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MEASURED PHYSICS PARAMETERS IN A ZONED FAST URANIUM CARBIDE CORE, ASSEMBLY 4Z

by

R. A. Karam, A. Ancarani,
F. H. Helm, W. Y. Kato,
G. W. Main, and G. K. Rusch

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Reactor Physics Division

December 1966

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I. SUMMARY

Assembly 4Z of ZPR-6 was a zoned cylindrical core with a central region having the composition of a 2600-liter carbide core, Assembly 5. Assembly 4Z was designed so that the neutron-energy spectrum and the importance spectrum at the center are the same as those for Assembly 5. The major objective associated with the zoned core is to test, in conjunction with the full-scale system, how well reactor-physics parameters for Assembly 5 can be predicted from the zoned-core measurements.

The following conclusions may be made on the basis of the experimental results:

1. The critical mass (1101 kg) was predicted within 1-2% with several different cross-section sets.
2. The measured neutron-energy spectrum is in general agreement with the calculated spectrum. Spectral indices obtained with fission ratios of different materials also support this conclusion.
3. Fission-rate traverses showed that the application of diffusion theory to zoned cores with narrow regions results in softer spectra of the high-energy groups. Results from a transport code, SNARG, were in better agreement with measurements.
4. ELMOE averaging is a valuable tool for producing averaged group cross sections for the asymptotic spectra, provided such spectra exist. ELMOE averaging for a composition of a driver zone produces cross sections characteristic of a core of that composition, not in the configuration of the driver zone but in some configuration where criticality is achieved by iteration on buckling. The subsequent use of such cross sections in a narrow driver zone, where an asymptotic spectrum cannot be achieved, gives poor approximation of the true spectrum.

5. Whereas the spatial distribution of the real fluxes was better predicted with transport than with diffusion calculations, the opposite is true for the importance of fission neutrons,

$$\sum_i \chi_i \phi_i^\dagger$$

The reasons for this are presently under investigation.

6. The sodium-void coefficient was found to be positive $[+2.3 \times 10^{-5} (\Delta k/k)/\text{kg}]$ at the center of Assembly 4Z. It decreased as a function of the axial distance from the center, and its value became zero at about 30 cm from the center and negative beyond that point. Calculations with first-order perturbation theory with a one-dimensional code overestimate the sodium-void coefficient by about 60%. Differences in k calculations with and without the sodium are in agreement with measurements.

7. The normalized denominator of the perturbation expression for Assembly 4Z, as obtained from the measured quantity,

$$\sum_i \chi_i \phi_i^\dagger,$$

and the calculated quantity,

$$\sum_i (\nu \Sigma_f) \phi_i,$$

by SNARG, was 6.772×10^5 . This value for Assembly 5 is 6.500×10^5 . Thus, the predicted values for the worth measurements at the center of Assembly 5 will be 4% greater than those at the center of Assembly 4Z.

II. INTRODUCTION

Assembly 4Z was a zoned core with a central region having the composition of a 2600-liter carbide reactor, Assembly 5. Assembly 4Z was designed so that the neutron-energy spectrum and the importance spectrum at the center are the same as those for Assembly 5. The central region had a radius of 41.5 cm. The other regions that comprised Assembly 4Z were the buffer, the driver, and the reflector, with outer radii of 47.4, 54.5, and 91.2 cm, respectively. The axial height of the core, i.e., the central, buffer, and driver regions, was 121.9 cm. The axial height of the core and reflector was 181.9 cm.

The major objective associated with the zoned core is to test, in conjunction with the full-scale system, how well reactor-physics parameters for Assembly 5 can be predicted from the zoned-core measurements.

The technique of simulating large cores with smaller zoned cores, if successful, would provide the advantage of reducing substantially the fuel inventory required for the study of large cores. In addition, there is considerable saving in the time and effort required to obtain the characteristics of full-size systems when such systems are simulated by the smaller zoned cores.

The accuracy with which reactivity coefficients (such as the sodium-void and Doppler coefficients and the reactivity worths of samples of various materials introduced into the central region) can be predicted for the full-size core from measurements performed in the zoned core, depends on: (1) the accuracy of the calculated importance-weighted integral of the source of fission neutrons in the entire volume of each assembly and (2) the accuracy of the spectral matching at the center of each system. The following relation is used to predict the values of the coefficients for the full-size system:

$$W_F = (W_Z)(S_Z/S_F), \quad (1)$$

where

W_F is the predicted worth of a sample in the full-size core,

W_Z is the measured worth of the same sample in the zoned core,

and

$$S = \int_V \left[\sum_i (\nu \Sigma_f)_i \phi_i \right] \left[\sum_i \chi_i \phi_i^\dagger \right] dV,$$

where V is the volume of the reactor and, the subscripts F and Z refer to the full-size and zoned cores, respectively.

Equation 1 results from perturbation theory and it is valid provided the reactivity-worth measurements are made at a position in the zoned core where the neutron energy and spatial distributions of the real and adjoint fluxes are the same as those in the full-size core. Thus, to successfully extrapolate from zoned to full-size systems, it is necessary to obtain an accurate value of S_Z/S_F . At present, S_Z/S_F can be obtained only from theoretical considerations. (Later in this report, a possible method for experimentally determining S_Z and/or S_F will be discussed.) Since there will always be some uncertainties in the calculation of S_Z/S_F , the magnitude of such uncertainties may be estimated with the proper types of measurements. The experimental program for Assembly 4Z comprised many different types of measurements, each of which was designed to either provide specific answers or show certain trends in the discrepancies between the analytical and experimental results.

The measurements can be divided into seven categories: (1) critical mass; (2) neutron energy and spatial distributions of the real fluxes; (3) spatial distribution of the importance of fission neutrons; (4) sodium-void coefficient mapping and the radial reactivity worth of U^{235} , U^{238} and graphite; (5) reactivity worths of a large number of materials measured at the center and at 16.5 cm radially from the center in the midplane; (6) Rossi-alpha measurements; and (7) Doppler-effect measurements.

The measurement of critical mass is the usual straightforward type. Information about the neutron-energy spectrum was obtained by three different methods: (1) direct measurement of the spectrum in the energy range of 1 keV to 1 MeV by the proton proportional counter;¹ (2) spectrum measurement with Li^6F solid-state spectrometer;¹ and (3) fission ratios of U^{238} , U^{234} , and Pu^{239} to U^{235} . The fission ratios were obtained as a function of the radius and the axial height at the midplane of the core. These fission ratios were also measured at the center of Assembly 4Z after the sodium in the central region was removed. Fission rates of U^{235} and U^{238} were also obtained along the axial and radial dimensions. Fine-structure fission rates across the heterogeneous composition of drawers were also obtained.

The radial and axial spatial distributions of the importance of the fission neutrons were measured by using neutrons released from the spontaneous fission of a Cf^{252} source. In these measurements it was assumed that the fission spectrum of Cf^{252} is similar to that of the fissile materials in the core.

The sodium-void coefficient was measured as a function of radius at the midplane by removing small quantities (approximately 1 kg) of sodium.

Another set of measurements consisted of voiding 5-cm cans of sodium in the central 18 drawers in both halves at the midplane in the axial direction. This 5-cm region was later extended to 10 cm. Consequently, the sodium was put back into this region, and the next 10-cm cans were replaced by empty cans. This procedure was continued throughout the axial direction. Finally, all the sodium in the central 18 drawers was voided. The heterogeneity effects on the sodium-void coefficients were also investigated by removing the sodium when it was placed next to U^{238} and again when it was next to graphite. These effects were studied by repeating the above procedure.

The reactivity worths of a large number of materials were measured relative to void at the center and at 16.5 cm radially from the center in the midplane. The samples were for the most part $5 \times 5 \times 2.54$ cm. Size effects of a few samples were also investigated. Some of these measurements were repeated after the sodium in the central region was removed.

Rossi-alpha measurement, from which prompt-neutron lifetime can be inferred, was made. This measurement was performed at the center of Assembly 4Z with two fission chambers and a time analyzer.

The extensive Doppler-effects study performed on Assembly 4Z is reported elsewhere.²

Finally, the reactivity worths of different materials in Assembly 5 are predicted. These predictions are made by obtaining a value for S_Z/S_F , based on measurements and calculation.

All the calculations reported here were done with Cross-section Sets 224³ and 801.⁴ The cross sections in Set 224 were ELMOE-averaged⁵ for typically large carbide cores, and those in Set 801 were ELMOE-averaged for Assembly 4Z. More refined calculations using the MC² code⁶ to generate cross sections for Assemblies 4Z and 5 will be performed in the near future.

III. COMPOSITION AND CRITICAL MASS

A. Composition

Assembly 4Z was a cylindrical core with four regions: central, buffer, driver, and reflector, with experimental outer radii of 41.5, 47.4, 54.5, and 91.2 cm, respectively. The axial height of the core, i.e., the central, buffer, and driver regions, was 121.9 cm. The axial height of the core and reflector was 181.9 cm. The central region in Assembly 4Z had

the same material composition as Assembly 5, the full-size core.

Table I lists the atomic concentrations of all the materials in both assemblies. Figure 1 shows a plane view of the midsection of Assembly 4Z. Each square in Fig. 1 represents one unit matrix and is 5.5 x 5.5 cm. The matrix tubes are Type 304 stainless steel and have a nominal wall thickness of 0.10 cm. The drawers into which the reactor materials are loaded are also stainless steel (Type 304) with a cross-sectional area of 25.8 cm². These drawers have three equal sides with the top side left open to allow the materials to be loaded. The wall thickness of the drawers is 0.081 cm.

112-7479

Fig. 1. Plane View of Midsection of ZPR-6 Assembly 4Z

TABLE I. Atomic Densities in Assemblies 5 and 4Z^a

Material	Atomic Density, x 10 ⁻²⁴ , in Assembly 5	Atomic Density, x 10 ⁻²⁴ , in Assembly 4Z			Atomic Density, x 10 ⁻²⁴ , in Reflector
	Core Region, ^b r ₀ = 75.60 cm	Region ^b			(Common to both assemblies ^c)
		Central, r ₀ = 41.531 cm	Buffer, r ₀ = 47.445 cm	Driver, r ₀ = 54.517 cm	
U ²³⁵	0.00153	0.00153	0.00004546	0.006564	0.000081
U ²³⁸	0.01056	0.01056	0.02172	0.005243	0.03993
Na	0.00920	0.00920	0.000	0.00920	0.000
C	0.01293	0.01293	0.030163	0.01293	0.000
Fe	0.00904	0.00904	0.00697	0.00904	0.004322
Ni	0.00113	0.00113	0.00087	0.00113	0.00054
Cr	0.002386	0.002386	0.001384	0.002386	0.00114

^aThe volume of Assembly 5 is 2600 liters, and that of Assembly 4Z is 1100 liters.

^b r_o is the outer radius.

^cThe outer radius of the reflector of Assembly 5 is 102.6 cm. This is equivalent to a 27-cm-thick reflector.

The dual-purpose control rods shown in Fig. 1 are essentially loaded in the same way as the rest of the drawers, except that these drawers were 50.8 cm long instead of the normal 60.96 cm, and they contained 0.158 cm less depleted uranium (0.2% enrichment). ZPR-6 has 10 dual-purpose control rods (five per half) and 12 safety rods or blades (six per half). The control rods are dual-purpose in that each may be operated for control if so desired and all will be driven out of the reactor upon a scram signal. The safety rods consisted of blades, 1 cm thick containing about 230 g of B^{10} . The rest of the space in a matrix unit that was not occupied by the safety blade was loaded with core materials.

Five basic types of drawers were used in Assembly 4Z. Three types were associated with the central zone, one type in the buffer region, and one type in the driver zone. Figure 2 shows four types of drawers; the first three types were used in a cyclic fashion in the central zone, and the last type was a typical driver drawer. A typical buffer drawer is shown in Fig. 3. The criteria followed in designing these drawers are:

1. The spacing between U^{235} plates be as uniform as possible;
2. The sodium cans be surrounded by U^{238} plates (this is considered the worst condition for the sodium-void coefficient); and
3. The rest of the materials be distributed as uniformly as possible.

The same three types of drawers used in the central zone will be used in the core of Assembly 5.

Fig. 2. Basic Types of Drawers in ZPR-6 Assembly 4Z

Fig. 3. Buffer Drawer

B. Critical Mass

The measured critical mass after the core was cylinderized was 1101 kg. This value of the critical mass does not include heterogeneity

effects. Table II compares the calculated critical mass by various cross-section sets with measurement.

TABLE II. Comparison of Calculated and Measured
Critical Mass (kg) of ZPR-6 Assembly 4Z

Critical Mass, kg	Code and Cross-section Set				
	MACH 1, Set 224, 22 groups	MACH 1, Set 801, 25 groups	RE-122, Set 635, 16 groups	RE-122, Set 149, 4 groups	SNARG-54, Set 224, 22 groups
Calculated ^a	1107	1098 ^b	1103	1153	1074 ^c
Measured	1101				

^aCalculations were done in cylindrical geometry.

^bSet 801 was ELMOE-averaged for Assembly 4Z.

^cIn this calculation, a fictitious absorption cross section was added to account for axial leakage.

C. Heterogeneity Effects

Heterogeneity effects on the critical mass were investigated by:

1. Measuring the worth of U^{235} as a function of the thickness of the plates in the central zone (see Section VII);
2. The relative position of U^{238} plates to U^{235} plates (see Sections IV and VI); and
3. The effect of having a continuous column of core material in the axial direction.

The effect of plate thickness of U^{235} on reactivity was found to be small (see Table XVI in Section VII and Table XII in Section VI) and within the experimental uncertainties.

The effect of placing U^{238} next to U^{235} plates resulted in a positive reactivity change. This positive reactivity change is due to the increase in U^{238} fission because of the enhanced exposure of U^{238} to fission neutrons released in the U^{235} plates before these neutrons reached the equilibrium spectrum. This phenomenon can be seen in the fine-structure fission-rate traverse of U^{238} (Section IV), and it seems to be limited to the vicinity of the interface because the increase in U^{238} fissions disappeared at the face of the 0.318-cm U^{238} plate that is away from the U^{235} plate (see Figs. 11 and 12 in Section IV). Experimentally this effect was tested by increasing the inter-facial contact in 18 drawers at the center of the core from 9 (see Fig. 2) to 18.

In this test, the relative positions of the U^{235} plates and sodium cans were unchanged; only the U^{238} and graphite plates were exchanged. The reactivity change that resulted from this test was $+7.6$ lh ($1.6 \times 10^{-4} \Delta k/k$). Based on the homogenized case, it is estimated that half of this amount may be associated with the heterogeneity effect of the U^{238} plates being in contact with the U^{235} plates. From these measurements it is estimated that this type of heterogeneity effect in the entire central zone is about $0.1\% \Delta k/k$. This is equivalent to about 5 kg of U^{235} .

The effect of having a continuous column of a material in the axial direction was investigated by rotating every other 5-cm section 90°

112-7481

Fig. 4. Top View of Six Central Drawers Modified for Measurement of Heterogeneity Effect. General appearance:

Lined portions are sections of normal loading (vertical plates); crossed portions are sections where loading has been rotated 90° (horizontal plates).

in the front 30 cm of six drawers at the center of Assembly 4Z. Figure 4 shows how the core materials were rearranged for this test. The reactivity change that resulted from this experiment was rather small, and it was within the experimental error.

Heterogeneity effects were not studied in the driver zone except for the fine-structure fission rate of U^{238} . These results (see Fig. 11 in Section IV) showed that the heterogeneity effects due to the relative position of U^{238} with respect to U^{235} are about the same order of magnitude as those in the central zone. The heterogeneity effect due to the thickness of the U^{235} plates is perhaps more positive in the driver than in the central region because the neutron spectrum in the driver is harder. Based on the study of D. Meneghetti,⁷ this effect is estimated to be of the order of $0.5\% \Delta k/k$. Thus the total heterogeneity effects can be as large as $0.6\% \Delta k/k$ or 24 kg of U^{235} .

Assembly 4Z was designed with a relatively large central zone ($r = 41.52$ cm). This design was chosen so that the neutron spectrum in the central region is matched to that of the full-size core over a large volume. It is certainly not an optimum design from the point of view of critical mass, since the anticipated difference between the critical masses of Assemblies 4Z and 5 is only 500 kg of U^{235} . A zoned core that is smaller than Assembly 4Z can be designed, and indeed such a core has already been made and is presently being assembled on the ZPR-9 facility.⁸ It is designated as ZPR-9 Assembly 11, and its critical mass is anticipated to be 500 kg of U^{235} . Assembly 11 has the same composition in the central zone as Assembly 4Z but with a radius of 19.3 cm.

IV. NEUTRON SPECTRUM, FISSION RATIOS, AND FISSION RATES

A. Spectrum Measurements

Bennett measured the neutron spectrum in the energy range of 1 keV to 1 MeV with a proton-recoil counter. The results are reported in Ref. 1, together with spectrum measurements obtained with Li^6F solid-state spectrometer. These measurements show a general agreement with the calculated spectrum with Cross-section Set 801.

*SIGNAL CABLE 93Ω MICRODOT CABLE. TEFLON INSULATION 0.130 diam WITH MICRODOT TYPE S-93 CONNECTOR

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112-7535 Rev. 1

Fig. 5. Small Surface-barrier Solid-state Counter Used for Fission-ratio Traverses

B. Fission Ratios

Fission ratios of U^{234} , U^{238} , and Pu^{239} to U^{235} were measured at the midplane as a function of the axial direction and at $1\frac{3}{4}$ in. from the midplane in the radial direction. These measurements were obtained with a surface-barrier solid-state detector. Figure 5 shows the counter arrangement used. The counter was driven with a stepping motor attached to a screw lead with practically no backlash. Position indication was reproducible to within 0.002 cm.

The axial traverse of the fission ratios was performed in Drawer 23-23 at the center of Assembly 4Z. (The numbers after a drawer refer to row and column, respectively, in the matrix; see Fig. 1). The two 1.27-cm sodium cans in Drawer 23-23 were replaced with two 0.635-cm cans to provide space for the counter-traverse mechanism. Figure 6 shows the loading of this drawer. The results of these measurements are shown in Table III.

112-7360

Fig. 6. Axial Traverse Drawer

TABLE III. Fission Ratios as Function of Axial Distance

Axial Position, cm from Center ($r = 0$)	Measured Fission Ratios			
	U^{234}/U^{235}	U^{238}/U^{235}	$P_{U^{239}}/U^{235}$	U^{238}/U^{234}
0.0	0.187 ± 0.005	0.0256 ± 0.0008	0.901 ± 0.024	0.136 ± 0.006
10.0	0.179 ± 0.005	0.0261 ± 0.0008	0.876 ± 0.023	0.146 ± 0.006
20.0	0.178 ± 0.005	0.0263 ± 0.0008	0.854 ± 0.023	0.147 ± 0.006
30.0	0.173 ± 0.005	0.0266 ± 0.0009	0.875 ± 0.025	0.154 ± 0.006
40.0	0.168 ± 0.005	0.0262 ± 0.0009	0.863 ± 0.026	0.155 ± 0.006
50.0	0.165 ± 0.005	0.0235 ± 0.0009	0.850 ± 0.026	0.142 ± 0.006
56.3	0.160 ± 0.005	0.0231 ± 0.0008	0.857 ± 0.027	0.144 ± 0.006

The fission ratios in the radial direction were measured in the same manner as the traverse in the axial direction, except in this case the traverse was through Drawers 23-23--13-23 in the vertical direction. Figure

A - ALUMINUM TRAVERSE TUBE 9/16 o.d. x 0.020 WALL

■ C ▨ DEPLETED U □ Na CAN ▩ FUEL

IN VOIDED CORE ALL Na CANS WERE REPLACED WITH 1/4 in. FRAMES

112-7363

Fig. 7. Radial (vertical) Traverse Drawer

7 shows the modification that was made in the core materials to provide space for the counter. Note that 1.27 cm of depleted uranium was used to hold the 0.635-cm sodium cans in place at the front of each drawer. The results of these measurements as well as the calculated values with transport (S_4 approximation) and diffusion codes, using the same cross-section set (224), are given in Table IV.

TABLE IV. Fission Ratios as a Function of Radial Distance

Radial Position, cm from Center ($Z = 4.345$ cm)	Measured Fission Ratios				Calculated Fission Ratios ^a			
	U^{234}/U^{235}	U^{238}/U^{235}	$P_{U^{239}}/U^{235}$	U^{238}/U^{234}	U^{234}/U^{235}	U^{238}/U^{235}	$P_{U^{239}}/U^{235}$	U^{238}/U^{234}
0.0	0.172 ± 0.004	0.0238 ± 0.0007	0.967 ± 0.025	0.138 ± 0.004	0.222 (0.221)	0.0267 (0.0266) ^b	0.943 (0.941)	0.120 (0.120)
15.0	0.168 ± 0.004	0.0230 ± 0.0008	0.964 ± 0.025	0.137 ± 0.004	0.222 (0.021)	0.0266 (0.0265)	0.942 (0.941)	0.120 (0.120)
30.0	0.163 ± 0.004	0.0232 ± 0.0008	0.956 ± 0.025	0.142 ± 0.005	0.220 (0.219)	0.0263 (0.0261)	0.940 (0.939)	0.119 (0.119)
40.0	0.174 ± 0.004	0.0228 ± 0.0008	0.934 ± 0.025	0.131 ± 0.005	0.223 (0.213)	0.0242 (0.0249)	0.930 (0.933)	0.108 (0.116)
50.0	0.241 ± 0.007	0.0377 ± 0.0012	1.020 ± 0.027	0.156 ± 0.008	0.324 (0.314)	0.0491 (0.0439)	1.052 (1.040)	0.151 (0.139)
55.0	-	0.0395 ± 0.0015	1.047 ± 0.030	-	-	0.0418 (0.0397)	1.062 (1.050)	-

^aValues in parantheses were obtained by diffusion theory with Set 224; the other values were obtained with the same cross-section set by transport S_4 calculations.

^bThis ratio was also obtained by diffusion theory with Set 801.

Comparison of the results in Table III at $r = 0$, $Z = 0$, and those in Table IV at $r = 0.0$, $Z = 4.445$ cm, reveals that the depleted uranium spacer used in the radial traverse affected the measurements. It is seen from the fission ratios of U^{234} and U^{238} to U^{235} that the upper portion of the energy spectrum became softer because of the spacer. It is further noticed from the fission ratio, Pu^{239}/U^{235} , that the spacer hardened the lower-energy portion of the spectrum. Comparison of the measurements at $r = 0$, $Z = 0$, in Table III with the calculations in Table IV shows that the calculated U^{238}/U^{235} fission ratio is in good agreement with measurements. The U^{234}/U^{235} fission ratio indicates that the calculated spectrum that is pertinent to this ratio is harder than measurements indicate. The same is true of the Pu^{239}/U^{235} fission ratio. However, since the introduction of a small depleted-uranium sample affects the measurements, the heterogeneous arrangement of the core material within a drawer can produce effects as large as the discrepancies between the calculations and the measurements or even larger.

Table IV indicates that the transport calculations give a harder spectrum in the driver ($r = 50$ cm) than diffusion calculations. This is reasonable because the leakage term in diffusion theory is overestimated and affects the higher portion of the energy spectrum. The large discrepancy in the fission ratio at $r = 50$ cm is due to the substantial perturbation introduced in the driver to allow for the insertion of the probe. Figure 8 shows how the driver material was rearranged. That the discrepancy between measurement and calculation of the fission ratios at $r = 50$ cm is not real can be seen from Fig. 9, in which the normalized absolute fission ratio is compared to the normalized relative fission ratio as obtained from activation of U^{238} and U^{235} foils. Also included in Fig. 9 is the calculated fission ratio by transport and diffusion codes with Cross-section Set 224.

Fig. 8. Arrangement of Driver Drawer to Allow Space for Fission Counter

Fig. 9. Comparison of Calculated and Measured Fission Ratio of U^{238}/U^{235}

C. Fission Ratios with Sodium Removed from Central Region

Fission ratios were also measured at the center of Assembly 4Z after the sodium in the entire central region was removed. Table V shows the results, with the calculated values obtained with diffusion theory and Set 224. Note that the removal of sodium from the entire central zone did not affect the U^{238}/U^{235} fission ratio. The voiding of the sodium did affect the U^{234} and Pu^{239} to U^{235} fission ratios. This is seen from the higher fission ratios obtained with the sodium removed indicating the expected hardening of the spectrum. The magnitude of this hardening, however, is not as large as Set 224 predicts.

TABLE V. Fission Ratios at Center, $r = 0$, $Z = 0$,
with Sodium Removed from Central Region

	Measured	Calculated (Diffusion Theory and Set 224)
U^{238}/U^{235}	0.0262 ± 0.0008	0.0298
U^{234}/U^{235}	0.214 ± 0.005	0.254
Pu^{239}/U^{235}	0.987 ± 0.024	0.983

The two principal sources of error in the fission-ratio measurements are in:

1. Determining the amount of material present in a counter; and
2. Measuring the fission rates to a desired degree of accuracy.

Except for those materials with an extremely low fission rate, the first factor is the predominant source of error.

The disintegration rate of the sample determines the amount of material present in a counter. The disc containing the fissionable material is counted in a low-geometry alpha counter whose accuracy is such that the disintegration rate can be determined to within $\pm 0.1\%$. An isotopic analysis of the material with an overall accuracy of better than 0.02% is obtained from a mass spectrometer. The analyses for the materials used are shown in Table VI. Using the known disintegration rates for each isotope, we can calculate a disintegration rate per unit weight for the material. Dividing the measured disintegration rate by the calculated value gives the amount of material on the disc. The principal source of error in this method arises from the half-lives that are used to calculate the disintegration rates. The half-lives that were used are shown in Table VII. The relative error in the mass is then determined from the formula

$$\sigma_m = \sqrt{\sigma_1^2 + \sigma_2^2}$$

where

σ_1 is the standard deviation in the calculated disintegration rate, and

σ_2 is the standard deviation in the experimental disintegration rate.

The errors quoted in Tables III, IV, and V are the total propagated errors.

TABLE VI. Isotopic Concentrations of Fissionable Materials

Isotope	Concentration, w/o				
	U ²³⁴	U ²³⁵	U ²³⁸	Pu ²³⁹	Pu ²⁴⁰
U ²³⁴	79.45	15.16	5.39	-	-
U ²³⁵	0.93	93.14	5.91	-	-
U ²³⁸	-	0.014	99.986	-	-
Pu ²³⁹	-	-	-	99.922	0.028

TABLE VII. Half-lives Used in Analysis

Material	Half-life, yr
U ²³⁴	$(2.475 \pm 0.016) \times 10^5$
U ²³⁵	$(7.13 \pm 0.16) \times 10^8$
U ²³⁸	$(4.50 \pm 0.01) \times 10^9$
Pu ²³⁹	$(2.435 \pm 0.005) \times 10^4$

D. Resonance Detector (Mn⁵⁵)

Manganese has several resonances in the energy range 350 eV to about 100 keV. Its use as a detector depends on the self-shielding that occurs when the resonance neutrons are absorbed in the outer foils of a stack of foils. This technique was used in Assembly 3 of ZPR-6,⁹ and it was repeated in Assembly 4Z. This detector

will also be used in Assembly 5 to verify whether or not the low-energy spectrum in Assembly 4Z is the same as that in Assembly 5.

The experiment consisted of stacking 39 manganese foils of 98% purity (1% copper and 1% nickel) and 1.597-cm diameter. This stack comprised a central section consisting of nineteen 10-mil foils and two outer layers, one on each side of the central section, consisting of ten 1-mil foils. The stack was irradiated at the center of Assembly 4Z in the space provided for the fission counter (see Fig. 7). This stack was placed against the drawer material, 5 cm from the front of Drawer 23-23. After irradiation, the foils were reduced in size from 1.587 to 0.953-cm diameter and then counted. The size was reduced to eliminate edge effects. The specific activity of the foils is plotted as a function of depth from one end in Fig. 10. The depth of the valley in this plot is a measure of the spectrum softness. If the depth is measured by taking the ratio of the specific activity at the edge to that at the center, this ratio is 1.82. This ratio for Assembly 3 of ZPR-6 was 1.66,

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Fig. 10. Manganese Activation in a Stack of Foils

calculations in cylindrical geometry were made using the diffusion-theory code MACH-1¹⁰ with Cross-section Sets 224 and 801 and the transport-theory code SNARG¹¹ with Cross-section Set 224.

All the foils used in the measurements were 0.0125 cm (5 mils) thick and had a diameter of 1.27 cm (1/2 in.) and 0.95 cm (3/8 in.) for U^{238} and U^{235} , respectively. They were placed vertically between adjacent material plates inside the core drawers and horizontally on top of U^{238} blocks in the reflector. The foils were located in the stationary half of the assembly; for the radial traverses they were placed 7.62 cm (3 in.) from the interface to eliminate the influence of the front walls of the drawers and of the residual gap between the halves.

Axial traverses were taken at the core center, at the boundary of the central zone, and in the middle of the driver. Radial traverses were taken in horizontal, vertical, and diagonal directions (at 45° to the horizontal diameter). This was done to detect any distortion caused by the orientation of the material plates to the core radius and by the irregular shape of the boundaries between the various zones.

To study the influence of boundaries on the flux distribution, most foils were placed at the outer edge of the central zone and in the buffer and driver regions where the flux gradient was expected to be highest.

The fine structure of the U^{238} fission rates was measured across a sequence of three drawers (23-21 to 23-23) in the core center. The same was done across six consecutive drawers (23-28 to 23-33) to cover the outer edge of the central zone and the buffer and driver regions. The results are shown in Figs. 11 and 12.

indicating the spectrum was harder in Assembly 3. Figure 10 indicates that the specific activity of the foil next to the core material is lower than that on the other side. This is due to the fact that there are fewer resonance neutrons in this region.

E. Fission-rate Traverses with U^{235} and U^{238}

The accuracy of the calculated spatial distribution of the real fluxes in Assembly 4Z was examined by measuring fission-rate traverses axially and radially with U^{235} and U^{238} foils. One-dimensional homogeneous

112-7477

Fig. 11. Fine Structure of U^{238} Fission Rate in Central Region

112-7541

Fig. 12. Fission Rate of U^{238} Foils, Placed between Plates in ZPR-6 Assembly 4Z

The fine structure of the U^{235} fission rate (Fig. 13) was measured in a sequence of three drawers (23-25 to 23-27). A detailed mapping of the radial traverse for U^{235} is shown in Fig. 14. Included in Fig. 14 are the

calculated fission rates by transport and diffusion theory using two cross-section sets (224 and 801). The transport as well as the diffusion calculations with Set 224 are in better agreement with the measurements than Set 801. Cross-section Set 801, even though it was ELMOE-averaged⁵ for Assembly 4Z, does not predict the fission rates across the buffer and driver regions as well as Set 224. This is probably due to the inapplicable ELMOE averaging in the narrow driver region.

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Fig. 13. Fine Structure of U^{235} Foils, Placed between Plates in ZPR-6 Assembly 4Z

The fission rate of U^{238} as a function of radius is shown in Fig. 15, which indicates that the transport calculations give better agreement with measurements. In obtaining the measured fission rates, it was necessary to average over the fine structure measurements in order to make meaningful comparison with the homogeneous calculations.

The axial fission-rate traverses of U^{235} and U^{238} are given in Figs. 16 and 17, respectively. In both cases, the experimental results follow very closely a cosine distribution and yield a reflector savings of 16.1 cm when extrapolated to zero flux. It is significant to note that a reflector savings of 16.1 cm is independent of the region in which the measurements were made.

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Fig. 14. Radial Traverse of U^{235} Fission Rates in ZPR-6 Assembly 4Z

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Fig. 15. Radial Traverse of U^{238} Fission Rates in ZPR-6 Assembly 4Z

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Fig. 16. Axial Traverse of U^{235} Fission Rates in ZPR-6 Assembly 4Z

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Fig. 17. Axial Traverse of U^{238} Fission Rates in ZPR-6 Assembly 4Z

V. THE SPATIAL DISTRIBUTION OF THE IMPORTANCE OF FISSION NEUTRONS

A. Theory

The reactivity worth of a sample is usually calculated by perturbation theory in which adjoint fluxes play as important a role as the real fluxes. However, the calculated adjoint fluxes are usually taken for granted, and all the experimental work is aimed at verifying the calculations of the real fluxes. It was decided to check on the relative spatial distributions of the importance of fission neutrons by using a Cf^{252} source. The measurements are based on the following principles.

The change in reactivity, $\Delta k/k$, according to perturbation theory,¹² can be written as

$$\Delta k/k = \frac{\int_v (\Phi^\dagger)(\delta P - \delta L)(\Phi) dv}{\int_V (\Phi^\dagger)(P)(\Phi) dV}, \quad (2)$$

where

$\Phi^\dagger$ is a row matrix representing the adjoint fluxes,

Φ is a column matrix representing the real fluxes,

P is a square matrix representing the production of neutrons,

δP represents the change in P as a function of space,

δL represents the change in the losses of neutrons as a function of space,

v is the volume of the perturbed region,

and

V is the volume of the entire reactor.

If δL is ignored and the numerator of Eq. (2) is expanded into energy groups, and if it is assumed that the change in P is a Dirac δ function, then the numerator of Eq. (2) reduces to

$$\text{Numerator} = \int_v \left\{ \sum_i \chi_i \phi_i^\dagger \right\} \left\{ \delta(\mathbf{r} - \mathbf{r}') \left[\sum_i \delta(\nu \Sigma_f)_i \phi_i \right] \right\} dv. \quad (3)$$

Now suppose the reactor system is just critical, then Eq. (3) is zero and the power level is steady in time. At this point, suppose a point-neutron source that does not absorb any neutrons is introduced into the system at position r' . Assume that the neutron source emits S neutrons per unit time and that the energy grouping of the spectrum is such that

$$S = \sum_{i=1}^n S_i. \quad (4)$$

Upon the introduction of the source into the system, the power level increases linearly in time. This increase in power level may be equated to a pseudo-reactivity whose value depends on the power level at which the source was introduced. Similarly, the source neutrons in Group i can be assumed to be resulting from a pseudo-change in the fission cross sections; i.e.,

$$S_i \delta(r - r') = \chi_i \left[\sum_{i=1}^n \delta(\nu \Sigma_f)_i \phi_i \right] \delta(r - r'). \quad (5)$$

The right-hand side of Eq. (5) represents the net increase in the neutron population of Group i at r' due to a pseudo-change in the fission cross sections. Combining Eqs. (4) and (5), one gets

$$S \delta(r - r') = \left[\sum_{i=1}^n \delta(\nu \Sigma_f)_i \phi_i \right] \delta(r - r'). \quad (6)$$

It is implied in Eq. (6) that

$$\sum_{i=1}^n \chi_i = 1.$$

Substituting Eq. (6) into Eq. (2), one obtains

$$(\Delta k/k)' = \frac{\left(\sum_{i=1}^n \chi_i \phi_i^\dagger \right) (S)}{N.I.}, \quad (7)$$

where $(\Delta k/k)'$ designates the pseudo-reactivity change, and N.I. is the normalization integral that appears in the denominator of Eq. (2). It is seen from Eq. (7) that the importance of the fission neutrons at position r' is simply

$$\left[\sum_{i=1}^n \chi_i \phi_i^\dagger \right]_{r'} = (\Delta k/k)'_{r'} \frac{N.I.}{(S)_{r'}}, \quad (8)$$

provided the neutron source has the same spectrum as the fissile materials in the system. It is apparent from Eq. (8) that the relative distributions of

$$\sum_{i=1}^n \chi_i \phi_i^\dagger$$

as a function of space can be measured by measuring $(\Delta k/k)'$ at the same power level for the corresponding position of S and normalizing the results to some point in the reactor.

Another important feature of Eq. (8) is that if $(\Delta k/k)'_r$ is measured at a known absolute neutron source rate,

$$\sum_{i=1}^n (\nu \Sigma_f)_i \phi_i,$$

then N.I. can be obtained experimentally as

$$\text{N.I.} = S/(\Delta k/k)', \quad (9)$$

where

$$\sum_{i=1}^n \chi_i \phi_i^\dagger$$

has been normalized to 1. Note that N.I. is independent of where the source is placed. It is relatively simple to experimentally obtain the absolute fission rate, i.e.,

$$\sum_{i=1}^n (\Sigma_f)_i \phi_i,$$

but the quantity,

$$\sum_{i=1}^n (\nu \Sigma_f)_i \phi_i,$$

is generally not measurable inside the system. However, some average ν can be obtained from theoretically calculated group-dependent fluxes and from the empirically known dependence of ν on the neutron energy.

B. Measurements

The measurements were performed with a microgram quantity of Cf^{252} . The Cf^{252} source was housed in a container, a cross-sectional view of which is shown in Fig. 18. The container was attached to the drive mechanism that was used with the fission-ratio measurements. The core materials were rearranged, to allow the introduction of the container, in the same manner as was done for the fission ratios (Figs. 6-8). The effect of the container without the source but with a glass ampule similar to the source ampule on the system was checked by first noting the critical position of one control rod at a power level where the power coefficient was zero. (The power coefficient is defined here as the amount of pseudo-reactivity associated with the spontaneous fissions occurring in U^{238} , U^{235} , and any neutron emission resulting from interactions of the cosmic rays.) Later the power level was decreased to the level at which the measurements with Cf^{252} source were conducted, and the control rod was used to establish steady state. The difference between the two control-rod positions was noted. The container was then introduced into the system at the center of the core, and the power level was studied as a function of time. It was found that the container had no measurable effect on the system. The container was then moved in increments from the center toward the core boundary, in both the radial and axial directions, and again it was found that the container had no measurable effect.

Fig. 18. Californium Sample Holder

The same type of procedures were repeated with the Cf^{252} source. After the difference between the two control-rod positions was determined for the two power levels without Cf^{252} source ($\sim 5 \times 10^{-5} \Delta k/k$), the power level was decreased by withdrawing the control rod to such a level that after the Cf^{252} source was inserted into a specified location, a period in time of no less than 5 min would result before reaching the power level at which measurements were made. This procedure allowed enough time for any transients to die out. The linear rise in the power level was monitored by a fission chamber located at the driver-reflector interface (drawer location 14-27). The shortest distance between the Cf^{252} source and

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Fig. 19. Linear Rise in Power Level due to Cf^{252} Source (Radial Traverse at $r = 20.0$ cm)

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Fig. 20. Radial Traverse of Cf^{252} Neutron Importance

lindrical geometry. The results from diffusion calculations with Set 224 are in better agreement with measurements than those from transport calculations. This is exactly the opposite of what was found with the reaction rate of U^{238} (Fig. 15). The uncertainties in the measured points are within the circles.

The axial traverse of

$$\sum_{i=1}^n \chi_i \phi_i^\dagger$$

the chamber was in the radial traverse, and that was 23 cm. A typical set of data of the linear rise is shown in Fig. 19. The slope of the data shown in Fig. 19 is proportional to¹³

$$\sum_{i=1}^n \chi_i \phi_i^\dagger$$

By measuring the slope after linear rise, the dependence of $\Delta k/k$ on the power level is eliminated. If the ratio of the slope of the linear rise obtained at position x to the slope obtained at the center of the core is plotted as a function of position, a curve of the normalized

$$\sum_{i=1}^n \chi_i \phi_i^\dagger$$

versus position results.

The radial traverse of

$$\sum_{i=1}^n \chi_i \phi_i^\dagger$$

is shown in Fig. 20. For comparison, the calculated importance of the fission neutrons by transport and diffusion, one-dimensional codes with Sets 224 and 801 are included.

The calculations were done in cy-

is shown in Fig. 21. Included in this diagram are the results of diffusion calculations with Set 224 and a cosine distribution. The measured points are higher than theory predicts.

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Fig. 21. Axial Traverse of Cf^{252}
Neutron Importance

neutrons emitted in energy group i is

$$N(E) dE = 2.410E^{1/2} e^{-E/1.466} dE, \quad (10)$$

where E is in MeV. The fraction of

$$\chi_i = \frac{\int_{(E_i + 1)/1.466}^{E_i/1.466} (E')^{1/2} e^{-E'} dE'}{\int_0^{\infty} (E')^{1/2} e^{-E'} dE'} \quad (11)$$

The expression for χ_i is in the form of incomplete Γ -functions and can be evaluated directly from the tabulations of Karl Pearson.¹⁵ Table VIII lists the values of the χ_i 's for Cf^{252} as obtained from Eq. (11) and the data from Ref. 14. Table VIII also lists the values of the χ_i 's for U^{235} obtained by similar procedures. The values of the χ_i 's that are used in Set 224 are given for comparison. The shape of

$$\sum_{i=1}^n \chi_i \phi_i^{\dagger}$$

as a function of radius was not particularly sensitive to the set of χ_i 's used. The largest deviation between the sets was a fraction of a percent. Thus, it seems that the assumption of the similarity between the fission-neutron spectra of U^{235} and Cf^{252} is not a bad one.

TABLE VIII. Values of χ_i 's for Cf^{252} and U^{235}

Energy Group	E_L , ^a MeV	χ_i 's for Cf^{252}	χ_i 's for U^{235}	χ_i 's from Set 224
1	3.6788	0.1783	0.1270	0.13035
2	2.2313	0.2065	0.1990	0.21089
3	1.3534	0.2198	0.2266	0.23223
4	0.82085	0.1678	0.1834	0.17941
5	0.49787	0.1056	0.1200	0.11487
6	0.30197	0.0605	0.0704	0.064840
7	0.18316	0.0310	0.0374	0.034097
8	0.11189	0.0152	0.0181	0.017125
9	0.06738	0.0073	0.0089	0.0083959
10	0.04087	0.0038	0.0042	0.0040697
11	0.02478	0.0019	0.0022	0.0019532
12	0.01503	0.0010	0.0013	9.3328×10^{-4}
13	0.0091188	0.0005	0.0008	4.3952×10^{-4}
14	0.0043073	0.0003	0.0005	2.6677×10^{-4}
15	0.0026126	0.0005	0.0002	6.8147×10^{-5}
16	0.0020347	0.0005	0.0002	1.8459×10^{-5}
17	0.0012341	0.0005	0.0002	2.2379×10^{-5}
18	9.6112×10^{-4}	0.0005	0.0002	6.1189×10^{-6}
19	5.8295×10^{-4}	0.0005	0.0002	7.1596×10^{-6}
20	2.7536×10^{-4}	0.0005	0.0002	4.4062×10^{-6}
21	1.0130×10^{-4}	0.0005	0.0002	1.5641×10^{-6}
22	2.902×10^{-5}	0.0005	0.0002	3.9265×10^{-7}

^a E_L is the lower boundary of the energy grouping.

Throughout the analysis the number of fissions that occurred due to the interaction of neutrons with Cf^{252} was ignored because it was considered that one microgram of Cf^{252} was too small to influence the system.

VI. SODIUM-VOID COEFFICIENT AND THE RADIAL WORTH OF CORE MATERIALS

A. Sodium-void Coefficient

The sodium-void coefficients were mapped in both the axial and radial directions. The measurements in the axial direction comprised voiding sections 4 in. long in 18 drawers at the center of the core, 9 drawers per half, and later all the sodium in the 18 drawers was removed. The voiding was achieved by replacing the sodium-filled Type 304 stainless-steel cans with empty cans that had the same weight. Each of the 18 central drawers was loaded with 5.08-cm cans in the front 10.16 cm of the drawer, i.e., toward the midplane, and the rest of the 50.8 cm of the drawer was loaded with 10.16-cm cans. The normal materials arrangement is shown in Fig. 2. Note that such arrangement placed the sodium next to depleted uranium. The measurements were started by removing the front 5 cm of sodium in each drawer. This section was then 10 cm long, 5 cm in each half, and it was designated as Section A. The next 5 cm of sodium were removed without putting back the first 5-cm cans. This section, designated as Section B, was 20 cm long, 10 cm per half, and it overlapped Section A. The empty stainless-steel cans in Section B were replaced with sodium-filled cans, and the next section, C, was voided. If the axial coordinate is designated by the letter Z with origin at the center of the core, then Section C extends from $Z = 20 - 10$ cm in one-half of the matrix and $Z = -20 - (-10)$ cm in the other half. Similarly, Sections D, E, F, and G extended from $Z = 30.5 - 20$ cm and $Z = -30.5 + 20$ cm, $Z = 40.6 - 30.5$ cm and $Z = -40.6 + 30.5$ cm, $Z = 50.8 - 40.6$ cm and $Z = -50.8 + 40.6$ cm, and $Z = 61.0 - 50.8$ cm and $Z = -61.0 + 50.8$ cm, respectively. In all cases, the voiding was confined to the particular section. In a different experiment, all the sodium in Sections B, C, D, E, F, and G was removed.

The reactivity worth of the sodium was obtained by the difference method with the sodium in and out through an auxiliary, calibrated, non-scrammable control rod. This rod was used because the reproducibility of the rod position was accurate to about 0.002 cm. To bring the system to criticality, the dual-purpose rods were driven in to a preset position without reversing the direction of driving. This procedure eliminated the "back-latch" effect that existed with the dual-purpose rods. The non-scrammable rod was then used to achieve steady state. The uncertainty in the measurements was of the order of 0.1-0.2 Δk . This uncertainty is based on the reproducibility of the reference runs. The reference runs were performed before and after every run. The largest contribution to the uncertainty in the measurements originated from opening and closing the halves.

The measurements obtained with the normal loading (Fig. 2) are listed in Table IX. Also listed are the calculated values with one-dimensional diffusion code (MACH 1) and Set 224. The main difference between the sets

of calculations is that in the case of the axial direction the real and adjoint fluxes in the radial direction are assumed to have a zero-order Bessel-function distribution, whereas in the case of the radial direction the fluxes are assumed to have a cosine distribution in the transverse direction. Neither type of calculation is adequate to represent the problem. However, the calculations in the radial direction are perhaps better even though the agreement with measurements is worse. The reason for this is that the reaction rate in the axial direction obeys a cosine distribution (Figs. 16 and 17) but the fluxes in the radial direction are not a zero-order Bessel function. A more serious problem in the calculation of the sodium-void coefficient is the use of first-order perturbation theory in which the fluxes are assumed to remain constant before and after the perturbation.

TABLE IX. Sodium-void Coefficient in Axial Direction,
Sodium Next to U^{238} (Normal Loading)

Section ^a	Amount of Sodium Removed, g	Measurements, $1h^b$		Calculations, MACH 1, Set 224, $1h^b$	
		Worth of Sample	Worth/kg	Axial Direction ^c	Radial Direction ^d
A	886.32	$+0.95 \pm 0.10$	+1.07	+1.015	+1.657
B	1772.64	$+1.70 \pm 0.10$	+0.96	+1.774	+3.233
C	1954.80	$+1.70 \pm 0.10$	+0.87	+1.182	+2.601
D	1954.80	$+0.00 \pm 0.15$	0.00	+0.502	+1.440
E	1954.80	-1.47 ± 0.15	-0.75	-0.320	-0.0542
F	1954.80	-3.30 ± 0.15	-1.69	-1.092	-1.632
G	1954.80	-3.95 ± 0.15	-2.02	-1.360	-3.028
BCDEFG	-	-5.40 ± 0.10	-	+1.335	+2.560

^aSee p. 32 for explanations of sections.

^b1% $\Delta k/k = 468$ $1h$.

^cMACH 1 is a one-dimensional diffusion code. Therefore, in the calculations in the axial direction it is assumed that the core is uniform in composition in the radial direction. Since this assumption would constitute a different assembly than Assembly 4Z, the results given above were multiplied by a normalization factor, which was arrived at by taking the ratio of the denominator of the perturbation expression for this assembly to that of Assembly 4Z. The real and adjoint fluxes were normalized to the same intensity at the center of the core.

^dThe calculations were done in the radial direction for Assembly 4Z. The real and adjoint fluxes were assumed to have a cosine distribution in the axial direction.

The same measurements described above were performed when the sodium was adjacent to graphite instead of depleted uranium as in the previous case. The three basic types of drawers shown in Fig. 2 were reloaded in the manner shown in Fig. 22. The rearrangement was limited to exchanging the U^{238} plates with graphite plates. The relative positions of the U^{235} plates and the sodium cans were the same as before. The results are given in Table X. Comparison between the results in Tables IX and X reveals that the sodium-void coefficient is more negative when the sodium is placed next to graphite than when placed next to depleted uranium. This is

understandable because the absence of the moderating effect of sodium increases the number of fissions in U^{238} when this material is adjacent to the sodium cans. The total worth of the sodium was calculated with MACH 1 and Set 224 by the difference in the k method. Inherent in the calculations is the assumption that the material in the core was homogeneous. The calculated value for the worth of sodium in Sections B, C, D, E, F, and G was -7.93 lh. The average of the value shown in Table IX and that in Table X is -7.35 lh. The agreement between measurements and calculations in this case seems to be good. Inherently, k calculations for the sodium-void coefficient are better than first-order perturbation calculations.

112-7362

Fig. 22. Material Arrangement in the Three Basic Drawers for the Sodium-void Coefficient

TABLE X. Sodium-void Coefficient in Axial Direction, Sodium Next to Graphite

Section ^a	Amount of Sodium Removed, g	Measurements, lh ^b	
		Worth of Sample	Worth/kg
A	886.32	+0.83 ± 0.10	+0.79
B	1772.64	+1.35 ± 0.10	+0.62
C	1954.80	+0.70 ± 0.10	+0.40
D	1954.80	-0.50 ± 0.10	-0.32
E	1954.80	-1.90 ± 0.00	-0.97
F	1954.80	-3.18 ± 0.20	-1.63
G	1954.80	-4.40 ± 0.20	-2.25
BCDEFG	-	-9.30 ± 0.20	-

^aSee p. 32 for explanation of sections.

^b1% $\Delta k/k$ = 468 lh.

B. Radial Worth of Core Materials

The reactivity worth of removing samples of sodium, U^{235} , U^{238} , and graphite was measured as a function of radius in the central, buffer, and driver regions and for the U^{238} in the reflector region as well. The sodium measurements

112-7538

Fig. 23. Radial Sodium-void Coefficient in ZPR-6 Assembly 4Z

comprised removal of the front 20-cm cans from six drawers per half, three drawers from the right and three drawers from the left of the center of the core. For example, in one measurement the following drawers were voided in the manner just described: 22-13, 23-13, and 24-13 on one side, and 22-33, 23-33, and 24-33 on the other side. As can be seen from Fig. 1, the three drawers on one side are on top of each other about the midplane of Assembly 4Z. The results are shown in Fig. 23 and Table XI. Calculated results with MACH 1, Set 801, are also shown in Fig. 23. Another set of calculations using the real and adjoint fluxes from one-dimensional, 22-group transport (S_4) code (SNARG) with a diffusion-perturbation subroutine

are also shown in Fig. 23. In this hybrid type of calculation, Set 224 was used. It is seen that the shape of the sodium-void coefficient is in fair agreement with measurements.

TABLE XI. Radial Sodium-void Coefficient

Matrix Position ^a of Sample Removed	Weight of Sample, g	Effective Radius, cm	Measurements, I_h^b	
			Worth of Sample	Worth/kg
22-22, 23-22, 24-22 22-24, 23-24, 24-24	1281.40	8.30	$+1.16 \pm 0.20$	+0.905
22-19, 23-19, 24-19 22-27, 23-27, 24-27	1248.80	24.20	$+1.710 \pm 0.20$	+0.568
22-16, 23-16, 24-16 22-30, 23-30, 24-30	1259.0	40.76	$+1.66 \pm 0.30$	+1.320
22-14, 23-14, 24-14 22-32, 23-32, 24-32	1227.0	48.82	$+1.60 \pm 0.30$	+1.30
22-13, 23-13, 24-13 22-33, 23-33, 24-33	1227.0	54.34	-5.20 ± 0.30	-4.24

^aSee p. 32 for explanation of matrix position.

^b1% $\Delta k/k = 468 I_h$.

The reactivity loss due to the removal of $5 \times 2.54 \times 0.158$ -cm plates of U^{235} was measured in the manner described in the previous paragraph. The results are plotted in Fig. 24 and tabulated in Table XII. It is seen from Fig. 24 that the Set 224 results obtained with diffusion theory are in better agreement with measurements than those with the same set and transport theory. This behavior is similar to what was found with the spatial distribution of an importance of fission neutrons (Fig. 20).

Fig. 24
Worth of U^{235} as a Function of Radius

112-7539

TABLE XII. Worth of U^{235} as a Function of Radius

Matrix Position ^a of Sample Removed	Weight of Sample, g	Effective Radius, cm	Measurements, I_h^b	
			Worth of Sample	Worth/kg
23-23	209.52	1.74	10.50 ± 0.10	50.11
23-19, 23-27	209.52	22.72	9.30 ± 0.10	44.38
23-16, 23-30	209.52	39.29	7.65 ± 0.10	36.51
23-14, 23-32	137.04	48.09	3.90 ± 0.10	28.38
23-14, 23-32	137.04	49.68	3.50 ± 0.10	25.53
23-13, 23-33	86.24	55.20	2.30 ± 0.10	26.66

^aSee p. 32 for explanation of matrix position.

^b $1\% \Delta k/k = 468 I_h$.

Similar measurements were performed with depleted uranium and graphite. For these cases, the $5 \times 5 \times 0.317$ -cm plates in a column were replaced by $5 \times 2.54 \times 0.317$ -cm plates so that the rest of the materials in a drawer retained their relative positions. The results for U^{238} and those for graphite are given in Figs. 25 and 26 and in Tables XIII and XIV.

112-7540

Fig. 25. Normalized Worth of U^{238} as a Function of Radius

112-7537

Fig. 26. Normalized Worth of Graphite as a Function of Radius

TABLE XIII. Measured Worth of Depleted Uranium as a Function of Radius

Matrix Position ^a of Sample Removed	Weight of Sample, g	Effective Radius, cm	Measurements, I_h ^b	
			Worth of Sample	Worth/kg
23-23	2676.0	1.59	-5.40 ± 0.20	-2.02
23-20	2676.0	16.56	-5.40 ± 0.20	-2.02
23-17	2676.0	33.12	-4.10 ± 0.20	-1.53
23-15	3558.0	44.16	-4.80 ± 0.20	-1.35
23-14	1813.4	51.72	-1.00 ± 0.20	-0.55
23-12	88308.0	60.72	+19.00	+0.215
07-23	88308.0	88.32	$+0.00 \pm 0.20$	0.00

^aSee p. 32 for explanation of matrix position.^b1% $\Delta k/k = 468 I_h$.

TABLE XIV. Measured Worth of Graphite as a Function of Radius

Matrix Position ^a of Sample Removed	Weight of Sample, g	Effective Radius, cm	Measurements, I_h ^b	
			Worth of Sample	Worth/kg
22-22, 23-22, 24-22 22-24, 23-24, 24-24	991	5.52	-0.10 ± 0.20	-0.10
22-19, 23-19, 24-19 22-27, 23-27, 24-27	991	22.08	-0.14 ± 0.2	-0.14
21-30, 22-30, 23-30, 24-30 21-16, 22-16, 23-16, 24-16	1651	39.33	$+0.03 \pm 0.20$	+0.018
21-15, 22-15, 23-15, 24-15 21-31, 22-31, 23-31, 24-31	1651	44.16	$+2.71 \pm 0.2$	+1.65
21-15, 22-15, 23-15, 24-15 21-31, 22-31, 23-31, 24-31	1651	45.89	$+2.32 \pm 0.20$	+1.40
21-14, 22-14, 23-14, 24-14 21-32, 22-32, 23-32, 24-32	1651	47.95	$+0.40 \pm 0.20$	+0.24
21-14, 22-14, 23-14, 24-14 21-32, 22-32, 23-32, 24-32	1651	52.44	-10.23 ± 0.20	-6.19

^aSee p. 32 for explanation of matrix position.^b1% $\Delta k/k = 468 I_h$.

VII. CENTRAL-WORTH MEASUREMENTS

A. Material Reactivity Worth at Center of Core

The central worths of different samples were measured at the center of Assembly 4Z by the following technique:

1. Criticality was achieved with dual-purpose rods and with the sample changer in the "in" position. The sample changer consisted of a drawer having the same composition as the central zone of the core with a voided front section (5 x 5 x 2.54 cm). Using an attached drive mechanism, the operator can insert or remove this drawer from the core remotely. The voided front section of this drawer was used for the sample whose worth was to be measured.

2. After criticality was achieved at the proper power level (that level which had zero power coefficient), the sample changer was withdrawn from the core, thus causing the neutron density to decrease. The drawer was worth about 30 lh. By means of the nonscrammable rod, the equivalent of about 10 lh was added to the system. The nonscrammable rod was equipped with a precision position indicator (0.002 cm). The power level was decreased to a point where, upon insertion of the sample changer to its original position, a waiting period of not less than 5 minutes resulted before the reactor reached that power level at which the reactor period measurement was initiated. The reactor period was measured "on line" by feeding the data directly to the DDP-24 computer¹⁶ for about 5 minutes.

3. This procedure was repeated several times. The differences in reactivity between the runs with and without a sample gave the worth of a sample.

The 10-lh excess reactivity added could have been obtained through a dual-purpose rod. However, if at any time during the series of runs it were found that a sample was worth more than the 10 lh, it would have been necessary to move the dual-purpose rod. Since there was some "backlash" associated with these rods, it was decided to use the nonscrammable rod.

At some times during the runs and especially after a weekend, a systematic drift in the reference runs was observed. This drift was of the order of 0.01 lh. Whenever such drift was observed, the samples whose worths were 1.0 lh or greater were measured. The uncertainty in these measurements was considered to be the same as the size of the drift. The sample whose worth was small, especially the sodium, was measured when the drift was absent, and this measurement was repeated at least ten times. The uncertainty in these measurements was based on the square root of the variance,

$$\sigma = \left[\sum_{i=1}^n (\chi_i - \bar{\chi})^2 / (n - 1) \right]^{1/2},$$

in the data, and it was propagated according to the theorem of propagation of error to account for the fact that some samples were in stainless-steel cans.

The results of the measurements are given in Table XV. It is seen that the sample size effect is negligible for U^{238} and stainless steel.

TABLE XV. Central Worths of Various Materials in ZPR-6 Assembly 4Z

Sample	Weight, g	Weight (and Composition) of Can, g	Measurements, Ih ^b		
			Gross Worth	Net Worth	Worth/kg
Cd	499.50	26.97 (SS)	-4.598	-4.553 ± 0.010	-9.115
W	1108.0	-	-5.742	-5.742 ± 0.010	-5.182
Na	51.38	56.42 (SS)	-0.144 ± 0.002	-0.049 ± 0.005	-0.954
Be	114.308	-	+1.107	+1.107 ± 0.010	+9.689
C	103.0	-	+1.194	+0.194 ± 0.010	+1.883
SS (304)	499.60	-	-0.849	-0.849 ± 0.010	-1.699
U^{238}	1153.80	-	-3.857	-3.857 ± 0.010	-3.343
Li	28.30	57.03 (SS)	-0.691	-0.595 ± 0.010	-21.024
SS Can	72.610	-	-0.122	-0.122 ± 0.002	-1.680
Al	166.90	-	-0.297	-0.297 ± 0.013	-1.779
B ₄ C	59.62	64.70 (SS)	-6.520	-6.410 ± 0.025	-107.511
UO ₂	106.5	20.1 (Al)	-0.370	-0.334 ± 0.015	-3.136
EuO	34.2	20.6 (Al)	-2.472	-2.436 ± 0.025	-71.225
GdO	59.2	20.2 (Al)	-1.164	-1.128 ± 0.025	-19.054
NaCl	58.175	26.1 (Al)	-0.150	-0.114 ± 0.015	-1.959
DyO	104.70	20.1 (Al)	-2.062	-2.026 ± 0.015	-19.350
Ni	546.0	-	-1.148	-1.148 ± 0.010	-2.102
Fe	488.0	-	-0.739	-0.739 ± 0.007	-1.510
Nb	481.2	30.10 (SS)	-3.842	-3.791 ± 0.010	-7.878
Zr	406.0	-	-0.650	-0.650 ± 0.005	-1.601
U^{238}	37.073	48.97 (SS)	-0.228	-0.141 ± 0.022	-3.800
U^{235}	{ 17.95 (total) 16.73 (25)	72.61 (SS)	+0.674	+0.803 ± 0.022	+44.735
Polyethylene	59.15	-	+19.751	+19.751 ± 0.010	+333.913
Mo	599.0	-	-3.415	-3.415 ± 0.010	-5.701
V	183.67	55.30 (SS)	-0.231	-0.133 ± 0.010	-0.72
Ta	924.70	27.54 (SS)	-11.956	-11.907 ± 0.200	-12.87
B ¹⁰	29.29	55.73 (SS)	-23.438	-23.331 ± 0.200	-796.552
Y	81.50	20.1 (Al)	+0.030	+0.066 ± 0.010	-0.810
Pu ²³⁹	117.157	7.053 (Al)	+8.400	+8.446 ± 0.200	+72.091
Pu ²⁴⁰	5.52	20.06 (SS)			
U^{233}	{ 146.708 (97.15% U^{233})	{ 5.286 (Al) 19.977 (SS)	+11.60	11.643 ± 0.200	+79.362

^aSome but not all samples were enclosed in cans of various materials.

^bMeasurements are relative to void. 1% $\Delta k/k = 468$ Ih.

The reactivity worth of U^{235} as a function of plate thickness was studied by measuring the worth of a foil, 0.0127 cm thick, and then stacking some of these foils together and measuring the worth again. The results

are given in Table XVI. Within experimental error, the thickness of the plate had negligible effect on its worth.

TABLE XVI. Worth of U^{235} as Function of Thickness at Center of Core

No. of Samples of Size 2 x 2 x 0.005 in.	Weight, g	Measurements, 1h ^a	
		Net Worth	Worth/kg
1	4.511	+0.226 ± 0.020	50.10 ± 4.40
2	9.358	+0.401 ± 0.020	42.85 ± 2.14
3	13.759	+0.611 ± 0.020	44.41 ± 1.45
6	27.628	+1.246 ± 0.030	45.10 ± 1.10
12	55.365	+2.493 ± 0.030	45.03 ± 0.54
23	105.77	+4.855 ± 0.010	45.90 ± 0.10

^a1% $\Delta k/k$ = 468 1h.

The calculated values of the worth of the samples are given in Section B below.

B. Worth Measurements at 16.5 cm Radially from the Center at the Midplane and Comparison with Calculations

The types of measurements described above were performed at 16.5 cm radially from the center in the midplane. The results of these measurements and the calculated values by Sets 224 and 801 are given in Table XVII. Also, some of the values listed in Table XV are given in Table XVI for convenient comparison.

TABLE XVII. Comparison of Measured and Calculated Worths of Various Materials at Center and at 16.5 cm from Center Radially in ZPR-6 Assembly 42^a

Sample (Weight, g)	Net Measured Worth ^b per Sample, 1h		Calc Worth with Set 224, 1h		Calc Worth with Set 801, 1h		$(x_0)_{\text{meas}}$ ^c	$(x_0)_{\text{Set 224}}$ ^c	$(x_e)_{\text{meas}}$ ^c	$(x_0)_{\text{meas}}$ $(x_0)_{\text{Set 224}}$
	At Center of Core	At 16.5 cm from Center	At Center of Core	At 16.5 cm from Center	At Center of Core	At 16.5 cm from Center				
U^{235} (13.869)	+0.611 ± 0.020	+0.568 ± 0.030	+0.664	+0.627	+0.757	+0.692	+1.000	+1.000	+1.000	1.000
U^{238} (1153.8)	-3.857 ± 0.010	-4.212 ± 0.014 ^d	-4.018	-3.77	-4.470	-4.140	-6.310	-6.051	-7.415	1.043
Pu^{239} (117.157)	+8.446 ± 0.200	-	+7.988	-	+9.386	+8.604	+13.820	+12.030	-	1.149
U^{233} (146.71)	+11.643 ± 0.200	-	+12.831	-	-	-	+19.055	+19.323	-	0.986
Na (51.38)	-0.049 ± 0.005	-0.032 ± 0.007	-0.092	-0.079	-0.064	-0.053	-0.080	-0.138	-0.056	0.580
Fe (488.0)	-0.739 ± 0.007	-0.663 ± 0.014	-0.954	-0.886	-1.079	-0.969	-1.209	-1.437	-1.167	0.841
Ni (546.0)	-1.148 ± 0.010	-1.013 ± 0.004	-1.650	-1.540	-1.709	-1.518	-1.879	-2.485	-1.783	0.756
SS-304 (499.6)	-0.849 ± 0.010	-0.767 ± 0.020	-1.224	-1.073	-	-	-1.389	-1.843	-1.350	0.754
C (103.0)	+0.194 ± 0.010	+0.181 ± 0.051	-0.154	-0.140	+0.068	+0.080	+0.317	-0.232	+0.318	-1.366
Be (114.31)	+1.107 ± 0.010	+1.098 ± 0.040	+0.352	+0.324	-	-	+1.811	+0.530	+1.933	3.417
Al (166.90)	-0.297 ± 0.013	-0.257 ± 0.020	-0.469	-0.435	-0.419	-0.371	-0.486	-0.706	-0.452	0.688
B^{10} (29.29)	-23.339 ± 0.200	-20.60 ± 0.20	-26.60	-24.596	-	-	-38.198	-42.587	-36.267	0.897
$(CH_2)_n$ (59.15)	+19.751 ± 0.010	+17.78 ± 0.10	+9.879	+9.405	-	-	+32.325	+14.878	-31.302	2.173
W (1108.0)	-5.742 ± 0.010	-5.268 ± 0.004	-6.441	-6.040	-	-	-9.397	-10.541	-9.274	0.891
Mo (599.0)	-3.415 ± 0.010	-3.062 ± 0.020	-3.797	-3.527	-4.324	-3.926	-5.589	-5.143	-5.390	0.990
Nb (481.2)	-3.791 ± 0.010	-3.543 ± 0.022	-4.742	-4.441	-	-	-6.204	-7.141	-6.238	0.869
Ta (924.70)	-11.907 ± 0.200	-9.98 ± 0.20	-13.890	-13.037	-	-	-19.487	-20.918	-17.570	0.931
B_4C (59.62)	-6.410 ± 0.025	-11.50 ± 0.20	-7.829	-7.342	-	-	-10.490	-11.790	-20.246	0.890
Zr (406.0)	-0.650 ± 0.005	-0.594 ± 0.002	-0.614	-0.566	-	-	-1.064	-0.925	-1.046	1.150

^aCalculations were done using perturbation theory with MACH 1, a one-dimensional diffusion code.

^bMeasured relative to void. 1% $\Delta k/k$ = 468 1h.

^c x is defined as the worth of a material to that of U^{235} . The subscripts 0 and e refer to positions $r = 0, Z = 0$, and $r = 16.5$ cm and $Z = 0$, respectively.

^dThis sample was different from the one used at the center.

The discrepancy between the calculated values with Sets 801 and 224 originates from two sources: (1) the underestimation in the buffer and driver regions of the spatial distribution of the real and adjoint fluxes obtained with Set 801, as can be seen from Figs. 14, 15, and 20; and (2) the difference in the spectrum and the group cross sections. The first source

of error accounts for 8% of the discrepancy. The reason for this is that the denominator of the perturbation term as obtained from Set 801 is 8% lower than that from Set 224. The spectra obtained with both cross-section sets at the center of Assembly 4Z are compared in Table XVIII.

TABLE XVIII. Comparison of the Neutron Spectrum Obtained with Cross-section Sets 224 and 801 at Center of ZPR-6 Assembly 4Z

Group	E_L^a MeV	ϕ_{center}	
		Set 224	Set 801
1	3.6788	0.014245	0.01498
2	2.2313	0.02730	0.02839
3	1.3534	0.04671	0.04859
4	0.82085	0.073186	0.07448
5	0.49787	0.10492	0.10345
6	0.30197	0.13371	0.12750
7	0.18316	0.12722	0.12778
8	0.11189	0.11594	0.11376
9	0.06738	0.09992	0.09719
10	0.04087	0.07454	0.07890
11	0.02478	0.060713	0.05991
12	0.01503	0.047056	0.04737
13	9.1188×10^{-3}	0.03040	0.03155
14	4.3073×10^{-3}	0.02634	0.02333
15	2.6126×10^{-3}	0.002145	0.004215
16	2.0347×10^{-3}	0.005566	0.005093
17	1.2341×10^{-3}	0.005178	0.007260
18	9.6112×10^{-4}	0.002508	0.002173
19	5.8295×10^{-4}	0.002323	0.002337
20	2.7536×10^{-4}	0.000692	0.001344
21	1.0130×10^{-4}	0.000186	0.000328
22	2.902×10^{-5}	0.000085	0.000027
23	1.371×10^{-5}		0.0000037
24	3.059×10^{-6}		0.0000072
25	0.6825×10^{-6}		0.0000084

^a E_L is the lower energy boundary of each group.

The data in Columns 9 and 10 of Table XVII reveal that the neutron spectrum at the center is the same as that at 16.5 cm from the center. The last column depicts the accuracy of the calculations. The deviation from unity in this column reflects the magnitude of errors in the cross sections and the neutron spectrum. The cross-sections sets used in this analysis do not seem to do justice to materials having low atomic numbers. The calculated values of the worths of the samples, shown in Columns 4 through 7 of Table XVII, are, for the most part, larger than the measured values.

The effect of the thickness of the U^{235} sample on its worth was also tested at 16.5 cm radially from the center in the midplane. The results are given in Table XIX.

TABLE XIX. Worth of U^{235} as a Function of Thickness at 16.5 cm Radially from the Center

Sample	Weight, g	Measurements, Ih^a	
		Net Worth	Worth/kg
3 (5x5x0.0127 cm)	13.869	+0.568 ± 0.030	40.95 ± 2.16
6 (5x5x0.0127 cm)	27.719	+1.164 ± 0.030	41.99 ± 1.08
12 (5x5x0.0127 cm)	55.029	+2.367 ± 0.040	43.03 ± 0.73

$a1\% \Delta k/k = 468 Ih$.

Table XIX shows that the thickness of the U^{235} sample may have a positive effect on $\Delta k/k$; however, the uncertainty in the measurements is such that we cannot unequivocally make this conclusion.

C. Central-worth Measurements with the Sodium Removed from the Central Zone of Assembly 4Z

The same type of measurements described above were reported, with fewer samples, after the sodium was removed from the entire central zone. The results, as well as the calculated values with MACH 1, Set 224, are given in Table XX.

TABLE XX. Central Worths of Various Samples with Sodium Voided in the Central Zone of ZPR-6 Assembly 4Z

Sample (Weight, g)	Measurements, Ih^a			Calculations (MACH 1, Set 224)
	Gross Worth ^b	Net Worth	Worth/kg	Worth of Sample, Ih^a
Na (51.38)	-0.125 ± 0.002	-0.035 ± 0.008	-0.681	-0.094
SS (72.18)	-0.115 ± 0.006	-0.115 ± 0.006	-1.593	-0.139
C (103.0)	+0.042 ± 0.008	+0.042 ± 0.008	+0.407	-0.097
Be (114.31)	+0.788 ± 0.008	+0.788 ± 0.008	+6.893	+0.558
Polyethylene (13.902)	+3.583 ± 0.006	+3.583 ± 0.006	+263.487	+2.459
U^{235} (27.719)	+1.032 ± 0.004	+1.126 ± 0.005	+40.620	+1.284
U^{238} (1153.80)	-3.191 ± 0.004	-3.191 ± 0.004	-2.766	-3.435
B^{10} (29.29)	-19.172 ± 0.010	-19.252 ± 0.010	-657.138	-21.798

$a1\% \Delta k/k = 468 Ih$.

^bSome samples were encased in stainless-steel cans.

VIII. ROSSI-ALPHA MEASUREMENTS

Rossi-alpha was measured at the center of Assembly 4Z by the usual technique.¹⁷⁻¹⁹ Two decay constants were observed in the decay of the prompt-neutron chains. If the dependence of the neutron chains on time is represented by

$$f(t) = BG + A_1 e^{\alpha_1 t} + A_2 e^{\alpha_2 t},$$

where BG represents the uncorrelated counts, then the experimental values of BG, A_1 , α_1 , A_2 , and α_2 as obtained from the least-squares fit by the variable metric method²⁰ are listed in Table XXI.

TABLE XXI. Decay Constants of the Prompt-neutron Chains and Their Associated Constants in ZPR-6 Assembly 4Z

BG	$(3.860 \pm 0.004) \times 10^{-4}$
A_1	$(2.687 \pm 0.425) \times 10^{-5}$
α_1	$(2.911 \pm 1.35) \times 10^5$
A_2	$(5.263 \pm 0.128) \times 10^{-5}$
α_2	$(2.753 \pm 0.127) \times 10^4$

Based on the asymptotic Rossi-alpha and a calculated β_{eff} of 0.00688, the lifetime of prompt neutron is found to be 2.52×10^{-7} sec. The calculated lifetime with MACH 1, Set 224, was 2.106×10^{-7} sec. The calculated lifetime with SNARG and Set 224 was 1.972×10^{-7} sec.

IX. COMMENTS ON THE CALCULATIONS AND PREDICTION OF S_Z/S_F

A. Comments on the Calculations

We have tried whenever possible to verify the calculational techniques and, to a certain degree, the accuracy of the group-dependent cross sections with the experimental results. Two cross-section sets were used in the analysis: (1) Set 224 uses 22 groups, and the cross sections of the isotopes used in this set have been ELMOE-averaged⁵ for typically large carbide cores; and (2) Set 801 uses 25 groups, and the cross sections in this set have been ELMOE-averaged for Assembly 4Z in the central zone and in the driver region. Both sets were used with a one-dimensional diffusion code, MACH 1. Set 224 was also used with a one-dimensional transport code, SNARG, in the S_4 approximation, with a fictitious absorption term to account for axial leakage. The following remarks about the theoretical results are made on the basis of comparison with experimental results:

1. The critical mass was predicted with a fair degree of accuracy, regardless of the cross-section set used (see Table II).

2. The calculated spectra with Sets 801 and 224 are in reasonable agreement with measurements.¹ Reasonable agreement between the calculated and measured fission ratios was also obtained in the central zone of Assembly 4Z.

3. The fission-rate traverses with U^{238} and U^{235} showed that the application of diffusion theory to zoned cores with narrow regions results in softer spectra in the high-energy groups. Use of transport theory is more appropriate (see Fig. 15). Figure 15 indicates that the ELMOE averaging in the driver zone, where the asymptotic spectrum characteristic of the driver cannot be achieved, predicts the spectrum poorly. ELMOE averaging would be very appropriate when such technique could be applied to a multiregion core instead of a one-region core.

4. Whereas the spatial distribution of the real fluxes is better predicted with transport than with diffusion calculations, the opposite is true for the importance of fission neutrons,

$$\sum_i^n \chi_i \phi_i^\dagger \quad (\text{see Fig. 20}).$$

The reasons for this phenomenon are presently under investigation.

5. First-order perturbation theory was used in the calculations of the worths of the samples and the sodium-void coefficients. The basis for using first-order perturbation theory are not rigorous and are perhaps questionable. For example, consider the definition of k as given in Ref. 12,

$$k = \frac{[\phi^\dagger P \phi]}{[\phi^\dagger L \phi]}, \quad (12)$$

where the square brackets represent integration over volume and energy. Assuming linearity on both sides of Eq. (12), one gets

$$\frac{\delta k}{k} = \frac{[\phi^\dagger(\delta P - \delta L) \phi] + [\phi^\dagger(\delta P - \delta L) \delta \phi] + [\delta \phi^\dagger(\delta P - \delta L) \phi]}{k[\phi^\dagger L \phi + \phi \delta L \phi + \phi^\dagger L \delta \phi + \phi^\dagger L \delta \phi + \delta \phi^\dagger L \phi + \delta M \delta L \phi + \delta \phi^\dagger L \delta P]}. \quad (13)$$

First-order perturbation ignores all terms involving $\delta \phi$ and $\delta \phi^\dagger$. In cases like the sodium-void coefficient, $[\delta \phi^\dagger(\delta P - \delta L) \phi]$ and $[\phi^\dagger(\delta P - \delta L) \delta \phi]$ can be of the same order of magnitude as $[\phi^\dagger(\delta P - \delta L) \phi]$. Even in the case of the samples used (5 x 5 x 2.54 cm) in the central worth measurements, it is questionable if these terms can be ignored. The denominator of Eq. (12) may be replaced by $[\phi^\dagger P \phi]$ for critical systems since the quantities other than the first term are generally much smaller than $k[\phi^\dagger L \phi]$.

B. Prediction of S_Z/S_F

The denominator of the perturbation expression for Assembly 4Z was calculated by the MACH 1 code with Sets 801 and 224, by the SNARG code with Set 224, and by using the experimental results for the importance of fission neutrons (Fig. 20) and the calculated sources of fission neutrons

$$\sum_{i=1}^n (\nu \Sigma_f)_i \phi_i$$

from the SNARG code. The results are given in Table XXII.

TABLE XXII. Calculated Values of S_Z for Assembly 4Z Code and Cross-section Set

	MACH 1, Set 224	MACH 1, Set 801	SNARG, Set 224	$\sum_i (\nu \Sigma_f)_i \phi_i$ from SNARG and Measured $\sum_i \chi_i \phi_i^\dagger$
S_Z^a	6.588×10^5	6.126×10^5	6.448×10^5	6.772×10^5

$^a S_Z$ is defined as $\int_V \left[\sum_i \chi_i \phi_i^\dagger \right] \left[\sum_i (\nu \Sigma_f)_i \phi_i \right] dV$. The values listed are

normalized so that $\sum_i \chi_i \phi_i^\dagger = 1$ and $\sum_i (\nu \Sigma_f)_i \phi_i = 1$ at the center of the core.

The difference in S_Z between the results from MACH 1 with Set 224, and SNARG with the same set is only 2.2%, even though there is a considerable difference between the quantity $\sum \chi_i \phi_i^\dagger$ obtained from the two codes (see Fig. 20). The reason for this is that diffusion theory underestimates

$$\sum_i (\nu \Sigma_f)_i \phi_i,$$

but gives reasonably accurate

$$\sum_i \chi_i \phi_i^\dagger.$$

The opposite is true with transport theory (see Figs. 15 and 20). For this reason, the measured importance of the fission neutrons and the calculated source of fission neutrons from SNARG were used to obtain S_Z .

The calculated value of S_F , the denominator of the perturbation expression for the full-size core, is 6.500×10^5 . This value was obtained from MACH 1 and Set 224. Thus, the quantity $S_Z/S_F = (6.772 \times 10^5)/(6.500 \times 10^5) = 1.041$. In other words, the central worth measurements in Assembly 5, the full-size core, will be 4% greater than those in Assembly 4Z. In arriving at this ratio, we assumed that the difference in S_Z as obtained from the semiempirical method and from MACH 1 is due to the presence of various zones in Assembly 4Z. This assumption is justified since for large cores, diffusion and transport theories give the same critical mass.²¹

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