Final Report
Strength and Stress Analysis
for
Waste Tank Structures
at
Hanford, Washington

MASTER

by
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FINAL REPORT

Abstract

A review of the efforts to develop methods of analyses for the underground reinforced concrete tanks is presented. The transition from thin shell to finite element programs with the problems of proper constitutive relationships is considered. A method of analysis is developed using a one-dimensional non-linear constitutive relationship for the three principal stress and orthotropic material reference axes. Constant Poisson's ratios and modulus of rigidity were used in the orthotropic stress-strain relationship. The non-linear problem is solved by successive approximations using a series of linear solutions.

Results are presented comparing the analyses and experimental results from the Wiss, Janney and Elstner model at 1.25 times working load and at ultimate load. Stress results for the 241-A prototype waste tanks at Hanford are presented for the static working and ultimate loads. These results indicate an ultimate load factor for structural integrity of 6.2 times the soil load over the dome. This load factor for static loading is judged to be more than adequate relative to the ACI criteria. Thermal stress fractures of the concrete plus stress corrosion cracking of the inner liner present the only mechanism for loss of containment.

Results from the earthquake analysis for OBE and DBE loadings indicate no additional risk of containment loss beyond the risk associated with static loading.

I. Introduction

This report summarizes the results of the program "Stress and Strength Analysis of Waste Tank Structures", Contract CA 502, conducted for the Atlantic Richfield Hanford Company, Hanford, Washington. The time period for this effort is June 1969 to April 1971.

The Hanford waste tanks were designed to contain radioactive materials in an aqueous solution at elevated temperatures. Storage is by underground reinforced concrete tanks with $3/8$ inch steel liners. The waste tanks are buried with a six to eight foot earth cover. The reinforced concrete tanks are composed of thin shell elements with one exception, a haunch region in the dome. The elliptical dome is placed on a right circular cylinder and the cylinder rests on a varying thickness flat slab.

The steel liner forms the primary barrier to leakage with the reinforced concrete being the structural element for sustaining soil loads and capping the steel liner. If the steel liner is breached, for example by stress corrosion, the reinforced concrete element becomes the primary container and its containment integrity is of importance. Part of the impetus for this program was to attempt to define the containment capabilities of the reinforced concrete shells under various load conditions. In addition, since some leakage had been detected around a limited number of waste tanks and the stress analysis program was to attempt to explain how leakage could occur through the reinforced concrete shells. Thus, the objectives of the program centered on defining the states of stress in the reinforced concrete shells when loaded by the soil, aqueous solution, vapor pressures and temperature.

II. Method of Analysis for Static Loading

1. Thin Shell Program - A thin shell program with orthotropic material properties and Fourier series loading capabilities was adopted to the problem of the Hanford Waste tank analysis. The program was selected because of asymmetrical loading capabilities, transient thermal gradient possibilities and multi-layered orthotropic material capabilities. A description of this computer program and the theory is given in the Air Force Flight Dynamics Laboratory, Technical Report AFFDL-TR-68- 144 published March, 1969. In conjunction with the use of the program analyses were developed for the non-linear response of a concrete section subjected to thrust and moment assuming a fixed set of stress resultants in each principal (s, θ) direction. This is presented in the Appendix.

The thin shell program was modified to perform an analysis of the foundation and slab reactions in order to determine the soil loading function and boundary conditions for the cylindrical shell. This involved the writing of the subroutine "Soilreact".

The stress-strain relationships for a general orthotropic material are given in the AFFDL publication as:

$$\epsilon_{11} = \sigma_{11}/E_1 - \nu_{12} \sigma_{22}/E_2 - \nu_{13} \sigma_{33}/E_3 + a_1 T$$

$$\epsilon_{22} = -\nu_{21} \sigma_{11}/E_1 + \sigma_{22}/E_2 - \nu_{23} \sigma_{33}/E_3 + a_2 T$$

$$\epsilon_{33} = -\nu_{31} \sigma_{11}/E_1 - \nu_{32} \sigma_{22}/E_2 + \sigma_{33}/E_3 + a_3 T$$

$$2 \epsilon_{12} = \sigma_{12}/G_{12}$$

$$2 \epsilon_{23} = \sigma_{23} / G_{23}$$

$$2 \epsilon_{13} = \sigma_{13} / G_{13}$$

With the additional assumption that the $\frac{h}{p} \ll 1$, or thickness to radius of curvature is small with respect to one, for an orthotropic thin shell the material parameters become:

$$B_{11} = \frac{E_1}{1 - \nu_{12} \nu_{21}}$$

$$B_{12} = \frac{\nu_{12} E_1}{1 - \nu_{12} \nu_{21}} = \frac{\nu_{21} E_2}{1 - \nu_{12} \nu_{21}}$$

$$B_{22} = \frac{E_2}{1 - \nu_{12} \nu_{21}}$$

$$\sigma_{11} = B_{11} \epsilon_{11} + B_{12} \epsilon_{22} + A_1 T$$

$$\sigma_{22} = B_{12} \epsilon_{11} + B_{22} \epsilon_{22} + A_2 T$$

$$\epsilon_{33} = \nu_{31} = \nu_{32} = \frac{1}{E_3} = a_3 = 0$$

Analyses were performed using the computer program to compute the stress resultants and separate subroutines for calculation of stress in steel and concrete along with the non-linear strain response of the concrete section to the stress resultants.

The method of solution using the thin shell program consisted in creating a multi-layered model with shell parts sub-divided into segments which contained variable thickness, radius of curvature (when required) and elastic parameters. These variables within each segment are read

in by use of a function generator routine. Initial elastic (isotropic) material parameters were selected to initiate an incremental load analysis. At each load increment the stress resultants were then used with STIFF1 to calculate the concrete and steel stresses and the strains in the layers (four in all). The layer stresses from the program with the strains calculated from STIFF1 were then used to modify the elastic parameters for convergence to a solution. The solution at a given load level was assumed when the assumed elastic parameters agreed with the calculated elastic parameters. Usually convergence occurred with four analyses at a given load level. The orthotropic axes were assumed to correspond to the n-s axes of the shell (and directions of principal reinforcement). Tensile fracture of the concrete and non-linear response of the concrete and/or steel are calculated in the subroutine STIFF1.

A problem in use of this thin shell program is the assumption of $\frac{h}{\rho} \ll 1$ which is in question in the haunch region where $\frac{h}{\rho} \approx \frac{1}{3}$. Difficulties were encountered in determining the input of $\frac{1}{\rho}$ for the function generator. Insignificant physical changes in this input led to large variations in the N_{θ} stress resultant. Attempts to stabilize these results by using different segments and direction of integration in the haunch region were not successful. Although results for orthotropic analyses using this program were reported there still remained questions as to the N_{θ} stress resultant in the haunch region. These problems in using a thin shell program led to a decision to use a thick shell finite element program.

2. Thick Shell Program - A thick shell program, ASOLID, a modified 1969 version of 1965 program written by E. Wilson was selected to

perform the analyses. It is similar to the SAASII program for axisymmetric solids written by Jones and Crose for the Space and Missile Systems Organization, Air Force Systems Command (Reference 2). Both are finite element programs with orthotropic, nonlinear material properties capabilities. However, the nonlinear material properties are those associated with a bi-linear effective stress-effective strain relationship which is not suitable for concrete. Hence, use of previously written routines STIFF1 with additional routines required were employed for prediction of strains and ultimate stress resultants, given the stress resultants from the finite element program. The stress resultants were obtained by integration of the element stresses performed along normals to the concrete reinforcement. The ASOLID program specifies orthotropic material properties in the principle directions as:

$$\begin{bmatrix} \sigma_{nn} \\ \sigma_{ss} \\ \sigma_{\theta\theta} \\ \sigma_{ns} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 \\ c_{12} & c_{22} & c_{23} & 0 \\ c_{13} & c_{23} & c_{33} & 0 \\ 0 & 0 & 0 & c_{44} \end{bmatrix} \begin{bmatrix} \epsilon_{nn} \\ \epsilon_{ss} \\ \epsilon_{\theta\theta} \\ \epsilon_{ns} \end{bmatrix} - \begin{bmatrix} \tau_{nn} \\ \tau_{ss} \\ \tau_{\theta\theta} \\ 0 \end{bmatrix}$$

or

$$\begin{bmatrix} \epsilon_{nn} \\ \epsilon_{ss} \\ \epsilon_{\theta\theta} \\ \epsilon_{ns} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_n} & -\frac{\nu_{ns}}{E_n} & -\frac{\nu_{n\theta}}{E_n} & 0 \\ -\frac{\nu_{ns}}{E_n} & \frac{1}{E_s} & -\frac{\nu_{s\theta}}{E_s} & 0 \\ -\frac{\nu_{n\theta}}{E_n} & -\frac{\nu_{s\theta}}{E_s} & \frac{1}{E_\theta} & 0 \\ 0 & 0 & 0 & \frac{1}{G_{ns}} \end{bmatrix} \begin{bmatrix} \sigma_{nn} \\ \sigma_{ss} \\ \sigma_{\theta\theta} \\ \sigma_{ns} \end{bmatrix} + A \begin{bmatrix} \alpha_n \\ \alpha_s \\ \alpha_\theta \\ 0 \end{bmatrix}$$

Fig. 1

The principal n-s coordinates are oriented at an angle β to the R-axis. Concrete is a non-linear isotropic material (until tensile fracture occurs) with varying stiffnesses in the principal stress directions. For these analyses the E_θ , E_s were varied element by element by using the strains computed in the STIFF1 routine and the stresses from the finite element program since $\sigma_{nn} \approx 0$ the E_n values was held constant. Tensile fracture of the concrete is incorporated, if present, in the STIFF1 computations and in the finite element program G_{ns} , $v_{n\theta}$ and $v_{s\theta}$ are set equal to zero if tensile fracture occurs in the element. Concrete and steel stresses are computed in the STIFF1 routine.

The stress strain curves for concrete are illustrated in Fig. 1 and are from Reference 3. Plotted also are the assumed stress strain curves for use in STIFF1. It may be noted that for a continuum concrete

the Poisson's ratio was assumed to be 0.2. Obviously, this is only an approximation of the non-linear values, especially near and after the ultimate stress when micro fractures cause voids and affect Poisson's ratio.

Since general constitutive relationships were not and are not available in literature the approach was to use the non-linear uniaxial stress-strain relationships in the θ and s directions with Poisson's ratio equal to 0.2. A state-of-the-art report is given in Reference 4. A lower bound tensile strength was assumed for the concrete (60 psi) as determined from a Weibull analysis of tensile test specimens and previously reported. Reference 5 discusses the problems of predicting tensile fracture in concrete. For compressive (shear) fracture a maximum compressive strain assumption of 0.003 was used. For ultimate stresses an octahedral shear - octahedral normal stress representation works well, however, for fracture the maximum strain assumption appears to better represent experimental data.

The method of solution was to obtain a stress-strain relationship for the non-linear material response by computing the strains from STIFF1 and modifying the elastic parameters E_s and E_θ (E_n remains constant) such that

$$\begin{bmatrix} \epsilon_{nn} \\ \epsilon'_{ss} \\ \epsilon'_{\theta\theta} \\ \epsilon_{ns} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_n} & -\frac{\nu_{ns}}{E_n} & -\frac{\nu_{n\theta}}{E_n} & 0 \\ -\frac{\nu_{ns}}{E_n} & \frac{1}{E'_s} & -\frac{\nu_{s\theta}}{E'_s} & 0 \\ -\frac{\nu_{n\theta}}{E_n} & -\frac{\nu_{s\theta}}{E'_s} & -\frac{1}{E'_\theta} & 0 \\ 0 & 0 & 0 & \frac{1}{G_{ns}} \end{bmatrix} \begin{bmatrix} \sigma_{nn} \\ \sigma_{ss} \\ \sigma_{\theta\theta} \\ \sigma_{ns} \end{bmatrix} + A \begin{bmatrix} \alpha_n \\ \alpha_s \\ \alpha_\theta \\ 0 \end{bmatrix}$$

where the prime strains and elastic parameters are computed by use of the stresses from the finite element program and STIFF1. When the computed elastic parameters agreed with the assumed elastic parameters the solution was deemed valid for the load magnitude. Inherent is the assumption that the axes of orthotropy coincide with the s - θ axes of the shell and the reinforcement. Specific cases for other than this assumption are given in the SASS II paper.

III. Results of Stress Analysis - Static Loading

A. Comparison of Analysis and Wiss, Janney, Elstner and Associate Model Results.

The model studies performed by Wiss, Janney, Elstner and Associates have been reported and the intent herein is to compare the results of the model study with the method of analysis outlined in the previous section. Specifically, a comparison of measured and predicted steel stresses is the most rigorous method of evaluating a method of analysis. Comparison of measured and predicted deflections does not provide as critical a test since the stresses are functions of the derivatives of displacement fields.

Figure 2 illustrates a plot of the nodal points for the model analysis. A minimum of four and a maximum of seven elements were used through the wall thickness. Five hundred and sixty-four nodal points with four hundred and fifty-five elements created the geometry for the model analysis.

Figures 3, 4 and 5 provide a comparison of stress results from the Wiss, Janney and Elstner model report and the method of analysis presented herein. The comparison is presented for a 1200 psf loading condition

Figure 2 - Plot of Nodal Points for WJE Model Analysis

Figure 3 - NS (Radial) Stress Distribution for WJE Model
(Outer Surface)

Figure 4 - Hoop (θ) Stress Distribution for WJE Model
(Outer Surfaces)

Figure 5 - Hoop (a) Stress Distribution for WJE Model
(Inner Surfaces)

with the predicted stresses at ultimate load also noted. Considering the variables present in the analysis and those in the model the writer is of the opinion the agreement of stress prediction and stress measurement is good. Among the variables assumed for the analysis that could differ from the actual values in the model are:

1. Placement of the steel in thin sections. A difference of 0.1" in steel placement could lead to a thirty per cent and greater difference in stress results.
2. Tensile fracture strength. A sixty psi lower bound strength was assumed for analysis purposes. For the model volume and stress gradients perhaps a mean tensile fracture strength of 300 psi would be appropriate.
3. The concrete stiffness possibly varied in the cylinder and domes from the measured values obtained from specimens.
4. Effect of surrounding concrete on strain gage measurements.
5. Yield strength of wire reinforcement greater than 30,000 psi.

Limits on time did not allow an investigation of the effect of these variables in the stress results.

The ultimate load results for the experimental model was 5400 psf (5.6 times working load of earth cover) while the analysis predicted an ultimate load of 4100 psi (4.3 times working load). The difference is -24%. However, the strength of the concrete in the cylinder of the experimental model was measured at 28 days as 2860 psi and 4920 psi in the dome. Subsequent to the first series of tests for the experimental model the concrete was subjected to steam during the thermal test. This certainly increased the concrete strength in both cylinder and dome, however, no data for this strength increase are available. If the strength

increase was to approximately 3900 psi the predicted results would agree with the experimental model results. An assumption of this magnitude increase is appropriate for the concrete and method of curing used. The analysis predicted fracture at the same point and in the same mode as occurred in the model. Of additional interest is the fact that the tensile fractures in the model were, as assumed for the analysis, parallel and perpendicular to the reinforcing steel directions.

B. Analysis results for 241-A Waste Tanks.

Stress results for the 241-A non-linear "orthotropic" analyses are illustrated in Figures 7, 8, 9 and 10. Figure 6 is a plot of the nodal points. The analyses for the 241-A waste tank were performed for the dome and cylinder with sliding boundary conditions at the cylinder base. Although data were created for analyzing the total structure and soil foundation the number of nodal points and elements exceeded the program capacity. The latter was being increased at the time the project was terminated. A total of 564 nodal points and 455 elements were used for modeling the waste tank dome and cylinder.

Results for the service or working loads indicate (Figures 7 and 8) the steel stresses are well within the allowables of 20,000 psi in both the meridional (s) and circumferential (θ) directions. The maximum stress is +15,000 in the outside meridional steel at the juncture of dome and cylinder.

Concrete compressive stresses are a maximum of 1220 psi at the juncture of cylinder and dome (inside surface) and outside this region vary from 100 to 500 psi.

Through thickness tensile fractures generated by the N_θ stress resultant are present in the haunch region. The fractures are parallel

Figure 6 - Plot of Nodal Points for 241-A Analysis

Figure 7 - 241-A s (Radial) Direction Reinforcing
Steel Stresses at Working Load

Figure 8 - 241-A θ (Circumferential) Direction
Reinforcing Steel Stresses at Working Load

Figure 9 - 241-A s Direction Reinforcing
Steel Stresses at Ultimate Load
50,000 psi Yield Stress

Figure 10 - 241-A θ Direction Reinforcing Steel Stresses at Ultimate Load
50,000 psi Yield Stress

to the meridions and extend from radius of 376" on the dome to a point on the cylinder approximately 30" below the dome-cylinder juncture. These tensile fractures should be less than .001" in width and not visible to the naked eye.

In general the waste tanks are well within the ACI-318 Code specifications for maximum allowable concrete and steel stresses.

The footing and cylinder adjacent to the footing were not included in these analyses, however, previous results for static loading have indicated the potential problems of leakage that may be encountered due to thermal loading.

Ultimate load results from these analyses are illustrated in Figures 9 and 10. Fracture of the concrete was indicated at the cylinder-dome juncture and this determined the ultimate load capacity. The concrete fracture was indicated as a shear fracture induced by compressive stresses with simultaneous yielding of the reinforcing in both sides of the shell. The shear fracture criterion for the concrete was a maximum normal compressive strain of 0.003. The fracture is due to the M_s moment in the meridian direction plus the axial force, N_s . Yielding of the reinforcing steel in the N_θ direction preceeding the fracture is indicated by Figure 10. The analysis was performed by incrementing the soil loads over the dome (maintaining constant lateral loads on Cylinder) to the ultimate load of 5900 psf at the crown (Z-axis). This is 6.2 times the working load, that is, the soil load on the dome. This factor differs from the experimental model for a number of reasons among which are:

1. The experimental model used a 30,000 psi yield steel, which was assumed for the analysis, vs. the 50,000 assumed for the

241-A analysis.

2. A non-linear rather than a linear relationship exists between model and prototype for nonlinear materials response.
3. Lateral soil pressures were incorporated in the 241-A analysis and not in the experimental model analysis.

The collapse mode indicated by the experimental model and the analyses herein should not lead to a catastrophic condition with respect to containment.

In both, these analyses and the experimental model, the footing has not been included as an element of the structure. It is possible that the footing may possess a lower ultimate load capacity than indicated by the model or these analyses. The ultimate load capacity, if limited by the footing, would lead to a critical condition for containment at ultimate load.

Generally, the ultimate load design criteria of AISC and ACI lead to load factors of 1.7 to 2.0 against collapse. For temporary loadings such as wind and earthquake the load factors reduce to approximately 1.3. Load factors for the Hanford waste tanks far exceed these criteria.

IV. Earthquake Analysis

1. Analysis Method and Input Data

The purpose of the earthquake investigation was to obtain data on the dynamic response of a buried waste tank to a step velocity dynamic loading and use this general result to solve the dynamic stress response of the vessel for earthquake pulse inputs, the latter by means of a Duhamel integration. The waste tanks are of rotationally symmetric geometry; however, because the earthquake motion is assumed to be caused by plane waves whose direction of propagation is transverse to the waste tank axis of symmetry (i.e., nonsymmetric loading is present), a three-dimensional analysis is needed.

DYNAX is a dynamic, finite element computer code (developed at IITRI) which is capable of solving dynamic elasticity problems involving non-axisymmetric loadings of arbitrarily shaped solids (and/or shells) of revolution. To apply this computer code, the shell structure and surrounding media are modeled by means of finite elements (Fig. 11). The dynamic loading is applied by specifying the displacements along the outside surface of the finite element model. The dynamic loading generates stress waves which propagate to the structure and interact with it. The stress resultants, displacements, velocities and accelerations in the structure are computed and postprocessed in the form of punched cards. Appendix B contains the governing equations for the finite element method and dynamic loading method used in the DYNAX program. These were provided by IITRI along with the punched card output.

The material constants specified for the soil were:

Rayleigh wave speed, $C_R = 1970$ ft/sec

Poisson's ratio, $\nu = 0.42$

Fig. 11 Finite Element Model

For an isotropic media, the shear wave speed can be computed from the Rayleigh equation (Ref. B2)

$$\left(\frac{c_R}{c_2}\right)^6 - 8 \left(\frac{c_R}{c_2}\right)^4 + c_R^2 \left(\frac{24}{c_2^2} - \frac{16}{k^2 c_2^2}\right) - 16 \left(1 - \frac{1}{k^2}\right) = 0$$

which yields $c_2 = 2083$ ft/sec. The dilatational wave speed is computed by Eq. (B2), which gives $c_1 = 5609$ ft/sec.

The incident angle was chosen so that the ratio of the vertical to horizontal velocities $\frac{\dot{w}}{\dot{u}}$ is $4/3$. This yielded an incident angle $\theta_1 = 31$ deg.

The incident wave was arbitrarily assigned an intensity A_1 of - 1000. Using Eqs. (B9 through B11) the magnitudes of the reflected waves were then computed to be

$$A_2 = + 792.8$$

$$B = - 273.8$$

and the angle of the reflected shear wave

$$\theta_2 = 71.3 \text{ deg.}$$

The magnitudes of the resulting velocities at the origin, as given by Eqs. (B13) are

$$\begin{aligned} \dot{u} &= - 0.156 H(t) \quad \text{ft/sec} \\ \dot{w} &= + 0.206 H(t) \quad \text{ft/sec} \end{aligned} \tag{15}$$

therefore the response to a unit vertical step velocity can be obtained by multiplying all output quantities (i.e., either displacement, velocity, acceleration or stress resultant) by a factor of $(1/0.206) = 4.845$.

The stresses are constant between each wave, and are given in the table on the following page for the three regions indicated in Fig. 1.

Stress	Region		
	I	II	III
σ_{xx}	-3226 lb/ft ²	-668 lb/ft ²	-1243 lb/ft ²
σ_{zz}	-2775 lb/ft ²	-575 lb/ft ²	0
σ_{xz}	424 lb/ft ²	760 lb/ft ²	0

2. Results from DYNAX

The soil and surrounding media were represented by a finite element model consisting of 197 elements and 200 nodes shown in Fig. 11. As can be seen from the figure, the boundary of the media is a minimum of 180 ft. beyond any point of the structure. The response of the model is only valid for the time needed for a wave to travel from the structure to the edge of the media and back. This is because the wave reflected off the edge of the media is not present in the actual problem and of course would lead to large errors in the solution if the program continued to run. Therefore, the model was used to obtain the structural response over a 0.0625 sec duration. The time integration was performed in 250 time steps in increments of 0.25 msec.

The DYNAX investigation was limited to providing information on the response of the structure to the dynamic step loading described in Appendix B. The following response variables were provided as functions of time: N_{ss} , $N_{\theta\theta}$, M_{ss} , $M_{\theta\theta}$. Each stress resultant was provided at 15 points of the waste tank at both $\theta = 0$ and $\theta = 90$ deg and consists of 24 points at intervals of 0.0025 sec starting from the instant at which

the incident wave impinges the vessel.

In addition to these records, the displacement, velocity and acceleration of the center of the shell at $\theta = 0$ were plotted during the computer run. These results are shown in Figs. 12, 13 and 14. The primary purpose of this output is to check whether the response of the waste tank reaches a steady state. If the steady state is not reached, the response functions cannot be used to generate response to pressure waves with arbitrary time histories over time ranges greater than the time range used in DYNAX. As can be seen from Fig. 14, from $t = 0.032$ sec the acceleration oscillates about zero, which indicates that the waste tank has for all practical purposes reached a steady state. These oscillations are of a type that generally occur in these computer simulations and are undoubtedly caused by numerical and approximation errors.

A second observation is that the shell (waste tank) response to the soil stress wave is within 0.015 secs. and being this rapid a peak or maximum acceleration segment over a short time interval may be removed from an acceleration record and used in the convolution formula to obtain the maximum waste tank stresses. This, in fact, obviates the need to integrate and compute stresses for the entire acceleration record. This is in contrast to the need for integrating from five to fifty seconds of the acceleration records to obtain above ground structure responses.

3. Stress Results

Waste Tank Stresses for .16g Earthquake (OBE) The acceleration record used was taken from the paper "Simulated Earthquake Motions" by P. C. Jennings, G. W. Hausner and N. C. Tsai, California Institute of Technology, April, 1968. The soil-waste tank system response in terms

Fig. 12 Radial Displacement Trajectory (for "Response Node" in Fig. 11)

Fig.13 Radial Velocity Trajectory (for "Response Node" in Fig.11)

Fig. 14 Radial Acceleration Trajectory (for "Response Node" in Fig. 11)

of the eight stress resultants was obtained for a 0.135 second period by means of the Duhamel integration. See Fig. 15 for that portion of the total record used in this analysis.

Results from the Duhamel integration were then used as input for calculation of the steel and concrete stresses in the waste tanks due to the earthquake loading. Stress computations were performed at each of the fifty-four time increments for the fifteen station points in the waste tank in the circumferential and meridional directions, inside and outside surfaces. (6480 point stress results).

Dome of Waste Tanks Six output points each at 0° and 90° to the wavefront were used to define the stresses in the dome. Three of the six were located in the region where the dome joints the cylinder (haunch region). In the circumferential direction the waste tank reinforcing steel stresses are indicated to be less than 5000 psi at all points for 0° and 90° to the wave front. The concrete stresses are correspondingly low in value. Linear superposition of the earthquake stresses onto the dead load stresses for the final resultant stress in the circumferential direction yields stress values less than 12,000 psi. This is well below any working stress values suggested by structural codes (usually 20,000 psi is recommended).

In the meridional direction all reinforcing steel stresses are less than 5,000 psi with the following exception. At the dome-cylinder juncture there is indicated a maximum stress of 16,500 tension on the inside steel and + 11,200 tension on the outside steel. These stresses in the earthquake model are created by the angle of the wave front and the large Poisson's ratio of the soil (.43) relative to the axial stiffness of the waste tank shell. For this analysis the common node points of the solid

soil elements and waste tank shell elements must have the same displacement field. In other words, the soil elements were elastically connected to the shell elements and the wave front caused vertical expansion of the soil by Poisson's ratio and this phenomenon with the angle of the wave front created tension in the cylinder wall of the waste tank. This occurred at 0° as well as 90° to the soil stress wave.

Since in the actual case the only connection between the waste tank and soil is friction, judgement must be used in interpretation of these stress values. It is the writer's opinion that these tensile steel stresses should be interpreted as being less than 5000 psi. It is not conceivable that the soil stress in the step-velocity pulse and the attendant vertical expansion due to Poisson's ratio could create, through friction, surface tractions in the order of 25,000 lb/ft.

Cylinder Output from four station points each at 0° and 90° were used to define the stresses. All reinforcing steel stresses, with one exception, were computed as less than 5000 psi. The one exception occurred in the cylinder approximately eight feet down from the tangent line to the dome. This indicated maximum tensile stress of 16,900 psi in the outside steel is again caused by the approximation in the model, elastic connections between the soil solid elements and the waste tank shell elements. This stress must be interpreted as being of lower value, in the writer's opinion, less than 5000 psi. For the 5000 psi earthquake stress the final stresses, dead load plus earthquake, add to the value of 17,000 psi.

Footing and Slab Stresses Computer output from five station points each at 0° and 90° to the wave front were obtained. All reinforcing steel stresses in the slab and footing were computed as being less than 5000 psi.

Waste Tank Stresses for 0.25g Earthquake (DBE)

The earthquake

acceleration record used for these results is reproduced in Fig. 15.

The maximum acceleration is 8.1 ft/sec^2 (.25g) in the vertical direction and 6.0 ft/sec^2 (.187g) in the horizontal direction. The record length used for computation of waste tank response is 0.135 secs.

All the results discussed herein were obtained by linear superposition of the static load orthotropic stresses (temperature plus soil loading) and the earthquake stresses for the total solution.

Dome Region (0° to Wave Front) The maximum reinforcing stress is 17,500 psi tension on the exterior at the dome-cylinder juncture and is attained at time approximately 0.13 secs. of the acceleration record. The earthquake stress is 4500 psi tension with the static load stress being 13,000 psi tension. Interpretation of this stress appears to indicate that it is due to additional dome loading created by the reflected p-wave and q-wave (shear wave) plus tension due to Poisson's ratio expansion of the soil.

All other reinforcing steel stresses are less than 5000 psi at 0° to the wave front.

Dome Region (90° to Wave Front) Again the maximum stresses are tension and occur at the dome-cylinder juncture. The total (static plus earthquake) stress is 25,500 psi and is on the outside re-steel of the dome. It is possible that the soil Poisson's ratio effect may be the determining factor in this as well as the previous stress magnitude at 0° . The large vertical lateral expansion of the soil and its elastic connection to the waste tank leads to tensile stresses in the cylinder as well as the cylinder-dome juncture.

The 25,500 psi tensile stress is probably greater than the actual

EARTHQUAKE C-1

Figure 15

Acceleration, velocity and displacement
for earthquake C-1

stress but it still is within the allowable stress (40,000 psi) for the "maximum possible" earthquake.

Cylinder (0° and 90° to Wave Front) The cylinder is subjected to membrane tension along the entire length and, again, this is due to the large Poisson's ratio expansion of the soil in the vertical direction as the stress wave passes. The maximum total (static plus earthquake) stress is 39,000 psi tension in the reinforcing steel and is at 90° to the wave front.

Slab Region (0° and 90° to Wave Front) The maximum total re-steel stress in the slab is less than 15,000 psi and mainly consists of the static load tensile stress in the footing under the cylinder wall.

V. Conclusions

Static Loading

The method of analysis presented herein uses a finite element program to obtain stress resultants with an initial assumption of elastic parameters. These parameters are then modified by calculating the strains associated with the stress resultants and use of the element stresses. This is done until convergence between assumed and calculated elastic parameters agree. The strain computations are done in a routine external to the finite element program. This routine assures the principal stress axes and principal orthotropic material axes are normal and parallel to the reinforcement in the waste tanks. Results from the experimental model confirm these assumptions for the cylindrical shell and dome.

Comparison of stress results from the experimental model and analyses gave good qualitative agreement and in most regions good quantitative agreement for a 1200 psf loading, a loading that induced the use of

orthotropic elastic parameters in the analysis. The ultimate load prediction was 24 per cent lower than that exhibited by the experimental model. However, it is suggested that steam curing of the model concrete could be responsible for this difference.

Working load analyses for the 241-A waste tank indicated the concrete and steel stress are well within those recommended by the ACI. Ultimate load results for the 241-A revealed a load factor against collapse of 6.2. The ultimate load analysis was based on the soil load over the dome increasing without any corresponding increase in other force and temperature loading parameters. This ultimate load factor of 6.2 is three to four times those recommended for ordinary structures. Although the foundation footing was not included in these analyses previous results indicate the footing should exhibit a load factor of 4.0 or greater. However, fracture of the footing would probably lead to loss of fluid containment integrity whereas fracture of the cylinder does not.

Finally, the working stresses appear to be well within the recommended by ACI with the ultimate load capacity greater than that recommended by ACI. The one containment problem with these older waste tanks (before 241AY) is the thermal fracturing that occurs in the footing perimeter. These fractures can lead to leakage if the inner liner is breached, for example, by stress corrosion cracking.

OBE Loading

The results obtained from the earthquake analysis indicate the waste tanks to be adequately designed for the OBE earthquake design criteria. There is uncertainty concerning the tensile stresses in the cylinder, but, even taken at face value, these stresses will not cause additional risk

in loss of structural or containment integrity of the waste tanks during an earthquake of .16g vertical and .12g horizontal acceleration.

DBE Loading

A general comment with respect to the stresses in all regions of the waste tank is that the Poisson's ratio expansion of the soil creates dominating membrane tension in the cylinder and in the cylinder-dome juncture while the remaining regions are well within allowable elastic stresses. The maximum stress of 39,000 psi in the cylinder is within the allowable stress of 40,000 psi for the maximum possible earthquake, however, the writer is of the opinion this is misleading as to the actual safety factor, the actual factor of safety against loss of structural integrity for static plus DBE loading would be at least four. In addition there should be no additional risk of containment integrity due to this loading.

Respectfully submitted,

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Appendix A

The purpose of this appendix is to illustrate the development of equations that lead to unidimensional relations between stress resultants, concrete and steel stresses and strains in the shell cross-section.

1. Assumptions for Equations:

Figure 1 illustrates a stress-strain curve for concrete indicating the non-linear relationship as a basis for these equations. The assumptions made, based on such a curve, are:

- a. Plane cross-sections remain plane.
- b. The stress-strain relationship for compression is:

$$\sigma_c = .85 \sigma'_c \left[2\epsilon_c/\epsilon_o - (\epsilon_c/\epsilon_o)^2 \right] \text{ - - - - - (1)}$$

- c. The fracture strength of the concrete is attained at a strain value of .003. This corresponds to a limiting stress value for compressive stress for a unique strain-stress relationship.
- d. The modulus of elasticity is constant in tension and corresponds to the initial modulus in compression.
- e. The tensile strength is fixed in value to a "lower bound strength" and when exceeded the tensile fracture propagates to the neutral axis. From Figure 2 the stress-strain relationship for steel is taken as:

$$f. \quad \sigma_s = E\epsilon \text{ for } \epsilon \leq \sigma_y/E \text{ - - - - - (2)}$$

$$g. \quad \sigma_s = \sigma_y + (\epsilon - \epsilon_y) E_p$$

2. Nomenclature:

A_s = Area of tensile steel (inside face)

A'_s = Area of comp. steel (outside face)

Figure 1 - Concrete $\sigma_c - \epsilon_c$ Curve

Figure 2 - Steel $\sigma_s - \epsilon_s$ Curve

b = Width of rectangular section
 d = Overall depth of section
 E_c = Initial modulus of elasticity of concrete = $1,000 \sigma'_c$
(secant modulus)
 E_s = Secant modulus of elasticity of steel
 E_p = Plastic modulus of elasticity of steel
 ϵ_c = Strain in concrete at top, bottom, or center of section with
subscripts T, B and Ca, respectively
 ϵ_{cmax} = Limiting compressive strain in concrete
 ϵ_o = Strain in concrete at maximum compressive stress
 ϵ_s = Strain in steel (tensile steel)
 ϵ'_s = Strain in steel (compressive or outer face steel)
 ϵ_t = Limiting tensile strain in concrete
 ϵ_y = Strain in the concrete fiber at a distance y_d , from top
 $\bar{\epsilon}$ = Strain at the central axis of cross-section (where "N" is acting)
 e_c = ϵ_c / ϵ_o
 e_s = ϵ_s / ϵ_o
 e_t = ϵ_t / ϵ_o
 σ_c = Stress in concrete fiber at top, bottom or center of section
with subscripts T, B, or Ca, respectively
 σ_s = Stress in tensile or inner steel
 σ'_s = Stress in compressive or outer steel
 kd = Depth of neutral axis from top fiber
 y_d = Depth of any concrete fiber from the top of section
 αd = Depth of inner (tensile) steel from top of section
 γd = Depth of outer (compressive) steel from top of section
 θ' = Central angle of curvature
 θ = θ' / ϵ_o

ω = Curvature = $\theta \epsilon_o / d$

θ' / d = Angle per unit length

Δk = Increment in the value of k

M = Bending moment at section

N = Thrust at section

p = A_s / bd

p' = A'_s / bd

M' = $Ne = M + N (\alpha - 1/2)d$ (moment about tensile steel of external resultants)

N' = $N / \sigma'_c bd$

σ'_c = Cylinder strength of concrete

3. Equations:

Figures 3 and 4 illustrate the geometry that needs to be considered for the development of the equations.

From Figure 3 $\theta' = d/R = \omega d$ - - - - - (3)

$$\text{and } \epsilon_y = \frac{R\theta' - [R - (k-y)d]\theta'}{R\theta'} = (k-y) d/R = (k-y)\theta'$$

$$\epsilon_s = \frac{[(\alpha - k) d + R]\theta'}{R\theta'} = (\alpha - k)\theta'$$

$$\epsilon'_s = \frac{R\theta' - [R - (kd-yd)]\theta'}{R\theta'} = (k-\gamma)\theta'$$

Let $e_y = \epsilon_y / \epsilon_o$, $e_s = \epsilon_s / \epsilon_o$, $e'_s = \epsilon'_s / \epsilon_o$, $\theta = \theta' / \epsilon_o = \omega d / \epsilon_o$

The total compressive force is: $c \int_0^k \sigma_c bd dy + A'_s E_s e'_s \epsilon_o$

Figure 3

Element of length " d " (depth of section) at N.A. shown in curvature diag. Shaded area is neglected when $\epsilon_{cB} > \epsilon_t$ (cracked case).

Figure 4

and upon substitution with simplification:

$$C = .85 \sigma_c bd (\theta k^2 / 3) (3 - \theta k) + A'_s E'_s \epsilon_o (k - \gamma) \theta \quad - - - - \quad (4)$$

The total tensile force in the concrete is: $T_c = \int_k^1 \sigma_c bd dy$

and substituting $f_c = -1.7 \sigma'_c e_y$

$$T_c = .85 \sigma'_c bd (1 - k^2) \theta$$

$T_c = 0$ if tensile strength of concrete exceeded in outer fiber.

The tensile force developed by the reinforcing steel is:

$$T_s = A_s E_s \epsilon_o (\alpha - k) \theta$$

From equilibrium on the cross-section with no tensile cracks:

$$-N = \int -.85 \sigma'_c bd k^3 / 3 \theta^2 - \left[.85 \sigma'_c bd (1 - 2k) + \epsilon_o bd \{ E_s p (\alpha - k) - E'_s p' (k - \gamma) \} \right] \theta$$

$$\text{or } N' = \frac{-N}{\sigma'_c bd} = c_1 \theta^2 + c_2 \theta \quad - - - - - \quad (5)$$

$$c_1 = -.85 k^3 / 3$$

$$c_2 = - \left[0.85 (1 - 2k) + \frac{\epsilon_o}{\sigma'_c} \{ E_s p (\alpha - k) - E'_s p' (k - \gamma) \} \right]$$

When tensile cracks are present:

$$N' = \frac{-N}{\sigma'_c bd} = c_1 \theta^2 + c_3 \theta \quad - - - - - \quad (6)$$

$$c_3 = - \left[.85 k^2 + \frac{\epsilon_o}{\sigma'_c} \{ E_s p (\alpha - k) - E'_s p' (k - \gamma) \} \right]$$

From considerations of equilibrium the bending moment developed by the concrete may be calculated as:

$$M_{c1} = .85 \sigma_c bd^2 \left[\theta k^2 \left(\frac{3\alpha - 5k}{3} \right) - \theta^2 k^3 \left(\frac{4\alpha - k}{12} \right) \right]$$

for tensile force in the concrete and with no tensile fractures:

$$M_c = M_{Tc} = .85 \sigma'_c b d^2 \frac{\theta}{3} (3\alpha - 2 + 3k - 6\alpha k - 3k^2 \alpha - k^3)$$

The moment of the compressive steel force about the tensile steel is:

$$M_{T's} = A'_s E'_s \epsilon_o (k - \gamma) (\alpha - \gamma) \theta d$$

The moment of the axial force about the steel is:

$$M_N = -N (\alpha - 1/2) d$$

$$\therefore M + M_N = M_{cl} + M'_{Ts} - M_{Tc}$$

Upon substitution and solving for the applied moment M' when no tensile fractures are present:

$$M' = \frac{M - N(\alpha - 1/2)d}{\sigma'_c b d^2} = C_4 \theta^2 + C_5 \theta \quad \text{--- (7)}$$

$$C_4 = -.85 k^3 \left(\frac{4\alpha - k}{12} \right)$$

$$C_5 = \frac{0.85}{3} \{ 6k^2 \alpha + 6k\alpha - 3k - 3\alpha + 2 - 4k^3 \} + \frac{E'_s \epsilon_o}{\sigma'_c} p' (k - \gamma) (\alpha - \gamma)$$

When tensile fractures are present:

$$M' = C_4 \theta^2 + C_6 \theta \quad \text{--- (8)}$$

$$C_6 = .85 \left(\frac{3k^2 \alpha - 5k^3}{3} \right) + \frac{E_s \epsilon_o}{\sigma'_c} p' (k - \gamma) (\alpha - \gamma)$$

Equations 5 or 6 and 7 or 8 are then used to eliminate ' θ ' and a fifth or sixth degree equation in terms of k is obtained. Upon solving for ' k ' the concrete and steel stresses are obtained. Various other conditions ($k > 1$, N being tensile) lead to first, fourth, fifth and sixth degree equations.

Appendix B

I. Dynamic Loading

1. Governing Equations

The free field loading on the containment vessel was generated by an incident pressure wave, followed by a reflected pressure wave and a reflected shear wave especially chosen so that the stress free surface conditions are satisfied. As shown in Fig. B2, the incident and reflected pressure wave fronts are inclined at an angle θ_1 to the free surface normal, while the reflected shear wave front is inclined at an angle θ_2 . Both wavefronts extend infinitely downward.

The propagating material is assumed to be an isotropic, elastic solid. The wave propagation problem is then governed by the equations of motion (in potential form) (Ref. B1).

$$\begin{aligned}\nabla^2 \phi - \frac{1}{c_1^2} \ddot{\phi} &= 0 \\ \nabla^2 h - \frac{1}{c_2^2} \ddot{h} &= 0\end{aligned}\tag{B1}$$

where a superscript dot denotes a time derivative and ϕ , h represent the pressure and shear wave potentials. The dilatational wave (p-wave) velocity c_1 and the shear wave (s-wave) velocity c_2 are given by

$$\begin{aligned}c_1^2 &= \frac{\lambda + 2\mu}{\rho} \\ c_2^2 &= \frac{\mu}{\rho} = \frac{(1-2\nu) c_1^2}{2(1-\nu)}\end{aligned}\tag{B2}$$

where λ and μ are the Lamé constants, ν is Poisson's ratio and ρ the density of the material. The relationship between the displacements and the potentials are given by

Fig.B2 Dynamic Loading

Fig.B3 Sign Conventions on Shell Element

$$u = \frac{\partial \varphi}{\partial x} + \frac{\partial h}{\partial z} \quad (B3)$$

$$w = \frac{\partial \varphi}{\partial z} - \frac{\partial h}{\partial x}$$

The stresses are given in terms of the potential functions by

$$\sigma_{zx} = \mu \left(2 \frac{\partial^2 \varphi}{\partial x \partial z} - \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} \right) \quad (B4a)$$

$$\sigma_{zz} = \lambda \nabla^2 \varphi + 2\mu \left(\frac{\partial^2 \varphi}{\partial z^2} - \frac{\partial^2 h}{\partial x \partial z} \right) \quad (B4b)$$

$$\sigma_{xx} = \lambda \nabla^2 \varphi + 2\mu \left(\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 h}{\partial x \partial z} \right) \quad (B4c)$$

Any wave form must satisfy the governing equations (B1) and (B2) and the traction free stress conditions on the free surface

$$\sigma_{xz} \Big|_{z=0} = \sigma_{zz} \Big|_{z=0} = 0 \quad (B5)$$

2. Step Wave Function

For purposes of making the solution amenable to arbitrary pulse shape inputs, a waveform with a step velocity (acceleration impulse) was chosen. For convenience, let us define the following functions by:

$$\delta(x) = 0 \quad \text{if } x \neq 0; \quad \int_{-\infty}^{\infty} \delta(x) dx = 1$$

$$H(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

$$L(x) = \begin{cases} x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases} \quad (B6)$$

$$Q(x) = \begin{cases} \frac{1}{2}x^2 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

where δ , H and L are the Dirac delta function, step function and ramp function respectively. A solution to Eqs. (B1) can be shown to be given by

$$\begin{aligned} \phi &= A_1 Q \left[k_1 (x \cos \theta_1 - z \sin \theta_1) + t \right] \\ &\quad + A_2 Q \left[k_1 (x \cos \theta_1 + z \sin \theta_1) + t \right] \\ h &= B Q \left[k_2 (x \cos \theta_2 + z \sin \theta_2) + c_2 t \right] \end{aligned} \quad (B7)$$

where

$$k_1 = \frac{1}{c_1}, \quad k_2 = \frac{1}{c_2} \quad (B8)$$

The free surface conditions (B5) then yield the following values for θ_2 , A_2 and B

$$\cos \theta_2 = \frac{c_2}{c_1} \cos \theta_1 \quad (B9)$$

$$A_2 = \frac{A_1}{\Delta} \left[\sin 2\theta_1 \sin 2\theta_2 - (2 \cos^2 \theta_1 - \kappa^2) \cos 2\theta_2 \right] \quad (B10)$$

$$B = \frac{A_1}{\kappa^2 \Delta} \left[2 \sin \theta_1 (\kappa^2 - 2 \cos^2 \theta_1) \right]$$

where

$$\Delta = \sin 2\theta_1 \sin 2\theta_2 - (\kappa^2 - 2 \cos^2 \theta_1) \cos 2\theta_2 \quad (B11)$$

$$\kappa = \frac{c_1}{c_2} = \sqrt{\frac{2(1-\nu)}{1-2\nu}}$$

As can be seen from these relationships, the relative intensities of the waves depend only on the incident angle of the pressure wave and Poisson's ratio.

3. Velocities and Accelerations

The velocities associated with the potential solutions (B7) are

$$\begin{aligned}
\dot{u} = & A_1 k_1 \cos \theta_1 H \left[k_1 (x \cos \theta_1 - z \sin \theta_1) + t \right] \\
& + A_2 k_1 \cos \theta_1 H \left[k_1 (x \cos \theta_1 + z \sin \theta_1) + t \right] \\
& + B k_2 \sin \theta_2 H \left[k_2 (x \cos \theta_2 + z \sin \theta_2) + t \right]
\end{aligned}
\tag{B12}$$

$$\begin{aligned}
\dot{w} = & A_1 k_1 \sin \theta_1 H \left[k_1 (x \cos \theta_1 - z \sin \theta_1) + t \right] \\
& + A_2 k_1 \sin \theta_1 H \left[k_1 (x \cos \theta_1 + z \sin \theta_1) + t \right] \\
& - B k_2 \cos \theta_2 H \left[k_2 (x \cos \theta_2 + z \sin \theta_2) + t \right]
\end{aligned}$$

The formulas for the accelerations are identical with the exception that the H-function is replaced by the δ -function.

At the origin, the velocities are given by

$$\begin{aligned}
\dot{u} &= k_1 \left[(A_1 + A_2) \cos \theta_1 + B k \sin \theta_2 \right] H(t) \\
\dot{w} &= k_1 \left[(A_2 - A_1) \sin \theta_1 - B k \cos \theta_2 \right] H(t)
\end{aligned}
\tag{B13}$$

From above it can be seen that at the origin the velocity is a single step wave wherein its magnitude depends on the intensity of the original wave, the angle of inclination and Poisson's ratio.

4. General Response

Since the input wave is a velocity step at the origin, the structure's response to this wave can be used to determine the response of the structure to incident pressure waves inclined at an angle θ_1 with arbitrary time histories. Let the structural response to the step input be denoted by $r(t)$. If the arbitrary pressure wave results in a velocity $g(t)$ at the origin, then the response of the structure $s(t)$ to the arbitrary pressure wave can be computed by means of a convolution formula of the form (Ref.B2)

$$s(t) = r(t) \dot{g}(0) + \int_0^t r(t-\tau) \ddot{g}(\tau) d\tau \quad (B14)$$

Consequently for the determination of the response, only the step response $r(t)$ and the acceleration record at the origin, $\ddot{g}(t)$, are needed. It should be noted that the arbitrary response obtained in this manner is based on the assumption that the dynamic loading is caused by an incident pressure wave inclined at angle θ_1 followed by the reflected pressure and shear waves. The actual wave pattern in an earthquake is far more complex, for it involves these waves inclined at various angles plus the long time Rayleigh wave effect.

II. Basis of Finite Element Method

The computer program DYNAX was developed for the harmonic analysis of rotationally symmetric structures and their surrounding media. It is basically a dynamic extension of Dunham's static finite element analysis of solids of revolution (Ref. B3). The detailed finite element derivation is much too lengthy to present here, thus, only a brief sketch of the method is given.

The first step in the finite element method is to approximate the continuous media with a finite number of elements (regions) that have common joints (nodal points). Since the code is restricted to arbitrarily shaped solids of revolution the basic geometrical form of these elements is selected to be a triangular cross section torus or a quadrilateral cross section torus. The total solid media can be constructed from any combination of these two kinds of elements. The joints are the junctions located at the corners of each triangle or quadrilateral as viewed in an r - z (radial-axial) slice (e.g., Fig.11) of a cylindrical coordinate system.

Although cross-sectional plots, such as Fig. 11, give the appearance that the elements are two-dimensional, they are actually three dimensional since each element cross section is revolved about the axis of revolution to form a toroidal element. To describe the behavior of the finite element system, a displacement field, expanded in a Fourier series in the θ -direction is assumed within the m^{th} element of the form (e.g., Ref. B3)

$$\begin{aligned} u_r^m(r, z, \theta, t) &= \sum_{n=0}^N u_{rn}^m(r, z, t) \cos(n\theta) \\ u_z^m(r, z, \theta, t) &= \sum_{n=0}^N u_{zn}^m(r, z, t) \cos(n\theta) \\ u_\theta^m(r, z, \theta, t) &= \sum_{n=0}^N u_{\theta n}^m(r, z, t) \sin(n\theta) \end{aligned} \quad (\text{B15})$$

where r, z, θ = the radial, axial, and angular cylindrical coordinates, t = time, n = the mode number, $N + 1$ = the number of modes and the coefficients $u_{rn}^m, u_{zn}^m, u_{\theta n}^m$ are the unknown distributions of space and time. If we assume an approximate displacement field for the unknown coefficients which is, say, linear in the coordinates r and z throughout the toroidal element, we can then express Eq. (B15) in terms of the actual physical Fourier components of the displacements, $\{u_{on}^m(t)\}$ of corners of the m^{th} element. Thus Eq. (B15) can be written as

$$\begin{aligned} u_r^m(r, z, \theta, t) &= \sum_{n=0}^N f_1(r, z, \{u_{on}^m(t)\}) \cos(n\theta) \\ u_z^m(r, z, \theta, t) &= \sum_{n=0}^N f_2(r, z, \{u_{on}^m(t)\}) \cos(n\theta) \end{aligned}$$

$$u_{\theta}^m(r, z, \theta, t) = \sum_{n=0}^N f_3(r, z, \{u_{on}^m(t)\}) \sin(n\theta) \quad (B16)$$

for the m^{th} toroidal element. The unknowns in the problem are no longer a set of unknown functions of space and time but rather a finite set of unknown functions of time $\{u_{on}^m(t)\}$ (three per unconstrained node per mode). The functions of f_1, f_2, f_3 are linear in the elements of $\{u_{on}^m(t)\}$. Since a linear displacement field is used together with a harmonic expansion, it is possible to maintain compatibility of displacements along contiguous element interfaces by matching their nodal point displacements.

A set of equations like Eq. (B16) can be written for each toroidal element in the continuum discretization.

A lumped-parameter idealization of the actual solid in which the mass properties of the system are separated from the elastic properties of the system, is employed. A physical lumped-mass approximation is made wherein a weighted fraction of the total mass of each element is lumped at the element's nodal points.

The differential equations of motion can be established in a manner analogous to that described in Ref.B3 with the exception that we must construct the Lagrangian (kinetic minus potential energy) of the system, rather than the potential energy alone as in the case of statically loaded problems. The basic approach is to first write the Lagrangian of the axisymmetric solid in terms of a Ritz series by first computing the Lagrangian of each finite element and summing over all elements. The Lagrangian of one element may be expressed as $L^m = T^m - V^m$ (kinetic minus potential energy) or

$$L^m = \left(\int_B \frac{1}{2} \rho \dot{u}_i \dot{u}_i dv \right) - \left(U - \int_B f^i u_i dv - \int_{S'} p^i u_i ds \right) \quad (B17)$$

where

$$U = \int_B \frac{1}{2} \tau^{ij} \epsilon_{ij} dv = \text{strain energy} \quad m = \text{the element label (total of } M \text{ elements)}$$

$$B = \text{body region of toroidal element} \quad \tau^{ij} = \text{stress tensor}$$

$$dv = \text{volume element differential} \quad \rho = \text{mass per unit volume}^*$$

$$S' = \text{portion of the surface on which stresses are prescribed} \quad \epsilon_{ij} = \text{strain tensor}$$

$$ds = \text{surface element differential} \quad f = \text{body force vector field}$$

$$p^i = \text{prescribed surface tractions on } S' \quad u_i = \text{displacement vector field}$$

The linear elastic constitutive equations ($\tau^{ij} = C^{ijk} \epsilon_k$), the assumed displacement field (Eq. B16), the strain displacement relationships in cylindrical coordinates and the nodal lumped masses are substituted into Eq. (B17). Since the θ -dependence is known explicitly in Eq. (B17), the integrals may be carried out directly. Further, as a consequence of the orthogonality of the trigonometric functions on the interval $0 \leq \theta \leq 2\pi$, double sums over the modes resulting from products of displacement appearing in Eq. (B17) reduce to single summations.

Summing over all toroidal elements, we can express the total Lagrangian of the system in terms of the unknown Fourier components

$$\{u_{on}^m(t)\} \quad T = \sum_{m=1}^M \sum_{n=0}^N L^m(\{u_{on}^m(t)\}, \{\dot{u}_{on}^m(t)\}) \quad (B18)$$

Next, Hamilton's variational principal, which states that the true solution is the one for which the variational operation

* For nodal lumped masses, the integral expression for T^m reduces to a finite summation over the element nodes.

$$\delta \int_{t_1}^{t_2} L_T = 0 \quad (B19)$$

is met for any two fixed end points t_1 and t_2 , is used.

The unknown Fourier components are treated as the dynamic generalized variables, e.e., $\{q\} = \{u_{on}^m(t)\}$, $\{\dot{q}\} = \{\dot{u}_{on}^m(t)\}$ and the Lagrange equations of motion equivalent to Eq. (B19)

$$\frac{d}{dt} \left(\frac{\partial L_T}{\partial \dot{q}_i} \right) - \frac{\partial L_T}{\partial q_i} = 0 \quad (B20)$$

are employed (q_i are the elements of $\{q\}$). Performing the operation indicated by Eq. (B20) eventually leads to a differential equation for each Fourier term of the form*

$$[M] \{\ddot{U}_n\} + [K_n] \{U_n\} = \{F_n\} \quad n = 0, 1, \dots, N \quad (B21)$$

where $[M]$ is the diagonal mass matrix; $\{U_n\}$ is an ordered vector of all the unknown displacement variables, $\{u_{on}^m(t)\}$, for the entire finite element assemblage; $[K_n]$ is the stiffness matrix; and $\{F_n\}$ is the Fourier component vector of the applied nodal forces.

The equations of motion (Eq. B21) are integrated by a noniterative version of the Newmark-Beta method (Ref. B4). The integration is formulated as the solution of the linear algebraic equations

$$[\bar{K}_n] \{U_n(t+\Delta t)\} = \{\bar{F}_n(t+\Delta t)\} \quad (B22)$$

where the effective stiffness and load are defined as

$$[\bar{K}_n] = [K_n] + \frac{1}{\beta(\Delta t)^2} [M]$$

*The standard notations (for matrices, $[]$, column vectors, $\{ \}$, and for time differentiation, $()$, are used in the explanation of the integration method.

and

$$\begin{aligned} \{ \bar{F}_n(t+\Delta t) \} &= \{ F_n(t+\Delta t) \} + [M] \left(\frac{1}{\beta(\Delta t)^2} \{ U_n(t) \} \right. \\ &\quad \left. + \frac{1}{\beta\Delta t} \{ \dot{U}_n(t) \} + \frac{1-2\beta}{2\beta} \{ \ddot{U}_n(t) \} \right) \end{aligned}$$

The parameter β must be within the range $0 \leq \beta \leq 1/4$. For $\beta = 1/4$ the method is infinitely stable and since Eq. (B22) is noniterative there are no convergence problems.

The solution to Eq. (B22) is obtained by the Choleski decomposition method. The decomposition is performed once for each value of n and may be reused during each incremental time step of the integration process.

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