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RESEARCH AND DEVELOPMENT PROGRAM

A STUDY OF COMPLETE LOSS OF COOLANT FLOW
IN THE YANKEE REACTOR

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WESTINGHOUSE ELECTRIC CORPORATION
ATOMIC POWER DEPARTMENT

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YAEC-83

Research And Development Program

A STUDY OF
COMPLETE LOSS OF COOLANT FLOW IN THE YANKEE REACTOR

by

J. M. Gallagher Jr.
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For The Yankee Atomic Electric Company
Under Research and Development Subcontract
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ABSTRACT

The complete loss of primary coolant flow accident caused by the instantaneous loss of power to all four pumps was investigated. The principal objective was to determine the elapsed time from the initiation of the pump failure to the occurrence of bulk boiling at the outlet of the hot channel. Inherent was the determination of operating conditions necessary to eliminate the hot channel bulk boiling, these being added inertia in the primary coolant loop and scram delay time. Both the 392 MW core and the 482MW core were included in the investigation. The results show that additional inertia is required as well as scram if bulk boiling is to be eliminated.

As a result of several of the assumptions made, the investigation was limited to a study of that region of the reactor core where heat transfer takes place between the fuel and the coolant; primary system components such as the pressurizer and steam generator were excluded. The film heat transfer coefficient was varied as a function of flow rate but not as a function of coolant temperature, thus boiling phenomena were not included. Distributed parameter effects were incorporated in equations which describe the core thermal kinetics by an approximation technique. An approximation technique is also presented which allows the extrapolation of the results to the two pump failure case. Data for the investigation were obtained by the use of a general purpose dc analogue computer.

FOREWORD

One of the basic objectives in the design of a nuclear power plant is to provide a reactor-plant system which is safe under all conceivable operational accidents. Loss of reactor coolant flow during full power operation is one of the many operational accidents which must be considered. The pump power sources in the Yankee plant have been divided into 3 groups so as to virtually eliminate the occurrence of a complete loss of coolant flow. If loss of two or more of the four pumps occurs, a scram signal is initiated which causes release of the neutron absorbing control rods. The time delay between the loss of pump power and reactor shutdown is a function of the speed of both the control rod drive and the reactor control system. The purpose of this report is to examine the thermal conditions which exist in the core after complete loss of pump power. The effects of variation of scram delay time and inertia in the coolant system are examined in detail.

One of the major limitations of the study described herein is that it is limited to one phase flow. Since the Yankee reactor is an open lattice of fuel rods, a redistribution of coolant flow will take place when bulk boiling occurs. A knowledge of the flow redistribution was not available when the study was made. Consequently, an examination of the case which involved bulk boiling was not performed. The results are therefore applicable only to the point at which the reactor outlet temperature reaches the saturation temperature of the system.

In spite of this limitation, the work has been quite valuable from many standpoints.

First, the conclusion may be drawn that scram delay time is not an extremely sensitive parameter because of the large amount of heat energy stored in the fuel. During full power operation roughly 6 full power seconds are stored in the fuel, so that an increase in scram delay time of 0.5 sec increases the energy which must be dissipated by less than 10%. Second, the results of the study indicated that a conservative ground rule prohibiting steady state operation in a manner which might lead to hot channel bulk boiling during transients may impose an unnecessarily severe restriction on the reactor core and system design. This has led to an intensive study in cooperation with the Mechanical, Hydraulic and Thermal Design Section of Reactor Engineering directed toward the development of computational procedures for determining local thermal conditions during boiling in the reactor core. As a result, a digital computer program describing flow redistribution during boiling has been written for the IBM-704. In addition, methods have been developed permitting the calculation of fuel and clad temperatures as a function of time at a point of localized film boiling within the core. It is expected that these new analytical procedures will permit a realistic evaluation of thermal conditions during boiling in the core and will make possible an extension of this study of loss of pump power to include consideration of bulk boiling in the hot channel.

1. INTRODUCTION

1.1 Statement of Problem

Some of the most important problems in the design of a nuclear reactor power system arise from the need to prevent thermal damage to the reactor core even under the most unlikely accident condition. One such type of accident is that which could be caused by complete loss of coolant flow in the reactor core. The prerequisite for a complete loss of coolant flow accident in the Yankee Reactor is the improbable event that essentially simultaneous removal of pump power occurs for all four primary loop pumps. This event has been termed improbable since the four pump motors are divided into three groups, two of which contain one pump each, and a third group containing two pumps. Each group is fed from a separate source of electrical power. One of the three power supply sources consists of a transformer fed from the main generator leads, the two other transformers being connected to separate, incoming, 115 kv lines. For the purposes of this study, the assumption has been made that all three sources of electrical pump power fail simultaneously and hence that all four pumps fail. At the time of pump power failure, the reactor core is assumed to be operating at full power in a steady state condition. When pump power is lost, the rate of coolant flow decreases in some manner as determined by the initial conditions,

the kinetic energy of the primary system, the number of pumps that failed (in this case all four) and the power dissipation caused by friction losses in the primary loop.

The physical mechanism of the accident and the related thermal and neutron kinetics of the reactor core are as follows. As coolant flow decreases, the core outlet temperature increases and the film heat transfer coefficient decreases. This increase in average coolant temperature causes a decrease in reactivity as a result of the negative temperature coefficient of reactivity associated with the moderator-coolant region. A decrease in reactivity causes a reduction of reactor power level. The specific power of the reactor, the fuel pellet diameter, and the fuel thermal conductivity* are such that a high thermal gradient is established in the fuel when the reactor is at full power. Hence, the average fuel temperature is well above the average coolant temperature. When a loss of coolant flow occurs, this large amount of heat energy stored in the fuel tends to maintain a relatively high heat flux at the moderator-clad interface while the film heat transfer coefficient is decreasing. These two effects result in a rapid approach to a burnout condition within some region of the core.

1.2 Scope of Study

This study deals with the transient behavior of the neutron and thermal kinetics of the core for the time interval after complete loss of coolant flow has occurred. System components external to the core such as the pressurizer and steam generator are not included in the study. The reasons for the system configuration used are discussed in detail in Section 2.12.

* Uranium dioxide fuel has a relatively low thermal conductivity as compared to metallic reactor fuels.

All physical properties of the core materials were assumed to be constant and all, except fuel thermal conductivity, were evaluated at the average temperature corresponding to the particular material region in question, where the average was taken over the whole core. The value for thermal conductivity of the fuel was that corresponding to the maximum radial-average fuel temperature at the geometric center of the core. The film heat transfer coefficient was defined as a function of the mass velocity but not as a function of bulk water temperature. Consequently, the data presented in this report has full significance only before the time at which bulk boiling occurs. This temperature dependence has been omitted because of the lack of knowledge of the flow behavior during bulk boiling. A further limitation on the results arises from the fact that consideration of both flow redistribution and the effect of voids on the neutron kinetics was omitted.

The scram mechanism was incorporated as a time delay of adjustable duration after the initiation of the flow coastdown, with rod motion beginning at the termination of the delay.

1.3 Criterion for Evaluation of Results

In the loss of coolant flow accident studies, the assumption was made that no thermal damage to the reactor core would be experienced if no bulk boiling occurred at the outlet of the core hot channel at any time during the accident. This is a conservative assumption adopted because a detailed knowledge is not now available for the redistribution of flow within the core when bulk boiling occurs. Without this knowledge, the assumption must be made that a flow redistribution would cause burnout to occur within some

region in the core. Therefore, the primary criterion for evaluation of results is whether the hot channel outlet water reached a temperature corresponding to bulk boiling. The elapsed time at which such an event occurred is also of importance since the heat flux at the surface of the clad has decreased with time. The longer the time required to reach bulk boiling, the lower the maximum heat flux and the less the likelihood that burnout would occur. The time to reach bulk boiling is thus an additional criterion for evaluation of results.

A further criterion is the time dependence of the hot channel outlet temperature transient. If the temperature went above the value for bulk boiling, reached a maximum, and then decreased to a value below bulk boiling, the time interval when the temperature was above bulk boiling was important. If this time interval was short, the probability of burnout was decreased.

These secondary criteria require the interpretation of the results beyond the limitations established by the assumptions upon which the system model was formulated. Such interpretations of data, however, are very important in that by this means results are opened to more thorough scrutiny, i.e. assumptions are then re-evaluated and system models are formulated which yield answers that were not previously obtainable. Examples of evaluations of this type are the considerations of the loss of two pumps by Bournia¹ and by the authors in Section 3.6.

1. Bournia, A. "Studies of Thermal Behavior under Loss of Pump Power Transient Conditions", YAE-72, August 1958.

1.4 Principal Conclusions Derived from Study

The results obtained from this study show that for the 392 MW Yankee core with complete loss of coolant flow, bulk boiling occurred at the outlet of the hot channel in approximately one second after the initiation of the accident for the condition of instantaneous scram. As a result of the relatively low thermal conductivity and the large heat storage of the fuel material, the effect upon the outlet temperature caused by scrambling the reactor is negligible for this initial portion of the transient. These results are presented in Fig. 1.1.

When stored mechanical energy is added to the primary loop, the flow decay rate is slower with the result that the increase in hot channel temperature is also much slower. When the reactor is scrammed, the increase in hot channel temperature can be kept below the bulk boiling point provided, of course, that sufficient energy has been added to the system. Typical transient behavior with the addition of energy are shown in Fig. 1.2.

The reader should bear in mind that all assumptions, results, and conclusions in this report, except for Section 3.6, are based upon the complete loss of coolant flow condition. In Section 3.6, a method is outlined which leads to the evaluation of partial loss of flow incidents. The input data for the evaluation is obtained from the computer study of the complete loss of flow accident.

Figure 1.1 Hot channel outlet temperature and average core outlet temperature as a function of time after initiation of pump failure for the 392 MW core and no added energy.

Figure 1.2 Hot channel outlet temperature and average core outlet temperature as a function of time after initiation of pump failure for the 392 MW core and an added energy of 8 MW sec.

2. DEVELOPMENT OF MODEL

2.1 Assumptions Employed in Formulation of Model

2.11 Introduction

In the study of any complex system numerous assumptions must be made in the process of formulation of a suitable mathematical system description or model. These assumptions are made in order to satisfy one or more of the following three needs:

- a. When an effect can be demonstrated to be of minor consequences as concerns the particular solution to be obtained, the effect can be omitted from the model by the incorporation of the proper assumptions.
- b. When the physical mechanism that accompanies a given phenomenon is not understood fully enough to permit the direct formulation of a descriptive set of differential equations, assumptions must be made that will result in an appropriate description of the mechanism.
- c. When the nature of the equations describing the system are such that they are not adaptable to the means of solution that is to be employed, assumptions are made that will remove the obstacles that are present.

A thorough understanding of the problem solved and a true evaluation of the data and results and conclusions drawn from the data cannot be obtained without first having a knowledge of the assumptions used in the formulation of the system description.

2.12 Configuration of System

The problem of interest in this report is primarily the transient behavior of the hot channel outlet temperature for a complete loss of coolant flow accident. In this case, the coolant flow deceleration is such that a particular particle of fluid located at the outlet of the steam generator at the initiation of the flow coastdown would not reach the reactor core inlet until after an elapsed time greater than the time for which the solution is desired. If flow coastdown is initiated only when the system has been at an unperturbed condition sufficiently long to establish proper steady state conditions, the core inlet temperature can be assumed to be constant. Consequently, if the operation of the pressurizer is assumed to be such that the primary coolant pressure remains constant during the transient and if the physical characteristics that determine the flow coastdown characteristics are assumed to remain constant at their initial values, there is no need for the inclusion of the steam generator in the system description. Therefore, the reactor core can be separated from the primary loop for the purposes of this problem and a system description formulated that involves only phenomena within the active reactor core volume.

2.13 Nonlinear and Distributed Parameters

The transient description of the reactor core is composed of two main physical systems, the power generation system or neutron kinetics, and the heat removal system or thermal kinetics. These two systems are bilaterally coupled, that is, the transient behavior of each is a function of the past history of the other. Each of these systems has space as well as time as an independent variable. The spatial variables associated with these two systems are required

in order to incorporate the following two effects. The first is the power generation or macroscopic spatial fission distribution determined from the geometrical configuration of the core and the distribution of materials within the core. The second is the detailed thermal or microscopic distribution within the core. This distribution is determined by both the power generation distribution and the heat transfer properties of the detailed material distribution within the core. The geometrical configuration of the detailed material distribution within the core has been assumed to be describable by a repeated structure of a representative configuration of the materials that compose the core. Such a representative configuration is hereafter referred to as a unit cell.

Spatial variables also exist within the unit cell because of the necessity for identifying the different material regions which compose the cell, specifically the fuel, airgap, clad, and moderator-coolant material regions. Within each material region the equations governing the dependent variables, i.e. temperature and neutron density, are functions of space as well as time. The physical properties of the material within each region and the conditions at the boundaries between regions are functions of the dependent variables. A complete mathematical description of both the heat generation and heat removal system would be of the form of nonlinear partial differential equations coupled by other nonlinear relationships. The complete solution to such a set of equations is a formidable task and may also be an unfeasible one.

In order to reduce the problem to one solvable by conventional analog computer techniques and a reasonable amount of equipment, as many as possible of the nonlinear factors were removed and the distributed parameter or spatial

effects were approximated. The assumption was made that all physical properties of the reactor core materials remained constant during the loss of coolant flow accident. The principle implication of this assumption was that the parameters associated with the thermal kinetics for the fuel region, the air gap, and the clad region remained constant. The assumption was also made that the film heat transfer coefficient was a function of coolant temperature. This assumption eliminated the inclusion of any boiling phenomena. In retrospect, this appears to be the most important assumption in restricting the applicability of the results. A more detailed discussion of the boiling problem is presented in Sections 2.2 and 3.2.

2.2 Mathematical Techniques Employed in Formulation of Model

2.21 Core Sectionalization and Unit Cell Definitions

The amount of effort required to formulate the system description can be reduced if the reactor core is assumed to be composed of a number of geometrically identical volumes each of which contains a fuel, air gap, clad, and moderator-coolant material region. As stated previously, a single volume of this type will be termed a unit cell. For the purposes of this study the radial boundary of the unit cell is located at a spatial position where the radial flow of heat energy in the moderator-coolant region is zero. This boundary condition restricts the coolant flow to definite channels and consequently does not allow a redistribution of flow from one cell to adjacent cells. The axial boundary of the unit cell can be located at the axial extremities of the core region or at an intermediate point within the reactor core if the core is to be subdivided into axial sections. A section as used here implies a

subdivision of an homogenized core in that each of the materials contained within the core, excluding control rods, is proportioned to the sections in the same manner as the volume of each section is proportioned to the total core volume. For the purposes of this study, the core was subdivided into four sections by an equal axial division and a radial division in which 45% of the volume was contained within the inside radial sections.

The section can be interpreted as a collection of unit cells of identical physical properties. Except for the fuel thermal conductivity, these properties are evaluated at the average values, over the sections, of the dependent variables involved. The same number of heterogeneous regions are contained in a section as in a unit cell. The values for the dependent variables for a section are given by the average of the corresponding dependent variable values for each unit cell within the section. For example, the average temperature of the fuel for the section is given by the average of the average fuel temperature for each unit cell within the section. Likewise, the maximum fuel temperature for a section is the average of the maximum fuel temperatures for each unit cell within the section.

The dependent variables for the separate spatial regions within each unit cell can also be described in terms of a value at a particular position, such as the maximum fuel temperature, or as an average over a given region such as the average fuel temperature. These values are weighted by the unit cells' position in the entire matrix of cells in such a manner that each dependent variable assumes the proper proportions with respect to the corresponding dependent variable in the adjacent cells.

When a particular unit cell or small collection of cells is of interest, conversion factors are provided to convert values of dependent variables for a section to corresponding values for the cells of interest. Examples of these are the hot channel factors which convert average values for the core to maximum values in the hot channel. In a study such as this, an inquiry as to why the hot channel was not studied directly might well be made. The primary difficulty with this approach was that not enough was known about the thermal and hydrodynamic transient phenomena that occur in the hot channel or the behavior of parameters to permit an explicit formulation of the hot channel. Furthermore, since the Yankee core has an open flow lattice, the term hot channel actually refers to a volume of indefinite extent located at the radial center of the core instead of a single channel. The approach used in this study was to include this hot channel volume inside a larger volume, to evaluate the physical properties on the basis of the average values for the dependent variables within this volume or section, and then to assume that the hot channel conversion factor predicted the behavior of various dependent variables within the hot channel volume.

The standard assumptions of neglecting the effect of axial heat conduction in the coolant region and the use of channeled slug flow were also employed. Reactivity effects caused by temperature changes were given by temperature coefficients, and diffusion theory was used for all neutron kinetic relationships.

The method of approximation for the distributed parameter effects is discussed in general in the following section and in detail in Appendix B.

For purposes of this study the assumptions were made that all power sources varied in a manner given by the neutron kinetic equations and that all power sources were contained within the fuel region. An investigation of the individual power sources was performed in order to check these assumptions. The total contribution of the gamma heating to the steady state core power output was found to be 12%. The total was composed of 9.7% in the fuel region, 1.5% in the stainless steel region and 0.8% in the water region. Since the introduction of heat sources into the stainless steel and water regions would complicate the analog simulation and also since the magnitudes of these sources were found to be small, the assumption was made that all gamma heat sources were located in the fuel region. Heating effects caused by processes such as slowing down of fast neutrons by a moderating material were also considered. At most, the contribution to the total core power would be 2% from the "slowing down" source and hence the power source could be lumped into the fuel region with a fair degree of accuracy.

An investigation showed that approximately 7% of the steady state power would remain as a "constant" heat source generated by long-lived fission products of various decay schemes. The source would be "constant" for 20 to 30 seconds after the loss of flow occurred since the overall decay rate of these fission products is long compared to the time interval in question. Since the mechanisms that determine the severity of a complete loss of flow accident in a Yankee type reactor are affected only to a slight degree by the "constant" heat source, the assumption was made that all sources behaved like neutrons and therefore could be simulated as such by means of the neutron kinetics equations.

2.22 Methods of Approximation

As was stated in Section 2.13, the equations which define the dependent variables within the unit cell are functions of space as well as time. The spatial functional relationships were incorporated in the mathematical description for the thermal kinetic system by two techniques. One method employed lumped parameter approximations while the other used an approximation technique for the inversion of transformed functions. The later technique is referred to as the moment method of approximation².

The axial spatial variable for the fuel material region as well as the axial and radial spatial variables for the clad material region were eliminated by the use of lumped parameter approximations. In this approximation, the power generation functional relationship is assumed to be given by the product of three functions each of which is a function of one of the three independent variables; namely the radial coordinate, the axial coordinate and time. The effect of the control rod motion upon the power distribution is given by weighting functions which are defined in Appendix C. The temperatures within each unit cell are thus axial averages for the unit cell and the differential equations that describe the thermal kinetics have two independent variables, radius and time. In the air gap and clad materials, the assumption of a linear radial gradient was used since the thickness of these two adjacent regions was very small. Thus, a lumped parameter representation for the clad material was employed. The equations for the fuel region were solved by an approach similar to that used by Carslaw and Jaeger³ with the exception that

2. Cunningham, J. P.; Flaherty, F. T., Jr. and Gallagher, J. M., "Dynamic Behavior of Homogeneous-Circulating-Fuel Reactor Systems", M.I.T. Cambridge, Mass., DACL Research Memorandum No. 7457-2 - June, 1957.
3. Carslaw, H. S. and Jaeger, J. C., "Conduction of Heat in Solids", Oxford Press, 1947.

an approximation technique was employed to determine a transfer function representation from the transformed solutions. These transfer functions gave the response of a dependent variable at some given position in space; this response results from a disturbance applied at a different given position in space. An example of such a transfer function is the relationship governing the change in the radial average fuel temperature caused by a change in the fuel temperature at the edge of the fuel region. The procedure for obtaining the approximation is called the moment method of approximation. The transfer functions obtained from this approximation are given in Appendix B.

2.3 Mathematical Description for System Model

This section presents, without derivation, the equations used in the analog computer study of the complete loss of flow accident study for the Yankee Power Reactor. All parameters used in these equations are defined in the Glossary, Appendix A. The values for the parameters as they appear in the equations are listed in Tables D.1 and D.2, Appendix D, for both the 392 MW core and the 482 MW core. The nonlinear functions employed in the mathematical description are given in Appendix C. The symbolic definitions for the dependent variables are given with the appropriate equation when first introduced. The dependent variables have been assumed to be describable in terms of an initial steady state value, a function of space only, denoted by the superscript o and a transient term, a function of space and time, denoted by the superscript $*$. The initial values for the dependent variables are given in Table D.3, Appendix D. The derivations of the equations with the assumptions required, in addition to those discussed in the preceding sections, are presented in Appendix B. For clarity of presentation, the fuel region equations have been given in terms of

the Laplace transform variable s instead of the conventional differential equation form with the time variable t . All other equations are given in differential equation form. In the following thermal kinetic equations the subscript j denotes the j^{th} section. The neutron kinetic equations were written for the total power generation in the core.

2.31 Thermal Kinetic Equations

2.31.1 Fuel Region

The average fuel temperature in the j^{th} section is given by the relationship

$$\overline{T_{fj}^*}(s) = \frac{1}{\frac{1}{2} \tau_f s + 1} T_{faj}^*(s) + \frac{\tau_f}{2 (mc_p)_{fj}} \frac{1}{\frac{2}{3} \tau_f s + 1} q_{cj}^*(s) \quad (1)$$

where T_{faj}^* is the change in the surface temperature of the fuel rod for the j^{th} section and q_{cj}^* is the change in the power generation for the j^{th} section. Appendix A contains a complete list of symbols along with their definitions and units.

The change in the fuel temperature at the surface of the fuel region for the j^{th} section is given as

$$T_{faj}^*(s) = \frac{1}{(UA)_{gj}} \frac{1}{\left(\frac{1}{2} \tau_f + \tau_a\right) s + 1} q_{cj}^*(s) + \frac{\frac{1}{2} \tau_f s + 1}{\left(\frac{1}{2} \tau_f + \tau_a\right) s + 1} \overline{T_{ssj}^*}(s) \quad (2)$$

where $\overline{T_{ssj}^*}$ is the change in the average clad temperature for the j^{th} section.

2.31.2 Stainless Steel or Clad Region

The average clad temperature for the j^{th} section is given by

$$(mc_p)_{ssj} \frac{d\bar{T}_{ssj}}{dt} = (UA)_{gj} (T_{faj} - \bar{T}_{ssj}) - (UA)_{wj} (\bar{T}_{ssj} - \bar{T}_{wj}) \quad (3)$$

where $(UA)_{wj}$ is a function of the flow rate w/w^0 and where $\bar{T}_{wj}$ is the average water temperature for the j^{th} section. In this and the following equations the dependent variables have been expressed as the total quantity rather than just the transient term since multiplication by a time variant parameter must be performed.

2.31.3 Coolant-Moderator Region

The average coolant-moderator temperature for the j^{th} section is given by

$$(mc_p)_{wj} \frac{d\bar{T}_{wj}}{dt} = (UA)_{wj} (\bar{T}_{ssj} - \bar{T}_{wj}) - (wc_p)_{wj} (T_{woj} - T_{wij}) \quad (4)$$

In this equation the parameter w is a function of time which was precalculated by the procedure outlined in Appendix C. The outlet temperature for the j^{th} coolant-moderator section is given by the relationship

$$T_{woj} = 2 \bar{T}_{wj} \left(t - \frac{\tau_{DCj}}{2} \right) - T_{wij} (t - \tau_{DCj}) \quad (5)$$

where τ_{DCj} is the time delay through the j^{th} section of the core. The parameter τ_{DCj} is a function of time since the flow rate is a function of time.

2.32 Neutron Kinetic Equations

The prompt power generation is given by the relationship

$$\frac{dq_c^*}{dt} = \frac{\delta k}{\ell} (q_c^o + q_c^*) - \frac{\beta}{\ell} q_c^* + \sum_{i=1}^3 \lambda_i \psi_i^* \quad (6)$$

where a three neutron group approximation was used. In this equation q_c^* is the change in the prompt power generation in the core and ψ_i^* is the delayed power generation. The delayed power generation for the i^{th} group is obtained from the equation

$$\frac{d\psi_i^*}{dt} = \frac{\beta_i}{\ell} q_c^* - \lambda_i \psi_i^* \quad (7)$$

The power generation for the j^{th} section is

$$\left. \begin{aligned} q_{cj}^* &= W_{qj}^o \mathcal{W} q_c^*, \quad j = 2 \text{ or } 4 \\ \text{and} \\ q_{cj}^* &= W_{qj}^o (2 - \mathcal{W}) q_c^*, \quad j = 1 \text{ or } 3 \end{aligned} \right\} \quad (8)$$

In this equation $\mathcal{W}$ is a normalized statistical weight which is defined in Appendix C as a function of rod position and hence time.

The total reactivity δk is defined as

$$\delta k = \delta k_R + \delta k_w + \delta k_f \quad (9)$$

where δk_R is the reactivity change caused by rod motion, δk_w is the reactivity change caused by the moderator-coolant temperature change and δk_f is the reactivity change caused by the fuel temperature change. The normalized reactivity change caused by rod motion is given in Fig. C.1 of Appendix C. The reactivity change attributable to the temperature change

in both the moderator-coolant region and the fuel region is

$$\delta k_w + \delta k_f = \mathcal{H} \left[\alpha_w (W_{p2}^0 \bar{T}_{w2}^* + W_{p4}^0 \bar{T}_{w4}^*) + \alpha_f (W_{p2}^0 \bar{T}_{f2}^* + W_{p4}^0 \bar{T}_{f4}^*) \right] \\ + (2 - \mathcal{H}) \left[\alpha_w (W_{p1}^0 \bar{T}_{w1}^* + W_{p3}^0 \bar{T}_{w3}^*) + \alpha_f (W_{p1}^0 \bar{T}_{f1}^* + W_{p3}^0 \bar{T}_{f3}^*) \right] \quad (10)$$

In this equation W_{p1}^0 through W_{p4}^0 represents the initial values of the statistical weights for each section of the core. These weights are discussed further in Appendix C.

2.4 Parameters Used in the Interpretation of Computer Data

The use of the unit cell method of analysis yielded results which were the average variable values within a given section. Since localized effects were desired in the application of the results to a core safety analysis, hot channel factors were used to determine these localized effects e.g., the outlet water temperature of the hot channel rather than the average outlet water temperature of the center top section determined the time at which bulk boiling occurred. The localized effects of primary importance were the hot channel water outlet temperature and maximum heat flux.

$F_{\Delta T}$ was the hot channel factor used to determine the enthalpy rise in the hot channel from data for an average channel. Since the average enthalpy rise in the center 45% of the core volume was obtained as a computer output and since the hot channel was within this volume, the hot channel factor was obtained for these sections.

$$F_{\Delta T_{1-2}} = \frac{\Delta H_{\max_{1-2}}}{\Delta H_{\text{avg}_{1-2}}} = F_{R_{1-2}} F_{E_1}$$

where $\Delta H_{\max_{1-2}}$ = maximum enthalpy rise through the center sections.

$\Delta H_{\text{avg}_{1-2}}$ = average enthalpy rise through the center sections.

F_{E_1} = engineering factor for enthalpy rise.

and $F_{R_{1-2}}$ = radial nuclear maximum to average power density ratio in the center sections.

A similar factor, $F_{Q_{1-2}}$, was used for the determination of the maximum heat flow from the computed output of the section average heat flow.

$$F_{Q_{1-2}} = \frac{q_{\max_{1-2}}}{q_{\text{avg}_{1-2}}} = F_{R_{1-2}} F_{Z_{1-2}} F_{E_2}$$

where F_{E_2} = engineering factor for heat flow.

$F_{Z_{1-2}}$ = nuclear axial maximum to average power density ratio for center sections.

$q_{\max_{1-2}}$ = maximum heat flow from stainless steel to water in center sections.

and $q_{\text{avg}_{1-2}}$ = average heat flow from stainless steel to water in center sections.

The numerical values for the factors are

$$F_{\Delta T_{1-2}} = 2.25 \quad \text{and}$$

$$F_{Q_{1-2}} = 3.48.$$

In order to determine the hot channel outlet temperature, the average outlet temperature of the center top section, as obtained from the analog computer solution, was combined with the hot channel factor, $F_{\Delta T_{1-2}}$, as follows

$$H_{in} + F_{\Delta T_{1-2}} (H_{out} - H_{in}) = H_{ohc}$$

where H_{in} = enthalpy of water entering bottom of core,
 H_{out} = enthalpy of average water leaving top center section,
 and H_{ohc} = enthalpy at outlet of hot channel.

From the evaluation of H_{ohc} , the temperature at the outlet of the hot channel was obtained. Hot channel boiling occurred when H_{ohc} reached the saturation enthalpy corresponding to 2000 psia.

In the interpretation of the heat transferred across the stainless steel-water interface, q_{sw1} , in terms of a maximum heat transfer rate, the assumption was made that the lower center section heat transfer rate followed the same time behavior pattern as the upper center section. Thus, the maximum heat transfer rate was obtained by application of the hot channel factor, $F_{Q_{1-2}}$, as follows

$$q_{max} = F_{Q_{1-2}} q_{sw1}$$

where q_{max} = maximum heat transfer rate from stainless steel to water,
 and q_{sw1} = average heat transfer rate in top center section.

To obtain maximum heat flux, the average heat transfer rate for section one, q_{sw1} , was divided by the heat transfer area of the section (22.5% of total area) and multiplied by $F_{Q_{1-2}}$.

3. RESULTS OF ANALOGUE COMPUTER STUDY

3.1 Introduction

In the analogue computer study of the complete loss of coolant flow accident, principal emphasis was placed upon an investigation of effects caused by variations in scram delay time and the amount of additional stored mechanical energy contained in the primary coolant loop. Scram delay time denotes the time interval between pump failure and the initiation of rod motion. Actual scram, in terms of reactivity inserted, occurs approximately 0.6 sec after the rods have started to move. This is shown by Fig. C.4. The amount of additional energy inserted into the primary coolant loop determines the rate at which the flow decays. The functional relationship for flow coastdown is given by Eq. C.3. Coastdown curves obtained from this relationship for different values of added energy are presented in Fig. C.5.

In this study data were obtained for both the 392 MW core and the 482 MW core. Other than the two previously mentioned parameters, the only parameter varied was the thermal conductivity of the fuel for the 392 MW core. The core was subdivided into four sections by an equal axial division and a radial division where 45% of the volume was contained within the inside radial sections. This radial division was incorporated to allow for parameter changes as a function of radius. However, because of the nature of the results obtained from the uniform core study and limited time available, the non-uniform core was not investigated.

The dependent variable of principal interest in this investigation was the outlet coolant temperature of the top inside section of the core. This variable was recorded on an x-y plotter in order to achieve a recorded accuracy

comparable to the computation accuracy. The hot channel outlet temperature was obtained from the top inside section outlet temperature, T_{wol} , by the conversion procedure outlined in Section 2.4.

Another dependent variable of importance was the rate of heat flow through the film boundary at the surface of stainless steel clad, q_{sw} . Only the heat flow in the top inside section was recorded for all runs. Since this is the region where burnout is most likely to occur, one of the main applications of these heat flow data were to provide input data for a hand calculation procedure that incorporated the effects of flow redistribution when bulk boiling in the hot channel was achieved. Such a procedure allows the extrapolation of the computer data to the bulk boiling condition in order to determine the relationship between burnout heat flux and actual heat flux. A study of this type has been performed for the two pump failure accident⁴. Here, as in the case of the outlet temperature, a hot channel conversion factor, F_Q , was required for proper interpretation of the data.

Other dependent variables such as fuel, clad, and coolant average temperatures for each of the four sections were also recorded. These recordings were employed primarily as a means of monitoring the computer operation. Typical curves are discussed in the following sections.

A list of cases studied is given in Table 3.1. In this table a scram delay time of infinity is used to denote the no scram case. The other times refer to the time at which rod motion is initiated. The "normal" case refers to a study of the four section core with parameters as tabulated in Appendix D.

4. Bournia, A. op. cit.

TABLE 3.1 LIST OF CASES STUDIED

a) 392 MW Core

Added Inertia I_A in MW-secs	Scram Delay seconds	Parameters
0	$\infty, 0$	Normal
2	$\infty, 0$	"
4	$\infty, 0, 1, 2$	"
8	$\infty, 0, 1, 2, 3, 5$	"
0	$\infty, 0$	One section core
0	$\infty, 0$	$k_f = \begin{cases} 1.26, j = 1 \text{ and } 2 \\ 2.28, j = 3 \text{ and } 4 \end{cases}$
2	$\infty, 0$	"
8	∞	"

b) 482 MW Core

Added Inertia I_A in MW-secs	Scram Delay seconds	Parameters
0	$\infty, 0$	Normal
8	$\infty, 0, 0.5, 1.0$	"
12	$\infty, 0, 1, 2$	"
20	$\infty, 0, 1, 2, 3$	"

3.2 392 MW Core Results

Before a detailed discussion of the computer data is undertaken, a review of the physical mechanism which accompanies a loss of coolant flow accident is in order. When a pump failure occurs the mass rate of coolant flow through the core decreases. As a result of the flow decrease a particular slug of water is in the core for a longer period of time and will thus receive more thermal energy and increase in temperature with respect to the full flow value. This increase in moderator-coolant temperature causes a decrease in reactivity which in turn tends to cause a decrease in power generation. When power generation decreases the fuel temperature also decreases. A decrease in fuel temperature, through the effect of the Doppler coefficient, causes an increase in reactivity. Hence the reduction in reactivity as a result of the moderator-coolant temperature rise is somewhat offset by the fuel temperature change and the power decrease is smaller. An additional effect that tends to reduce the fuel temperature change, and thus the previously mentioned associated reactivity increase, is the decrease in film heat transfer coefficient caused by the decrease in flow. However, the film heat transfer coefficient is much greater in magnitude than the effective heat transfer coefficient from fuel to clad. As a result the clad temperature rise, with respect to coolant temperature, will increase somewhat but no noticeable effect will appear in the fuel temperature transient. Plots of fuel, clad, and coolant average temperatures, for the case of no scram, $\tau_{DS} = \infty$, are shown in Fig. 3.1. The change in reactivity caused by the core temperature changes is shown in Fig. 3.2. The total reactivity change 4 seconds after loss of coolant flow was initiated is - 1%. This reactivity change is substantial enough to cause a large reduction

Figure 3.1 Average fuel, clad, and coolant temperatures for section 1 as a function of time after the initiation of pump failure for the 392 MW core with no added energy.

Figure 3.2 Reactivity changes as a function of time after the initiation of pump failure for the 392 MW core with no added energy and no scram.

in power generation (approximately a 60% reduction). At this same time the average fuel temperature rise above average moderator temperature has decreased about 20%. Plots of fuel, clad, and coolant average temperature for instantaneous scram, $\tau_{DS} = 0$, are shown in Fig. 3.1. This figure shows that the thermal lag in the fuel region is large enough, with respect to the flow decay time, that little beneficial effect is achieved by an instantaneous scram for the first three or four seconds after the loss of pump power. In fact, the average core water temperature continues to increase for the duration of the transient. This is to be expected since the flow has effectively dropped to zero before the fuel temperature has reached the water temperature. For the case of instantaneous scram the reactor power has been reduced by more than 90% at 4 seconds after loss of coolant flow, while the reduction in average fuel temperature rise above average coolant temperature is about 40% as compared to 20% for the no scram case.

When energy is added to the primary coolant loop, the flow decay rate is reduced. If enough energy is added, this decay rate becomes less than the thermal lag associated with the fuel heat transfer. Temperature curves for an added energy of 8 MW sec are shown in Fig. 3.3. Here the increase in average water temperature is much less than for $E_A = 0$. Consequently, for the case of no scram, $\tau_{DS} = \infty$, the change in reactivity is smaller which in turn results in a smaller change in power and fuel temperature. For the case of instantaneous scram, the flow rate is high enough* that a substantial amount of heat removal from the fuel has taken place. As a result of this higher heat removal

* After an elapsed time of 4 seconds, the flow for $E_A = 8$ MW-sec is approximately three times that for $E_A = 0$.

Figure 3.3 Average fuel, clad, and coolant temperatures for section 1 as a function of time after initiation of pump failure for the 392 MW core with an added energy of 8 MW sec.

the average fuel temperature has decreased by 60% compared with 40% for no added energy. The average coolant temperature decreases for the case of instantaneous scram and $E_A = 8$ MW-sec. This clearly shows that the rate of heat removal from the core as a result of coolant flow is greater than the rate of heat addition from the fuel region, or that the rate of flow decay is less than the thermal lag associated with the fuel.

Also shown in Fig. 3.3 is the effect upon the fuel temperature of a change in scram delay time intermediate in the range of 0 to ∞ seconds.

The change in the outlet temperature of the top center section is shown in Figs. 3.4 through 3.7. Instead of converting these curves to hot channel outlet temperatures a line has been drawn to indicate the temperature rise which corresponds to bulk boiling at the outlet of the hot channel. For this line an $F_{AT} = 3.36$ for the whole core or 2.25 for the center section was used. The data presented in this figure show that for all values of added energy studied bulk boiling occurred when the reactor was not scrammed. However, an added energy of approximately 4 MW-sec was sufficient to prevent bulk boiling with instantaneous scram. The allowable scram delay time is less than one second for an added energy of 4 MW-sec but is approximately two seconds for an added energy of 8 MW-sec. A scram delay of less than two seconds is within the capabilities of the Yankee Reactor control system.

The effect of added energy upon the decay rate of the heat flow through the film at the clad surface is shown in Figs. 3.8 and 3.9. For the case of no scram, Fig. 3.8, the addition of energy results in higher heat flow rates as should be expected since the power generation becomes more constant with time. With instantaneous scram, Fig. 3.9, the range of variation caused by added

Figure 3.4 Change in outlet temperature of section 1 as a function of time after initiation of pump failure for the 392 MW core with no scram and with added energy as a parameter.

Figure 3.5
Change in outlet temperature of section 1 as a function of time after initiation of pump failure for the 392 MW core with instantaneous scram and with added energy as a parameter.

Figure 3.6 Change in outlet temperature of section 1 as a function of time after initiation of pump failure for the 392 MW core with an added energy of 4 MW sec and with scram delay time as a parameter.

Figure 3.7 Change in outlet temperature of section 1 as a function of time after initiation of pump failure for the 392 MW core with an added energy of 8 MW sec and scram delay time as a parameter.

Figure 3.8 Heat flux in section 1 as a function of time after initiation of pump failure for the 392 MW core with no scram and with added energy as a parameter.

Figure 3.9

Heat flux in section 1 as a function of time after initiation of pump failure for the 392 MW core with instantaneous scram and with added energy as a parameter.

energy is much smaller. This means that for the instantaneous scram case the effect of a change in E_A on the burnout condition is predominantly a function of the effect of this change in E_A upon the temperature or enthalpy rise in the hot channel. For the no scram case the situation is not as clearly defined since an increase in E_A increases the heat flux at any given time but reduces the enthalpy rise in that hot channel at that same time. With parameter changes in scram delay, the heat flow from stainless to water follows the no-scram case for the initial portion of the transient and then tends towards the instantaneous scram curve in the latter portion of the transient. As a result, the effect upon q_{sw} of a change in scram delay time is very noticeable. Figure 3.10 illustrates this effect for $E_A = 8$ MW sec which was the largest value of added energy studied for the 392 MW core.

3.3 482 MW Core Results

The diameter of the fuel rod is smaller for the 482 MW core studied than for the 392 MW core, 0.262 in. compared to 0.290 in. Thus, the time constant associated with the fuel region is smaller, for the 482 MW core 5.46 seconds compared to 6.77 seconds. While the masses of each of the materials in the 482 MW core are greater than those for the 392 MW core, the heat transfer area between material regions is also greater with the result that the time constant associated with the thermal kinetics of any of the material regions is smaller for the 482 MW core.

The average coolant-moderator temperature is the same for each core design but the power output of the 482 MW core is considerably greater, thus the outlet temperature of the hot channel is greater and the allowable temperature rise in the hot channel is smaller. For the same $F_{\Delta T}$ as before the

Figure 3.10 Heat flux in section 1 as a function of time after initiation of pump failure for the 392 MW core with an added energy of 8 MW sec and with scram delay time as a parameter.

temperature rise at the outlet of the top inside section that corresponds to bulk boiling at the outlet of the hot channel is approximately 8.6°F. Plots of the upper center section outlet temperature, T_{wol}^* , are shown in Figs. 3.11 through 3.15 for various conditions of scram and added energy. A comparison of these curves with Figs. 3.4 through 3.7 shows that the temperature variations for the 482 MW core were larger than for the first core for identical values of scram delay time and added energy. The excursions are greater before and just after scram because of the larger amount of power generated in the reactor core. In the latter portion of the transient, the rate of drop of the outlet temperature is greater as a result of the lower time constants associated with the thermal kinetics of the core.

In order to eliminate hot channel bulk boiling with a scram delay time of approximately 2 seconds an added energy of 20 MW sec is needed. This compares with 8 MW sec for the 392 MW core where a scram delay time of 2 seconds or less was needed to prevent bulk boiling.

3.4 Effect of Core Sectionalization Upon Transient Response

The division of the core into four sections with identical physical properties for each section did not have a significant effect upon the answers for the case of no added energy. This is illustrated by Figs. 3.16 and 3.17 in which the use of four sections does not yield a significantly different time to bulk boiling or a different heat flux at boiling. Also the sectionalization has little effect upon the transient response of either dependent variable shown in the figures. In both of these figures the more adverse answers were obtained for the four section core configuration.

Figure 3.11

Change in outlet temperature of section 1 as a function of time after initiation of pump failure for the 482 MW core with no scram and with added energy as a parameter.

Figure 3.12 Change in outlet temperature of section 1 as a function of time after initiation of pump failure for the 482 MW core with instantaneous scram and with added energy as a parameter.

Figure 3.13 Change in outlet temperature of section 1 as a function of time after initiation of pump failure for the 482 MW core with an added energy of 8 MW sec and with scram delay time as a parameter.

Figure 3.14 Change in outlet temperature of section 1 as a function of time after initiation of pump failure for the 482 MW core with an added energy of 12 MW sec and with scram delay time as a parameter.

Figure 3.15 Change in outlet temperature of section 1 as a function of time after initiation of pump failure for the 482 MW core with an added energy of 20 MW sec and with scram delay time as a parameter.

Figure 3.16 Hot channel temperature response caused by complete loss of coolant flow with no added energy.

Figure 3.17 Maximum heat flux caused by complete loss of coolant flow.

The effect of a change in physical properties of the four section representation was examined by means of a variation of the thermal conductivity of the UO_2 fuel. This parameter change was used because a variation of the conductivity gave the greatest change in the thermal time constant associated with the fuel and hence the most significant change in answers. The conductivity of the inner sections was changed from 0.9 Btu/hr,ft, $^{\circ}\text{F}$ to 1.26 Btu/hr,ft, $^{\circ}\text{F}$ and that of the outer sections from 0.9 to 2.28 Btu/hr, ft, $^{\circ}\text{F}$ (see Appendix C). Even with these variations, the time constants of the fuel are such that the results are not significantly altered for the cases with no added energy.

For the case presented in Figures 3.16 and 3.17, an examination of the reactivity behavior showed that only 7% of the total reactivity change was attributable to the two outside radial sections. Since this was the only coupling that existed between radial sections and also since physical property changes would not cause significant variation in results, the outer sections could have been omitted for this study*.

In view of these facts and the results presented in Figs. 3.16 and 3.17, the conclusion can be drawn that the thermal lags in the radial direction within any section are much greater than the lags associated with the fluid flow.

These results also show that the incorporation of the actual control rod motion in terms of $\delta k/\text{sec}$ yields essentially the same results as a step scram delayed 0.6 second. Again this is true because of the large time constants associated with the radial thermal kinetics.

* For studies encompassing detailed control rod programs rather than blanket control, the inclusion of the outer sections becomes a necessity.

3.5 Effect of Variation in Enthalpy Rise Hot Channel Factor ($F_{\Delta T}$)

The extent to which $F_{\Delta T}$ may be modified due to design changes is not known at this time, but the effect of multi-region loading, etc. upon $F_{\Delta T}$ are being considered under the Research and Development Program. Therefore, it seems advisable to investigate the changes in the results caused by design modifications which lead to variations in $F_{\Delta T}$. The time to bulk boiling for the 392 MW core with no scram and instantaneous scram with no added energy are shown in Fig. 3.18 as a function of $F_{\Delta T}$. The calculations are based upon a 2000 psia system pressure. The results presented in Fig. 3.18 indicate that the time to bulk boiling increases as $F_{\Delta T}$ decreases; however, they do not indicate the probability of burnout occurring as a function of $F_{\Delta T}$. For example, if $F_{\Delta T} = 1.5$, the outlet of the hot channel reaches boiling at 3.7 seconds with no scram compared to 1 second with $F_{\Delta T} = 3.36$. The probability of burnout at 3.7 seconds is less than at 1 second due to the reduction of heat flux as time increases. In fact, whereas conditions at 1 second may be sufficient to cause burnout, the conditions operative at 3.7 seconds may not be.

3.6 Extrapolation of Computer Data to Obtain Results for Partial Loss of Flow

After the reduction and evaluation of the data obtained from the computer study for the complete loss of flow incident, results for partial loss of flow were desired. An approximate method that permits preliminary evaluation of partial loss of flow was developed for the determination of these transients. The method makes use of data from the complete loss of flow study and thus all basic assumptions are the same. In addition to these assumptions, the assumption is made that an instantaneous heat balance can be used which is not influenced by the past history of the transient behavior. By means of this

Figure 3.18 Time to bulk boiling at hot channel outlet as a function of the enthalpy rise hot channel factor, $F_{\Delta T}$.

assumption, the computer results for complete loss of flow with inertia added to the flow system are extrapolated to partial loss of flow incidents caused by flow coastdowns whose functional time dependencies are different. The temperature transients caused by a flow coastdown associated with the loss of two pumps, as shown in Fig. C.5, have been analyzed by means of this method and are shown in Figs. 3.19 and 3.20. The temperature transients for loss of four pumps are shown on these figures for comparative purposes.

For the 392 MW core (Fig. 3.19), a comparison of the loss of four pumps with the loss of two pumps shows that the time to bulk boiling increases from 1 second to 1.8 seconds for no scram and from 1.1 seconds to ∞ seconds for instantaneous scram. Similarly, for the 482 MW core (Fig. 3.20), the time to bulk boiling increases from 0.5 seconds to 1 second for both no scram and instantaneous scram. However, for the 482 MW core with instantaneous scram, the saturation enthalpy is exceeded for a time period of less than 1 second. This should tend to reduce the probability of fuel element burnout.

These results were obtained by an approximation procedure and thus only preliminary conclusions can be drawn for the loss of two pumps in the Yankee system. From the results one would infer that for the 392 MW core, a scram delay of two seconds* would cause a temperature transient resulting in bulk boiling in the hot channel. However, as a consequence of the probable short duration of this boiling, conditions may be such that burnout will not occur. The results for the loss of two pumps for the 482 MW core are less encouraging because with a scram delay of two seconds, bulk boiling occurs at

* This time delay was chosen since it is easily within the capabilities of the control system.

Figure 3.19 Change in outlet temperature of section 1 as a function of time after initiation of pump failure for the 392 MW core with loss of two and four pumps.

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Figure 3.20 Change in outlet temperature of section 1 as a function of time after initiation of pump failure for the 482 MW core with loss of two and four pumps.

1 second and would probably continue for a considerable length of time. Thus, while burnout may or may not occur for the 392 MW core with a two second scram delay, the probability of burnout under similar conditions is much greater for the 482 MW core.

4. SUMMARY AND CONCLUSIONS

The investigation discussed in this report consisted primarily of a study of the change in the reactor core outlet temperature caused by a reduction in primary coolant flow rate. The active region of the reactor core was isolated from the remainder of the primary loop system by the assumptions of a constant core inlet temperature, an ideal pressurizer (no change in system pressure during accident), and a precalculated flow coastdown curve. All physical properties of materials were assumed to remain constant with respect to time. As a result of this assumption, the film heat transfer coefficient did not vary as a function of coolant temperature and consequently the inclusion of any boiling phenomena was prohibited. This provided the most serious limitation upon the applicability of the data obtained from the study. However, the statement should be made here that the study discussed in this report was formulated on the basis that the principal constraint upon the core temperature behavior was that bulk boiling in the outlet of the hot channel was not permissible. This constraint was then used as the primary criterion for the evaluation of results. The time required to reach a condition of bulk boiling at the outlet of the hot channel following the initiation of a four pump failure (complete loss of pump power) is shown in Fig. 4.1. This figure presents results for both the 392 MW core and the 482 MW core. As shown by these curves, if no stored mechanical energy is added to the primary loop coolant, bulk boiling occurs very soon after the pump failure is initiated for both no scram and instantaneous scram. When energy is added, the time to bulk boiling increases and if the reactor is scrammed, the condition of bulk boiling can be eliminated provided

Figure 4.1 Time to bulk boiling in hot channel as a function of added energy.

that the scram delay time is below a certain value as shown in Fig. 4.1.

The principal conclusion to be drawn from this study is that bulk boiling occurs at the outlet of the hot channel in approximately 1 second after the complete loss of pump power for the 392 MW core and in approximately 1/2 second for the 482 MW core. As a result of the large effective thermal inertia associated with the oxide fuel, the action of scrambling the reactor is not sufficient to eliminate bulk boiling. Therefore, for a complete loss of coolant flow, stored mechanical energy must be added to the primary coolant in order to prevent bulk boiling, the amount being determined by the core design and power level and the delays associated with the scram instrumentation.

APPENDIX A
LOSS OF FLOW
GLOSSARY

<u>Symbol</u>	<u>Definition</u>	<u>Units</u>
a	Impeller loss coefficient	
A	General heat transfer area parameter specified by associated subscripts	ft ²
A ₁	1 + a	
b	Inside radius of stainless steel clad or Rotor windage loss coefficient	ft
c	Radius associated with the average stainless steel temperature at steady state	ft
c _p	General specific heat parameter specified by associated subscripts	Btu/lb-°F
D _e	Hydraulic radius of unit cell	ft
E _A	Stored mechanical energy	MW-sec
E _O	Total initial kinetic energy	ft-lb
F _{ΔT}	Enthalpy rise hot channel factor	
F _E	Engineering component in overall hot channel factor	
F _θ	Film drop hot channel factor	
F _R	Nuclear radial component in overall hot channel factor	
F _Q	Heat flux hot channel factor	

F_Z	Nuclear axial component in overall hot channel factor	
G	Mass velocity	lb/hr-ft ²
h	Film heat transfer coefficient, specified by associated subscripts	Btu/hr-ft ² -°F
H	Enthalpy of moderator-coolant	Btu/lb
k	General thermal conductivity parameter, specified by associated subscripts	Btu/hr-ft ² -°F
ℓ	Mean thermal-neutron lifetime in a finite medium	sec
L	Active core length	ft
m	General mass parameter, specified by associated subscripts	lbs
P_o	Total initial pumping power	ft-lb/sec
q_c	Power generation caused by neutron fissions	Btu/sec
q_{sw}	Power transferred across the stainless steel to water interface	Btu/sec
q''	Heat flux at stainless steel-water interface	Btu/hr-ft ²
r	Independent space variable	ft
s	Complex-frequency variable	sec ⁻¹
S	Source of thermal neutrons from fission	neutrons/cm ² -sec
T	General temperature variable, specified by associated subscripts and superscripts	°F
t	Independent time variable	sec

t_s	Time measured from initiation of control rod motion	sec
U	General heat transfer conductivity parameter, specified by associated subscripts	Btu/sec-°F-ft ²
v	Velocity of coolant-moderator	ft/sec
w_w	Mass rate of flow of coolant-moderator	lbs/sec
w_{qj}	Power fraction associated with j^{th} section	lbs/sec
w_{pj}	Statistical weight associated with j^{th} section	
$\mathcal{H}$	Normalized statistical weight	
z	Independent space variable	ft

<u>Symbol</u>	<u>Definition</u>	<u>Units</u>
α_f	Doppler coefficient of reactivity	$\Delta k/k - ^\circ F$
	or	
	Thermal diffusivity of fuel	ft^2/sec
α_p	Pressure coefficient of reactivity	$\Delta k/k \text{ psia}$
α_w	Moderator coefficient of reactivity	$\Delta k/k - ^\circ F$
β	Fraction of total fission neutrons in all delayed groups	
β_i	Fraction of total fission neutrons in the i^{th} delayed group	
δk	Total excess reactivity factor	$\Delta k/k$
δk_R	Available control rod worth	$\Delta k/k$
λ_i	Decay constant of i^{th} delayed neutron precursor group	sec^{-1}
μ	Fraction of total rod worth inserted	
	or	
	Viscosity of moderator-coolant at bulk temperature	$lb/hr-ft$
τ	General time constant, specified by associated subscripts	sec
τ_{DC}	Delay time through core for coolant	sec
τ_{DS}	Scram delay time	sec
τ_i	Time constant for i^{th} delayed neutron-precursor group, reciprocal of λ_i	sec
ϕ_f^*	Fast adjoint flux	$neutron/cm^2 \text{ sec}$
ψ_i	Power generator caused by i^{th} delayed-neutron precursor group	Btu/sec

Superscripts

o	Initial undisturbed steady state value
*	Transient response caused by application of disturbance
-	Spatial average

Subscripts

a	Value at $r = a$, edge of fuel region
c	Core
f	Fuel region
g	Air gap
hc	Hot channel
i	Inlet value
j	Denotes j^{th} section
j = 1	Section 1: Inside upper section
j = 2	Section 2: Inside lower section
j = 3	Section 3: Outside upper section
j = 4	Section 4: Outside lower section
o	Outlet value
ss	Stainless steel region
w	Coolant-moderator region

APPENDIX B

LOSS OF FLOW

DERIVATION OF THE MATHEMATICAL MODEL

This appendix presents a detailed discussion of the derivation of the mathematical model used for the loss of flow study. Since many of the results obtained here are applicable to other problems, this appendix has been used to present, in a formal manner, the thermal kinetic analysis of the fuel region based on a unit cell approach.

B.1 Derivation of Equations for Fuel Region

In the derivation of the fuel region thermal kinetic equations, the following assumptions were made:

- a. The radial component of the volumetric heat source is uniform. This assumption is justified by the fact that the reciprocal of the thermal diffusion length in uranium oxide multiplied by the fuel element radius is less than unity.
- b. The axial and radial components of both temperature and volumetric heat generation are separable and the axial components represented by average values over a given section. Because the axial fuel temperature gradient is very high relative to the axial water temperature gradient, the axial fuel temperature distribution can be assumed to be independent of the axial coolant temperature distribution during a transient.
- c. All physical constants for the fuel material remain constant during the transient.

The thermal diffusion equation for the fuel region can be written as

$$\frac{\partial^2 T_f(r,t)}{\partial r^2} + \frac{1}{r} \frac{\partial T_f(r,t)}{\partial r} + \frac{Q(t)}{k_f} = \frac{1}{\alpha_f} \frac{\partial T_f(r,t)}{\partial t} \quad (\text{B.1})$$

where $T_f(r,t)$ is the axial-average fuel temperature as a function of radius and time,

$Q(t)$ is the axial-average uniform-radial volumetric heat source in Btu/sec-ft³,

α_f is the thermal diffusivity of the fuel material in ft²/sec

and k_f is the fuel thermal conductivity in Btu/°F-ft²-sec.

The dependent variables in Eq. (B.1) are written as the sum of an initial steady state term, denoted by the superscript o, and a transient term, denoted by the superscript *, that is,

$$X(r,t) = X^o(r) + X^*(r,t)$$

where

$$X^*(r,t) = 0 \text{ for } t < 0$$

Thus, from Eq. (B.1)

$$\left[\frac{\partial^2 T_f^*(r,t)}{\partial r^2} + \frac{1}{r} \frac{\partial T_f^*(r,t)}{\partial r} + \frac{Q^*(t)}{k_f} \right] + \left[\frac{d^2 T_f^o(r)}{dr^2} + \frac{1}{r} \frac{dT_f^o(r)}{dr} + \frac{Q^o}{k_f} \right] = \frac{1}{\alpha_f} \frac{\partial T_f^*(r,t)}{\partial t} \quad (\text{B.2})$$

Since the coefficients of Eq. (B.2) are constant with time and since for time less than zero all transient terms are zero, Eq. (B.2) can be separated into the two relationships

$$\frac{d^2 T_f^o(r)}{dr^2} + \frac{1}{r} \frac{dT_f^o(r)}{dr} + \frac{Q^o}{k_f} = 0 \quad (B.3a)$$

and

$$\frac{\partial^2 T_f^*(r,t)}{\partial r^2} + \frac{1}{r} \frac{\partial T_f^*(r,t)}{\partial r} + \frac{Q^*(t)}{k_f} = \frac{1}{\lambda_f} \frac{\partial T_f^*(r,t)}{\partial t} \quad (B.3b)$$

Boundary conditions for Eq. (B.3a) are

$$T_f^o(r) = T_{fa}^o, \quad r = a$$

and

$$\left. \frac{dT_f^o(r)}{dr} \right|_{r=0} = 0.$$

Boundary conditions for Eq. (B.3b) are

$$T_f^*(r,t) = T_{fa}^*(t), \quad r = a$$

and

$$\left. \frac{\partial T_f^*(r,t)}{\partial r} \right|_{r=0} = 0$$

the initial condition for Eq. (B.3b) is $T_f^*(r,t) = 0$ for $t = 0$.

The solution to Eq. (B.3a) is straightforward and can be found in many references⁵. Equation (B.3b) can be solved in the following manner. The Laplace transform with respect to time can be written as

$$\frac{\partial^2 T_f^*(r,s)}{\partial r^2} + \frac{1}{r} \frac{\partial T_f^*(r,s)}{\partial r} - \frac{s}{\lambda_f} T_f^*(r,s) = - \frac{Q^*(s)}{k_f} \quad (B.4)$$

5. Glasstone, S., "Principles of Nuclear Reactor Engineering" New York, Van Nostrand, 1955. Sect. 11.86.

Solution to the homogeneous equation can be obtained by a change in independent variable

$$\chi = \sqrt{\frac{s}{\lambda_f}} r \quad (B.5)$$

the homogeneous equation has the form of a modified Bessel equation of order zero⁶, thus

$$T_f^*(r,s) = AI_0\left(\sqrt{\frac{s}{\lambda_f}} r\right) + BK_0\left(\sqrt{\frac{s}{\lambda_f}} r\right) + \frac{Q_c^*(s)}{(\rho c_p)_f s} \quad (B.6)$$

where ρ_f is the density of fuel material in lbs/ft³ and c_{pf} is the heat capacity in Btu/lb-°F. The constants A and B can be evaluated from the Laplace transform of the boundary conditions for Eq. (B.3b),

$$T_f^*(r,s) = \frac{I_0\left(\sqrt{\frac{s}{\lambda_f}} r\right)}{I_0\left(\sqrt{\frac{s}{\lambda_f}} a\right)} T_{fa}^*(s) + \left[\frac{I_0\left(\sqrt{\frac{s}{\lambda_f}} r\right)}{I_0\left(\sqrt{\frac{s}{\lambda_f}} a\right)} + 1 \right] \frac{q_c^*(s)}{(mc_p)_f s} \quad (B.7)$$

The volumetric heat generation has been changed to a total heat generation for the unit cell by use of the relationships

$$q_c^*(s) = V_f Q^*(s)$$

and

$$m_f = V_f \rho_f.$$

6. Hildebrand, F. B., "Advanced Calculus for Engineers" - New York, Prentice Hall, 1948 - Sect. 4.8.

Equation (B.7) is the transformed transient fuel temperature as a function of the radial variable. However, the temperature of interest is the average fuel temperature change which when multiplied by the Doppler Coefficient of reactivity yields the change in reactivity resulting from a change in fuel temperature. Here the coefficient of reactivity has been assumed to be that which pertains to the unit cell or the section in question and consequently the reactivity change is that attributable to the same volume. In order to determine the rate of heat flow from the fuel into the air gap, the gradient of the fuel temperature at the fuel region boundary is also required.

The transformed average fuel temperature is given by the relationship

$$\bar{T}_f^*(s) = \frac{\int_0^a 2 \pi r T_f^*(r,s) dr}{\pi a^2} \quad (B.8)$$

Substitution of Eq. (B.7) into (B.8) with subsequent integration yields

$$\bar{T}_f^*(s) = \frac{2 I_1 \left(a \sqrt{\frac{s}{\lambda_f}} \right)}{\left(a \sqrt{\frac{s}{\lambda_f}} \right) I_0 \left(a \sqrt{\frac{s}{\lambda_f}} \right)} T_{fa}^*(s) + \left[1 - \frac{2 I_1 \left(a \sqrt{\frac{s}{\lambda_f}} \right)}{\left(a \sqrt{\frac{s}{\lambda_f}} \right) I_0 \left(a \sqrt{\frac{s}{\lambda_f}} \right)} \right] \frac{q_c^*(s)}{(mc_p)_f s} \quad (B.9)$$

The rate of heat flow from the fuel region into the gap is given by the relationship

$$k_f A_f \left. \frac{\partial T_f^*(r,s)}{\partial r} \right|_{r=a} = k_g A_g \left. \frac{\partial T_g^*(r,s)}{\partial r} \right|_{r=a} \quad (B.10)$$

where $T_g^*(r,s)$ is the transformed radial temperature distribution in the gap.

If the gap material is assumed to have negligible heat capacity, the

instantaneous rate of heat flow into the gap is equal to the rate at which heat is removed from the gap. This relationship can be expressed as

$$-k_g A_g \left. \frac{\partial T_g^*(r,s)}{\partial r} \right|_{r=a} = (UA)_g \left[T_{fa}^*(s) - \bar{T}_{ss}(s) \right] \quad (B.11)$$

where k_g is the thermal conductivity of the gap in Btu/sec-ft-°F,
 A_g is the area of the fuel surface in ft²,
 $(UA)_g$ is the overall heat transfer conductivity from fuel interface
to average stainless steel temperature in Btu/sec-°F,
and $\bar{T}_{ss}(s)$ is the transformed average stainless steel temperature in °F.

The radial component of heat flow at the surface of the fuel region is, from Eq. (B.7)

$$k_f A_f \left. \frac{\partial T_f^*(r,s)}{\partial r} \right|_{r=a} = \frac{I_1 \left(a \sqrt{\frac{s}{\alpha_f}} \right)}{I_0 \sqrt{\frac{s}{\alpha_f}}} \left[T_{fa}^*(s) - \frac{q_c^*(s)}{(mc_p)_f s} \right] \quad (B.12)$$

Combination of Eqs. (B.10) through (B.12) results in

$$\begin{aligned} T_{fa}^*(s) = & \frac{2 I_1 \left(a \sqrt{\frac{s}{\alpha_f}} \right)}{(UA)_g \left(a \sqrt{\frac{s}{\alpha_f}} \right) I_0 \left(a \sqrt{\frac{s}{\alpha_f}} \right) + 2 (mc_p)_f s I_1 \left(a \sqrt{\frac{s}{\alpha_f}} \right)} q_c^*(s) \\ & + \frac{(UA)_g \left(a \sqrt{\frac{s}{\alpha_f}} \right) I_0 \left(a \sqrt{\frac{s}{\alpha_f}} \right)}{(UA)_g \left(a \sqrt{\frac{s}{\alpha_f}} \right) I_0 \left(a \sqrt{\frac{s}{\alpha_f}} \right) + 2 (mc_p)_f s I_1 \left(a \sqrt{\frac{s}{\alpha_f}} \right)} \bar{T}_{ss}^*(s) \end{aligned} \quad (B.13)$$

The transformed average fuel temperature, as given by the heat generation rate and the average stainless temperature, is obtained from the simultaneous solution of Eqs. (B.9) and (B.13). In order to incorporate the solution for the average fuel temperature into the remainder of the mathematical description for the system the time domain solution is required. This means that the inverse transforms of the relationships given by these two equations are needed. Conventional analog computer techniques are such that the only transformed functions which can be utilized directly are those describable by integral powers of the operator s and the delay function e^{-as} . An approximation method exists which when applied to functions of s of the form given in Eqs. (B.9) and (B.13) yields rational polynomials of a with the delay function as a multiplier, if applicable. The irrational functions in Eq. (B.9) and the first moment approximations are as follows:

If

$$H_1(s) = \frac{2 I_1 \left(a \sqrt{\frac{s}{\lambda_f}} \right)}{\left(a \sqrt{\frac{s}{\lambda_f}} \right) I_0 \left(a \sqrt{\frac{s}{\lambda_f}} \right)}, \quad (B.14)$$

the first moment approximation is

$$\left[H_1(s) \right]^{al} = \frac{1}{\frac{a^2}{8\lambda_f} s + 1}. \quad (B.15)$$

If

$$H_2(s) = \frac{1}{(mc_p)_f} \left[1 - \frac{2 I_1 \left(a \sqrt{\frac{s}{\lambda_f}} \right)}{\left(a \sqrt{\frac{s}{\lambda_f}} \right) I_0 \left(a \sqrt{\frac{s}{\lambda_f}} \right)} \right], \quad (B.16)$$

the first moment approximation is

$$\left[H_2(s) \right]^{al} = \frac{a^2 \rho_f}{8 m_f k_f} \frac{1}{\frac{a^2}{6 \lambda_f} s + 1} \quad (B.17)$$

Therefore, the approximation equation for the average fuel temperature in the j^{th} section can be written in the form

$$\bar{T}_{fj}^*(s) = \frac{1}{\frac{1}{2} \tau_f s + 1} T_{faj}^*(s) + \frac{\tau_f}{2 (mc_p)_{fj}} \frac{1}{\frac{2}{3} \tau_f s + 1} q_{cj}^*(s), \quad (B.18)$$

where

$$\tau_f = \left(\frac{a}{2} \right)^2 \left(\frac{pc_p}{k} \right)_f$$

In this equation $q_{cj}^*(s)$ is the transformed power generated in the j^{th} section.

In the time domain the power generation in the j^{th} section can be related to the total power generation rate in the core by the expression

$$q_{cj}^*(t) = W_{qj} \left(\frac{z}{l} \right) q_c^*(t)$$

where $W_{qj} \left(\frac{z}{l} \right)$ is the power statistical weight as a function of the section and the control rod position, for uniform rod motion.

The moment approximations for the fuel boundary equation are as follows.

If

$$H_3(s) = \frac{2 I_1 \left(a \sqrt{\frac{s}{\lambda_f}} \right)}{(UA)_g \left(a \sqrt{\frac{s}{\lambda_f}} \right) I_0 \left(a \sqrt{\frac{s}{\lambda_f}} \right) + 2 (mc_p)_f s I_1 \left(a \sqrt{\frac{s}{\lambda_f}} \right)}, \quad (B.20)$$

the first moment approximation is

$$\left[H_3(s) \right]^{a1} = \frac{1}{(UA)_g} \frac{1}{\left[\frac{(mc_p)_f}{(UA)_g} + \frac{1}{2} \frac{a^2}{2} \frac{pc_p}{k} \right]_f s + 1} \quad (B.21)$$

For the function

$$H_4(s) = \frac{(UA)_g a \sqrt{\frac{s}{\lambda_f}} I_0 a \sqrt{\frac{s}{\lambda_f}}}{(UA)_g a \sqrt{\frac{s}{\lambda_f}} I_0 a \sqrt{\frac{s}{\lambda_f}} + 2 (mc_p)_f s I_1 a \sqrt{\frac{s}{\lambda_f}}}, \quad (B.22)$$

the second moment approximation was employed since the first moment approximation omits the fact that there is no energy storage in the gap and thus, a change in the edge temperature of the stainless steel would immediately produce a change in the edge of fuel temperature. The energy storage in the stainless steel region has been incorporated by the use of a lumped parameter approximation as shown in the next section,

$$\left[H_4(s) \right]^{a2} = \frac{\frac{1}{2} \left(\frac{a}{2} \right)^2 \left(\frac{pc_p}{k} \right)_f s + 1}{\left[\frac{(mc_p)_f}{(UA)_g} + \frac{1}{2} \left(\frac{a}{2} \right)^2 \left(\frac{pc_p}{k} \right)_f \right] s + 1} \quad (B.23)$$

The fuel temperature at the boundary of the j^{th} section can thus be approximated by the equation

$$T_{faj}^*(s) = \frac{1}{(UA)_g} \frac{1}{\left(\frac{1}{2} \tau_f + \tau_a \right) s + 1} q_{cj}^*(s) + \frac{\frac{1}{2} \tau_f s + 1}{\left(\frac{1}{2} \tau_f + \tau_a \right) s + 1} \bar{T}_{ssj}^*, \quad (B.24)$$

where

$$a = \frac{(mc_p)_f}{(UA)_g}.$$

B.2 Derivation of Equations for the Clad and the Coolant-Moderator Regions

In this section, the assumption has been made that the spatial effects within the stainless steel region can be omitted since the temperature gradient is approximately linear. Spatial effects in the coolant-moderator boundary region adjacent to the stainless steel region are included to a first approximation by use of the Dittus-Boelter correlation for the film heat transfer coefficient. Thermal spatial effects within the coolant-moderator region were incorporated approximately by the definition of a bulk water temperature. The bulk water temperature as used with the preceding heat transfer correlation is given as the momentum heat capacity weighted fluid temperature averaged for the flow cross sectional area. The rate of change of heat storage in the stainless steel region is equal to the net rate of heat flow to the region.

Thus

$$(mc_p)_{ss} \frac{d \bar{T}_{ss}}{dt} = (UA)_g (T_{fa} - \bar{T}_{ss}) - (UA)_w (\bar{T}_{ss} - \bar{T}_w) \quad (B.26)$$

where $(mc_p)_{ss}$ is the heat storage capacity of the clad region,
 $(UA)_w$ is the overall heat transfer conductivity between the average stainless steel temperature and the bulk water temperature,
and $\bar{T}_w$ is the bulk water temperature in the moderator-coolant region.

The heat balance on the moderator coolant region is written as

$$(mc_p)_w \frac{d \bar{T}_w}{dt} = (UA)_w (\bar{T}_{ss} - \bar{T}_w) - (wc_p)_w (T_{wo} - T_{wi}) \quad (B.27)$$

where the coolant outlet temperature is given by the relationship

$$T_{wo}(t) = 2 \bar{T}_w(t - \frac{\tau_{DC}}{2}) - T_{wi}(t - \tau_{DC}), \quad (B.28)$$

where $(mc_p)_w$ is the heat storage capacity of the moderator-coolant region in Btu/°F,

$(wc_p)_w$ is the rate of flow of heat capacity in Btu/°F-sec.,

T_{wi} is the change in coolant inlet temperature in °F,

T_{wo} is the change in coolant outlet temperature in °F, and

τ_{DC} is the transport time in seconds through the coolant moderator region.

In addition,

$$\frac{\tau_{DC}}{2} \frac{d}{dt} \bar{T}_w(t - \frac{\tau_{DC}}{2}) + \bar{T}_w(t - \frac{\tau_{DC}}{2}) = \bar{T}_w(t) \quad (B.29a)$$

and

$$\tau_{DC} \frac{d}{dt} T_{wi}(t - \tau_{DC}) + T_{wi}(t - \tau_{DC}) = T_{wi}(t). \quad (B.29b)$$

B.3 Neutron Kinetic Equations

The relationship which specifies the time behavior of the average thermal neutron concentration, n , in the core as a function of the reactivity factor δk and the effective neutron lifetime is commonly written as

$$\frac{dn}{dt} = \frac{\delta k - \beta}{l} n + \sum_{i=1}^N \lambda_i C_i \quad (B.30)$$

In this equation, C_i is the concentration of the delayed neutrons emitted by the i^{th} group. The total fraction of delayed neutrons is β , the fraction of the delayed neutrons from the i^{th} group is β_i and the decay

constant for the i^{th} group is λ_i . The time behavior of the concentration for the i^{th} group of delayed neutron emitters is given by the relationship

$$\frac{dC_i}{dt} = \frac{\beta_i}{\ell} n - \lambda_i C_i \quad i = 1, 2, \dots, N \quad (\text{B.31})$$

The number N of delayed neutron emitter groups at the present time is usually assumed to be 6, with the exclusion of the photoneutron groups. For the purposes of some kinetic studies, groups can be combined to yield an approximate representation with a lesser number of groups. The parameter values for the approximation employed in this study are given in Appendix C.

If the dependent variables in equations (B.30) and (B.31) are expressed in terms of initial steady state values and transient values they can be written as

$$\frac{dn^*}{dt} = \frac{\delta k}{\ell} (n^0 + n^*) - \frac{\beta}{\ell} n^* + \sum_{i=1}^N \lambda_i C_i^* \quad (\text{B.32})$$

and

$$\frac{dC_i^*}{dt} = \frac{\beta_i}{\ell} n^* - \lambda_i C_{i0}^* \quad (\text{B.33})$$

The average thermal neutron concentration can be related to total core power generation by the expression

$$q_c \text{ [Btu/sec]} = \frac{3.1 \times 10^{-11} \left[\frac{\text{watts}}{\text{fission/sec}} \right]}{1.055 \times 10^3 \left[\frac{\text{watts}}{\text{Btu/sec}} \right]} V_c \Sigma_f v n \left[\frac{\text{fissions}}{\text{sec}} \right],$$

where V_c is the core volume for which n applies, v is the average neutron velocity, and Σ_f is the macroscopic fission cross section.

A delayed power generation is defined as

$$\psi_i \left[\frac{\text{Btu}}{\text{sec}} \right] = \frac{3.1 \times 10^{-11} \left[\frac{\text{watts}}{\text{fission/sec}} \right]}{1.055 \times 10^3 \left[\frac{\text{watts}}{\text{Btu/sec}} \right]} V_c \Sigma_f v C_i \left[\frac{\text{fission}}{\text{sec}} \right].$$

The neutron kinetic equations can therefore be written in terms of total core power generation as

$$\frac{d q_c^*}{dt} = \frac{\delta k}{\ell} (q_c^0 + q_c^*) - \frac{\beta}{\ell} q_c^{v*} + \sum_{i=1}^N \lambda_i \psi_i^* \quad (\text{B.34})$$

and

$$\frac{d \psi_i^*}{dt} = \frac{\beta_i}{\ell} q_c^* - \lambda_i \psi_i^*. \quad (\text{B.35})$$

APPENDIX C

LOSS OF FLOW

FUNCTIONS AND CONSTANTS FOR SYSTEM MODEL

Before analog computations could be performed for the Yankee system, the functions and constants associated with the mathematical formulation were evaluated. The purpose of this appendix is to present the constants and functional dependencies and also to specify the particular computational form employed. Within this appendix, the functions and constants are discussed in two groups: those which affect the neutron kinetic equations and those which affect the thermal kinetics. The reactor physics functions and constants were assumed to be the same for the 392 and 482 MW cores.

C.1 Reactor Physics Functions and Constants

C.1.1 Constants and Sources

C.1.1.1 Doppler Coefficient of Reactivity

Value:

$$\alpha_f = -8.5 \times 10^{-6} \frac{\Delta k}{k - \beta_F}$$

Source: At the time of this study, this was the "best" value for the hot Yankee core. However, an evaluation (confirmed after this study was completed) by Davis⁷ of Hanford indicates that the coefficient may be two or three times the magnitude used. The incorporation of the higher value in the study would not significantly affect the results because of the large thermal inertia of the fuel.

7. Davis, M. V., J. Appl. Phys., 2, 250 (1957).

C.11.2 Moderator Coefficient of Reactivity

Value:

$$\alpha_w = -2.7 \times 10^{-4} \frac{\Delta k}{k \text{ } ^\circ\text{F}}$$

Source: The value was obtained for the Yankee core at operating temperature with no boron or control rods.

C.11.3 Delayed Neutron Fractions and Time Constants

The data for the weighted fractions for the Beginning-of-life Yankee core (as given below) were used as a basis for the three group constants.

TABLE C.1

Delayed Neutron Constants⁸

Group	β_i	τ_i (sec)
1	0.000240	77.2
2	0.001412	31.6
3	0.001288	7.79
4	0.002826	3.03
5	0.000993	0.707
6	0.000231	0.274

$$\sum \beta_i = 0.00699$$

The six groups given in Table C.1 were combined in various ways into three groups, e.g., group I from groups 1, 2; group II from groups 3, 4 and group III from groups 5, 6. The response of the three group approximations to a step was compared to the response of six groups. The three group combination which best approximated the response of six groups over the time range

8. Keepin, G. R., et. al. Delayed Neutrons from Fissionable Isotopes of Uranium, Plutonium and Thorium, The Physical Review 4, 107 (1957).

from 0 to 20 seconds was then used as a model for the delayed neutron behavior. An investigation showed that a combination of 1, 2; 3, 4; and 5, 6 gave the best approximation. Thus, the values used were:

Group	β_i	τ_i (sec)
I (1,2)	0.001224	38.2
II (3,4)	0.004114	4.52
III (5,6)	0.001652	0.625
<hr/>		
$\Sigma \beta_i = 0.00699$		

Evaluation of the analog computer results indicated that the three group approximation was more than adequate.

C.11.4 Neutron Lifetime

Value:

$$\ell = 2.5 \times 10^{-5} \text{ sec.}$$

Source: Perturbation theory⁹ was employed to evaluate the neutron lifetime.

C.11.5 Available Control Rod Worth

Value:

$$\delta k_R (\text{max}) = 0.10 \Delta k/k$$

Source: The control rods were assumed to have a worth of 0.161 $\Delta k/k$, which is a value between black rods and black-plus-resonance rods. Of this total worth, 0.061 $\Delta k/k$ was required to hold down the hot, equilibrium Xe and Sm poisoned, beginning-of-life Yankee Reactor core. Thus, 0.10 $\Delta k/k$ was available for control. In the case of loss of flow, this control was in

9. Benedict, M. et al 1955 Nuclear Congress (Preprint 154).

the form of a scram.

The total available control rod worth was combined with the functional relation between fractional rod worth and time after scram initiation to obtain the $\Delta k/k$ inserted due to rod movement. Figure C.1 shows μ as a function of time after scram initiation.

C.12 Functions and Sources

C.12.1 Control Rod Drop

Functions:

$$\text{Rod movement} = 9 t_s^2$$

Source: The relation between the distance a rod moves and the time of insertion from initiation was evaluated by a method suggested by A. Bishop of WAPD. The method which employs an equivalent "g" acceleration, derives from a functional relationship which included rod weight, friction force, buoyancy, pressure drag and skin drag for the hot core with 100% flow.

- (a) free fall distance = 7 ft,
- (b) control rod effective volume = 0.802 ft^3 ,
- (c) rod weight = 400 lbs, and
- (d) friction force = 35 lbs.

The above numbers were chosen as representative of the parameter values during the control rod drop of a Yankee rod in final form. When these assumptions were coupled with standard analytical expressions for drag, pressure and buoyancy forces the functional relationship was obtained.

Figure C.2 shows the comparison between free fall and resisted fall. Since flow decayed and the water density decreased during the transient, the actual full rod insertion time was faster than predicted even though the side thrust due to flow redistribution was not considered. Hence, the negative

Figure C.1 Fraction of rod worth inserted as a function of time after initiation of scram.

Figure C.2

Amount of rod travel as a function of time after initiation of scram.

reactivity inserted because of a movement of the control rods occurred at a rate faster than that indicated by the functional relationship.

C.12.2 Fractional Control Rod Worth Inserted

Functional Dependence: Figure C.3 shows the fraction of total rod worth inserted (μ) as a function of the position of the rods within the core.

Source: Perturbation theory gave the results presented. For the analog solution, a function of time rather than position was desired for the fractional control rod worth inserted, μ . By combining the rod drop equation and the functional relation between μ and position, the desired relationship was obtained as presented in Figure C.1.

C.12.3 Statistical Weights

Functional Dependence: Table C.1 shows the statistical weights, W_{pj} , for the four region core as a function of control rod position.

Source: The statistical weights, W_{pj} , for the four region core were obtained for a radially uniform core with rods inserted $1/6$, $1/3$, $1/2$ and $2/3$ of the core length by the evaluation of

$$W_{pj} = \frac{\int_{\text{core}} V_i \phi_f^* S dV}{\int_{\text{core}} \phi_f^* S dV} \quad (C-1)$$

The axial calculations used WANDA¹⁰ ϕ_f^* and S while the radial part employed hand calculations with the assumption of the eigenfunction shape for a bare core.

10. WANDA - "A One-Dimensional Few Group Diffusion Equation Code for the IBM-704", WAPD-TM-28, November 1956.

Figure C.3 Fraction of total rod worth inserted, μ , as a function of the fraction of total length of rod inserted for core poisoned uniformly in the radial direction.

For the assumption that the rods were at the top of the core before initiation of scram, the statistical weights as a function of time after scram were obtained by combination of rod drop equation with statistical weight relation.

For the analog simulation, a function of time was again needed rather than a function of space. A combination of the relation between statistical weight and rod position with the rod drop equation gave a function of time. In Figure C.4 $W_{p2}/W_{p2}^0 = \mathcal{H}$ is plotted as a function of time. Since

$$\frac{W_{p2}}{W_{p2}^0} = \frac{W_{p4}}{W_{p4}^0}, \quad \frac{W_{p1}}{W_{p1}^0} = \frac{W_{p3}}{W_{p3}^0}, \quad \text{and } W_{p1} = 2 W_{p2}^0 - W_{p2}, \quad \text{all } W_{pi}'\text{'s were defined in}$$

terms of $\mathcal{H}$ and the initial values of the section statistical weights, W_{pj}^0 .

These functional relationships were used in the evaluation of the relative importance of the temperature coefficients of reactivity as the position of the control rods changed.

Figure C.4

Normalized statistical weight function of section 2, η , as a function of time after initiation of scram.

TABLE C.2

Statistical Weights ($W_{\rho i}$) and Power Fractions ($W_{q i}$) for
a Four Section Core Poisoned Uniformly in the Radial
Direction with Various Axial Control Rod Positions

Axial Control Rod Position (z/L)	Section 1		Section 2		Section 3		Section 4	
	W_{ρ}	W_q	W_{ρ}	W_q	W_{ρ}	W_q	W_{ρ}	W_q
0	.413	.334	.413	.334	.087	.166	.087	.166
1/6	.329	.284	.496	.384	.070	.141	.105	.191
1/3	.173	.189	.652	.479	.037	.094	.138	.238
1/2	.038	.088	.787	.580	.008	.044	.167	.288
2/3	.012	.059	.813	.609	.003	.029	.172	.303

C.2 Heat Transfer Functions and Constants

C.2.1 Constants and Sources

C.2.1.1 Unit Cell Dimensions

Values:	Cores	
	<u>482 MW</u>	<u>392 MW</u> ¹¹
Pellet Diameter (in)	0.262	0.290
Outside Diameter (in)	0.305	0.337
Tube Wall Thickness (in)	0.019	0.021
Active Fuel Length (ft)	8.5	7.5
Number of Rods	28,842	23,218
Rod Lattice (in)	0.380	0.425

Source: The mechanical design numbers above were evaluated from the core configurations presently envisioned.

C.2.1.2 Masses of Reactor Core Constituents

Values:	Cores	
	<u>482 MW</u>	<u>392 MW</u>
Fuel (lb)	57,700	50,100
Stainless Steel (lb)	14,570	12,630
Water (lb)	5,670	5,080

Source: The numbers above were calculated for the cores with unit cell dimensions as given in C.2.1.1¹². The water mass tabulated above is the active heat transfer mass which is 90% of the total core water. All core

11. Yankee Atomic Electric Company, Research and Development Program, Quarterly Progress Report, September 30, 1957, YAE-44.

12. Ibid.

water is not available for heat transfer because part of the water is adjacent to areas that are not producing heat, e.g. control rods.

C.21.3 Specific Heat

Values:	Cores	
	<u>482 MW</u>	<u>392 MW</u>
Fuel (Btu/lb-°F)	0.0738	0.0730
Stainless Steel (Btu/lb-°F)	0.15	0.15
Water (Btu/lb-°F)	1.18	1.18

Source: The specific heat of UO_2 was evaluated using the equation given in The Chemistry of Uranium¹³. The evaluation was made at the average fuel temperature. The stainless steel specific heat used was taken from commercial design data for stainless steel in the temperature range of 600-700°F. The specific heats of water were obtained from data supplied by E. McCabe of WAPD and correspond to the average core water temperature.

C.21.4 Thermal Conductivity

Values:	<u>482 MW and 392 MW Cores</u>
Fuel (Btu/hr-ft-°F)	0.90
Stainless Steel (Btu/hr-ft-°F)	11.1
Gap (Btu/hr-ft-°F)	0.16

Source: The above values are those used in the steady state thermal design of the Yankee Reactor by the WAPD Reactor Engineering Department. The fuel conductivity was computed by A. Bournia from data obtained in in-pile tests at NRX. The conductivity value obtained from the data

13. Katz, J. J. and Robinowitch, E. "The Chemistry of Uranium" (1951).

corresponds to the radial average fuel temperature at the core geometrical center of both Yankee reactor designs. The stainless steel conductivity corresponds to a temperature range of 600-700°F while that for the gap (He) corresponds to 900°F. Since the average fuel temperatures in the four sections were much lower than the temperature at which the conductivity of UO_2 was evaluated, the effect of a change in the value was examined.

For the center two sections which comprised 45% of the core volume, a value of 1.26 Btu/hr-ft-°F was used while the value in the outer sections was 2.28 Btu/hr-ft-°F. The values used were obtained by interpretation and analysis of the data given by Kingery¹⁴. The assumption was made that the functional dependence of conductivity on temperature was the same as that given by Kingery but with a constant of proportionality which "forced" the value to 0.9 Btu/hr-ft-°F at 2800°F.

C.21.5 Overall Heat Transfer Conductivity from Fuel Surface to Average Stainless Steel

Values:

	Cores	
	<u>482 MW</u>	<u>392 MW</u>
(UA) _g (Btu/hr-°F)	12.17 x 10 ⁶	9.50 x 10 ⁶

Sources: The heat transfer conductivity from the fuel edge to the average stainless steel used in the study was evaluated from steady state heat transfer considerations. The assumption was made that the combination of half of the clad resistance and the gap resistance were the components in the determination of the conductivity.

¹⁴. Kingery, J. R. J. of Am. Cer. Society 37, 108 (1954).

C.22 Functions and Sources

C.22.1 Power Fractions

Functional Dependence: Table C.1 shows the power fractions, W_{qj} , for the four section radially uniform core as a function of control rod position.

Source: The power fractions W_{qj} , for the four section core were evaluated by techniques similar to those employed in the evaluation of the statistical weights. The power fractions were obtained from

$$W_{qj} = \frac{\int_{\text{core}} V_i S dV}{\int_{\text{core}} S dV} \quad (C-2)$$

for various control rod positions. These power fractions determined the percentage of the total power generated within each section. The amount of analog equipment available was limited and thus certain approximations were necessary in order to accomplish the solution of the problem. One such assumption was that

$$\frac{W_{q2}}{W_{q2}^0} = \frac{W_{p2}}{W_{p2}^0}.$$

The above assumption was justifiable since the error introduced was small (the order of 3%) and thus did not distort the answers. Thus, since

$$\frac{W_{q2}}{W_{q2}^0} = \frac{W_{q4}}{W_{q4}^0}, \quad \frac{W_{q1}}{W_{q1}^0} = \frac{W_{q3}}{W_{q3}^0}, \quad \text{and } W_{q1} = 2 W_{q2}^0 - W_{q2}$$

the simulation was carried out using $\mathcal{N}$ which was generated in the determination of the statistical weights.

C.22.2 Mass Flow Rate

Functional Dependence: Figure C.5 shows the normalized flow rate, W/W^0 , as a function of time for the loss of four pumps with various amounts of added inertia, and the loss of two pumps with no added inertia.

Source: The equations used to determine the pump coastdown curves shown in Figure C.5 were developed by W. Lyman of WAPD and are contingent upon the following assumptions.

1. The rate of change of stored energy of the primary coolant and pump rotor is equal to the rate of energy dissipation in the various parts of the system.
2. The rate of energy dissipation because of piping friction and pump impeller losses is proportional to the cube of the coolant velocity.
3. The rate of energy dissipation because of pump rotor "windage" losses is proportional to the square of the pump rotor speed.
4. The speed of the pump rotor is proportional to the velocity of the coolant at all times during flow coastdown, and the kinetic energies of the coolant and pump rotor can be added.

Based upon these assumptions, the equation for the normalized flow is

$$\frac{W}{W^0} = \frac{b/A_1}{\left(\frac{A_1 + b}{A_1}\right) \left(\exp\left(\frac{b P_0 t}{2E_0}\right) - 1\right)} \quad (C-3)$$

where $A_1 = 1 + a$
 $a =$ Impeller loss coefficient
 $b =$ Rotor windage loss coefficient
 $E_0 =$ Total initial kinetic energy
and $P_0 =$ Total initial pumping power.

Figure C.5

Normalized primary coolant flow rate, W/W° , as a function of time after initiation of a four pump failure with various amounts of added energy, E_A .

For the Yankee system, the following constants were used

$$\begin{aligned} A_1 &= 1.177 \\ a &= 0.177 \\ b &= 0.09 \\ E_o &= 1.507 \times 10^6 \text{ ft-lb} \\ P_o &= 2.475 \times 10^6 \text{ ft-lb/sec.} \end{aligned}$$

C.22.3 Heat Transfer Conductivity from Average Stainless Steel to Average Water

Functional Dependence: The Dittus-Boelter heat transfer correlation was used in the evaluation of the film coefficient as a function of mass flow rate. The film resistance and half of the clad resistance were combined to form a time variant average stainless steel to average water heat transfer conductivity shown as a normalized function in Figure C.6.

Source: The methods used were the same as those employed in the steady state design of the Yankee Power Reactor. The effects of nucleate and film boiling were neglected in the determination of the heat transfer conductivity. Since uncertainties are involved in the evaluation of boiling heat transfer coefficients, the film heat transfer coefficients were conservatively evaluated from forced convection considerations.

The conductivity, $(UA)_w$, was determined by a combination of half the clad resistance and the film resistance.

$$(UA)_w = \frac{2 \pi L}{\left(\frac{.5}{k_s} \ln \frac{c}{b} + \frac{1}{hc} \right)} \left(\frac{\text{Btu}}{\text{hr}^\circ\text{F} - \text{unit cell}} \right) \quad (\text{C-4})$$

Figure C.6 Normalized heat transfer conductivity between stainless steel average temperature and primary coolant bulk water temperature, $(UA)_w / (UA)_w^0$, as a function of normalized flow rate, W/W^0 .

The film heat transfer coefficient was determined from

$$h = \frac{.023}{De} \left(\frac{De \cdot G}{\mu} \right)^{0.8} \left(\frac{C_p \mu}{k} \right)^{.4} \quad (\text{Dittus-Boelter}). \quad (\text{C-5})$$

Since the mass flow rate was time variant, the film coefficient was also a variable with time. The physical constants associated with the film coefficient correlation would also vary with time since the system temperatures were changing. However, all physical properties were assumed constant at values corresponding to a water temperature of 516°F and the pressure of 2000 psia. Combination of the relationships given above lead to equation C-4.

C.22.4 Time Delay Through Core

Functional Dependence: Figure C.7 shows the time variable delay associated with the flow of coolant through the core.

Source: The time delay through the core at any instant can be determined from the integral expression.

$$L = \int_t^{t + \tau_D(t)} v(t') dt' \quad (\text{C-6})$$

Figure C.7 Reciprocal of time delay through core, $1/\tau_{DC}$, as a function of time after initiation of pump failure through core for no added energy.

APPENDIX D

PRESENTATION OF COMPUTER DATA

The following tables present typical coefficient values and initial conditions that were used for the complete loss of flow computer study. The coefficient values are classified according to whether they occur in the thermal kinetics (D.1) or neutron kinetics equations (D.2) as given in Section 2. The initial conditions for each section are tabulated for steady state 100% power (D.3).

D.1 Thermal Kinetics

TABLE D.1 Coefficient Values for Thermal Kinetics

Parameter	Units	Section	Equation	482 MW Core	392 MW Core
τ_f	sec	-	1, 2	5.464	6.77
$(mc_p)_{fj}$	Btu/°F	1	1	948	832
		2		948	832
		3		1158	1017
		4		1158	1017
$(UA)_{gj}$	Btu/sec -°F	1	2, 3	760	594
		2		760	594
		3		929	726
		4		929	726
τ_a	sec	-	2	1.247	1.403
$(mc_p)_{ssj}$	Btu/°F	1	3	492	426
		2		492	426
		3		601	521
		4		601	521

Parameter	Units	Section	Equation	482 MW Core	392 MW Core
$(UA)_{wj}^{\circ}$	Btu/sec- $^{\circ}$ F	1	3, 4	5.108×10^3	3.706×10^3
		2		5.108×10^3	3.706×10^3
		3		6.243×10^3	4.53×10^3
		4		6.243×10^3	4.53×10^3
$(mc_p)_{wj}$	Btu/ $^{\circ}$ F	1	4	1505	1348
		2		1505	1348
		3		1840	1648
		4		1840	1648
$(wc_p)_{wj}^{\circ}$	Btu/sec- $^{\circ}$ F	1	4	5.01×10^3	5.01×10^3
		2		5.01×10^3	5.01×10^3
		3		6.13×10^3	6.13×10^3
		4		6.13×10^3	6.13×10^3
τ_{DCj}	sec	1	5	0.43	0.43
		2		0.43	0.43
		3		0.43	0.43
		4		0.43	0.43

D.2 Neutron Kinetics

TABLE D.2 Coefficient Values for Neutron Kinetics

Parameter	Units	Section	Equation	392 and 482 MW Cores
ℓ	sec	-	6, 7	2.5×10^{-5}
β	-	-	6	0.0699
β_{12}	-	-	7	0.01224
β_{34}	-	-	7	0.04115
β_{56}	-	-	7	0.01651
λ_{12}	sec ⁻¹	-	6, 7	1.60
λ_{34}	sec ⁻¹	-	6, 7	0.221
λ_{56}	sec ⁻¹	-	6, 7	0.0262
w_{qj}^o	-	1	8	0.334
		2		0.334
		3		0.166
		4		0.166
w_{pj}^o	-	1	10	0.413
		2		0.413
		3		0.087
		4		0.087
λ_f	$\Delta k/k - F$		10	-8.5×10^{-6}
λ_w	$\Delta k/k - F$		10	-2.7×10^{-4}

D.3 Initial Conditions

TABLE D.3 Initial Conditions for Variables

Variable	Section							
	1		2		3		4	
	482 MW	392 MW	482 MW	392 MW	482 MW	392 MW	482 MW	392 MW
$\bar{T}_{fj}$	1217	1288	1186.5	1263.2	788.7	819.6	776.3	809.5
T_{aj}	771.9	779	741.4	754.2	607.9	613.1	595.5	603
$\bar{T}_{sj}$	571.2	570	540.7	545.2	526.3	528.1	513.9	518
$\bar{T}_{wj}$	541.2	536.5	510.7	511.7	514.1	514.5	501.7	504.4
T_{wij}	526	524.1	495.5	499.3	507.9	509.4	495.5	499.3
T_{woj}	556.5	548.9	526	524.1	520.3	519.5	507.9	509.4
q_{cj}^o	15.26×10^4	12.415×10^4	15.26×10^4	12.415×10^4	7.585×10^4	6.17×10^4	7.585×10^4	6.17×10^4
q_{swj}^o	15.26×10^4	12.415×10^4	15.26×10^4	12.415×10^4	7.585×10^4	6.17×10^4	7.585×10^4	6.17×10^4

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