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ISOTOPIC CONCENTRATIONS FOR A TIME-VARYING IRRADIATION OF URANIUM OR PLUTONIUM

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S. B. Gunst

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Expressions for isotopic concentrations as functions of irradiation are derived for uranium and plutonium having any initial isotopic composition. Fourteen isotopes including nine heavy elements and five fission products are considered. The decay of certain isotopes with half lives in the range from two hours to 13.2 years is taken into account as is the specific time-varying irradiation history. The expressions presented are applicable to any irradiation described by a succession of steps during which the thermal flux and epithermal flux are constant for each step but may vary from step to step. Expressions are also presented for quantities related to the isotopic concentrations including (1) the number of fissions in each fissionable isotope during each irradiation step; (2) the corresponding megawatt days per ton (of fuel initially present); (3) the accumulated fissions and megawatt days per ton, both isotopic and total; (4) the thermal, epithermal, and total conversion ratios present at the end of each irradiation step; and (5) the fractional changes in both η and η_f as a result of irradiation and as functions of time after removal from the high flux. A code for the Bell Interpretive System on the IBM-650 digital computer is described for the calculation of the isotopic concentrations and the related quantities.

ISOTOPIC CONCENTRATIONS FOR A TIME-VARYING IRRADIATION OF URANIUM OR PLUTONIUM

S. B. Gunst

The calculation of long-term reactivity changes in reactor fuels requires the solution of differential equations for the concentrations of all isotopes significant to the reactivity. To simplify the solutions, assumptions are ordinarily made. For example, isotopes of short half life are commonly assumed to have infinite decay constants which implies that their daughter isotopes are formed instantly, isotopes of long half life are assumed to be stable, and the ratio of epithermal-to-thermal flux is assumed to be constant throughout irradiation. Under such conditions, the isotopic concentrations depend not upon the true, detailed irradiation history but upon the total time-integrated thermal flux and the constant epithermal-to-thermal flux ratio. They do not depend explicitly upon time but rather upon the flux-time product. The expressions for isotopic concentration are particularly simple and involve initial (pre-irradiation) concentrations only. An exposure of any magnitude is treated as a single flux-time step; and the isotopic concentrations as a function of irradiation are obtained by solutions for steps of various total flux-times.

Although solutions based on such assumptions* have been useful in theoretical calculations, there are serious limitations:

- 1) The assumption that neutron capture in U-238 yields Pu-239 instantly is shown (Ref 1) to introduce an error in the calculated reactivity for irradiated natural uranium that may be as large as 2%.
- 2) The assumption of a constant ratio for the epithermal-to-thermal flux is unrealistic in many cases—e.g., the blanket of the Shippingport reactor in which the spatial power distribution, particularly close to the seed, changes over the lifetime. This ratio may vary severalfold for irradiations of more than one cycle in a facility such as the Materials Testing Reactor (MTR) where there are changes in loading from cycle to cycle.
- 3) The concentrations of short-lived isotopes and their daughter isotopes are particularly sensitive to variations in the irradiation history which occur over time intervals comparable to

*Work done by J. C. Connor of Bettis Atomic Power Division on the calculation of isotopic changes in irradiated uranium fuel specimens, January 1958.

the half lives. The concentrations of the fission products Xe-135, Sm-149, and their precursors, for example, are strongly dependent upon the detailed history.

- 4) If irradiated specimens are measured in a critical facility such as the Reactivity Measurement Facility (RMF) (Ref 2) within a few hours after irradiation, decaying isotopes will influence the measurement as will the detailed irradiation history prior to removal from the high flux.

To circumvent such limitations, isotopic-concentration calculations should take into account the decay of certain short-lived isotopes as well as the true, detailed irradiation history. A number-density code (S0269 for the Bell Interpretive System on the IBM-650 digital computer) for uranium and/or plutonium is described which, for any thermal and epithermal flux history, computes the isotopic number densities for nine heavy isotopes in two chains (U-235, U-236, U-237, Np-237; U-238, Np-239, Pu-239, Pu-240, Pu-241), and five fission-product isotopes in two mass chains (I-235, Xe-135; Nd-149, Pm-149, Sm-149). Although the decay of 23.5-minute U-239 and 2-minute Te-135 (precursors of Np-239 and I-135, respectively) is assumed to be instantaneous, the code specifically provides for the decay of 6.75-day U-237, 2.33-day Np-239, 13.2-year Pu-241, 6.68-hour I-135, 9.13-hour Xe-135, 2-hour Nd-149, and 53-hour Pm-149. How each of these might influence the reactivity in any irradiation history of interest may be deduced from code results.

For this program, the irradiation history is described by successive steps during which the thermal and epithermal fluxes are constant for each step, but may vary from step to step. A step-wise history is essentially realized in irradiation facilities such as the MTR; and in principle, any irradiation history may be approximated as closely as desired by steps.

Initial number densities for each of the fourteen isotopes mentioned may have arbitrary magnitude since the expressions give the number densities at the end of a step based upon (1) the number densities at the start of the step, (2) the time duration of the step, (3) the thermal flux and epithermal-to-thermal flux ratio during the step, and (4) the neutron interaction parameters appropriate for the step. Number densities are computed for an irradiation history consisting of any number of successive steps.

ISOTOPIC NUMBER DENSITY AND FISSION EXPRESSIONS

For heavy elements, the general expression for the time rate of change in the concentration $N^{i,j}$ of an isotope having atomic number i and atomic weight j is

$$\frac{dN^{i,j}}{dt} = N^{i,(j-1)} \sigma_{\gamma}^{i,(j-1)} \phi + N^{(i-1),j} \lambda_{\beta}^{(i-1),j} - N^{i,j} \sigma_a^{i,j} \phi - N^{i,j} \lambda^{i,j} \quad (1)$$

where t is the time, ϕ is the thermal flux, λ_{β} is a disintegration constant for β^- emission, and λ is a total disintegration constant. Each concentration $N^{i,j}$ is the number of atoms of a given isotope per unit volume. The quantities $\sigma_{\gamma}^{i,j}$ and $\sigma_a^{i,j}$ are the total effective capture and absorption cross sections for isotope i,j and include thermal and epithermal effects. By definition,

$$\sigma_{\gamma}^{i,j} \equiv \bar{\sigma}_{\gamma}^{i,j} + \frac{\phi_R}{\phi} R_{\gamma}^{i,j} \quad (2)$$

and

$$\sigma_a^{i,j} \equiv \bar{\sigma}_a^{i,j} + \frac{\phi_R}{\phi} R_a^{i,j} + \delta^{i,j} \quad (3)$$

where the subscripts γ and a refer to capture and absorption, respectively, $\bar{\sigma}^{i,j}$ is the effective thermal cross section, $R^{i,j}$ is the effective epithermal (not epicadmium) resonance integral, ϕ_R is the epithermal flux per unit lethargy for an epithermal distribution which is inversely proportional to the energy, and $\delta^{i,j}$ accounts for fast fission under the provision that $\delta^{i,j} \phi$ gives the probability

for fast-fission per unit time and per atom. The expressions tacitly assume that fast fission, epithermal absorption, and epithermal capture are proportional to the thermal flux during each irradiation step, even though they may vary from step to step.

In Eq (1) the first term on the right accounts for production by neutron capture, the second for production by β^- decay, the third for loss by neutron capture, and the fourth for loss by decay. Production by other processes such as alpha emission, (n, 2n), (n, d), and (n, p) is considered to be negligible. In integral form, Eq (1) may be written

$$N^{i,j} = e^{-\psi^{i,j}t} \int \left[N^{i,(j-1)} \sigma_{\gamma}^{i,(j-1)} \phi + N^{(i-1),j} \lambda_{\beta}^{(i-1),j} \right] e^{\psi^{i,j}t} dt + ce^{-\psi^{i,j}t}, \quad (4)$$

where

$$\psi^{i,j} = \sigma_a^{i,j} \phi + \lambda^{i,j}, \quad (5)$$

and c is an integration constant which may be evaluated by imposing the initial conditions.

For fission products, a term for production by fission must be added, and Eq (4) takes the form

$$N^{i,j} = e^{-\psi^{i,j}t} \int \left[N^{i,(j-1)} \sigma_{\gamma}^{i,(j-1)} \phi + N^{(i-1),j} \lambda_{\beta}^{(i-1),j} + \sum_{u,v} y_{i,j}^{u,v} N^{u,v} (\sigma_a^{u,v} - \sigma_{\gamma}^{u,v}) \phi \right] e^{\psi^{i,j}t} dt + ce^{-\psi^{i,j}t}, \quad (6)$$

where the last term in the brackets is the production by fission in fissionable isotope u, v (of atomic number u and atomic weight v), having effective fission cross section $(\sigma_a^{u,v} - \sigma_{\gamma}^{u,v})$ and a yield $y_{i,j}^{u,v}$ for the fission-product isotope i, j from the fission of isotope u, v. The summation includes all fissionable isotopes u, v.

For the k^{th} irradiation step of duration t_k , subscripts k should be attached to the quantities in Eqs (4), (5), and (6) which, though constant during any one step, may vary from one step to another*. At the end of the k^{th} irradiation step, the number density is described by $N_k^{i,j}$; while at the start of the k^{th} step, it is designated $N_{k-1}^{i,j}$. The steps run successively, with no interruption for intervals during which the specimen is out-of-pile, since such intervals are described by steps for which the flux is zero.

The solutions of Eq (4) for the nine heavy isotopes are given in Eqs (A1) through (A9) in Appendix A. Fast fission is accounted for in U-238 only. To obtain expressions for the fission-product concentrations, it is first necessary to derive expressions for the number of fissions $F_k^{u,v}$ of isotope u, v during the k^{th} irradiation step. Exact expressions for $F_k^{u,v}$ are given in Eqs (A10) through (A13) in Appendix A.

The Eqs (A1) through (A13) illustrate the increasing complexity of the algebra in expressions for the higher isotopes. Exact expressions for fission-product concentrations would, therefore, become prohibitively long. Expressions of reasonable length may be obtained by approximating the production from fission. If the fission rate in each fissionable isotope is assumed to be constant during any irradiation step k, the summation term in Eq (6) becomes a constant and may be written

$$\sum_{u,v} y_{i,j}^{u,v} (F_k^{u,v}/t_k),$$

*Although the subscripts k were intentionally omitted in Eqs (4), (5), and (6) to avoid confusion, they are included in the equations in the appendices.

where $F_k^{u,v}/t_k$ is the average fission rate during the step. The number-density equations for the five fission products may then be derived readily and are given in Eqs (A14) through (A18) in Appendix A. Despite the assumption of constant fission rates during each irradiation step, the number densities for the fission products may, in principle, be computed as accurately as desired by dividing each step into any number of shorter steps. The expressions for the heavy isotopes and the number of fissions remain exact.

For a step k in which the thermal flux ϕ_k is zero, Eqs (A1) through (A18) reduce to Eqs (A19) through (A36).

QUANTITIES RELATED TO THE NUMBER DENSITIES

Solutions for the isotopic number densities and the number of fissions in each isotope facilitate the evaluation of other important quantities. The number of fissions during the k^{th} step leads to the energy density (isotopic or total) in megawatt days/ton of heavy isotopes (MWD/T). The accumulated MWD/T (isotopic or total) for all irradiation steps from the start of irradiation is then obtained trivially by addition. The expressions for MWD/T are given in Eqs (A37) through (A41) in Appendix A.

Expressions for the thermal, epithermal, and total conversion ratios are given in Eqs (A42) through (A44).

The regeneration factor η (the number of fission neutrons produced per thermal neutron absorbed) for the material within the fuel volume (including fission products but excluding cladding and moderator) is given in Eq (A45). The quantity ηf , where f is thermal utilization, is also given by Eq (A45) provided this expression now includes the effects of cladding and moderator. Since reactivity measurements may be made in a different facility than that in which specimens are irradiated, the neutron interaction parameters appropriate to the spectrum of the measurement facility (or any desired spectrum) may be used for each isotope in the expressions for η and ηf . Thus, the parameters may differ as desired from those appropriate for the irradiation history.

CODE DETAILS

Input and Output

The following information concerning the S0269 code appears in Appendix B:

- 1) Memory locations for the 166 input parameters needed and for the four quantities which describe each specific step in the irradiation history
- 2) Output locations
- 3) Reproduction of the code.

Almost all of the thousand available memory locations are used in the code. Although some consolidation could be accomplished by certain revisions, the operating time saved would not justify the effort required for the revision.

The four quantities which describe a step in the irradiation are an index number j whose function will be described, the elapsed time t of the step, the thermal-neutron flux ϕ , and the epithermal-to-thermal flux ratio ϕ_R/ϕ . The computer stops when there are no further irradiation-history steps to be read.

Parameter Adjustments Based on Concentrations

It was intended that the values of effective cross sections and resonance integrals used in the code calculations could be adjusted within the code to account for changes in these quantities as a result of irradiation-induced changes in isotopic concentrations. For this reason the subscript k for the k^{th} step is attached to these quantities in the expressions in the appendices. Although there is

nothing in the code to prevent incorporation of suitable adjustment methods, the lack of sufficient memory locations precludes such adjustment in this specific code. Appendix E gives an alternative adjustment method.

Calculated Output

Reactivity measurements on a specimen irradiated in a high-flux facility are made during periods in which the specimen is out-of-pile. Consequently, the reactivity transient (associated with Xe-135, Sm-149, Pu-239, and Pu-241) is present during the time of measurement*. The transient behavior of η and ηf following each cycle of irradiation is calculated for 20 selected cooling times, or times after discharge from the high flux, covering any time range desired. These transient calculations have no influence on the number densities computed for the true irradiation history, but do give the transient behavior which would be expected during any shutdown interval if the specimen were to remain out-of-pile throughout the period of this transient.

An index number j is assigned to each irradiation history step and is used to control certain code computations including the transient calculations:

- 1) For all values of j (and ϕ), the code punches (in locations 196 to 210) the accumulated flux-time summed over k steps and the isotopic number densities present at the end of the k^{th} step. It also punches (in locations 822 to 827) the elapsed time of the step, the thermal flux ϕ during the step, and the average isotopic fission rates during the step.
- 2) For j and ϕ both nonzero, used to describe the last irradiation step prior to specimen discharge for reactivity measurement, the code punches additional output as follows:
 - a) In locations 907 to 912, the code punches the accumulated flux-time summed over k steps with the corresponding number of fissions, both isotopic and total.
 - b) In locations 970 to 975, the code punches the accumulated flux-time summed over k steps and the corresponding energy density (MWD/T) for each isotope and all isotopes.
 - c) In locations 919 to 923, the code punches the accumulated flux-time summed over k steps, the epithermal-to-thermal flux ratio ϕ_R/ϕ for the k^{th} step only, and the thermal, epithermal, and total conversion ratios prevalent at the end of this step.
- 3) For $j \neq 0$ and $\phi = 0$, used to describe a shutdown interval during which reactivity measurements are taken, the code computes the isotopic number densities present at the end of the step and stores these for use in the subsequent irradiation history calculations.
 - a) For $j = 20$ and $\phi = 0$, the code also computes the reactivity transient for the 20 selected cooling times entered as input in locations 151 to 170. For each of these cooling times, the code punches (in locations 901 to 906) the accumulated flux-time summed over k steps, the epithermal-to-thermal flux ratio averaged over k steps, the accumulated MWD/T, the cooling time, the fractional change $\Delta\eta/\eta_0$ in the regeneration factor relative to a non-irradiated specimen, and the fractional change $\Delta(\eta f)/(\eta f)_0$. For the last of these cooling times, the code also punches (in locations 924 to 940) the accumulated flux-time summed over k steps and the isotopic number densities which would be present after this last cooling time. Thus, the so-called "asymptotic" number densities for a long cooling time may be available (for all isotopes except 13.2-year Pu-241).

*Although the "asymptotic" reactivity could be essentially attained by taking the measurements 200 or more hours after discharge, much irradiation time would be lost by imposing such a restriction since the down-time for refueling the MTR, for example, is usually only 50 to 100 hours. To measure the asymptotic reactivity, the specimen would remain out-of-pile during the succeeding irradiation interval.

- b) For $j = 1$ and $\phi = 0$, the code punches the data described in locations 901 to 906 and 924 to 940 for the last of the cooling times only. This case gives only "asymptotic" values for the number densities, $\Delta\eta/\eta_0$ and $\Delta(\eta f)/(\eta f)_0$.

Despite the many quantities calculated, the code is basically a number-density code for any specified irradiation history. The other quantities computed require only moderate additional computer time and are of particular interest to users of the code.

Round-off Errors

The floating-point decimal notation employed in the Bell Interpretative System permits no more than eight-digit significance in a single calculation. Round-off errors may, therefore, become appreciable, particularly in the case of subtraction of two quantities which are nearly equal. The possibility of encountering such difficulties in the present code has been investigated and the following conclusions made:

- 1) Calculations for the case of zero flux [expressions (A19) through (A36) in Appendix A] are satisfactory (round-off error, if any, is in seventh or eighth significant figure) for any step time equal to or greater than 100 seconds. This time is a practical lower limit for the time of a zero-flux step in irradiations of interest.
- 2) Calculations for the case of nonzero flux [expressions (A1) through (A18) in Appendix A] have round-off errors which depend upon the magnitude of the flux-time product. Provided each flux-time step exceeds $3 \times 10^{19} \text{ n/cm}^2$, the round-off error is negligible (estimated to be less than 0.2% for the isotopic concentrations for exposures in the range from 3×10^{19} to 10^{22} n/cm^2).

If the true irradiation history includes steps for which the flux-time product is less than $3 \times 10^{19} \text{ n/cm}^2$, the limitation (2) requires that the true history be somewhat modified. The S0264 Code for the Bell Interpretative System takes the true history as input* and computes a modified history in which the flux-time product of each step exceeds $3 \times 10^{19} \text{ n/cm}^2$. A brief description of this code appears in Appendix D.

Although the true history is modified in the S0264 Code, both the total elapsed time and total flux-time are preserved. Calculated concentrations for isotopes influenced primarily by long-term effects, therefore, would be expected to be nearly the same for both the modified and true irradiation histories; however, the calculated concentrations for isotopes influenced by short-term effects may not be the same. In particular, for true flux-time steps less than $3 \times 10^{19} \text{ n/cm}^2$ (40 hours in a flux of $2 \times 10^{14} \text{ n/cm}^2\text{-sec}$), the modified history will not give the true concentrations for the short-lived isotopes I-135, Xe-135, and Nd-149. The transient during a zero-flux step in the modified history is, therefore, reliable only if it is preceded by a true step in which the flux-time product exceeds $3 \times 10^{19} \text{ n/cm}^2$. This situation is realized in most experimental cases of interest.

Errors in Fission-Product Concentrations Introduced by the Use of Average Fission Rates

Although flux-time steps larger than $3 \times 10^{19} \text{ n/cm}^2$ give negligible round-off errors, much larger flux-time steps will introduce an error in the fission-product concentrations if the fission rate near the end of an irradiation step differs markedly from the average fission rate during the step, a situation which would occur if the concentration of a fissionable isotope were to change appreciably during the step. If an upper limit for a flux-time step of 10^{20} n/cm^2 is imposed, the error in

*Input for the S0264 Code may be obtained from the detailed irradiation history provided by the S0216 Code, input for which is obtained from the S0215 and S0226 Codes. The S0226 Code, written by E. D. McGarry, is used for the processing of flux-monitor measurements to give the average time-integrated thermal flux and the ratio of epithermal-to-thermal flux for each cycle of irradiation. The S0215 and S0216 Codes were written by J. C. Connor and provide the true irradiation history of a specimen.

fission-product concentrations, resulting from the assumption of constant fission rate during the step, is estimated to be less than 2%. This error would be virtually independent of the number of steps in an irradiation because the five specific fission products considered attain equilibrium concentrations essentially during each irradiation step.

The maximum flux-time step is an input parameter (location 146). If it is set at 10^{20} n/cm², the code divides larger flux-time steps into as many equal steps as are needed to make each step $\leq 10^{20}$ n/cm². Thus, each flux-time step within the code lies between 3×10^{19} and 10^{20} n/cm², although the input flux-time steps may have any magnitude greater than 3×10^{19} n/cm².

RECOMMENDED CODE REVISION TO MINIMIZE ROUND-OFF ERRORS

As described previously, round-off errors could be appreciable for a small flux-time step. To obviate this difficulty, a modified irradiation history is used. The true irradiation history could be used if the basic number density and fission expressions were rewritten. The expressions of Appendix A, which contain numerous differences of exponentials, all reduce to expressions containing simple summations. Expressions for the change in isotopic number density during the k^{th} step are derived, from which the number density at the end of the step may be obtained trivially as desired. This procedure is consistent with the objective of minimizing round-off errors. Expressions (C1) through (C45) in Appendix C are revised expressions corresponding to (A1) through (A36) in Appendix A.

A code for the Bell Interpretive System on the IBM-650 digital computer has been written based on the revised expressions. The machine-computation time proves to be long, and would be more reasonable if the code were written in machine language for a more advanced computer.

In the revised code, a convergence criterion for the summations requires that the n^{th} or last computed term contribute less than a given fraction to the sum (say 10^{-9} for eight-digit accuracy). Round-off errors associated with small flux-time steps become negligible. However, if a ψt term in a sum is large (>10), corresponding to a large flux-time or time step, round-off errors again may become troublesome because terms in a sum may become very large before the sum begins to converge. As shown in Eq (5), ψt includes both absorption and decay. Evidently, what is needed is a code which evaluates exponentials (see expressions of Appendix A) for large flux-time steps and all zero-flux steps, and evaluates summations (see expressions of Appendix C) for small flux-time steps. The preparation and use of such a code for the IBM-704, for example, would give high computational accuracy and reasonable running time. Number densities would be computed for the specific measured irradiation history rather than for a modified history as is done in the present S0269 code.

CONCLUSION

It may be argued that high precision in the calculation of number densities is not justified because of the many uncertainties in the fission and absorption parameters needed for the calculations. This may be true; however, if theory and experiment are to be compared, uncertainties in the theoretical calculations should depend only upon uncertainties in parameters and not upon errors introduced by code or computer limitations. Only then will it be meaningful to examine parameters and adjust their magnitudes in order to bring theory and experiment into agreement.

While the present S0269 code is very useful, it should be regarded as an interim code to be replaced by the superior code to be written, as described in the previous section, for a faster computer.

APPENDIX A

EXPRESSIONS FOR ISOTOPIC NUMBER DENSITY AND QUANTITIES DERIVABLE THEREFROM

Superscripts 25, 26, 27, 37, 28, 39, 49, 40, 41, I, Xe, Nd, Pm, and Sm refer to U-235, U-236, U-237, Np-237, U-238, Np-239, Pu-239, Pu-240, Pu-241, I-135, Xe-135, Nd-149, Pm-149, and Sm-149, respectively. For all decaying isotopes considered, total decay constants appear because $\lambda_\beta = \lambda$.

Heavy-Element Number Densities

$$N_k^{25} = N_{k-1}^{25} e^{-\psi_k^{25} t_k} \quad (A1)$$

$$N_k^{26} = \frac{N_{k-1}^{25} \phi_k^{25}}{\psi_k^{26} - \psi_k^{25}} \left[e^{-\psi_k^{25} t_k} - e^{-\psi_k^{26} t_k} \right] + N_{k-1}^{26} e^{-\psi_k^{26} t_k} \quad (A2)$$

$$N_k^{27} = \phi_k^{26} \left\{ \frac{N_{k-1}^{25} \phi_k^{25}}{\psi_k^{26} - \psi_k^{25}} \left[\frac{e^{-\psi_k^{25} t_k} - e^{-\psi_k^{27} t_k}}{\psi_k^{27} - \psi_k^{25}} - \frac{e^{-\psi_k^{26} t_k} - e^{-\psi_k^{27} t_k}}{\psi_k^{27} - \psi_k^{26}} \right] + \frac{N_{k-1}^{26}}{\psi_k^{27} - \psi_k^{26}} \left[e^{-\psi_k^{26} t_k} - e^{-\psi_k^{27} t_k} \right] \right\} + N_{k-1}^{27} e^{-\psi_k^{27} t_k} \quad (A3)$$

$$N_k^{37} = \lambda^{27} \left\{ \phi_k^{26} \left\{ \frac{N_{k-1}^{25} \phi_k^{25}}{\psi_k^{26} - \psi_k^{25}} \left[\frac{1}{\psi_k^{27} - \psi_k^{25}} \left(\frac{e^{-\psi_k^{25} t_k} - e^{-\psi_k^{37} t_k}}{\psi_k^{37} - \psi_k^{25}} - \frac{e^{-\psi_k^{27} t_k} - e^{-\psi_k^{37} t_k}}{\psi_k^{37} - \psi_k^{27}} \right) - \frac{1}{\psi_k^{27} - \psi_k^{26}} \left(\frac{e^{-\psi_k^{26} t_k} - e^{-\psi_k^{37} t_k}}{\psi_k^{37} - \psi_k^{26}} - \frac{e^{-\psi_k^{27} t_k} - e^{-\psi_k^{37} t_k}}{\psi_k^{37} - \psi_k^{27}} \right) \right] + \frac{N_{k-1}^{26}}{\psi_k^{27} - \psi_k^{26}} \left[\frac{e^{-\psi_k^{26} t_k} - e^{-\psi_k^{37} t_k}}{\psi_k^{37} - \psi_k^{26}} - \frac{e^{-\psi_k^{27} t_k} - e^{-\psi_k^{37} t_k}}{\psi_k^{37} - \psi_k^{27}} \right] \right\} + \frac{N_{k-1}^{27}}{\psi_k^{37} - \psi_k^{27}} \left[e^{-\psi_k^{26} t_k} - e^{-\psi_k^{37} t_k} \right] \right\} + N_{k-1}^{37} e^{-\psi_k^{37} t_k} \quad (A4)$$

$$N_k^{28} = N_{k-1}^{28} e^{-\psi_k^{28} t_k} \quad (A5)$$

$$N_k^{39} = \frac{N_{k-1}^{28} \phi_k^{28}}{\psi_k^{39} - \psi_k^{28}} \left[e^{-\psi_k^{28} t_k} - e^{-\psi_k^{39} t_k} \right] + N_{k-1}^{39} e^{-\psi_k^{39} t_k} \quad (A6)$$

$$N_k^{49} = \lambda^{39} \left\{ \frac{N_{k-1}^{28} \phi_k^{28}}{\psi_k^{39} - \psi_k^{28}} \left[\frac{e^{-\psi_k^{28} t_k} - e^{-\psi_k^{49} t_k}}{\psi_k^{49} - \psi_k^{28}} - \frac{e^{-\psi_k^{39} t_k} - e^{-\psi_k^{49} t_k}}{\psi_k^{49} - \psi_k^{39}} \right] \right. \\ \left. + \frac{N_{k-1}^{39}}{\psi_k^{49} - \psi_k^{39}} \left[e^{-\psi_k^{39} t_k} - e^{-\psi_k^{49} t_k} \right] \right\} + N_{k-1}^{49} e^{-\psi_k^{49} t_k} \quad (A7)$$

$$N_k^{40} = \phi_k^{49} \phi_k \left\{ \lambda^{39} \left[\frac{N_{k-1}^{28} \phi_k^{28}}{\psi_k^{39} - \psi_k^{28}} \left[\frac{1}{\psi_k^{49} - \psi_k^{28}} \left(\frac{e^{-\psi_k^{28} t_k} - e^{-\psi_k^{40} t_k}}{\psi_k^{40} - \psi_k^{28}} - \frac{e^{-\psi_k^{49} t_k} - e^{-\psi_k^{40} t_k}}{\psi_k^{40} - \psi_k^{49}} \right) \right. \right. \right. \\ \left. \left. - \frac{1}{\psi_k^{49} - \psi_k^{39}} \left(\frac{e^{-\psi_k^{39} t_k} - e^{-\psi_k^{40} t_k}}{\psi_k^{40} - \psi_k^{39}} - \frac{e^{-\psi_k^{49} t_k} - e^{-\psi_k^{40} t_k}}{\psi_k^{40} - \psi_k^{49}} \right) \right] \right. \\ \left. + \frac{N_{k-1}^{39}}{\psi_k^{49} - \psi_k^{39}} \left[\frac{e^{-\psi_k^{39} t_k} - e^{-\psi_k^{40} t_k}}{\psi_k^{40} - \psi_k^{39}} - \frac{e^{-\psi_k^{49} t_k} - e^{-\psi_k^{40} t_k}}{\psi_k^{40} - \psi_k^{49}} \right] \right\} \\ \left. + \frac{N_{k-1}^{49}}{\psi_k^{40} - \psi_k^{49}} \left[e^{-\psi_k^{49} t_k} - e^{-\psi_k^{40} t_k} \right] \right\} + N_{k-1}^{40} e^{-\psi_k^{40} t_k} \quad (A8)$$

$$N_k^{41} = \phi_k^{40} \phi_k \left\{ \phi_k^{49} \phi_k \left\{ \lambda^{39} \left[\frac{N_{k-1}^{28} \phi_k^{28}}{\psi_k^{39} - \psi_k^{28}} \left[\frac{1}{\psi_k^{49} - \psi_k^{28}} \left(\frac{1}{\psi_k^{40} - \psi_k^{28}} \left(\frac{e^{-\psi_k^{28} t_k} - e^{-\psi_k^{41} t_k}}{\psi_k^{41} - \psi_k^{28}} - \frac{e^{-\psi_k^{40} t_k} - e^{-\psi_k^{41} t_k}}{\psi_k^{41} - \psi_k^{40}} \right) \right. \right. \right. \right. \right. \\ \left. \left. - \frac{1}{\psi_k^{40} - \psi_k^{49}} \left(\frac{e^{-\psi_k^{49} t_k} - e^{-\psi_k^{41} t_k}}{\psi_k^{41} - \psi_k^{49}} - \frac{e^{-\psi_k^{40} t_k} - e^{-\psi_k^{41} t_k}}{\psi_k^{41} - \psi_k^{40}} \right) \right) \right. \right. \\ \left. \left. - \frac{1}{\psi_k^{49} - \psi_k^{39}} \left(\frac{1}{\psi_k^{40} - \psi_k^{39}} \left(\frac{e^{-\psi_k^{39} t_k} - e^{-\psi_k^{41} t_k}}{\psi_k^{41} - \psi_k^{39}} - \frac{e^{-\psi_k^{40} t_k} - e^{-\psi_k^{41} t_k}}{\psi_k^{41} - \psi_k^{40}} \right) \right. \right. \right. \right. \\ \left. \left. - \frac{1}{\psi_k^{40} - \psi_k^{49}} \left(\frac{e^{-\psi_k^{49} t_k} - e^{-\psi_k^{41} t_k}}{\psi_k^{41} - \psi_k^{49}} - \frac{e^{-\psi_k^{40} t_k} - e^{-\psi_k^{41} t_k}}{\psi_k^{41} - \psi_k^{40}} \right) \right) \right] \right\} \right\}$$

$$\begin{aligned}
& + \frac{N_{k-1}^{39}}{\psi_k^{49} - \psi_k^{39}} \left[\frac{1}{\psi_k^{40} - \psi_k^{39}} \left(\frac{e^{-\psi_k^{39} t_k} - e^{-\psi_k^{41} t_k}}{\psi_k^{41} - \psi_k^{39}} - \frac{e^{-\psi_k^{40} t_k} - e^{-\psi_k^{41} t_k}}{\psi_k^{41} - \psi_k^{40}} \right) \right. \\
& \left. - \frac{1}{\psi_k^{40} - \psi_k^{49}} \left(\frac{e^{-\psi_k^{49} t_k} - e^{-\psi_k^{41} t_k}}{\psi_k^{41} - \psi_k^{49}} - \frac{e^{-\psi_k^{40} t_k} - e^{-\psi_k^{41} t_k}}{\psi_k^{41} - \psi_k^{40}} \right) \right] \\
& + \frac{N_{k-1}^{49}}{\psi_k^{40} - \psi_k^{49}} \left[\frac{e^{-\psi_k^{49} t_k} - e^{-\psi_k^{41} t_k}}{\psi_k^{41} - \psi_k^{49}} - \frac{e^{-\psi_k^{40} t_k} - e^{-\psi_k^{41} t_k}}{\psi_k^{41} - \psi_k^{40}} \right] \\
& + \frac{N_{k-1}^{40}}{\psi_k^{41} - \psi_k^{40}} \left[e^{-\psi_k^{40} t_k} - e^{-\psi_k^{41} t_k} \right] \Bigg] + N_{k-1}^{41} e^{-\psi_k^{41} t_k} \quad (A9)
\end{aligned}$$

Number of Fissions in Each Fissionable Isotope during k^{th} Irradiation Step

$$F_k^{25} = (N_{k-1}^{25} - N_k^{25}) (\sigma_{ak}^{25} - \sigma_{\gamma k}^{25}) \phi_k / \psi_k^{25} \quad (A10)$$

$$F_k^{28} = (N_{k-1}^{28} - N_k^{28}) (\sigma_{ak}^{28} - \sigma_{\gamma k}^{28}) \phi_k / \psi_k^{28} \quad (A11)$$

$$\begin{aligned}
F_k^{49} = & (\sigma_{ak}^{49} - \sigma_{\gamma k}^{49}) \phi_k \left\{ \lambda^{39} \left[\frac{N_{k-1}^{28} \sigma_{\gamma k}^{28} \phi_k}{\psi_k^{39} - \psi_k^{28}} \left[\frac{1}{\psi_k^{49} - \psi_k^{28}} \left(\frac{1 - e^{-\psi_k^{28} t_k}}{\psi_k^{28}} - \frac{1 - e^{-\psi_k^{49} t_k}}{\psi_k^{49}} \right) \right. \right. \right. \\
& \left. \left. - \frac{1}{\psi_k^{49} - \psi_k^{39}} \left(\frac{1 - e^{-\psi_k^{39} t_k}}{\psi_k^{39}} - \frac{1 - e^{-\psi_k^{49} t_k}}{\psi_k^{49}} \right) \right] \right. \\
& \left. + \frac{N_{k-1}^{39}}{\psi_k^{49} - \psi_k^{39}} \left(\frac{1 - e^{-\psi_k^{39} t_k}}{\psi_k^{39}} - \frac{1 - e^{-\psi_k^{49} t_k}}{\psi_k^{49}} \right) \right] + \frac{N_{k-1}^{49}}{\psi_k^{49}} (1 - e^{-\psi_k^{49} t_k}) \right\} \quad (A12)
\end{aligned}$$

$$\begin{aligned}
F_k^{41} = & (\sigma_{ak}^{41} - \sigma_{\gamma k}^{41}) \phi_k \left\{ \sigma_{\gamma k}^{40} \phi_k \left[\sigma_{\gamma k}^{49} \phi_k \left\{ \lambda^{39} \left[\frac{N_{k-1}^{28} \sigma_{\gamma k}^{28} \phi_k}{\psi_k^{39} - \psi_k^{28}} \left\{ \frac{1}{\psi_k^{49} - \psi_k^{28}} \left[\frac{1}{\psi_k^{40} - \psi_k^{28}} \left(\frac{1 - e^{-\psi_k^{28} t_k}}{\psi_k^{28}} \right. \right. \right. \right. \right. \right. \right. \right. \right. \\
& \left. \left. \left. - \frac{1 - e^{-\psi_k^{41} t_k}}{\psi_k^{41}} \right) - \frac{1}{\psi_k^{41} - \psi_k^{40}} \left(\frac{1 - e^{-\psi_k^{40} t_k}}{\psi_k^{40}} - \frac{1 - e^{-\psi_k^{41} t_k}}{\psi_k^{41}} \right) \right] \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& - \frac{1}{\psi_k^{40} - \psi_k^{49}} \left(\frac{1}{\psi_k^{41} - \psi_k^{49}} \left(\frac{1 - e^{-\psi_k^{49} t_k}}{\psi_k^{49}} - \frac{1 - e^{-\psi_k^{41} t_k}}{\psi_k^{41}} \right) \right. \\
& \left. - \frac{1}{\psi_k^{41} - \psi_k^{40}} \left(\frac{1 - e^{-\psi_k^{40} t_k}}{\psi_k^{40}} - \frac{1 - e^{-\psi_k^{41} t_k}}{\psi_k^{41}} \right) \right) \\
& - \frac{1}{\psi_k^{49} - \psi_k^{39}} \left[\frac{1}{\psi_k^{40} - \psi_k^{39}} \left(\frac{1}{\psi_k^{41} - \psi_k^{39}} \left(\frac{1 - e^{-\psi_k^{39} t_k}}{\psi_k^{39}} - \frac{1 - e^{-\psi_k^{41} t_k}}{\psi_k^{41}} \right) \right. \right. \\
& \left. \left. - \frac{1}{\psi_k^{41} - \psi_k^{40}} \left(\frac{1 - e^{-\psi_k^{40} t_k}}{\psi_k^{40}} - \frac{1 - e^{-\psi_k^{41} t_k}}{\psi_k^{41}} \right) \right) \right. \\
& \left. - \frac{1}{\psi_k^{40} - \psi_k^{49}} \left(\frac{1}{\psi_k^{41} - \psi_k^{49}} \left(\frac{1 - e^{-\psi_k^{49} t_k}}{\psi_k^{49}} - \frac{1 - e^{-\psi_k^{41} t_k}}{\psi_k^{41}} \right) \right. \right. \\
& \left. \left. - \frac{1}{\psi_k^{41} - \psi_k^{40}} \left(\frac{1 - e^{-\psi_k^{40} t_k}}{\psi_k^{40}} - \frac{1 - e^{-\psi_k^{41} t_k}}{\psi_k^{41}} \right) \right) \right] \Bigg\} \\
& + \frac{N_{k-1}^{39}}{\psi_k^{49} - \psi_k^{39}} \left\{ \frac{1}{\psi_k^{40} - \psi_k^{39}} \left[\frac{1}{\psi_k^{41} - \psi_k^{39}} \left(\frac{1 - e^{-\psi_k^{39} t_k}}{\psi_k^{39}} - \frac{1 - e^{-\psi_k^{41} t_k}}{\psi_k^{41}} \right) \right. \right. \\
& \left. \left. - \frac{1}{\psi_k^{41} - \psi_k^{40}} \left(\frac{1 - e^{-\psi_k^{40} t_k}}{\psi_k^{40}} - \frac{1 - e^{-\psi_k^{41} t_k}}{\psi_k^{41}} \right) \right] \right. \\
& \left. - \frac{1}{\psi_k^{40} - \psi_k^{49}} \left[\frac{1}{\psi_k^{41} - \psi_k^{49}} \left(\frac{1 - e^{-\psi_k^{49} t_k}}{\psi_k^{49}} - \frac{1 - e^{-\psi_k^{41} t_k}}{\psi_k^{41}} \right) - \frac{1}{\psi_k^{41} - \psi_k^{40}} \left(\frac{1 - e^{-\psi_k^{40} t_k}}{\psi_k^{40}} - \frac{1 - e^{-\psi_k^{41} t_k}}{\psi_k^{41}} \right) \right] \right\} \\
& + \frac{N_{k-1}^{49}}{\psi_k^{40} - \psi_k^{49}} \left[\frac{1}{\psi_k^{41} - \psi_k^{49}} \left(\frac{1 - e^{-\psi_k^{49} t_k}}{\psi_k^{49}} - \frac{1 - e^{-\psi_k^{41} t_k}}{\psi_k^{41}} \right) - \frac{1}{\psi_k^{41} - \psi_k^{40}} \left(\frac{1 - e^{-\psi_k^{40} t_k}}{\psi_k^{40}} - \frac{1 - e^{-\psi_k^{41} t_k}}{\psi_k^{41}} \right) \right] \Bigg\} \\
& + \frac{N_{k-1}^{40}}{\psi_k^{41} - \psi_k^{40}} \left(\frac{1 - e^{-\psi_k^{40} t_k}}{\psi_k^{40}} - \frac{1 - e^{-\psi_k^{41} t_k}}{\psi_k^{41}} \right) \Bigg\} + N_{k-1}^{41} \left(\frac{1 - e^{-\psi_k^{41} t_k}}{\psi_k^{41}} \right) \Bigg\} \quad (A13)
\end{aligned}$$

Isotopic Number Density Expressions for the Fission Products

In the following, the summations over u, v pertain to U-235, U-238, Pu-239, and Pu-241; I, Xe, Nd, Pm, and Sm refer to I-135, Xe-135, Nd-149, Pm-149, and Sm-149.

$$N_k^I = \frac{1 - e^{-\psi_k^I t_k}}{\psi_k^I} \sum_{u,v} y_I^{u,v} F_k^{u,v} / t_k + N_{k-1}^I e^{-\psi_k^I t_k} \quad (A14)$$

$$N_k^{Xe} = \frac{1 - e^{-\psi_k^{Xe} t_k}}{\psi_k^{Xe}} \sum_{u,v} \frac{F_k^{u,v}}{t_k} (y_{Xe}^{u,v} + y_I^{u,v} \lambda^I / \psi_k^I) + \lambda^I \frac{e^{-\psi_k^I t_k} - e^{-\psi_k^{Xe} t_k}}{\psi_k^{Xe} - \psi_k^I} \left(N_{k-1}^I - \frac{1}{\psi_k^I} \sum_{u,v} y_I^{u,v} \frac{F_k^{u,v}}{t_k} \right) + N_{k-1}^{Xe} e^{-\psi_k^{Xe} t_k} \quad (A15)$$

$$N_k^{Nd} = \frac{1 - e^{-\psi_k^{Nd} t_k}}{\psi_k^{Nd}} \sum_{u,v} y_{Nd}^{u,v} F_k^{u,v} / t_k + N_{k-1}^{Nd} e^{-\psi_k^{Nd} t_k} \quad (A16)$$

$$N_k^{Pm} = \frac{1 - e^{-\psi_k^{Pm} t_k}}{\psi_k^{Pm}} \sum_{u,v} \frac{F_k^{u,v}}{t_k} (y_{Pm}^{u,v} + y_{Nd}^{u,v} \lambda^{Nd} / \psi_k^{Nd}) + \lambda^{Nd} \frac{e^{-\psi_k^{Nd} t_k} - e^{-\psi_k^{Pm} t_k}}{\psi_k^{Pm} - \psi_k^{Nd}} \left(N_{k-1}^{Nd} - \frac{1}{\psi_k^{Nd}} \sum_{u,v} y_{Nd}^{u,v} \frac{F_k^{u,v}}{t_k} \right) + N_{k-1}^{Pm} e^{-\psi_k^{Pm} t_k} \quad (A17)$$

$$N_k^{Sm} = \frac{1 - e^{-\psi_k^{Sm} t_k}}{\psi_k^{Sm}} \sum_{u,v} y_{Sm}^{u,v} F_k^{u,v} / t_k + \lambda^{Pm} \left\{ \frac{1}{\psi_k^{Pm}} \left[\frac{1 - e^{-\psi_k^{Sm} t_k}}{\psi_k^{Sm}} - \frac{e^{-\psi_k^{Pm} t_k} - e^{-\psi_k^{Sm} t_k}}{\psi_k^{Sm} - \psi_k^{Pm}} \right] \sum_{u,v} \frac{F_k^{u,v}}{t_k} \left(y_{Pm}^{u,v} + y_{Nd}^{u,v} \frac{\lambda^{Nd}}{\psi_k^{Nd}} \right) + \lambda^{Nd} \left(N_{k-1}^{Nd} - \frac{1}{\psi_k^{Nd}} \sum_{u,v} y_{Nd}^{u,v} \frac{F_k^{u,v}}{t_k} \right) \frac{1}{\psi_k^{Pm} - \psi_k^{Nd}} \left[\frac{e^{-\psi_k^{Nd} t_k} - e^{-\psi_k^{Sm} t_k}}{\psi_k^{Sm} - \psi_k^{Nd}} - \frac{e^{-\psi_k^{Pm} t_k} - e^{-\psi_k^{Sm} t_k}}{\psi_k^{Sm} - \psi_k^{Pm}} \right] + N_{k-1}^{Pm} \frac{e^{-\psi_k^{Pm} t_k} - e^{-\psi_k^{Sm} t_k}}{\psi_k^{Sm} - \psi_k^{Pm}} \right\} + N_{k-1}^{Sm} e^{-\psi_k^{Sm} t_k} \quad (A18)$$

Expressions for the Case of Zero Flux

For the special case of zero flux, expressions (A1) through (A18) reduce to the following:

$$N_k^{25} = N_{k-1}^{25} \quad (A19)$$

$$N_k^{26} = N_{k-1}^{26} \quad (A20)$$

$$N_k^{27} = N_{k-1}^{27} e^{-\lambda^{27} t_k} \quad (A21)$$

$$N_k^{37} = N_{k-1}^{37} + N_{k-1}^{27} \left(1 - e^{-\lambda^{27} t_k} \right) \quad (A22)$$

$$N_k^{28} = N_{k-1}^{28} \quad (A23)$$

$$N_k^{39} = N_{k-1}^{39} e^{-\lambda^{39} t_k} \quad (A24)$$

$$N_k^{49} = N_{k-1}^{49} + N_{k-1}^{39} \left(1 - e^{-\lambda^{39} t_k} \right) \quad (A25)$$

$$N_k^{40} = N_{k-1}^{40} \quad (A26)$$

$$N_k^{41} = N_{k-1}^{41} e^{-\lambda^{41} t_k} \quad (A27)$$

$$F_k^{25} = 0 \quad (A28)$$

$$F_k^{28} = 0 \quad (A29)$$

$$F_k^{49} = 0 \quad (A30)$$

$$F_k^{41} = 0 \quad (A31)$$

$$N_k^I = N_{k-1}^I e^{-\lambda^I t_k} \quad (A32)$$

$$N_k^{Xe} = \lambda^I \frac{e^{-\lambda^I t_k} - e^{-\lambda^{Xe} t_k}}{\lambda^{Xe} - \lambda^I} N_{k-1}^I + N_{k-1}^{Xe} e^{-\lambda^{Xe} t_k} \quad (A33)$$

$$N_k^{Nd} = N_{k-1}^{Nd} e^{-\lambda^{Nd} t_k} \quad (A34)$$

$$N_k^{Pm} = \lambda^{Nd} \frac{e^{-\lambda^{Nd} t_k} - e^{-\lambda^{Pm} t_k}}{\lambda^{Pm} - \lambda^{Nd}} N_{k-1}^{Nd} + N_{k-1}^{Pm} e^{-\lambda^{Pm} t_k} \quad (A35)$$

$$N_k^{Sm} = \frac{\lambda^{Pm} \left(1 - e^{-\lambda^{Nd} t_k}\right) - \lambda^{Nd} \left(1 - e^{-\lambda^{Pm} t_k}\right)}{\lambda^{Pm} - \lambda^{Nd}} N_{k-1}^{Nd} + N_{k-1}^{Pm} \left(1 - e^{-\lambda^{Pm} t_k}\right) + N_{k-1}^{Sm} \quad (A36)$$

Expressions for Energy Density in MWD/T

By definition,

$$\sum_{i,j} A^{i,j} N_o^{i,j} = 235 N_o^{25} + 236 N_o^{26} + 237 N_o^{27} + 237 N_o^{37} + 238 N_o^{28} + 239 N_o^{39} + 239 N_o^{49} + 240 N_o^{40} + 241 N_o^{41},$$

where the subscript o refers to the initial or pre-irradiation situation. Symbolically, let

Isotopic energy density = $E^{i,j}/m$ for the energy per unit mass (of fuel initially present) in the fission of isotope i, j.

Energy per fission = $Q^{i,j}$ for the fission of isotope i, j in units of Mev.

Then, the isotopic energy densities for the k^{th} irradiation step may be written

$$E_k^{25}/m = \frac{F_k^{25} Q^{25}}{\sum_{i,j} A^{i,j} N_o^{i,j}} (1.0140 \times 10^6) \text{ in MWD/T} \quad (A37)$$

$$E_k^{28}/m = \frac{F_k^{28} Q^{28}}{\sum_{i,j} A^{i,j} N_o^{i,j}} (1.0140 \times 10^6) \text{ in MWD/T} \quad (A38)$$

$$E_k^{49}/m = \frac{F_k^{49} Q^{49}}{\sum_{i,j} A^{i,j} N_o^{i,j}} (1.0140 \times 10^6) \text{ in MWD/T} \quad (A39)$$

$$E_k^{41}/m = \frac{F_k^{41} Q^{41}}{\sum_{i,j} A^{i,j} N_o^{i,j}} (1.0140 \times 10^6) \text{ in MWD/T} \quad (A40)$$

The total energy density from all isotopes during the k^{th} step is

$$E_k/m = \frac{\sum_{i,j} F_k^{i,j} Q^{i,j}}{\sum_{i,j} A^{i,j} N_o^{i,j}} (1.0140 \times 10^6) \text{ in MWD/T} \quad (A41)$$

which is the sum of (A37) through (A40). The summations of each of the expressions (A37) through (A41) over all k irradiation steps would give the accumulated isotopic and total energy densities.

Expressions for the Conversion Ratios

The thermal, epithermal, and total conversion ratios may be written, respectively, as follows:

$$(CR)_{th} = \frac{N_k^{28} \sigma_{\gamma k}^{28} + N_k^{40} \sigma_{\gamma k}^{40}}{N_k^{25} \sigma_{ak}^{25} + N_k^{49} \sigma_{ak}^{49} + N_k^{41} \sigma_{ak}^{41}} \quad (A42)$$

$$(CR)_f = \frac{N_k^{28} R_{\gamma k}^{28} + N_k^{40} R_{\gamma k}^{40}}{N_k^{25} \sigma_{ak}^{25} + N_k^{49} \sigma_{ak}^{49} + N_k^{41} \sigma_{ak}^{41}} (\phi_R / \phi)_k \quad (A43)$$

$$CR = \frac{N_k^{28} \sigma_{\gamma k}^{28} + N_k^{40} \sigma_{\gamma k}^{40}}{N_k^{25} \sigma_{ak}^{25} + N_k^{49} \sigma_{ak}^{49} + N_k^{41} \sigma_{ak}^{41}} \quad (A44)$$

which is equal to the sum of (A42) and (A43).

Expressions for the Regeneration Factor η and the Product ηf

The regeneration factor η or the number of neutrons produced per neutron absorbed in fuel material is defined by

$$\eta_k = \sum_{u,v} \nu^{u,v} (\bar{\sigma}_{ak}^{u,v} - \bar{\sigma}_{\gamma k}^{u,v}) N_k^{u,v} / \sum_{i,j} \bar{\sigma}_{ak}^{i,j} N_k^{i,j} \quad (A45)$$

where $\nu^{u,v}$ is the average number of fission neutrons produced per thermal fission of isotope u, v ; $\bar{\sigma}_{ak}^{u,v}$ and $\bar{\sigma}_{\gamma k}^{u,v}$ are effective thermal cross sections for absorption and capture; and the summations extend over all fissionable isotopes u, v and all absorbing isotopes i, j within the fuel volume. A term for fission-product poisons which may be written as a double summation over the k irradiation steps and the u, v fissionable isotopes is included implicitly in the denominator :

$$\sum_k \sum_{u,v} \bar{\sigma}_{FP,k}^{u,v} F_k^{u,v} ,$$

where $\bar{\sigma}_{FP,k}^{u,v}$ is the effective thermal absorption cross section per fission for fission products from the fission of isotope u, v --the so-called "barn per fission" quantity for each fissionable isotope. Since this quantity may vary with irradiation, the subscript k is included.

Expression (A45) is also used for the product ηf , if the summation in the denominator is interpreted to include terms for absorption in the moderator and structural materials.

If η_0 and $(\eta f)_0$ pertain to the situation prior to irradiation, expression (A45) may be used to evaluate

$$\frac{\Delta \eta}{\eta_0} = \frac{\eta_k - \eta_0}{\eta_0} \quad (A46)$$

and

$$\frac{\Delta(\eta f)}{(\eta f)_0} = \frac{(\eta f)_k - (\eta f)_0}{(\eta f)_0} . \quad (A47)$$

APPENDIX B

DETAILS OF THE S0269 CODE

Input Quantities and Their Locations in the Code

Location	Quantity		Location	Quantity	
001	N_o^{25}	Original number densities in cm^{-3}	026	σ_a^{25}	Effective thermal cross sections in cm^2
002	N_o^{26}		027	σ_a^{26}	
003	N_o^{27}		028	σ_a^{27}	
004	N_o^{37}		029	σ_a^{37}	
005	N_o^{28}		030	σ_a^{28}	
006	N_o^{39}		031	σ_a^{39}	
007	N_o^{49}		032	σ_a^{49}	
008	N_o^{40}		033	σ_a^{40}	
009	N_o^{41}		034	σ_a^{41}	
010	N_o^I		035	σ_a^I	
011	N_o^{Xe}	Accumulated isotopic fissions in cm^{-3}	036	σ_a^{Xe}	
012	N_o^{Nd}		037	σ_a^{Nd}	
013	N_o^{Pm}		038	σ_a^{Pm}	
014	N_o^{Sm}		039	σ_a^{Sm}	
015	$\sum_k F_k^{25}$	Decay constants in sec^{-1}	040	σ_γ^{25}	
016	$\sum_k F_k^{28}$		041	σ_γ^{26}	
017	$\sum_k F_k^{49}$		042	σ_γ^{27}	
018	$\sum_k F_k^{41}$		043	σ_γ^{37}	
019	λ^{27}		044	σ_γ^{28}	
020	λ^{39}		045	σ_γ^{39}	
021	λ^{41}		046	σ_γ^{49}	
022	λ^I		047	σ_γ^{40}	
023	λ^{Xe}		048	σ_γ^{41}	
024	λ^{Nd}				
025	λ^{Pm}				

Input Quantities and Their Locations in the Code (Cont)

Location	Quantity		Location	Quantity	
049	R_a^{25}	Epithermal resonance integrals in cm^2	073	Q^{25}	Energy per fission in Mev
050	R_a^{26}		074	Q^{28}	
051	R_a^{27}		075	Q^{49}	
052	R_a^{37}		076	Q^{41}	
053	R_a^{28}		077	y_I^{25}	Isotopic fission yields (fractional)
054	R_a^{39}		078	y_I^{28}	
055	R_a^{49}		079	y_I^{49}	
056	R_a^{40}		080	y_I^{41}	
057	R_a^{41}		081	y_{Xe}^{25}	
058	R_a^I		082	y_{Xe}^{28}	
059	R_a^{Xe}		083	y_{Xe}^{49}	
060	R_a^{Nd}		084	y_{Xe}^{41}	
061	R_a^{Pm}		085	y_{Nd}^{25}	
062	R_a^{Sm}		086	y_{Nd}^{28}	
063	R_γ^{25}		087	y_{Nd}^{49}	
064	R_γ^{26}		088	y_{Nd}^{41}	
065	R_γ^{27}		089	y_{Pm}^{25}	
066	R_γ^{37}		090	y_{Pm}^{28}	
067	R_γ^{28}		091	y_{Pm}^{49}	
068	R_γ^{39}	Factor for fast fission	092	y_{Pm}^{41}	
069	R_γ^{49}		093	y_{Sm}^{25}	
070	R_γ^{40}		094	y_{Sm}^{28}	
071	R_γ^{41}		095	y_{Sm}^{49}	
072	δ^{28}		096	y_{Sm}^{41}	

Input Quantities and Their Locations in the Code (Cont)

Location	Quantity		Location	Quantity	
097	σ_a^{25}		128	A^{25}	Atomic weight
098	σ_a^{26}		129	A^{26}	
099	σ_a^{27}		130	A^{27}	
100	σ_a^{37}		131	A^{37}	
101	σ_a^{28}		132	A^{28}	
102	σ_a^{39}		133	A^{39}	
103	σ_a^{49}		134	A^{49}	
104	σ_a^{40}		135	A^{40}	
105	σ_a^{41}		136	A^{41}	
106	σ_a^I	Effective thermal cross sections in cm^2			
107	σ_a^{Xe}				
108	σ_a^{Nd}				
109	σ_a^{Pm}				
110	σ_a^{Sm}				
111	σ_γ^{25}				
112	σ_γ^{26}				
113	σ_γ^{27}				
114	σ_γ^{37}				
115	σ_γ^{28}				
116	σ_γ^{39}	Used in the calculation of η			
117	σ_γ^{49}				
118	σ_γ^{40}				
119	σ_γ^{41}				
120	ν^{25}				
121	ν^{28}				
122	ν^{49}				
123	ν^{41}				
124	σ_{FP}^{25}	Neutrons per fission			
125	σ_{FP}^{28}				
126	σ_{FP}^{49}				
127	σ_{FP}^{41}				
		Fission product cross section per fission in cm^2			

Input Quantities and Their Locations in the Code (Cont)

Location	Quantity	
137	1.0140×10^6 MWD/T conversion factor	} Constants needed
138	115	
139	1	
140	20 in C-address	
141	$N_o^{H_2O}$	} Initial number densities in cm^{-3}
142	N_o^{Zr}	
143	$\bar{\sigma}_a^{H_2O}$	
144	$\bar{\sigma}_a^{Zr}$	
145		Address-set location
146	$(\phi t)_{max}$	Maximum flux-time step in n/cm^2
151 - 170		Transient cooling times in seconds
951	j_k	} Input for each step in the irradiation history
952	t_k	
953	ϕ_k	
954	$(\phi_R/\phi)_k$	

Output Quantities and Their Locations in the Code

Location	Quantity	
196	$\sum_k \phi_k t_k$	Accumulated flux-time in n/cm^2
197	N_k^{25}	} Isotopic number densities at end of k^{th} step in cm^{-3}
198	N_k^{26}	
199	N_k^{27}	
200	N_k^{37}	
201	N_k^{28}	
202	N_k^{39}	
203	N_k^{49}	
204	N_k^{40}	
205	N_k^{41}	
206	N_k^I	
207	N_k^{Xe}	

Output Quantities and Their Locations in the Code (Cont)

Location	Quantity	
208	N_k^{Nd}	Isotopic number densities at end of k^{th} step in cm^{-3}
209	N_k^{Pm}	
210	N_k^{Sm}	
822	t_k	Elapsed time of k^{th} step in sec
823	ϕ_k	Thermal flux for k^{th} step in n/cm^2 -sec
824	F_k^{25}/t_k	Average fission rates during k^{th} step in sec^{-1}
825	F_k^{28}/t_k	
826	F_k^{49}/t_k	
827	F_k^{41}/t_k	
901	$\sum_k \phi_k t_k$	Accumulated flux-time in n/cm^2
902	$\langle \phi_R / \phi \rangle$	Flux ratio averaged over all steps
903	$\sum_k (MWD/T)_k$	Accumulated MWD/T over all steps
904	t	Cooling time in seconds (same as input time taken from one of the locations 151 to 170)
905	$\Delta\eta/\eta_0$	Fractional change in η over all steps
906	$(\Delta\eta f)/(\eta f)_0$	Fractional change in ηf over all steps
907	$\sum_k \phi_k t_k$	Accumulated flux-time in n/cm^2
908	$\sum_k F_k^{25}$	Accumulated isotopic fissions in cm^{-3}
909	$\sum_k F_k^{28}$	
910	$\sum_k F_k^{49}$	
911	$\sum_k F_k^{41}$	
912	$\sum_k \sum_{u,v} F_k^{u,v}$	Accumulated total fissions in cm^{-3}
919	$\sum_k \phi_k t_k$	Accumulated flux-time in n/cm^2
920	$(\phi_R/\phi)_k$	Flux ratio for k^{th} step
921	$(CR)_{th}$	Thermal conversion ratio
922	$(CR)_f$	Epithermal conversion ratio
923	CR	Total conversion ratio

Output Quantities and Their Locations in the Code (Cont)

Location	Quantity	
924	$\sum_k \phi_k t_k$	<p>Pseudo number densities in cm^{-3} which would be obtained for a given cooling time. Punched on three cards, with the accumulated flux-time the first word on each card.</p>
925	N_c^{25}	
926	N_c^{26}	
927	N_c^{27}	
928	N_c^{37}	
929	$\sum_k \phi_k t_k$	
930	N_c^{28}	
931	N_c^{39}	
932	N_c^{49}	
933	N_c^{40}	
934	N_c^{41}	
935	$\sum_k \phi_k t_k$	
936	N_c^I	
937	N_c^{Xe}	
938	N_c^{Nd}	
939	N_c^{Pm}	
940	N_c^{Sm}	
970	$\sum_k \phi_k t_k$	Accumulated flux-time in n/cm^2
971	$\sum_k (\text{MWD}/T)_k^{25}$	<p>Isotopic and total MWD/T accumulated over all steps</p>
972	$\sum_k (\text{MWD}/T)_k^{28}$	
973	$\sum_k (\text{MWD}/T)_k^{49}$	
974	$\sum_k (\text{MWD}/T)_k^{41}$	
975	$\sum_k \sum_{u,v} (\text{MWD}/T)_k^{u,v}$	

Quantities in locations 700, 705, 706, and 976 are also punched for use as described in Appendix E.

Code Reproduction

The S0269 Code is reproduced on the following three pages.

1	147	4	+4	146	952	900	+4	951	263	195	+2	900	000	000	+0	201	414	913
2	171	6	+0	410	907	912	+3	197	721	721	+3	203	727	000	+1	000	721	721
3	177	6	+3	201	044	722	+3	204	047	000	+1	000	722	722	+3	201	067	723
4	183	6	+3	000	953	723	+4	722	721	921	+4	723	721	922	+1	000	921	923
5	189	2	+0	410	919	923	+0	203	000	941								
6	261	6	+0	400	001	146	+0	400	151	170	+0	005	251	001	+9	018	001	197
7	267	6	-3	128	001	691	+0	111	009	267	-1	691	692	692	+0	111	008	269
8	273	6	+0	400	950	953	+9	001	139	263	+9	002	951	261	+8	952	000	147
9	279	6	+0	201	280	947	-0	301	254	254	+0	006	947	001	+0	011	007	277
10	285	6	+2	199	000	000	+1	000	200	232	+3	255	202	234	+2	202	000	000
11	291	6	+3	257	206	238	+2	257	258	898	+2	023	022	000	+4	898	000	000
12	297	6	+3	258	207	000	+1	000	239	239	+3	259	208	240	+2	259	260	897
13	303	6	+3	000	024	000	+3	000	208	241	+3	260	209	000	+1	000	241	241
14	309	6	+2	139	260	000	+3	000	024	000	+2	242	000	242	+2	025	024	000
15	315	6	+2	139	260	000	+3	000	209	000	+1	000	242	000	+1	000	210	242
16	321	6	+9	002	231	217	+9	002	234	220	+9	006	237	223	+0	008	249	950
17	327	6	+9	004	191	824	+0	203	000	000	+0	006	247	001	+9	001	140	253
18	333	6	+0	090	334	253	+9	001	151	951	+0	080	394	950	+0	203	000	277
19	339	6	+9	002	197	229	+9	001	201	233	+9	001	204	236	+2	097	111	000
20	345	6	+2	103	117	000	+3	000	122	000	+3	000	235	000	+1	000	701	701
21	351	6	+3	000	237	000	+1	000	701	701	+9	001	252	702	-9	000	229	229
22	357	6	+0	203	000	359	+3	000	097	000	+1	000	702	702	+0	060	358	001
23	363	6	+0	202	030	366	+9	000	252	000	+0	203	000	367	+3	000	124	000
24	369	6	+0	011	004	362	+0	050	358	097	+0	050	366	124	+3	141	143	899
25	375	6	+1	000	702	899	+4	701	899	704	+0	454	000	000	+4	701	702	703
26	381	6	+9	002	703	705	+0	410	700	700	+0	410	705	706	+0	006	248	001
27	387	6	+2	000	139	905	+4	704	706	000	+2	000	139	906	+9	001	951	904
28	393	6	+0	454	000	000	+8	251	000	408	+0	005	247	000	+9	001	901	924
29	399	6	+9	005	233	930	+0	410	92									

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Current "Best" Values of Input Parameters for Irradiation at 46.1°C and Shutdown Transient Reactivity at 20°C

1	001 6	+3155000071	+0000000000	+0000000000	+0000000000	+4323050073	+0000000000
2	007 6	+0000000000	+0000000000	+0000000000	+0000000000	+0000000000	+0000000000
3	013 6	+0000000000	+0000000000	+0000000000	+0000000000	+0000000000	+0000000000
4	019 6	+1188300044	+3442000044	+1664800041	+2881700045	+2108400045	+9625000045
5	025 6	+3632100044	+5701000028	+5100000026	+0000000000	+1455000028	+2310000026
6	031 6	+6000000027	+9573000028	+2598000028	+1268500029	+7000000026	+2461000032
7	037 6	+0000000000	+0000000000	+5955000030	+9160000027	+5100000026	+0000000000
8	043 6	+1455000028	+2310000026	+6000000027	+2816000028	+2598000028	+3458000028
9	049 6	+9208000028	+2949000028	+0000000000	+4683000028	+2300000027	+0000000000
10	055 6	+2600000029	+8545000029	+2443000029	+0000000000	+2830000031	+0000000000
11	061 6	+0000000000	+6970000029	+3331000028	+2949000028	+0000000000	+4683000028
12	067 6	+2300000027	+0000000000	+1030000029	+8545000029	+1331000029	+2800000024
13	073 6	+2050000052	+2050000052	+2050000052	+2050000052	+6100000048	+6100000048
14	079 6	+5800000048	+5800000048	+2000000047	+2000000047	+2000000047	+2000000047
15	085 6	+1130000048	+2000000048	+1890000048	+2000000048	+0000000000	+0000000000
16	091 6	+0000000000	+0000000000	+0000000000	+0000000000	+0000000000	+0000000000
17	097 6	+5996000028	+5317000026	+0000000000	+1517000028	+2410000026	+6000000027
18	103 6	+9763000028	+2687000028	+1295300029	+7000000026	+2531000032	+0000000000
19	109 6	+0000000000	+5857000030	+9650000027	+5317000026	+0000000000	+1517000028
20	115 6	+2410000026	+6000000027	+2822000028	+2687000028	+3470000028	+2470000050
21	121 6	+2590000050	+2900000050	+3060000050	+5000000027	+5000000027	+5000000027
22	127 6	+5000000027	+2350000052	+2360000052	+2370000052	+2370000052	+2380000052
23	133 6	+2390000052	+2390000052	+2400000052	+2410000052	+1014300056	+1150000052
24	139 6	+1000000050	+0000000020	+5490000073	+9204000072	+3300000025	+1800000025
25	145 2	+0000000941	+1000000070				

Selected Cooling Times for Transient Calculations

26	151 6	+0000000000	+7200000053	+1440000054	+2520000054	+3600000054	+4320000054
27	157 6	+5760000054	+7200000054	+1080000055	+1440000055	+1800000055	+2520000055
28	163 6	+3600000055	+5400000055	+7200000055	+1080000056	+1800000056	+3600000056
29	169 2	+1080000057	+3600000057				

APPENDIX C

REVISED EXPRESSIONS FOR ISOTOPIC NUMBER DENSITY AND FISSION

Revised expressions which minimize round-off errors in code calculations for small flux-time steps are presented. Alternative expressions are given in some cases. The quantity $\Delta N_k^{i,j}$ is the change in number density of isotope i, j during the k^{th} irradiation step. In terms involving multiple summations, the summation $\sum_{a,b,\dots}$ means a sum over all values of $a, b, \dots$ from zero to n , subject to the condition that $a + b + \dots = n$. The expressions are exact.

General Expressions

$$\Delta N_k^{25} = N_{k-1}^{25} \sum_{n=1}^{\infty} \frac{t_k^n}{n!} \left(-\psi_k^{25} \right)^n \quad (C1)$$

$$\Delta N_k^{26} = N_{k-1}^{25} \phi_{\gamma k}^{25} \phi_k t_k \left[\sum_{n=0}^{\infty} \frac{t_k^n}{(n+1)!} \left(\psi_k^{26} - \psi_k^{25} \right)^n \right] e^{-\psi_k^{28} t_k} + N_{k-1}^{26} \sum_{n=1}^{\infty} \frac{t_k^n}{n!} \quad (C2)$$

$$\Delta N_k^{26} = N_{k-1}^{25} \phi_{\gamma k}^{25} \phi_k t_k \sum_{n=0}^{\infty} \frac{t_k^n}{(n+1)!} \sum_{a,b} \left(-\psi_k^{26} \right)^a \left(-\psi_k^{25} \right)^b + N_{k-1}^{26} \sum_{n=1}^{\infty} \frac{t_k^n}{n!} \left(-\psi_k^{26} \right)^n \quad (C3)$$

$$\Delta N_k^{27} = \phi_{\gamma k}^{26} \phi_k t_k \left\{ N_{k-1}^{25} \phi_{\gamma k}^{25} \phi_k t_k \left[\sum_{n=0}^{\infty} \frac{t_k^n}{(n+2)!} \sum_{a,b} \left(\psi_k^{27} - \psi_k^{26} \right)^a \left(\psi_k^{27} - \psi_k^{25} \right)^b \right] + N_{k-1}^{26} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+1)!} \left(\psi_k^{27} - \psi_k^{26} \right)^n \right\} e^{-\psi_k^{27} t_k} + N_{k-1}^{27} \sum_{n=1}^{\infty} \frac{t_k^n}{n!} \left(-\psi_k^{27} \right)^n \quad (C4)$$

$$\Delta N_k^{27} = \phi_{\gamma k}^{26} \phi_k t_k \left\{ N_{k-1}^{25} \phi_{\gamma k}^{25} \phi_k t_k \sum_{n=0}^{\infty} \frac{t_k^n}{(n+2)!} \sum_{a,b,c} \left(-\psi_k^{27} \right)^a \left(-\psi_k^{26} \right)^b \left(-\psi_k^{25} \right)^c + N_{k-1}^{26} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+1)!} \sum_{a,b} \left(-\psi_k^{27} \right)^a \left(-\psi_k^{26} \right)^b \right\} + N_{k-1}^{27} \sum_{n=1}^{\infty} \frac{t_k^n}{n!} \left(-\psi_k^{27} \right)^n \quad (C5)$$

$$\Delta N_k^{37} = \lambda^{27} t_k \left\{ \phi_{\gamma k}^{26} \phi_k t_k \left\{ N_{k-1}^{25} \phi_{\gamma k}^{25} \phi_k t_k \sum_{n=0}^{\infty} \frac{t_k^n}{(n+3)!} \sum_{a,b,c} \left(\psi_k^{37} - \psi_k^{27} \right)^a \left(\psi_k^{37} - \psi_k^{26} \right)^b \left(\psi_k^{37} - \psi_k^{25} \right)^c \right. \right.$$

$$\begin{aligned}
& + N_{k-1}^{26} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+2)!} \sum_{a,b} \left(\psi_k^{37} - \psi_k^{27} \right)^a \left(\psi_k^{37} - \psi_k^{26} \right)^b \Big\} \\
& + N_{k-1}^{27} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+1)!} \left(\psi_k^{37} - \psi_k^{27} \right)^n \Big\} e^{-\psi_k^{37} t_k} + N_{k-1}^{37} \sum_{n=1}^{\infty} \frac{t_k^n}{n!} \left(-\psi_k^{37} \right)^n \quad (C6)
\end{aligned}$$

$$\begin{aligned}
\Delta N_k^{37} = \lambda^{27} t_k \Big\{ & \varphi_{\gamma k}^{26} \phi_k t_k \left\{ N_{k-1}^{25} \varphi_{\gamma k}^{25} \phi_k t_k \sum_{n=0}^{\infty} \frac{t_k^n}{(n+3)!} \sum_{a,b,c,d} \left(-\psi_k^{37} \right)^a \left(-\psi_k^{27} \right)^b \left(-\psi_k^{26} \right)^c \left(-\psi_k^{25} \right)^d \right. \\
& + N_{k-1}^{26} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+2)!} \sum_{a,b,c} \left(-\psi_k^{37} \right)^a \left(-\psi_k^{27} \right)^b \left(-\psi_k^{26} \right)^c \Big\} \\
& + N_{k-1}^{27} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+1)!} \sum_{a,b} \left(-\psi_k^{37} \right)^a \left(-\psi_k^{27} \right)^b \Big\} + N_{k-1}^{37} \sum_{n=1}^{\infty} \frac{t_k^n}{n!} \left(-\psi_k^{37} \right)^n \quad (C7)
\end{aligned}$$

$$\Delta N_k^{28} = N_{k-1}^{28} \sum_{n=1}^{\infty} \frac{t_k^n}{n!} \left(-\psi_k^{28} \right)^n \quad (C8)$$

$$\Delta N_k^{39} = N_{k-1}^{28} \varphi_{\gamma k}^{28} \phi_k t_k \left[\sum_{n=0}^{\infty} \frac{t_k^n}{(n+1)!} \left(\psi_k^{39} - \psi_k^{28} \right)^n \right] e^{-\psi_k^{39} t_k} + N_{k-1}^{39} \sum_{n=0}^{\infty} \frac{t_k^n}{n!} \left(-\psi_k^{39} \right)^n \quad (C9)$$

$$= N_{k-1}^{28} \varphi_{\gamma k}^{28} \phi_k t_k \sum_{n=0}^{\infty} \frac{t_k^n}{(n+1)!} \sum_{a,b} \left(-\psi_k^{39} \right)^a \left(-\psi_k^{28} \right)^b + N_{k-1}^{39} \sum_{n=1}^{\infty} \frac{t_k^n}{n!} \left(-\psi_k^{39} \right)^n \quad (C10)$$

$$\begin{aligned}
\Delta N_k^{49} = \lambda^{39} t_k \Big\{ & N_{k-1}^{28} \varphi_{\gamma k}^{28} \phi_k t_k \sum_{n=0}^{\infty} \frac{t_k^n}{(n+2)!} \sum_{a,b} \left(\psi_k^{49} - \psi_k^{39} \right)^a \left(\psi_k^{49} - \psi_k^{28} \right)^b \\
& + N_{k-1}^{39} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+1)!} \left(\psi_k^{49} - \psi_k^{39} \right)^n \Big\} e^{-\psi_k^{49} t_k} + N_{k-1}^{49} \sum_{n=1}^{\infty} \frac{t_k^n}{n!} \left(-\psi_k^{49} \right)^n \quad (C11)
\end{aligned}$$

$$\Delta N_k^{49} = \lambda^{39} t_k \left\{ N_{k-1}^{28} \phi_k^{28} t_k \sum_{n=0}^{\infty} \frac{t_k^n}{(n+2)!} \sum_{a,b,c} \left(-\psi_k^{49} \right)^a \left(-\psi_k^{39} \right)^b \left(-\psi_k^{28} \right)^c \right. \\ \left. + N_{k-1}^{39} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+1)!} \sum_{a,b} \left(-\psi_k^{49} \right)^a \left(-\psi_k^{39} \right)^b \right\} + N_{k-1}^{49} \sum_{n=1}^{\infty} \frac{t_k^n}{n!} \left(-\psi_k^{49} \right)^n \quad (C12)$$

$$\Delta N_k^{40} = \phi_k^{49} t_k \left\{ \lambda^{39} t_k \left\{ N_{k-1}^{28} \phi_k^{28} t_k \sum_{n=0}^{\infty} \frac{t_k^n}{(n+3)!} \sum_{a,b,c} \left(\psi_k^{40} - \psi_k^{49} \right)^a \left(\psi_k^{40} - \psi_k^{39} \right)^b \left(\psi_k^{40} - \psi_k^{28} \right)^c \right. \right. \\ \left. \left. + N_{k-1}^{39} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+2)!} \sum_{a,b} \left(\psi_k^{40} - \psi_k^{49} \right)^a \left(\psi_k^{40} - \psi_k^{39} \right)^b \right\} \right. \\ \left. + N_{k-1}^{49} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+1)!} \left(\psi_k^{40} - \psi_k^{49} \right)^n \right\} e^{-\psi_k^{40} t_k} + N_{k-1}^{40} \sum_{n=1}^{\infty} \frac{t_k^n}{n!} \left(-\psi_k^{40} \right)^n \quad (C13)$$

$$\Delta N_k^{40} = \phi_k^{49} t_k \left\{ \lambda^{39} t_k \left\{ N_{k-1}^{28} \phi_k^{28} t_k \sum_{n=0}^{\infty} \frac{t_k^n}{(n+3)!} \sum_{a,b,c,d} \left(-\psi_k^{40} \right)^a \left(-\psi_k^{49} \right)^b \left(-\psi_k^{39} \right)^c \left(-\psi_k^{28} \right)^d \right. \right. \\ \left. \left. + N_{k-1}^{39} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+2)!} \sum_{a,b,c} \left(-\psi_k^{40} \right)^a \left(-\psi_k^{49} \right)^b \left(-\psi_k^{39} \right)^c \right\} \right. \\ \left. + N_{k-1}^{49} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+1)!} \sum_{a,b} \left(-\psi_k^{40} \right)^a \left(-\psi_k^{49} \right)^b \right\} + N_{k-1}^{40} \sum_{n=1}^{\infty} \frac{t_k^n}{n!} \left(-\psi_k^{40} \right)^n \quad (C14)$$

$$\Delta N_k^{41} = \phi_k^{40} t_k \left\{ \phi_k^{49} t_k \left\{ \lambda^{39} t_k \left\{ N_{k-1}^{28} \phi_k^{28} t_k \sum_{n=0}^{\infty} \frac{t_k^n}{(n+4)!} \right. \right. \right. \\ \left. \left. \sum_{a,b,c,d} \left(\psi_k^{41} - \psi_k^{40} \right)^a \left(\psi_k^{41} - \psi_k^{49} \right)^b \left(\psi_k^{41} - \psi_k^{39} \right)^c \left(\psi_k^{41} - \psi_k^{28} \right)^d \right. \right. \\ \left. \left. + N_{k-1}^{39} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+3)!} \sum_{a,b,c} \left(\psi_k^{41} - \psi_k^{40} \right)^a \left(\psi_k^{41} - \psi_k^{49} \right)^b \left(\psi_k^{41} - \psi_k^{39} \right)^c \right\} \right. \right.$$

$$\begin{aligned}
& + N_{k-1}^{49} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+2)!} \sum_{a,b} \left(\psi_k^{41} - \psi_k^{40} \right)^a \left(\psi_k^{40} - \psi_k^{49} \right)^b \Big\} \\
& + N_{k-1}^{40} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+1)!} \left(\psi_k^{41} - \psi_k^{40} \right)^n \Bigg] e^{-\psi_k^{41} t_k} + N_{k-1}^{41} \sum_{n=1}^{\infty} \frac{t_k^n}{n!} \left(-\psi_k^{41} \right)^n \quad (C15)
\end{aligned}$$

$$\begin{aligned}
\Delta N_k^{41} = & \varphi_{\gamma k}^{40} \phi_k t_k \left\{ \varphi_{\gamma k}^{49} \phi_k t_k \left\{ \lambda^{39} t_k \left\{ N_{k-1}^{28} \varphi_{\gamma k}^{28} \phi_k t_k \sum_{n=0}^{\infty} \frac{t_k^n}{(n+4)!} \right. \right. \right. \\
& \sum_{a,b,c,d,e} \left(-\psi_k^{41} \right)^a \left(-\psi_k^{40} \right)^b \left(-\psi_k^{49} \right)^c \left(-\psi_k^{39} \right)^d \left(-\psi_k^{28} \right)^e \\
& + N_{k-1}^{39} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+3)!} \sum_{a,b,c,d} \left(-\psi_k^{41} \right)^a \left(-\psi_k^{40} \right)^b \left(-\psi_k^{49} \right)^c \left(-\psi_k^{39} \right)^d \Big\} \\
& + N_{k-1}^{49} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+2)!} \sum_{a,b,c} \left(-\psi_k^{41} \right)^a \left(-\psi_k^{40} \right)^b \left(-\psi_k^{49} \right)^c \Big\} \\
& + N_{k-1}^{40} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+1)!} \sum_{a,b} \left(-\psi_k^{41} \right)^a \left(-\psi_k^{40} \right)^b \Bigg] \\
& + N_{k-1}^{41} \sum_{n=1}^{\infty} \frac{t_k^n}{n!} \left(-\psi_k^{41} \right)^n \quad (C16)
\end{aligned}$$

$$\begin{aligned}
\Delta N_k^{42} = & \varphi_{\gamma k}^{41} \phi_k t_k \left\{ \varphi_{\gamma k}^{40} \phi_k t_k \left\{ \varphi_{\gamma k}^{49} \phi_k t_k \left\{ \lambda^{39} t_k \left\{ \varphi_{\gamma k}^{28} \phi_k t_k N_{k-1}^{28} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+5)!} \right. \right. \right. \right. \\
& \sum_{a,b,c,d,e,f} \left(-\psi_k^{42} \right)^a \left(-\psi_k^{41} \right)^b \left(-\psi_k^{40} \right)^c \left(-\psi_k^{49} \right)^d \left(-\psi_k^{39} \right)^e \left(-\psi_k^{28} \right)^f \\
& + N_{k-1}^{39} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+4)!} \sum_{a,b,c,d,e} \left(-\psi_k^{42} \right)^a \left(-\psi_k^{41} \right)^b \left(-\psi_k^{40} \right)^c \left(-\psi_k^{49} \right)^d \left(-\psi_k^{39} \right)^e \Big\}
\end{aligned}$$

$$\begin{aligned}
& + N_{k-1}^{49} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+3)!} \sum_{a, b, c, d} \left(-\psi_k^{42} \right)^a \left(-\psi_k^{41} \right)^b \left(-\psi_k^{40} \right)^c \left(-\psi_k^{49} \right)^d \Bigg\} \\
& + N_{k-1}^{40} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+2)!} \sum_{a, b, c} \left(-\psi_k^{42} \right)^a \left(-\psi_k^{41} \right)^b \left(-\psi_k^{40} \right)^c \Bigg\} \\
& + N_{k-1}^{41} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+1)!} \sum_{a, b} \left(-\psi_k^{42} \right)^a \left(-\psi_k^{41} \right)^b \Bigg\} + N_{k-1}^{42} \sum_{n=1}^{\infty} \frac{t_k^n}{n!} \left(-\psi_k^{42} \right)^n \quad (C17)*
\end{aligned}$$

$$F_k^{25} = \frac{(\phi_{ak}^{25} - \phi_{\gamma k}^{25}) \phi_k}{\psi_k^{25}} \Delta N_k^{25} \quad (C18)$$

$$F_k^{28} = - \frac{(\phi_{ak}^{28} - \phi_{\gamma k}^{28}) \phi_k}{\psi_k^{28}} \Delta N_k^{28} = - \frac{\delta_k^{28} \phi_k}{\psi_k^{28}} \Delta N_k^{28} \quad (C19)$$

$$\begin{aligned}
F_k^{49} = & (\phi_{ak}^{49} - \phi_{\gamma k}^{49}) \phi_k t_k \left\{ \lambda^{39} t_k \left\{ \phi_{\gamma k}^{28} \phi_k t_k N_{k-1}^{28} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+3)!} \sum_{a, b, c} \left(-\psi_k^{49} \right)^a \left(-\psi_k^{39} \right)^b \left(-\psi_k^{28} \right)^c \right. \right. \\
& + N_{k-1}^{39} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+2)!} \sum_{a, b} \left(-\psi_k^{49} \right)^a \left(-\psi_k^{39} \right)^b \Bigg\} + N_{k-1}^{49} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+1)!} \left(-\psi_k^{49} \right)^n \Bigg\} \quad (C20)
\end{aligned}$$

$$\begin{aligned}
F_k^{41} = & (\phi_{ak}^{41} - \phi_{\gamma k}^{41}) \phi_k t_k \left\{ \phi_{\gamma k}^{40} \phi_k t_k \left\{ \phi_{\gamma k}^{49} \phi_k t_k \left\{ \lambda^{39} t_k \left\{ \phi_{\gamma k}^{28} \phi_k t_k N_{k-1}^{28} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+5)!} \right. \right. \right. \\
& \sum_{a, b, c, d, e} \left(-\psi_k^{41} \right)^a \left(-\psi_k^{40} \right)^b \left(-\psi_k^{49} \right)^c \left(-\psi_k^{39} \right)^d \left(-\psi_k^{28} \right)^e \\
& + N_{k-1}^{39} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+4)!} \sum_{a, b, c, d} \left(-\psi_k^{41} \right)^a \left(-\psi_k^{40} \right)^b \left(-\psi_k^{49} \right)^c \left(-\psi_k^{39} \right)^d \Bigg\} \Bigg\}
\end{aligned}$$

*Expression (C17), for the change in concentration of Pu-242 during the k^{th} irradiation step, has no corresponding expression in Appendix A because, in the exponential form of Appendix A, its length would be prohibitive. Equation (C17) illustrates the extension to the higher isotopes of the summation form of the expressions and is of practical value since isotopic plutonium measurements on the mass spectrometer give P-242 as well as the lighter plutonium isotopes.

$$\begin{aligned}
& + N_{k-1}^{49} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+3)!} \sum_{a,b,c} \left(-\psi_k^{41} \right)^a \left(-\psi_k^{40} \right)^b \left(-\psi_k^{49} \right)^c \Big\} \\
& + N_{k-1}^{40} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+2)!} \sum_{a,b} \left(-\psi_k^{41} \right)^a \left(-\psi_k^{40} \right)^b \Big\} + N_{k-1}^{41} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+1)!} \left(-\psi_k^{41} \right)^n \Big\} \quad (C21)
\end{aligned}$$

$$\Delta N_k^I = \sum_{u,v} y_I^{u,v} F_k^{u,v} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+1)!} \left(-\psi_k^I \right)^n + N_{k-1}^I \sum_{n=1}^{\infty} \frac{t_k^n}{n!} \left(-\psi_k^I \right)^n \quad (C22)$$

$$\begin{aligned}
\Delta N_k^{Xe} &= \sum_{u,v} y_{Xe}^{u,v} F_k^{u,v} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+1)!} \left(-\psi_k^{Xe} \right)^n \\
&+ \lambda^I t_k \left[\sum_{u,v} y_I^{u,v} F_k^{u,v} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+2)!} \sum_{a,b} \left(-\psi_k^{Xe} \right)^a \left(-\psi_k^I \right)^b \right. \\
&+ N_{k-1}^I \sum_{n=0}^{\infty} \frac{t_k^n}{(n+1)!} \sum_{a,b} \left(-\psi_k^{Xe} \right)^a \left(-\psi_k^I \right)^b \Big] \\
&+ N_{k-1}^{Xe} \sum_{n=1}^{\infty} \frac{t_k^n}{n!} \left(-\psi_k^{Xe} \right)^n \quad (C23)
\end{aligned}$$

$$\Delta N_k^{Nd} = \sum_{u,v} y_{Nd}^{u,v} F_k^{u,v} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+1)!} \left(-\psi_k^{Nd} \right)^n + N_{k-1}^{Nd} \sum_{n=1}^{\infty} \frac{t_k^n}{n!} \left(-\psi_k^{Nd} \right)^n \quad (C24)$$

$$\begin{aligned}
\Delta N_k^{Pm} &= \sum_{u,v} y_{Pm}^{u,v} F_k^{u,v} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+1)!} \left(-\psi_k^{Pm} \right)^n \\
&+ \lambda^{Nd} t_k \left[\sum_{u,v} y_{Nd}^{u,v} F_k^{u,v} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+2)!} \sum_{a,b} \left(-\psi_k^{Pm} \right)^a \left(-\psi_k^{Nd} \right)^b \right.
\end{aligned}$$

$$+ N_{k-1}^{Nd} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+1)!} \sum_{a,b} \left(-\psi_k^{Pm} \right)^a \left(-\psi_k^{Nd} \right)^b \Big] + N_{k-1}^{Pm} \sum_{n=1}^{\infty} \frac{t_k^n}{n!} \left(-\psi_k^{Pm} \right)^n \quad (C25)$$

$$\begin{aligned} \Delta N_k^{Sm} = & \sum_{u,v} y_{Sm}^{u,v} F_k^{u,v} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+1)!} \left(-\psi_k^{Sm} \right)^n \\ & + \lambda^{Pm} t_k \left\{ \sum_{u,v} y_{Pm}^{u,v} F_k^{u,v} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+2)!} \sum_{a,b} \left(-\psi_k^{Sm} \right)^a \left(-\psi_k^{Pm} \right)^b \right. \\ & + \lambda^{Nd} t_k \left[\sum_{u,v} y_{Nd}^{u,v} F_k^{u,v} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+3)!} \sum_{a,b,c} \left(-\psi_k^{Sm} \right)^a \left(-\psi_k^{Pm} \right)^b \left(-\psi_k^{Nd} \right)^c \right. \\ & + N_{k-1}^{Nd} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+2)!} \sum_{a,b,c} \left(-\psi_k^{Sm} \right)^a \left(-\psi_k^{Pm} \right)^b \left(-\psi_k^{Nd} \right)^c \Big] \\ & \left. + N_{k-1}^{Pm} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+1)!} \sum_{a,b} \left(-\psi_k^{Sm} \right)^a \left(-\psi_k^{Pm} \right)^b \right\} \\ & + N_{k-1}^{Sm} \sum_{n=1}^{\infty} \frac{t_k^n}{n!} \left(-\psi_k^{Sm} \right)^n \quad (C26) \end{aligned}$$

Expressions for Zero-Flux Case

For the special case of zero flux, expressions (C1) through (C26) reduce to the following:

$$\Delta N_k^{25} = 0 \quad (C27)$$

$$\Delta N_k^{26} = 0 \quad (C28)$$

$$\Delta N_k^{27} = N_{k-1}^{27} \sum_{n=1}^{\infty} \frac{t_k^n}{n!} \left(-\lambda^{27} \right)^n \quad (C29)$$

$$\Delta N_k^{37} = -\Delta N_k^{27} \quad (C30)$$

$$\Delta N_k^{28} = 0 \quad (C31)$$

$$\Delta N_k^{39} = N_{k-1}^{39} \sum_{n=1}^{\infty} \frac{t_k^n}{n!} \left(-\lambda^{39} \right)^n \quad (C32)$$

$$\Delta N_k^{49} = -\Delta N_k^{39} \quad (C33)$$

$$\Delta N_k^{40} = 0 \quad (C34)$$

$$\Delta N_k^{41} = N_{k-1}^{41} \sum_{n=1}^{\infty} \frac{t_k^n}{n!} \left(-\lambda^{41} \right)^n \quad (C35)$$

$$\Delta N_k^{42} = 0 \quad (C36)$$

$$F_k^{25} = 0 \quad (C37)$$

$$F_k^{28} = 0 \quad (C38)$$

$$F_k^{49} = 0 \quad (C39)$$

$$F_k^{41} = 0 \quad (C40)$$

$$\Delta N_k^I = N_{k-1}^I \sum_{n=1}^{\infty} \frac{t_k^n}{n!} \left(-\lambda^I \right)^n \quad (C41)$$

$$\Delta N_k^{Xe} = \lambda^I t_k N_{k-1}^I \sum_{n=0}^{\infty} \frac{t_k^n}{(n+1)!} \sum_{a,b} \left(-\lambda^{Xe} \right)^a \left(-\lambda^I \right)^b + N_{k-1}^{Xe} \sum_{n=1}^{\infty} \frac{t_k^n}{n!} \left(-\lambda^{Xe} \right)^n \quad (C42)$$

$$\Delta N_k^{Nd} = N_{k-1}^{Nd} \sum_{n=1}^{\infty} \frac{t_k^n}{n!} \left(-\lambda^{Nd} \right)^n \quad (C43)$$

$$\Delta N_k^{Pm} = \lambda^{Nd} t_k N_{k-1}^{Nd} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+1)!} \sum_{a,b} \left(-\lambda^{Pm} \right)^a \left(-\lambda^{Nd} \right)^b + N_{k-1}^{Pm} \sum_{n=1}^{\infty} \frac{t_k^n}{n!} \left(-\lambda^{Pm} \right)^n \quad (C44)$$

$$\Delta N_k^{Sm} = \left(\lambda^{Pm} t_k \right) \left(\lambda^{Nd} t_k \right) N_{k-1}^{Nd} \sum_{n=0}^{\infty} \frac{t_k^n}{(n+2)!} \sum_{a,b} \left(-\lambda^{Pm} \right)^a \left(-\lambda^{Nd} \right)^b - N_{k-1}^{Pm} \sum_{n=1}^{\infty} \frac{t_k^n}{n!} \left(-\lambda^{Pm} \right)^n \quad (C45)$$

APPENDIX D

MODIFIED FLUX-TIME CODE S0264

Purpose

This code converts the detailed irradiation history for a particular irradiation sample (the output of the S0216 Code) into an approximate irradiation history. Such modification is necessary in order to minimize round-off errors in the S0269 Code used for isotopic number densities and other quantities derived from these densities. Specifically, a group of steps in the detailed irradiation history of a sample is converted into a single zero-flux step followed by a single nonzero-flux step. The total elapsed time and the total flux-time for all steps in the group are preserved in the conversion to the two equivalent steps.

Input

The output of the S0216 Code, which gives the detailed irradiation history steps, should have t_k in seconds in location 951, ϕ_k in neutrons/cm²-sec in location 952, and j_k in location 953. Inspection of the detailed irradiation history data permits grouping steps of irradiation so that each group corresponds to a total exposure of 3×10^{19} n/cm² or more (approximately 40 hours in a flux of 2×10^{14} n/cm²-sec). The different charges within a cycle at the MTR frequently comprise suitable groups. Each group is run as a separate problem. The input consists of the group of cards (each containing t_k , ϕ_k , and j_k), preceded by a card on which the equivalent thermal flux for the group (usually the maximum flux) is punched in location 852, and the measured epithermal-to-thermal flux ratio (ϕ_R/ϕ from the S0226 Code) is punched in location 853.

Example: Group A (Charge A), Cycle 106

	<u>Quantity</u>	<u>Location No.</u>		<u>A</u>	<u>B</u>	<u>C</u>		
First card	{	ϕ_{eq}	852	+2	000	000	064	Equivalent thermal flux
		ϕ_R/ϕ	853	+5	000	000	048	Epithermal-to-thermal flux ratio
Cards of group	{	t_k	951	+4	000	000	054	Elapsed time of step
		ϕ_k	952	+1	900	000	064	Thermal flux of step
		j_k	953	+0	000	000	001	Index number for step

General Comments

- 1) One problem consists of one group only.
- 2) The first input step (951 to 953) of the group must be a zero-flux step (all zeroes in 952). Otherwise, an unconditional stop will occur with all 9's displayed. The last step should be a nonzero-flux step.

Output

For each input step of the group exclusive of the first, two output cards are punched with four words per card as follows:

Odd cards	950	j = dimensionless index, the value of j on the first input card (zero-flux card)
	951	t = equivalent time at zero flux (seconds)
	952	$\phi = 0$
	953	ϕ_R/ϕ = epithermal-to-thermal flux ratio (same as in 853)

Even cards	{	950	j = dimensionless index, the value of j on the input card most recently read
		951	t_{eq} = equivalent time at equivalent flux
		952	ϕ_{eq} = equivalent thermal flux in n/cm^2 -sec (same as in 852)
		953	ϕ_R/ϕ = epithermal-to-thermal flux ratio (same as in 853)

Use of the Output as Input to the S0269 Code

Only the last two output cards are used as input for the S0269 Code. The input deck for the S0269 Code is made up of the last two output cards from the successive groups (described previously and run as separate problems in the S0264 code). In the input deck, as now assembled, the first card for a given cycle in the MTR (which will be a zero-flux card) should be repunched so that j , in location 950, contains either 001 or 020 in the C-address. In the S0269 Code, these values of j will trigger the computation of η for either one cooling time (the last of 20 read as input into the S0269 Code) or for all 20 cooling times (from which the transient behavior of η is obtained).

The input deck for the S0269 Code should be modified further by removing the first card (zero-flux card) and placing it at the end of the deck. This will preclude the computation of η as a function of cooling time for the case prior to any irradiation and will provide for such computation for the case following the last irradiation.

APPENDIX E

S0306 CODE, EXTENSION OF S0269 CODE FOR IRRADIATION HISTORY STEPS OTHER THAN THE FIRST

Purpose

The S0306 Code is identical to S0269 except that it cannot be used for the initial irradiation step. This code permits computations for later steps in the irradiation history without repeating the earlier steps and may be used for one or more irradiation history steps. Since cross sections and resonance integrals for the various isotopes are read as input, they may be altered in successive problems to account for their variations with concentration or to account for self-shielding.

Instructions

Code instructions are identical to those of S0269 except for the following:

<u>Location</u>	<u>Word in S0269</u>	<u>Word in S0306</u>
267	- 3 128 001 691	+ 0 454 000 000
268	+ 0 111 009 267	+ 0 454 000 000
269	- 1 691 692 692	+ 0 454 000 000
270	+ 0 111 008 269	+ 0 454 000 000
271	+ 4 137 000 700	+ 0 400 700 700
381	+ 9 002 703 705	+ 0 400 705 706
382	+ 0 410 700 700	+ 0 400 901 903
383	+ 0 410 705 706	+ 0 400 971 976

Input

Input cards for locations 001 to 146, 151 to 170, 700, 705 to 706, 901 to 903, and 971 to 976 must precede the irradiation history cards in locations 950 to 953. The locations 950 to 953 contain the input for one step in the irradiation history. Successive steps may be run by stacking appropriate input in these locations. Input for 001 to 014 is obtained from the output for the preceding irradiation history step of S0269 (or S0306, depending upon which code was used for the preceding step) in the nearest 197 to 210 locations preceding the last punched 901 output card or cards.* Input for 015 to 018 is obtained from the output of S0269 (or S0306) in the nearest 908 to 911 locations, preceding the last punched 901 output. Input for 019 to 146 is identical to that used in S0269, unless certain adjustments to cross sections or resonance integrals for concentration are to be made as the irradiation history proceeds. Input for locations 151 to 170 is identical to that used in S0269. Input for locations 700 and 705 to 706 is obtained from the first two output cards of S0269. Input for locations 901 to 903 is identical to the last such output of S0269 (or S0306). Input for locations 971 to 976 is identical to the last such output of S0269 (or S0306), and immediately precedes the data in 908 to 911 previously identified.

Output

The output of the S0306 Code is identical to the output of the S0269 Code, except that the first two output cards of S0269 (locations 700 and 705 to 706) are used as input and are not included in the output.

*There may be as many as 20 successive 901 cards.

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