psi_1(\underline{r})}{\int d\underline{r} \delta \phi(\underline{r}, t) \psi_1(\underline{r})} \quad (200.37)$$

$$\langle \beta_x \rangle = \frac{\int d\underline{r} y_x \Sigma_f(\underline{r}) \delta \phi(\underline{r}, t) \psi_1(\underline{r})}{\int d\underline{r} \delta \phi(\underline{r}, t) \psi_1(\underline{r})} \quad (200.38)$$

$$\langle \gamma_x \rangle = \frac{\int d\underline{r} N_{x_0}(\underline{r}) \sigma_x \delta \phi(\underline{r}, t) \psi_1(\underline{r})}{\int d\underline{r} \delta \phi(\underline{r}, t) \psi_1(\underline{r})} \quad (200.39)$$

$$\langle A \rangle = \frac{\int d\underline{r} [\sigma_x \phi_0(\underline{r}) + \lambda_x] \delta N_x(\underline{r}, t) \psi_1(\underline{r})}{\int d\underline{r} \delta N_x(\underline{r}, t) \psi_1(\underline{r})} \quad (200.40)$$

For flux variations in the first overtone only, and with the assumption that the variations in the iodine and xenon concentrations are in that same mode, the coefficients $\langle\beta_i\rangle$, $\langle\beta_x\rangle$, $\langle\gamma_x\rangle$ and $\langle A\rangle$ in equations (200.32) and (200.33) are time-independent. In a digital simulation of this mode of oscillation, equations (200.32) and (200.33) would be solved by assuming $\langle\delta\phi\rangle$ to be constant in time during a time-step length τ . Equations (200.32) and (200.33) can then be solved exactly, with solutions

$$\langle I \rangle \Big|_{t+\tau} = a_{11} \langle I \rangle \Big|_t + a_{13} \langle \delta\phi \rangle \Big|_t \quad (200.41)$$

$$\langle X \rangle \Big|_{t+\tau} = a_{22} \langle X \rangle \Big|_t + a_{21} \langle I \rangle \Big|_t + a_{23} \langle \delta\phi \rangle \Big|_t \quad (200.42)$$

where

$$\begin{aligned} a_{11} &= e^{-\lambda_i \tau} \\ a_{13} &= \frac{(1 - e^{-\lambda_i \tau})}{\lambda_i} \langle \beta_i \rangle \\ a_{22} &= e^{-\langle A \rangle \tau} \\ a_{21} &= \frac{\lambda_i (e^{-\lambda_i \tau} - e^{-\langle A \rangle \tau})}{\langle A \rangle - \lambda_i} \\ a_{23} &= (\langle \beta_i \rangle + \langle \beta_x \rangle - \langle \gamma_x \rangle) \frac{(1 - e^{-\langle A \rangle \tau})}{\langle A \rangle} \\ &\quad - \langle \beta_i \rangle \frac{(e^{-\lambda_i \tau} - e^{-\langle A \rangle \tau})}{\langle A \rangle - \lambda_i} \end{aligned} \quad (200.43)$$

Since the oscillations being considered are in the mode ψ_1 , $\delta N_i(\underline{r}, t)$ and $\delta N_x(\underline{r}, t)$ behave as $e^{\omega t} \psi_1(\underline{r})$. Thus, equations (200.34) and (200.35),

$$\langle I \rangle \Big|_{t+\tau} = e^{\omega\tau} \langle I \rangle \Big|_t \quad (200.44)$$

$$\langle X \rangle \Big|_{t+\tau} = e^{\omega\tau} \langle X \rangle \Big|_t \quad (200.45)$$

Substitution of equations (200.44) and (200.45) into equations (200.41) and (200.42) yields the equations

$$(a_{11} - e^{\omega\tau}) \langle I \rangle + a_{13} \langle \delta\phi \rangle = 0 \quad (200.46)$$

$$(a_{22} - e^{\omega\tau}) \langle X \rangle + a_{21} \langle I \rangle + a_{23} \langle \delta\phi \rangle = 0 \quad (200.47)$$

where the weighted quantities are evaluated at time t . Quantity $\langle I \rangle$ can be eliminated from equations (200.46) - (200.47), and therefore

$$\langle X \rangle = C \langle \delta\phi \rangle \quad (200.48)$$

where

$$C = \frac{\frac{a_{21} a_{13}}{a_{11} - e^{\omega\tau}}}{a_{22} - e^{\omega\tau - \lambda}} = a_{23} \quad (200.49)$$

Now in equation (200.48), $\langle X \rangle$ and $\langle \delta\phi \rangle$ are the integrated weighted excess xenon and flux distributions at time t , and C must therefore be independent of the time step length τ . That this is so can be seen by multiplying the equation (200.23) for the excess flux by

$\psi_1(\underline{r})$ and integrating over the core volume. From equation (200.49), ω is clearly a function of the time-step length τ . If the limit of equation (200.49) is taken as $\tau \rightarrow 0$, and ω_0 is defined equal to ω as $\tau \rightarrow 0$, after using equations (200.43), equation (200.49) simplifies to:

$$C = \frac{\langle \beta_i \rangle \lambda_i - (\langle \gamma_x \rangle - \langle \beta_x \rangle) (\lambda_i + \omega_0)}{(\langle A \rangle + \omega_0) (\lambda_i + \omega_0)} \quad (200.50)$$

By eliminating C from equations (200.49) and (200.50), a relationship can be obtained between ω , τ and ω_0 . Equations (200.49) and (200.50) can be written in the following quadratic form:

$$e^{2\omega\tau} + be^{\omega\tau} + c = 0 \quad (200.51)$$

$$\omega_0^2 + b' \omega_0 + c' = 0 \quad (200.52)$$

Note that ω and ω_0 are complex numbers, with the real part equal to the stability index defined in section B. The roots of equations (200.51) and (200.52) are, respectively:

$$e^{\omega\tau} = -\frac{b}{2} \pm i\sqrt{\frac{4c - b^2}{4}} \quad (200.53)$$

$$\omega_0 = -\frac{b'}{2} \pm i\sqrt{\frac{4c' - b'^2}{4}} \quad (200.54)$$

where the square roots in the above equations are real quantities.

Noting that for z a complex number ($z = x + iy$), $\left| e^z \right|^2 = e^{2x}$, equations (200.53) and (200.54) yield

$$e^{2\tau R(\omega)} = c \quad (200.55)$$

$$R(\omega_0) = -\frac{b'}{2} \quad (200.56)$$

where $R(z)$ is the real part of z .

Substituting the expressions for c and b' in equations (200.55) and (200.56), eliminating C between the equations, and using equations (200.43) one obtains:

$$\begin{aligned}
 2\tau R(\omega) &= e^{-(\lambda_i + \langle A \rangle) \tau} + \left[2R(\omega_0) + \langle A \rangle + \lambda_i \right] \\
 &\times \left[\frac{\langle \beta \rangle (e^{-\lambda_i \tau} - e^{-\langle A \rangle \tau})}{\langle A \rangle - \lambda_i} - \frac{(\beta_i + \langle \beta_x \rangle - \langle \gamma_x \rangle) (e^{-\lambda_i \tau} - e^{-(\lambda_i + \langle A \rangle) \tau})}{\langle A \rangle} \right] \\
 &\times [\langle \gamma_x \rangle - \langle \beta_x \rangle]^{-1}
 \end{aligned} \tag{200.57}$$

Equation (200.57) is similar to equation (4.8) in reference 1, except that weighted average quantities appear in equation (200.57), while at arriving at this equation Davison assumed that the coefficients in equations (200.24) and (200.25) were independent of space. By using a perturbation argument, Davison has shown^{/1} that, in the case where the quantity $\sigma_x \phi_0$ in equation (200.25) was allowed to depend on position, while retaining the assumption that $y_1 \Sigma_f$, $y_x \Sigma_f$ and N_x are constant in space, his equation (4.8) was still valid, if an average weighted value of $\sigma_x \phi_0$ was used in the equation. The derivation above, however, shows that, for the particular mode of oscillation considered, equation (200.57) is valid for arbitrary spatial dependence of the above quantities, provided properly weighted average quantities are used.

Davison^{/1} has shown that equation (200.57) can be further simplified if the following conditions hold:

$$\begin{aligned}
 \frac{\lambda_i}{\langle A \rangle} &\ll 1 \\
 \lambda_i \tau &\ll 1
 \end{aligned} \tag{200.58}$$

^{/1} B. Davison, "Correction for Non-Zero Time Step in the Numerical Simulation of Spatial Oscillations of Xenon in a Nuclear Reactor, AECL-1292 (1961).

$$\left| \tau R(\omega) \right| \ll 1$$

With the assumptions (200.58), equation (200.57) becomes

$$R(\omega) = R(\omega_0) \frac{(1 - e^{-\langle A \rangle \tau})}{\langle A \rangle \tau} - \frac{\lambda_i}{2} \left[1 + e^{-\langle A \rangle \tau} - \frac{2(1 - e^{-\langle A \rangle \tau})}{\langle A \rangle \tau} \right] \quad (200.59)$$

Equation (200.59) is in a very useful form, since the observed $R(\omega)$ is related to the true, or extrapolated $R(\omega_0)$ through the dimensionless variable $\langle A \rangle \tau$. Equation (200.59) can therefore be thought of as a universal relationship between $R(\omega)$ and $R(\omega_0)$, in terms of a reduced time-step length $\tau_r = \langle A \rangle \tau$. Note that in the more exact equation (200.57), $R(\omega)$ and $R(\omega_0)$ are related not only through the reduced time-step length τ_r , but also through the actual time-step length τ , and the quantities $\langle \beta_i \rangle$, $\langle \beta_x \rangle$, $\langle \gamma_x \rangle$, and $\langle A \rangle$.

Figure 200.20 is a comparison of equations (200.57) and (200.59), where typical parameters for a large PWR were used in equation (200.57). In using equation (200.57), the weighted average quantities were approximated by volume average values. As shown below, this is a good approximation when the oscillation is restricted to the first overtone. It appears from Figure 200.20 that the approximation (200.59) to equation (200.57) is not particularly good for a typical PWR design. However, the suggestion of Barss^{/1} has been followed and equation (200.59) was used as a correlation between $R(\omega)$ and $R(\omega_0)$, with $\langle A \rangle$ a fitting parameter. This approach is justified on two grounds. First it appears from Figure 200.20 that equation (200.59) can be a good approximation to equation (200.57) if an effective value of $\langle A \rangle$ is used in the equation. Second, in view of the approximations made in arriving at equation (200.57) or (200.59), these equations, when applied to an actual digital simulation of xenon oscillations, may be expected to yield a good qualitative description of the behavior of the solutions, but should not be expected to yield accurate quantitative results. Thus, the use of $\langle A \rangle$ as a fitting parameter seems indeed indicated.

^{/1} W. M. Barss, "Digital Simulation of Xenon Instability in Reactors", AECL-1226. (1961)

Figure 200.20 Comparison of Exact and Approximate Correlations of the Stability Index as a Function of Time Step Length

3. Development of a Correlation for the Reduced Time Step Length $\langle A \rangle \tau$

A series of numerical calculations were performed with program ZEST in order to develop a correlation for the reduced time-step length τ_r , or equivalently, the quantity $\langle A \rangle$ in equation (200.59). It is useful, however, to study the definition of $\langle A \rangle$ given in equation (200.40) in the derivation of equation (200.59). For oscillations in the first overtone, if one assumes $\delta N_x(\underline{r}, t)$ to be proportional to $e^{\omega t} \psi_1(\underline{r})$, then equation (200.40) reduces to

$$\langle A \rangle = \sigma_x \frac{\int d\underline{r} \phi_o(\underline{r}) \psi_1^2(\underline{r})}{\int d\underline{r} \psi_1^2(\underline{r})} + \lambda_x \quad (200.60)$$

Now if the approximation is made

$$\frac{\int d\underline{r} \phi_o(\underline{r}) \psi_1^2(\underline{r})}{\int d\underline{r} \psi_1^2(\underline{r})} \approx \frac{\int d\underline{r} \phi_o(\underline{r})}{\int d\underline{r}} = \bar{\phi}_o \quad (200.61)$$

then equation (200.60) can be written

$$\langle A \rangle = \sigma_x \bar{\phi}_o + \lambda_x \quad (200.62)$$

Equation (200.61) is exact for a flat flux, while for a cosine distribution, with flat fluxes in the other dimensions, and with ψ_1 the first geometric overtone, the error in equation (200.61) is less than 7%.

From equation (200.62), it is evident that a correlation for $\langle A \rangle$ of the form

$$\langle A \rangle = a + b \bar{\phi}_o, \quad (200.63)$$

where a and b are fitted parameters, should be sought for. In addition to the appealing simplicity of the correlation (200.63), it is physically intuitive that the error introduced by a finite time-step length should be a function primarily of the flux level, and only secondarily of flux

distribution, core size, etc., since it is the assumption of a constant flux level in equations (200.24) and (200.25) during the duration of the time step which introduces an error in the solution.

In addition to the calculations shown in Figure 200.17, for the standard 11-ft core with a flattened power distribution and fuel temperature feedback, two additional cores were studied which had significantly different characteristics from that of Figure 200.17. The results of the calculations, for time-step lengths of one, two, and four hours, are shown in Figures 200.21 and 200.22. Both sets of calculations are for a 10-ft core, 2.6 a/o enriched, with uniform loading and no temperature feedback. Figure 200.21 corresponds to an average power density of 84.5 w/cc ($\bar{\phi}_0 = 4.5 \times 10^{13}$), and Figure 200.22 corresponds to an average power density of 106 w/cc ($\bar{\phi}_0 = 5.6 \times 10^{13}$). The former core is stable, while the later is unstable.

Figure 200.23, is a plot of equation (200.59). The results of the numerical calculations shown on Figures 200.17, 200.21 and 200.22, were graphically fitted to equation (200.59) and the best fits are shown in Figure 200.23. In arriving at these results, the stability index, which is equivalent to $R(\omega)$ or $R(\omega_0)$ was calculated from equation (200.1). It can be seen from Figure 200.23, that the numerical results can be very well fitted by an expression of the form (200.59). Moreover, it is important to realize that the 11-ft core and one of the 10-ft cores analyzed were both operated at the same average power density of 84.5 w/cc. For both of these cases, the best fit to equation (200.59) yields the same value for the parameter $\langle A \rangle$, as can be observed from Figure 200.23. The second 10 ft core was operated at 106 w/cc. Therefore it appears, based on the above calculations, that $\langle A \rangle$ can be well represented by an expression of the form (200.63). From the data of Figure 200.23, $\langle A \rangle$ can be expressed by

$$\langle A \rangle = .253 + .0348 \times 10^{-13} \bar{\phi}_0 \quad (200.64)$$

Figure 200.21 Variation of Xenon-Induced Spatial Stability Characteristics with Time Step Length in Digital Calculations (84.5 w/cc)

Figure 200.22 Variation of Xenon-Induced Spatial Stability Characteristics with Time-Step Length in Digital Calculations (106 w/cc)

Figure 200.23 Comparison of Correlation (200.59) and Digital Calculations

In terms of the average power density p , for an average enrichment of 2.6 a/o, $\langle A \rangle$ is given by

$$\langle A \rangle = .253 + .00186 p \quad (200.65)$$

where p is in units of w/cc.

Using the above correlation for $\langle A \rangle$, Figure 200.24 was generated, showing the variation of apparent (observed) stability index as a function of time-step length, for an average power density of 84.5 w/cc. For different power densities, Figure 200.23 should be used together with equations (200.64) or (200.65).

It is worthwhile to note the functional behavior shown on Figure 200.24. In general, a core which is unstable to the first overtone, will be predicted to be relatively more stable by digital calculations with a non-zero time-step length. The error in the observed stability index introduced by the finite time-step length increases as the stability index becomes more positive. There is a region in the stability plane, for stability indices in the neighborhood of $-.02$, where the results are relatively insensitive to the length of the time-step. For large and negative stability indices (very stable cores), the use of a finite time-step length leads to a conservative estimate of stability. Finally, the error introduced by the use of a non zero time-step length becomes relatively larger as the average flux level is increased.

Figure 200.24 is useful in explaining two apparent, but nevertheless incorrect, conclusions reached from results of digital calculations.

- a) As a result of numerical calculations where the time step length was varied, Strawbridge in reference 1 has concluded that the damping factor was relatively insensitive to the length of the time step. Analysis of his results, however,

/1 L. E. Strawbridge, E. C. Allard, and C. P. Bhalla, "Xenon-Induced Core Instabilities", WCAP-3269-48 (1965)

Figure 200.24 Effect of Time Step Length on Spatial Stability Index

shows that the calculations which involved varying the time-step length were for a stable core, in the region of the stability plane where, as was observed above, the error introduced by a non-zero time-step length is a minimum. Indeed the results of reference 1 tend to confirm the theoretical predictions of Figure 200.24.

- b) In section C of this report, two-dimensional calculations were described, which seemed to predict that there existed a particular value of the average fuel enrichment at which the core was most unstable, lower and larger values of enrichment leading to relatively more stable characteristics. One of the reasons given for this behavior in section C was related to the use of a non-zero time-step length in the calculations. It is now clear from Figure 200.23 that errors due to a finite time-step length can indeed lead to such an apparent behavior. The calculations in section C were for an unstable core. As a result of the variation in enrichment, at constant power level, the average thermal flux level, and therefore $\langle A \rangle$, were different in all calculations. From Figure 200.23 it is observed that for positive values of the stability index, and for the same value of time step length τ , a digital calculation can predict a more stable behavior for a core which is in truth relatively more unstable, if the average flux level is different.

Finally, although results based on the analysis presented here are very promising, equations (200.64) and (200.65), and Figure 200.24, should be considered preliminary in nature. Further calculations are being carried out to further refine the correlations.

/1 L. E. Strawbirdge, E. C. Allard, and C. P. Bhalla, "Xenon-Induced Core Instabilities", WCAP-3269-48 (1965)

F. The Effect of Small Perturbations on Core Stability

The linearized modal expansion theory of spatial xenon stability, with linear feedback, predicts no threshold which is related to perturbation size. That is, a core which is predicted to be unstable, no matter how close to the stability threshold, will undergo divergent (if small) flux oscillations following any perturbation, regardless of how small in magnitude. In an operating reactor, the presence of inherent fluctuations in the neutron flux (noise) may somewhat alter this conclusion. Indeed, an input perturbation of magnitude smaller than the magnitude of such fluctuations would not be expected to cause any change in the behavior of the system.

A series of one-dimensional ZEST calculations were performed in an effort to obtain some quantitative results related to the effect of small perturbations. In a digital simulation with a diffusion theory code, pseudo-fluctuations, or a pseudo noise field, are introduced by round-off errors in the machine, as well as by the nature of the eigenvalue calculation itself, which is usually terminated when the pointwise fluxes are converged to within a criterion ϵ . The use of a non-zero time-step also introduces errors in the calculations and, therefore, pseudo flux fluctuations. This last effect was discussed in section E.

A 12-ft uniform slab with an average enrichment of 2.6 a/o and operated at an average power density of 84.5 w/cc was selected for analysis. Temperature feedback was neglected. This core, as shown on Figure 200.18, is spatially unstable. A first series of calculations was performed for which the following procedure was followed. Following the establishment of equilibrium conditions, a perturbation of zero magnitude was inserted into the core. In the eigenvalue calculation of the flux distribution following insertion of the perturbation, and in subsequent time-steps, the source guess, which normally would be the converged source distribution from the previous eigenvalue calculation (see section G), was uniformly perturbed by about 3%. The new converged flux distribution was thus somewhat different from the initial eigenvalue calculation, although within

the convergence criterion ϵ . This difference in converged flux distribution can be seen on Figure 200.25, at time $t = -1$ hour. Following the removal of the (zero magnitude) perturbation, the core was followed at regular intervals for a period of 220 hours. In doing this, different time-step lengths (two hours and one hour, respectively), and different values of the pointwise flux convergence criterion ($\epsilon = 10^{-4}$ and $\epsilon = 10^{-5}$, respectively) were used. The results of these calculations are shown on Figure 200.25. In every case, divergent flux oscillations were excited, even though the size of the input perturbation was zero. Moreover, both the growth rate and the period of oscillations are very sensitive to the value of the convergence criterion and of the time-step length. Figure 200.26 shows, on an extended scale, the resulting oscillation for a time-step length of one hour, and a convergence criterion of 10^{-4} .

A second series of calculations was performed, where the source guess for each eigenvalue calculation was not perturbed, and thus corresponded to the converged source distribution from the previous eigenvalue calculation. Again both time-step length and pointwise convergence criteria were varied. The results of the calculations are displayed on Figure 200.27. Since the source guess to the eigenvalue calculations was not perturbed, the pseudo-fluctuations arising from insertion of a zero perturbation in this set of calculations were much smaller in magnitude than for the calculations of Figure 200.25. Figure 200.27 shows that all calculations tend to converge to the same limit, apparently the true solution; and that with time-step lengths of one hour the flux does display an oscillatory behavior, albeit convergent.

From the limited numerical calculations described herein, it is possible to deduce at least one conclusion; namely, that very small perturbations of the flux in an otherwise unstable core, can excite divergent oscillations. Moreover, it appears that the pseudo-fluctuations introduced by the use of a non-zero convergence criterion, and a non-zero time-step length, affects significantly the observed stability characteristics.

Figure 200.25 Digital Simulation of Xenon-Induced Spatial Oscillations with Very Small Perturbation (Perturbed Source Guess)

Figure 200.26 Digital Simulation of Xenon-Induced Spatial Oscillations with Very Small Perturbation
(Perturbed Source Guess - Condensed Power Density Scale)

Figure 200.27 Digital Simulation of Xenon-Induced Spatial Oscillations with Very Small Perturbation (Normal Source Guess)

G. Description of Program ZEST - A One-Dimensional Two-Group Diffusion Theory Code for Xenon Spatial Oscillation Studies

This section includes a brief description of program ZEST, with emphasis on those characteristics of interest to the studies reported herein. Program ZEST was specifically designed for, and tailored to, xenon oscillation studies. The program solves the two-group, one-dimensional (slab, cylindrical, spherical) neutron diffusion equations, coupled with the differential equations describing the time dependence of the local I-135 and Xe-135 atom concentrations. The difference form of the diffusion equations is solved as in program SPARTA, using a many line relaxation procedure^{/1}. The local feedback effects of fuel temperature and moderator density are included in the flux calculation. The variation of the fast removal and absorption cross sections with fuel temperature, and of the fast removal and absorption, and the thermal absorption, cross sections with moderator density are accounted for.

The I-135 and Xe-135 concentrations are obtained from an exact analytical solution of the differential equations, assuming constancy of the fluxes during a time step. At the end of each time step, the output edit includes, in addition to the converged eigenvalue, the spatial distributions of fast and thermal fluxes, fission source, power density, Xe-135 atom concentration, fuel temperature, and moderator density.

The excitation of an oscillation is fully automated in that the following set of calculations is always performed by the program:

- a. the computation of steady-state conditions, with equilibrium Xe-135 concentration
- b. a flux calculation with a poison inserted into the core and with the equilibrium Xe-135 concentration; the location and strength of the perturbation are on input data
- c. calculation of the I-135 and Xe-135 concentrations at a specified time after insertion of the perturbation; this time period is an input datum

^{/1} C. G. Poncelet, "Solution of the Multigroup Space-Time Diffusion Equations" Proceedings of the International Conference on Research Reactors and Reactor Mathematics and Computation", Mexico City (1967)

- d. a flux calculation with the poison inserted and the Xe-135 concentration obtained in (c)
- e. a flux calculation with the poison removed, and the Xe-135 concentration obtained in (c)

This series of calculations is followed by a specified number of time-steps; the duration of these time steps is constant and is an input datum.

The main justification of one-dimensional calculations, as opposed to multi-dimensional calculations, is obviously based on machine time requirements. It should be noted, however, that for the mode of oscillation of interest to this study, one-dimensional calculations in slab geometry do afford a relatively realistic picture. As has been observed from two-dimensional calculations^{/1,2}, the mode most readily excited in the x-y plane corresponds to oscillations across a diameter. Such oscillations can be simulated in one dimensional slab geometry by using a reflected slab whose length corresponds to the actual length along a diameter of the core. The results of a ZEST calculation for the standard 11-ft core with no temperature feedback are shown on Figure 200.28. These results can be compared with the results of the two-dimensional calculations for the same core, shown on Figure 200.19 (four-hour time steps). Note the close similarity between the one- and two-dimensional results.

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- /1 M. J. O'Boyle, et al, "Control of Xenon Instabilities in Large PWR's Quarterly Progress Report for the Period Ending December 31, 1966" WCAP-3680-2 (1966)
 - /2 M. J. O'Boyle, et al, "Control of Xenon Instabilities in Large PWR's Quarterly Progress Report for the Period Ending March 31, 1967" WCAP-3680-3 (1967)

Figure 200.28 One Dimensional Simulation of Azimuthal (Diametral) Oscillations

EUXE-300

REMEDIAL CONTROL PROCEDURES

There was no significant progress on this task during the report period.

EUXE-400

THREE-DIMENSIONAL ANALYSIS

This task was inactive during this report period.