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COMPILATION AND CORRELATION OF EXPERIMENTAL, HYPERSONIC, STAGNATION POINT CONVECTIVE HEATING RATES

by L. L. PERINI

THE JOHNS HOPKINS UNIVERSITY • APPLIED PHYSICS LABORATORY

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FOREWORD

The work reported herein was conducted by the Aeronautics Division of the Applied Physics Laboratory for the Atomic Energy Commission Safety Branch, Space Nuclear Systems Office. Activities of the Aeronautics Division are under the general supervision of Dr. William H. Avery. The Project Engineer for the Aerospace Nuclear Safety Program is James C. Hagan.

ABSTRACT

Experimental shock-tube data for the aerodynamic heating at the stagnation point on spherical bodies at superorbital and suborbital flight velocities (U_0) have been compiled from a number of sources and compared to theoretical predictions. It was found that the empirical correlation given by Detra, Kemp and Riddell is remarkably close to a least-squares fit of the data. It is therefore recommended that their correlation be used for continuum flow theoretical predictions of the laminar hypersonic convective stagnation-point heating rate to spherical bodies in the region, $U_0 = 6000$ to $58,000$ ft/sec.

SUMMARY

Experimental data on stagnation-point heat-transfer rates on spherical bodies from published sources have been compiled. The data were gathered from ten publications that encompassed five facilities -- four shock-tube facilities and a ballistic range. The total of ~ 500 data points encompassed a simulated flight velocity (U_0) range of 7,600 to 57,900 ft/sec, a simulated altitude range of 20 to 170 K ft, and nose radii from 0.125 to 2.5 in. A statistical analysis of the data indicated a normal distribution, implying a lack of facility or experimentalist bias.

Various correlations based on numerical studies and experimental data are discussed and compared. Uncertainties as to transport properties and/or velocity gradients required for some of the correlations negates somewhat their use in a priori predictions of heating rates. A least-squares curve fit of all of the previously mentioned experimental data to the function form $\dot{q}_w \sqrt{R_N/P_{T_S}} = C (U_0/26,000)^m$ results essentially in the Detra, Kemp and Riddell correlation equation, which was originally based on AVCO shock-tube data that were limited to speeds up to $U_0 = 27,200$ ft/sec. It is therefore concluded that the Detra, Kemp and Riddell correlation is applicable throughout the continuum hypersonic regime, and that this correlation provides the best available estimate of the laminar, convective, stagnation-point heat transfer rate on spherical bodies; however, one should be cognizant of radiative, nonequilibrium, ablation and non-continuum phenomena when attempting to predict the heating rate to a re-entry configuration.

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NOMENCLATURE

C	=	Constant, see Eq. (21)
$\bar{D}$	=	Average deviation
g	=	Acceleration of gravity, ft/sec ²
h_d	=	Average atomic dissociation energy times the atom mass fraction - see Ref. 21
h_s	=	Stagnation point (total) enthalpy
h_w	=	Wall enthalpy
h_{w300}	=	Wall enthalpy evaluated at T = 300°K
k	=	Flow dimensionality; k = 0 planar flow, or 1 axisymmetric flow
Le	=	Lewis number
m	=	Constant, see Eq. (21)
M_0	=	Free-stream Mach number
n	=	Total number of data points
N	=	See Eq. (14)
Nu	=	Nusselt number
P_1	=	Initial shock-tube pressure
Pr	=	Prandtl number
P_{T_S}	=	Stagnation point total pressure
q_0	=	Free-stream dynamic pressure = $\rho_0 U_0^2 / 2$
$\dot{q}_w$	=	Convective heat transfer rate
R	=	Universal gas constant
Re	=	Reynolds number
R_N	=	Nose radius
S	=	Distance along body surface

T_0	=	Reference temperature
T_{tS}	=	Stagnation-point total temperature
T_w	=	Wall temperature
U_e	=	Velocity at edge of boundary layer
U_0	=	Free-stream velocity
U_s	=	Shock tube shock velocity
Z	=	Altitude
β	=	Velocity gradient parameter $= \frac{R_N}{U_0} \frac{dU_e}{dS}$
γ	=	Ratio of specific heats
σ	=	Standard deviation, RMS
μ_w	=	Viscosity evaluated at wall conditions
ρ_e	=	Density at edge of boundary layer, stagnation value herein
ρ_0	=	Free-stream density
ρ_{SL}	=	Sea-level density
ρ_w	=	Density at wall conditions

I. INTRODUCTION

The Applied Physics Laboratory is aiding the Atomic Energy Commission in the evaluation of safety aspects of spacecraft employing nuclear power sources. Toward this goal, knowledge of the stagnation-point heating rate and ablation losses is a fundamental requirement for aerothermodynamic studies. The ablation models and supporting experimental data were documented in a previous memorandum.¹ A large body of experimental and theoretical studies have been conducted over the past decade on the laminar hypersonic stagnation point heating rates over spherical bodies. It is the purpose of this memorandum to compile the experimental results and attempt to select the theoretical or experimental correlation that best represents the data. The experimental data are presented in Section II, the theoretical studies are discussed in Section III and the theoretical predictions are compared to the experimental data in Section IV.

II. EXPERIMENTAL DATA

Convective stagnation-point heating rate measurements have been compiled from ten sources which cover a simulated free stream velocity (U_0) range of 7600 to 57,900 ft/sec and a simulated altitude, Z , range of 20,000 to 170,000 feet. Approximately 500 data points have been collected for hemispherical nose shapes ranging in nose radii, R_N , from 0.125 in. to 2.50 in. To facilitate comparisons $\dot{q}_w \sqrt{R_N / P_{T_S}}$ has been utilized as the heating rate parameter where P_{T_S} is the stagnation pressure and $\dot{q}_w$ is the heat

transfer rate. Since a portion of the data was not reported in this form it was necessary to evaluate this parameter from the published results. Toward this end it became necessary to evaluate P_{T_S} and $(dU_e/dS)_S = 0$ from

$$P_{T_S} = 2q_0 = \rho_0 U_0^2 \quad (1)$$

$$\beta = \frac{R_N}{U_0} \left(\frac{dU_e}{dS} \right)_{S=0} = \left(2 \frac{\rho_0}{\rho_e} \right)^{\frac{1}{2}} \quad (2)$$

Equation (1) was checked against the real-gas, normal-shock solution^{2,3} and found to yield excellent agreement. Equation (2) is restricted to a spherical nose shape, because the Newtonian pressure distribution was assumed in the derivation.

The overwhelming majority of the data (97%) were measured in shock-tube facilities. One test series utilized a ballistic range and reported 15 data points. A short description of each of the data sources follows.

a) Rose and Stack,⁴ AVCO, 1958.

The tests were performed at AVCO Research Laboratory and utilized a combustion-driven, 1.5-in.-inside-diameter shock tube. The models were 0.25-in.-radius hemisphere-cylinders. Values of $\dot{q}_w$ vs U_0 were taken from Fig. 12 of the paper. The corresponding altitudes varied with velocity for a given P_1 (initial shock tube pressure) and were taken from Fig. 5 of the paper in order to evaluate P_{T_S} from Eq. (1). The majority of the heat-transfer measurements were taken with platinum calorimeter gages. Thin heat-transfer gauges were utilized in a few tests and showed reasonable agreement with the platinum calorimeter data.

b) Offenhartz, Weisblatt and Flagg,⁵ AVCO, 1962.

Eleven data points were presented for a blunted cylinder having a nose radius of 0.425 in. and cylinder radius of 0.1875 in. The AVCO 1.5-in.-I.D., combustion-driven shock tube was utilized. The data were presented as $\dot{q}_w$ vs the nondimensional stagnation enthalpy h_s/RT_0 . We have used these data as plotted by Hoshizaki,⁶ who gave $\dot{q}_w \sqrt{R_N/P_{T_S}}$ vs U_0 coordinates. Heat-transfer measurements were obtained with standard calorimeter heat transfer gauges.

c) Hoshizaki,⁶ Lockheed, 1962.

Data (32 points) were presented for two models with nose radii of 0.25 and 0.50 inches, on $\dot{q}_w \sqrt{R_N} / \sqrt{\beta R_N P_{T_S}} / U_0$ vs U_0 coordinates (Fig. 12 of the reference). The facility was a 3 inch inside diameter shock tube located at Lockheed Missiles and Space Corp. The velocity gradient was evaluated from Eq. (2) in conjunction with perfect-gas, $\gamma = 1.4$, Newtonian assumptions for ρ_0/ρ_e (see Eq. (10) in Section III). The data of Offenhartz et al.⁵ were also given in Fig. 12⁶ and used herein.

d. Rose and Stankevics,⁷ AVCO, 1963.

An AVCO electric-arc-driven, 6-in.-I.D. shock tube and platinum-coated calorimeter gauges were used. The models were hemispherical cylinders of 0.25- and 0.50-in. nose radii. The data (95 points) were taken from their Fig. 14, where $\dot{q}_w \sqrt{R_N}$ vs U_0 was presented (the nitrogen data points were not used). The corresponding altitudes were taken from their Fig. 2.

e) Nerem,⁸ Ohio State Univ., 1964.

Hemispherical cylindrical models with nose radii of 0.25, 0.50, and 1.0 in. were tested in an arc-driven, 4-in.-I.D. shock tube. Heat transfer rates were measured with platinum-foil-calorimeter gauges. Data (93 points) for $\dot{q}_w$, simulated altitude, and simulated flight velocity were taken from Tables I-III of their report.

f) Compton and Cooper,⁹ Ames, 1965.

The tests were conducted in the Ames Prototype Hypervelocity Free-Flight Facility -- a ballistic range. The sabot-held models were 0.125-in.-radius aluminum hemispheres, and the time of onset of melting was used as a measure of the stagnation-point heating rate. A plot of $\dot{q}_w \sqrt{R_N/P_{T_S}}$ vs U_0 was presented for the 15 data points.

g) Rose and Stankevics,¹⁰ AVCO, 1965.

Sphere-cone models of 0.25-, 0.50- and 0.75-in. radii were tested in an arc-driven, 6-in.-I.D. shock tube (same as Ref. 7). Heat-transfer rates were measured with infrared gauges coated with aluminum, gold, Nichrome and carbon. The data (45 points) were presented in their Fig. 14 as $\dot{q}_w \sqrt{R_N}$ vs U_0 . The equivalent altitude of 140,000 ft, as stated by the authors, was used to evaluate P_{T_S} from Eq. (1).

h) Horton and Babineaux,¹¹ JPL, 1967.

A 0.5-in.-radius hemispherical model was tested in the JPL arc-driven, 6-in.-I.D. shock tube. Heat-transfer rates were measured with platinum-strip-calorimeter gauges. The data (11 points) were taken from their Fig. 11, where $\dot{q}_w \sqrt{R_N/P_{T_S}}$ vs U_0 was presented.

i) DeRienzo and Pallone,¹² AVCO, 1967.

Previously unpublished AVCO data, taken by Weisblatt, H. and Rushton, G. H. (private communication to DeRienzo and Pallone) were presented for 0.187-in.-radius hemispherical cylinders.* The data (10 points) are given in their Fig. 11 as $\dot{q}_w \sqrt{R_N/P_{T_S}}$ vs U_0 .

j) Livingston and Williard,¹⁴ JPL, 1971.

The tests were conducted in an arc-driven, 6-in.-I.D. shock tube (same as Ref. 11). Data were presented (nine points) for a 2.5-in.-radius hemispherical model, and it was indicated that radiative flux may not have been negligible. An additional nine data points were presented from a previous study (Williard¹⁵) for a 0.5-in.-radius hemispherical model. The data were taken from their¹⁴ Fig. 8, where $\dot{q}_w \sqrt{R_N/P_{T_S}}$ vs shock velocity U_s was presented. As suggested by the authors, the simulated free-stream velocity was evaluated from $U_0 = 1.37 U_s$.

Stagnation-point heating rate data (9 points) were published by Scala and Warren¹⁶ in 1962 and indicated a factor of two increase above then available theories for $U_0 > 32,000$ ft/sec. The data and a subsequent article of Gruszczyski and Warren¹⁷ were the incentive for studies on the effect of the calorimeter heat transfer gauge material (e.g., Refs. 9 and 10)

*Results are also presented for a cylinder oriented perpendicular to the flow (6 data points) and show a 30% reduction in $\dot{q}_w$ below the hemispherical cylinders as expected due to the geometry, i.e., $\dot{q}_w$ for a planar flow is $1/\sqrt{2}$ times the axisymmetric value, for the same β as noted from Eq. (4) in Section III. The results of Ref. 13 show a 6% difference in β between a sphere and a cylinder in crossflow.

on measurements. The studies concluded that there was no effect of surface material used. The Warren data have been neglected in this compilation since they have not been verified by subsequent studies.

III. THEORETICAL STUDIES

Many theoretical studies of stagnation-point heating have been published since the work of Sibulkin¹⁸ in 1952. It is not our intent to review in detail these many correlations and/or numerical solutions. The basic assumptions inherent in the majority of them are that the flow is laminar, steady, and nonradiating, and that the boundary layer equations are applicable. These assumptions result in a set of partial differential equations (encompassing conservation of momentum, energy and mass), which are reduced to ordinary differential equations by employing similarity assumptions (e.g., Levy transformation¹⁹) and are solved numerically based on various assumptions concerning the transport and thermodynamic properties. The numerical solutions are then curve-fitted and compared with experimental data.

In the solutions the velocity gradient is treated as an independent parameter and appears in the expression for $\dot{q}_w$. A clear exposition of the procedure can be found in the excellent paper by L. Lees.¹⁹ The basic equations, subject to the above assumptions, have been well established and differences in results are evidently due to differences in transport properties,²⁰ thermodynamic properties, numerical techniques, and/or correlation methods. In hypersonic flow the evaluation of $\dot{q}_w$ has been treated in two

distinct regimes -- suborbital and superorbital (orbital speed $\approx 26,000$ ft/sec) -- which is a classification dictated by the chemistry. In the superorbital regime air must be treated as a complex thermodynamic state with ionized, molecular and atomic species. In the suborbital regime the binary approximation can be employed, since only molecular and atomic species are present (i.e., no ionization), thereby considerably simplifying numerical calculations.

a) Suborbital Regime

In the suborbital velocity regime the correlation formula of Fay and Riddell²¹ (F&R) is considered the most reliable,* and is widely used,

$$\dot{q}_w = 0.76 P_r^{-0.6} (\rho_w \mu_w)^{0.1} (\rho_e \mu_e)^{0.4} (h_s - h_w) \sqrt{\left(\frac{dU_e}{dS} \right)_{S=0}} \\ \times \left[1 + (Le^{0.52} - 1) h_d / h_s \right] \quad (3)$$

where ρ_e and μ_e are evaluated for the flow external to the boundary layer at stagnation conditions and h_d is the average atomic dissociation energy times the atom mass fraction in the external flow. The correlation is based on numerical solutions for spherical shapes employing Sutherland's equation for viscosity²³ and equilibrium air from the Hilsenrath and Beckett²⁴ calculations. The Lewis number Le and Prandtl number Pr were treated as independent parameters. Calculations for a frozen assumption were performed and it was concluded that "for a Lewis number not too far from unity, there is little difference in heat transfer for the frozen as opposed to the equilibrium

*An earlier version of this study²² utilized $[1 + 0.45 (Le - 1) h_d / h_s]$ for the last factor. For $Le = 1.4$ the factors are equivalent.

boundary layer." It was also found that with $Pr = 0.71$, $Le = 1.4$, and Sutherland's viscosity equation, Eq. (3) gave "good agreement" with the experimental shock tube data of Rose and Stark.⁴ Evaluation of the velocity gradient used to obtain the good agreement was not discussed, i.e., whether the gradient was computed assuming a real gas or a perfect gas.

Also utilized in a number of studies is the Lees equation¹⁹ for a cold wall based on $Le = 1$ and $\rho\mu = \text{constant}$ across the boundary layer,

$$\dot{q}_w = \frac{1}{2} \frac{k}{2^2} Pr^{-\frac{2}{3}} (\rho_e \mu_e)^{\frac{1}{2}} h_s \sqrt{(dU_e/ds)_{s=0}} \quad (4)$$

where $k = 0$ for planar flow and $k = 1$ for axisymmetric flow. Evaluating

P_{T_S} from Eq. (1), Lees equation can be written for axisymmetric flow, $Pr = 0.71$ and $h_s = U_0^2/2$ as

$$\dot{q}_w \sqrt{\frac{R_N}{P_{T_S}}} = 315.9 \times 10^4 \left[\frac{\rho_e}{\rho_0} \frac{\mu_e}{U_0} \beta \right]^{\frac{1}{2}} U_0^2 \quad (5)$$

For simple engineering calculations the empirical correlation of Detra, Kemp and Riddell²⁵ (D,K & R) is also employed. The relation is based on the experimental shock tube data of Rose and Stark⁴ (see Section II a) above),

$$\dot{q}_w = \frac{17600}{\sqrt{R_N}} \sqrt{\frac{\rho_0}{\rho_{SL}}} \left(\frac{U_0}{26000} \right)^{3.15} \frac{h_s - h_w}{h_s - h_w}_{300}, \quad \frac{\text{BTU}}{\text{ft}^2\text{-sec}} \quad (6)$$

where ρ_{SL} is a sea level reference density and h_{w300} is the wall enthalpy evaluated at 300°K. As shown in Section III this relation fits the Rose and Stark data with a standard deviation* of 13.3%. It should be noted that the data upon which the correlation is based extend to $U_0 = 26000$ ft/sec.

In addition to the above relations others have been in vogue, notably the Scala²⁶ and the reference enthalpy²⁷ methods. These methods will not be discussed in this report.

In order to evaluate and compare Eqs. (5) and (6), we have selected a trajectory defined by $P_{TS} = 1$ atm. Using real-gas, equilibrium, normal-shock tabulations^{2,3} the resulting total conditions behind the shock as a function of U_0 are presented in Table I. These results can be used to facilitate the comparison of techniques as follows. Employing the Sutherland viscosity, Eq. (2) for β and the properties in Table I the square root factor in Eq. (5) can be considered to be a constant, equal to $1.421 \times 10^{-4} \text{ lb}^{\frac{1}{2}}\text{-sec-ft}^{\frac{3}{2}}$ with an accuracy of $\pm 8\%$ between velocities of 10,000 to 30,000 ft/sec. Lees' relation, Eq. (5), can then be written as

$$\dot{q}_w \sqrt{R_N/P_{TS}} \cong 449 (U_0/26,000)^2 \text{ BTU}/(\text{ft}^{\frac{3}{2}}\text{-sec-atm}^{\frac{1}{2}}) \quad (7)$$

*The standard deviation is defined below by Eq. (23).

A comparison can be made with the Fay and Riddell (F&R) result by assuming a cold wall ($h_s \gg h_w$), $Le = 1$, $\rho_e \mu_e = \rho_w \mu_w$ and $Pr = 0.71$. The F&R correlation is then 5.9% greater than the Lees' equation for the same assumptions on β and μ_e . The F&R correlation can then be written as:

$$\dot{q}_w \sqrt{R_N/P_{T_S}} \cong 475 (U_0/26,000)^2 \text{ BTU}/(\text{ft}^{3/2}\text{-sec-atm}^{1/2}) \quad (8)$$

The D,K & R correlation, Eq. (6), may be compared to the latter two expressions by choosing $\rho_{SL} = 0.07647 \text{ lb/ft}^3$, by using Eq. (1) for P_{T_S} and by assuming a cold wall ($h_s \gg h_w = h_{w300}$). It can then be written as:

$$\dot{q}_w \sqrt{R_N/P_{T_S}} = 638.8 (U_0/26,000)^{2.15} \text{ BTU}/(\text{ft}^{3/2}\text{-sec-atm}^{1/2}) \quad (9)$$

Comparing the latter three equations, Eqs. (7-9), it is noted that the Lees' equation and the F&R correlation yield approximately the same values for $\dot{q}_w$ when the same assumptions are employed, e.g., $Le = 1$. When the Lees' equation is compared to the D,K & R correlation it is found that the ratio of the two is given by $1.423 (U_0/26,000)^{0.15}$. This factor amounts to 1.4 at $U_0 = 26,000$ and 1.23 at $U_0 = 10,000 \text{ ft/sec}$.

In order to show the effect of viscosity and velocity gradient assumptions, Eq. (5) was evaluated for a) Sutherland's viscosity and a real gas β , b) Hansen's²⁸ viscosity and a real-gas β , and c) Sutherland's viscosity and a perfect-gas evaluation of β . The density ratio in Eq. (5) was taken from Table I in all cases. Equation (2) was utilized to evaluate

the velocity gradient and for the perfect-gas, Newtonian assumption,

$\rho_0/\rho_e \rightarrow (\gamma-1)/(\gamma+1)$ as $M_0 \rightarrow \infty$. Therefore,

$$\beta_{M \rightarrow \infty} = \sqrt{\frac{2(\gamma-1)}{\gamma+1}} = 0.578 \quad (10)$$

for $\gamma = 1.4$. The results are compared in Fig. 1. It is seen that the velocity gradient assumption employed in evaluating $\dot{q}_w$ has a strong influence. For the cases considered herein the perfect-gas β is $\approx 30\%$ greater than the real-gas β at $U_0 = 26,000$ ft/sec.

Utilization of the Fay and Riddell correlation or the Lees equation requires knowledge of both the velocity gradient β and the transport properties (i.e., μ , Pr and Le), none of which is accurately known for a dissociating gas. The D,K & R correlation, Lees' equation and the F&R correlation are all functionally similar, i.e., $\dot{q}_w \sqrt{R_N/P_{T_S}} \propto U_0^2$.

b) Superorbital

Hoshizaki⁶ presented the following correlation equation based on numerical solutions employing Hansen's transport and thermodynamic properties:

$$\dot{q}_w \sqrt{R_N} = 2.59 \times 10^4 \beta^{\frac{1}{2}} \left(\frac{\rho_0}{g} \right)^{\frac{1}{2}} \left(\frac{U_0}{10,000} \right)^{3.19} \left(1 - \frac{h_w}{h_s} \right) \quad (11)$$

Assuming a cold wall and using Eqs. (1) and (2), this can be rewritten as

$$\dot{q}_w \sqrt{R_N/P_{T_S}} = 965.78 \left(\frac{U_0}{26,000} \right)^{2.19} \left(2 \frac{\rho_0}{\rho_e} \right)^{\frac{1}{2}} \frac{\text{BTU}}{\text{ft}^{3/2} \text{-sec-atm}^{\frac{1}{2}}} \quad (12)$$

Fay and Kemp²⁹ utilized a 'binary diffusion' model for a simple N₂-only gas and computed their own thermodynamic and transport properties. For equilibrium assumptions and a cold wall, the correlation to the numerical solutions is given as:

$$\dot{q}_w \sqrt{R_N} = 600 \frac{Nu}{Re^{\frac{1}{2}}} P_{T_S}^{\frac{1}{2}} \frac{U_0}{24,000} \left(\frac{P_{T_S}}{\rho_e} \right)^{\frac{1}{2}} \frac{BTU-ft^{\frac{1}{2}}}{ft^2-sec} \quad (13)$$

where R_N is in feet, P_{T_S} is in atmospheres, ρ_e is in amagats* and

$$\begin{aligned} \frac{Nu}{Re^{\frac{1}{2}}} &= 0.47 \text{ for } 10,000 < U_0 < 24,000 \text{ ft/sec} \\ \frac{Nu}{Re^{\frac{1}{2}}} &= 0.47 \left(\frac{U_0}{24,000} \right)^{-N} \text{ for } 24,000 < U_0 < 60,000 \text{ ft/sec} \\ N &= 0.83 - 0.11 \log_{10} P_{T_S} - 0.02 \left(\log_{10} P_{T_S} \right)^2 \end{aligned} \quad (14)$$

Again this can be written as

$$\dot{q}_w \sqrt{\frac{R_N}{P_{T_S}}} = \frac{1471.82}{1.0833^N} \left(\frac{U_0}{26,000} \right)^{2.5-N} \left(2 \frac{\rho_0}{\rho_e} \right)^{\frac{1}{2}} \frac{BTU}{ft^{3/2}-sec-atm^{\frac{1}{2}}} \quad (15)$$

Correlation equations for frozen-composition assumptions were also presented showing increased rates over the equilibrium values. It was concluded based on physical reasoning, by the authors,²⁹ "that the heat

* Density ratioed to the density at standard temperature and pressure, 1 amagat = 2.45×10^{-3} slug/ft³.

transfer parameters in air and nitrogen are practically the same at identical flight conditions."

Pallone and Van Tassell³⁰ presented correlations for numerical solutions of N_2 and air gas models using, respectively, transport properties of Yos³¹ and Hansen.²⁸ It was concluded that a nitrogen model was adequate for preliminary design calculations. In a recent study DeRienzo and Pallone¹² extended the Pallone and Van Tassell results to $U_0 = 70,000$ ft/sec and employed modified Yos transport properties to agree with Maecker's³² high-temperature thermal conductivity data. A correlation equation was not presented; however, tabular values of $Nu/Re^{\frac{1}{2}}$ are given for $P_{T_S} = 1$ atm and $R_N = 1$ ft. Figure 2 shows variations with velocity of $Nu/Re^{\frac{1}{2}}$ for the Hoshizaki, Fay and Kemp, and DeRienzo and Pallone studies discussed above at $P_{T_S} = 1$ atm. The Fay and Kemp values were found from Eq. (14), the DeRienzo and Pallone values were taken from Table I of their paper,¹² and the Hoshizaki values were found from

$$\frac{Nu}{\sqrt{Re}} = \frac{\dot{q}_w P_{rw}}{\left(\rho_w \mu_w \frac{dU_e}{dS} \right)^{\frac{1}{2}} h_s} \quad (16)$$

and a correlation equation from Hoshizaki's paper^{*}

$$\dot{q}_w = 4.32 \times 10^4 \left[\rho_w \mu_w \frac{dU_e}{dS} \right]^{\frac{1}{2}} \left(\frac{T_w}{533} \right)^{0.2} \left(\frac{U_0}{10^4} \right)^{1.69} \frac{BTU}{ft^2 \cdot sec} \quad (17)$$

* Eq. (17), taken from Hoshizaki's paper, was further approximated by Hoshizaki in deriving Eq. (11).

to give, for $T_w = 300^\circ\text{K}$,

$$\text{Nu}/\sqrt{\text{Re}} = 7.53/U_0^{0.31} \quad (18)$$

As may be noted from Fig. 2 there is considerable deviation over the majority of the velocity regime between the three studies, reaching a peak difference of 42% at $U_0 = 24,000$ ft/sec.

Using the tabular results in Table I for density ratio at $P_{T_S} = 1$ atm, it is found that $(\beta)^{\frac{1}{2}} = (2\rho_0/\rho_e)^{\frac{1}{2}}$ can be approximated by $\beta^{\frac{1}{2}} = 0.594$ for $U_0 > 26,000$ ft/sec. Equations (12) and (15) can then be rewritten as;

Hoshizaki:

$$\dot{q}_w \sqrt{R_N/P_{T_S}} = 573.5 (U_0/26,000)^{2.19} \text{ BTU}/(\text{ft}^{3/2}\text{-sec-atm}^{\frac{1}{2}}) \quad (19)$$

Fay and Kemp:

$$\dot{q}_w \sqrt{R_N/P_{T_S}} = 817.7 (U_0/26,000)^{1.67} \text{ BTU}/(\text{ft}^{3/2}\text{-sec-atm}^{\frac{1}{2}}) \quad (20)$$

As in the suborbital regime it is found that $\dot{q}_w/\sqrt{R_N/P_{T_S}}$ is proportional to the free-stream velocity raised to a power. This functional relationship will be utilized to correlate the experimental data in Section IV.

IV. RESULTS OF LEAST-SQUARES FIT OF DATA

For the present study the experimental data presented in Section II were curve-fitted by a standard least-squares technique (see Appendix), assuming a functional relationship of the form

$$\dot{q}_w \sqrt{R_N/P_{T_S}} = C (U_0/26,000)^m \quad (21)$$

This is the general form of the relation between heating rate and velocity that was determined by a number of the previous investigations as discussed in Section III. The constants that result from a least-squares fit of approximately 500 data points, to this expression, are $C = 637.4$ and $m = 2.124$. When compared to the D,K & R correlation, Eq. (9), the results are remarkably similar. The D,K & R correlation constants are 638.8 and 2.15 for C and m , respectively. The standard deviation for the least squares fit correlation is $\sigma = 16.3\%$ whereas the D,K & R correlation value is $\sigma = 16.5\%$. The deviation is defined as

$$\sigma = \left[\frac{1}{n} \sum \left(1 - \frac{\dot{q}_w}{\dot{q}_{w_i}} \right)^2 \right]^{\frac{1}{2}} \cdot 100 \quad (22)$$

where n is the total number of data points, $\dot{q}_w$ is the heat transfer rate evaluated from the correlation and $\dot{q}_{w_i}$ is the experimental data point at the same velocity. A comparison of the least squares fit and D,K & R correlation is shown in Fig. 3. Also shown are the heating rates from the Hoshizaki and Fay and Kemp correlations given by Eqs. (19) and (20), respectively. The distribution with velocity of the experimental data is shown in Fig. 4, where it is noted that 66% of the data points fall in the range, $U_0 = 20,000$ to $40,000$ ft/sec. Figure 5 shows a distribution of the deviation as measured from the D,K & R correlation. Superimposed on the data is the normal distribution function with a $\sigma = 17\%$. The $\sigma = 17\%$ corresponds to a standard deviation defined by

$$\sigma = \left[\frac{1}{n} \sum \left(1 - \frac{q_{wi}}{q_w} \right)^2 \right]^{\frac{1}{2}} \times 100 \quad . \quad (23)$$

This form is felt to be more applicable than Eq. (22) for practical usage, although the actual value is only slightly changed from that given by Eq. (22). As noted from Fig. 5, the data are essentially normally distributed.

A tabulation of the data sources and pertinent conditions is given in Table II. Also tabulated are the standard deviation as computed from Eq. (23) and the average deviation $\bar{D}$ defined as

$$\bar{D} = \frac{1}{n} \sum \left(1 - \dot{q}_{wi} / \dot{q}_w \right) \quad (24)$$

for each group of data.

The distribution of the standard deviation σ (computed from Eq. (23)) with velocity is shown in Fig. 6, where σ has been computed for 5000-ft/sec velocity increments. The suborbital regime has a lower σ than the super-orbital regime. For data below $U_0 = 25,000$ ft/sec, $\sigma = 14.1\%$, whereas for data above $U_0 = 25,000$ ft/sec, $\sigma = 18.5\%$, approximately a 32% increase in σ . This is not surprising, since the data below 25,000 ft/sec are almost exclusively from the Rose and Stark tests, and the data above 25,000 ft/sec represent data from a number of different facilities, hence more scatter.

Figure 7 and 8 show plots of the data and the results from the D,K & R correlation, Eq. (9). The symbols are defined in Table II. A composite plot showing all the data, along with the D,K & R curve is shown in Fig. 9.

V. CONCLUSIONS

Based on the experimental data presented above it is recommended that the Detra, Kemp and Riddell (D,K & R) correlation, Eq. (6) or (9), be utilized for theoretical predictions of the laminar, hypersonic, convective stagnation-point heating rate for spherical nose shapes from $U_0 = 6000$ ft/sec to $U_0 = 57,000$ ft/sec. Although the D,K & R relation is empirical, it does approximately represent a least-squares fit of the available (~ 500 data points) shock-tube data from a number of facilities. The majority of the experimental investigations that reported data indicated that the flow was in equilibrium and that radiative effects were negligible. No attempt was made to assess these statements. The simulated altitude regime ranged from 20,000 to 170,000 ft; however, one should be cognizant of radiative, nonequilibrium, ablation, and non-continuum phenomena when attempting to predict the stagnation point heating rate for a re-entry configuration. The data are normally distributed about the D,K & R relation, which is a necessary, although not sufficient, criterion of random scatter. The standard deviation of the shock-tube data from the D,K & R relation is $\sigma = 17\%$.

Other correlations and/or theoretical results are available; however, values of velocity gradient and/or transport properties are somewhat uncertain, and it would appear that values should be selected to agree with the D,K & R correlation.

For body shapes other than spherical shapes for which the D,K & R relation has been derived, it is recommended that the theoretical result of Lees be utilized, as determined from Eq. (4),

$$\frac{\dot{q}_w}{\dot{q}_{w_s}} = \frac{k}{2} \sqrt{\frac{\beta}{\beta_s}} \quad (25)$$

where the subscript s denotes the sphere value and $\dot{q}_{w_s}$ is the value of heating rate from the D,K & R correlation. Experimental verification of this procedure was indicated in the data of DeRienzo and Pallone (see footnote, p. 5) between a spherical and cylindrical body. The velocity gradients ratio should be based on experimental data (e.g., Refs. 13, 33, 34 and 35). It should be noted that experimental values for β are based on thermally-perfect-gas test conditions (i.e., no dissociation) and that real-gas values for a highly dissociated gas may be significantly different, i.e., see Table I. Utilization of the above recommended approach, Eq. (25), in conjunction with the D,K & R correlation equation, Eq. (9), may be more realistic as it assumes that the velocity gradient ratio of two bodies is the same for a real gas and a perfect gas.

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Table I Trajectory Conditions Defined by
 $P_{T_S} = 1 \text{ atm, Real Gas.}^{2,3}$

U_0 K ft/sec	Z, Alt. K ft	T_{t_S} °R	$\frac{\rho_e}{\rho_0}$	$\sqrt{\beta}^*$
10	110	3265	8.52	0.696
15	130	5131	10.67	0.658
20	140	6436	13.30	0.623
26	150	7460	15.94	0.595
30	160	8675	16.13	0.593
40	180	12,870	15.74	0.597
50	190	15,110	16.58	0.589

* $\sqrt{\beta} = 2 (\rho_0/\rho_e)^{1/4}$

Table II. Tabulation of Data Sources

Figure	Symbol	R_N inches	P_1 cm H _g	Z K ft	Number Points	Deviations*		ΔV K ft/sec	Reference Source
						σ %	D %		
7	•	0.26	10.0	20-25	34	13.6	4.9	7.6-13.5	4 (AVCO)
7	+	0.26	1.0	64-80	83	14.2	- 5.3	10.1-27.2	4 (AVCO)
7	○	0.26	0.1	111-127	46	11.3	0.0	13.6-26.4	4 (AVCO)
7	*	0.25	0.1	~ 110	5	14.1	8.9	23.9-35.4	7 (AVCO)
7	△	0.25	0.025	~ 140	44	14.1	8.1	29.5-53.9	7 (AVCO)
7	□	0.50	0.1	~ 110	21	11.9	7.5	25.6-40.2	7 (AVCO)
7	○	0.50	0.025	~ 140	25	15.2	8.2	29.3-54.5	7 (AVCO)
7	◆	0.25	0.025	~ 140	1	16.5	16.5	44.2	10 (AVCO)
7	◇	0.50	0.025	~ 140	24	15.7	- 5.8	31.4-49.7	10 (AVCO)
7	◊	0.75	0.025	~ 140	20	18.1	- 9.2	30.3-47.7	10 (AVCO)
8	■	2.50	0.025	UNK	9	24.1	-17.0	25.5-36.6	14 (JPL)
8	×	0.50	0.025	UNK	9	12.0	-10.3	25.7-27.8	14 (JPL)
8	▲	0.50	0.025	UNK	11	12.2	- 6.4	22.4-33.7	11 (JPL)
8	○	0.187	0.025	UNK	10	10.2	6.3	46.2-57.9	12 (AVCO)
8	◆	0.125	N/A**	UNK	15	21.2	19.8	35.8-42.4	9 (AMES)
8	△	1.0	0.10	~ 110	16	18.0	- 6.2	27.5-35.9	8 (OHIO ST.)
8	◇	1.0	0.02	145	8	20.3	-10.3	26.8-38.1	8 (OHIO ST.)
8	○	1.0	0.006	170	32	19.3	0.2	19.9-40.4	8 (OHIO ST.)
8	□	0.5	0.10	110	15	17.3	- 7.2	21.1-35.6	8 (OHIO ST.)
8	+	0.5	0.02	145	17	26.6	-12.0	23.5-37.1	8 (OHIO ST.)
8	*	0.5	0.006	170	5	20.3	4.2	26.0-30.4	8 (OHIO ST.)
8	○	0.425	0.05	UNK	6	4.5	- 2.8	33.1-35.0	5 (AVCO)
8	●	0.425	0.025	UNK	5	15.2	- 4.7	36.4-38.6	5 (AVCO)
8	•	0.50	0.05-0.3	UNK	18	22.3	12.4	22.0-36.6	6 (LOCKHEED)
8	★	1.0	0.05-0.3	UNK	14	35.4	-31.0	23.7-35.5	6 (LOCKHEED)

*Deviations measured from Detra, Kemp and Riddell correlation.

**Ballistic range.

Fig. 1 COMPARISON OF VISCOSITY AND VELOCITY GRADIENT ASSUMPTIONS ON LAMINAR CONVECTIVE STAGNATION POINT HEATING RATE

Fig. 2 COMPARISON OF THEORETICAL RESULTS FOR CONVECTIVE HYPERSONIC STAGNATION POINT HEATING RATES

Fig. 3 COMPARISON OF CORRELATIONS TO A LEAST SQUARES FIT OF EXPERIMENTAL DATA

Fig. 4 DISTRIBUTION OF EXPERIMENTAL DATA WITH VELOCITY

Fig. 5 DISTRIBUTION OF THE AVERAGE DEVIATIONS OF THE DATA.

Fig. 6 DISTRIBUTION OF THE STANDARD DEVIATION OF THE DATA WITH VELOCITY

Fig. 7 COMPARISON OF EXPERIMENTAL DATA FROM REFS. 4, 7 AND 10
WITH THE DETRA, KEMP AND RIDDELL CORRELATION

Fig. 8 COMPARISON OF EXPERIMENTAL DATA FROM REFS.5, 6, 8, 9, 11, 12 AND 14 WITH THE DETRA, KEMP AND RIDDELL CORRELATION

Fig. 9 COMPOSITE PLOT COMPARING EXPERIMENTAL DATA WITH THE DETRA, KEMP AND RIDDELL CORRELATION

APPENDIX

Least-Squares Technique

The functional form of the equation assumed is given by:

$$\dot{q}_w \sqrt{R_N/P_{T_S}} = C (U_0/26,000)^m \quad . \quad (A-1)$$

Eq. (A-1) can be reduced to a linear form by taking logarithms,

$$\ln \left(\dot{q}_w \sqrt{R_N/P_{T_S}} \right) = \ln(C) + m \ln (U_0/26,000) \quad . \quad (A-2)$$

Defining

$$\begin{aligned} y &\equiv \ln \left(\dot{q}_w \sqrt{R_N/P_{T_S}} \right) \\ a_0 &\equiv \ln(C) \\ a_1 &\equiv m \\ x &\equiv \ln (U_0/26,000) \end{aligned} \quad (A-3)$$

Eq. (A-2) can then be represented as:

$$y = a_0 + a_1 x \quad (A-4)$$

It was decided to minimize the function

$$R = \left[\sum_i \left(1 - \frac{a_0 + a_1 x_i}{y_i} \right) \right]^2 \quad (A-5)$$

Application of the minimization conditions

$$\frac{\partial R}{\partial a_0} = \frac{\partial R}{\partial a_1} = 0 \quad (A-6)$$

and solving the resulting two equations for a_0 and a_1 , yields

$$a_0 = \frac{\sum_i \frac{x_i}{y_i^2} \sum_i \frac{x_i}{y_i} - \sum_i \frac{x_i^2}{y_i^2} \sum_i \frac{1}{y_i}}{\left(\sum_i \frac{x_i}{y_i^2} \right)^2 - \sum_i \frac{x_i^2}{y_i^2} \sum_i \frac{1}{y_i^2}} \quad (\text{A-7})$$

$$a_1 = \frac{\sum_i \frac{1}{y_i} - a_0 \sum_i \frac{1}{y_i^2}}{\sum_i \frac{x_i}{y_i^2}} \quad (\text{A-8})$$

The root-mean-square deviation is then found from

$$\sigma = \sqrt{\frac{1}{n} \sum_i \left[1 - \frac{a_0 + a_1 x_i}{y_i} \right]^2} \quad (\text{A-9})$$