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EBR-II REACTOR OPERATION SIMULATOR:

Volume I. A Description of EROS

by

**J. J. Kaganove, E. M. Dean,
and A. V. Campise**

APPLIED TECHNOLOGY

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EBR-II Project

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FOREWORD

This report is the first of two volumes on EROS.
Volume II (in preparation) will be a users' manual.

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ABSTRACT

The basic objective of the EROS code is to provide the necessary and sufficient dynamic simulation of normal and off-normal EBR-II reactor operating conditions.

EROS solves the space-independent reactor-kinetics equations and computes the temperature-induced reactivity feedback in the system by solving a set of equations representing the spatial heat and mass transfer. Up to 20 channels or 1000 feedback equations are available for modeling a reactor-system feedback network. Trips can be initiated by delayed signals from instruments that sense any quantity calculated--e.g., power inverse period, reactivity, or temperature.

I. INTRODUCTION

Control, stability, and safety characteristics of the basic LMFBR concept are of prime importance to the eventual acceptance of this reactor type by the public and private utility industry. Development of digital dynamic simulation techniques is deemed critical in acquiring an in-depth understanding of the normal and off-normal behavior of the LMFBR. Development of such simulations has had a long history of successes in structuring of generalized simulators¹ to be used in studies of dynamic behavior of reactor systems. The development of the Atomics International Reactor Simulator (AIROS)²⁻⁴ is a principal example of a successful generalized simulator for such dynamic studies. This generalized simulator was used as the technical basis for the further development of a specialized dynamic simulator to be used in the kinetics studies of the Experimental Breeder Reactor No. II (EBR-II) system--EBR-II Reactor Operation Simulator (EROS).

A. Technical Rationale

The technical logic and theoretical basis of EROS are summarized in the basic objective of this development effort:

"To provide the necessary and sufficient dynamic simulation of normal and off-normal EBR-II reactor operating conditions, in various irradiation core loadings, to permit a continuing plant kinetics and safety surveillance of the dynamic behavior of the reactor system and to permit a continuing evaluation of the EBR-II plant protective system."

The primary incentive in digital programming of the EROS code was to obtain a dynamic simulation of the interrelationships between the neutronic-thermohydraulic characteristics of various irradiation core loadings and the irradiation tests and driver-fuel elements in the reactor.

Modifications and improvements have been completed on AIROS to simulate the complex dynamic conditions in each EBR-II core loading.

Major accomplishments in this area consisted of providing:

1. A 1000-node heat-transfer capability and allowing for reactivity feedback from 20 different channels.
2. A steady-state temperature-iteration subroutine to adjust the temperature-dependent physical properties in a 1000-node heat-transfer model of EBR-II.
3. Two input tables for each of the 20 feedback channels for separate axial reactivity-worth curves for temperature-induced reactivity coefficients for fuel and sodium.
4. A model to simulate the dynamic behavior of the control-rod-bank effect in EBR-II.
5. An input option to read in a 500-point reactivity-data table to simulate nonlinear reactivity insertions.
6. An input option to read in a 100-point data table of time versus relative flow rate.
7. A Lagrangian interpolation subroutine to interface input-data tables with the variable time-step characteristics of EROS kinetics.
8. A transfer-function analysis subroutine.
9. Additional plant-protective-system trips and response time for computed reactor power, reactor period, and reactor flow.
10. A new format for tables of physical properties as functions of temperature.

11. Extensive CRT film output of computed dynamic-simulation results.

12. An extensive improvement in the basic digital solutions and error-criteria tests to differential equations, thereby decreasing the running times by a factor of 2-10 depending on the type of problem (i.e., loss of coolant flow or reactivity insertions).

13. An option to input afterglow-heating data (100-point table) in terms of reactor operating and specimen-cooling time.

14. A method to simulate the nonlinear behavior of the dynamic feedback network associated with bowing of reactor subassemblies with a uranium blanket or stainless steel reflector.

15. Extensive two- and three-dimensional displays of calculated transient conditions in the reactor core on CRT film output.

16. A pseudochannel stacking option to allow for geometry and material changes in the direction of coolant flow.

The effort outlined above constitutes the present progress of the EBR-II Project in dynamic simulation. In view of the complexity of EBR-II irradiation core loadings, the present effort remains flexible to allow for reemphasis as dictated by EBR-II programmatic requirements.

B. Dynamic Modeling

The EBR-II plant is composed principally of a sodium-cooled reactor, an associated process system, and a power plant with a design thermal-power capability of 62.5 MWt. The EBR-II core was designed to be fueled with either ^{235}U or plutonium. It is now fueled predominantly with ^{235}U . EBR-II was originally designed as an engineering facility to determine the overall feasibility of this type of reactor for use in central-station power plants, but recent emphasis has been on its capabilities for irradiation in a fast-neutron flux.

Using EBR-II as an irradiation facility has involved testing various types of fast-reactor fuel elements that may change the dynamic characteristics of the core. Therefore, the continued safe and reliable operation of EBR-II requires a constant surveillance of dynamic characteristics. This surveillance includes:

1. Kinetics surveillance.
2. Evaluations of the plant protective system.
3. Safety analysis.

1. Kinetics Surveillance

A continuing kinetics surveillance is maintained at EBR-II through rod-drop and oscillator techniques. The reactor is periodically tested by rod-drop and oscillator experiments with a stainless steel drop rod or a boron oscillator in place of a standard control rod. A digital data-acquisition system is used to reduce the data, and an inverse-kinetics digital program is used to compute the reactivity feedback associated with the power variation recorded from an out-of-core ion chamber. The feedback data constitute the basic experimental results relating to reactor stability and to dynamic characteristics of a core loading.

Various modeling techniques have been used for understanding the characteristics of the feedback data from various EBR-II core loadings. One approach uses EROS for the dynamic simulation of the reactor core and rod-drop and/or oscillator experiments. Radial and axial heat-transfer nodes are set up to compute the temperature distributions in "averaged" fuel elements in various rows of the reactor core and inner blanket. Prompt- and delayed-neutron constants are used as input to the basic space-independent reactor-kinetics equations to solve the power variation resulting from a given experiment. A closed-loop reactivity-feedback network is maintained, and temperature changes in the fuel, coolant, and other feedback components are associated with the weighted and localized temperature coefficients of reactivity. The temperature-induced reactivity changes are used to compute the total reactivity feedback associated with a given experiment.

2. Evaluations of Plant Protective System

Assurance of safety requires a continuing surveillance of the dynamic characteristics of all systems in the reactor plant. This type of surveillance includes studies of the reliability, redundancy, and independent performance of all EBR-II plant-control and protective systems under emergency operating conditions.

Studies of this type are directed specifically at understanding the response of the current EBR-II plant protective system (PPS) to malfunctions of components.

The basic objective of this type of dynamic study is formulating a firm technical basis for all PPS setpoints and instrument responses in circuits involving reactor period, power level, coolant flow, and temperature. This formulation is based on maintaining transient temperatures of all materials within a safe range during occurrence of assumed malfunctions. The EROS program has been tailored to this type of dynamic study; thus it provides a detailed simulation of the EBR-II closed-loop behavior.

3. Safety Analysis

The continued safe and reliable operation of the EBR-II requires a constant surveillance of these dynamic characteristics. The required surveillance involves a study of the EBR-II reactor and primary-coolant system in terms of the neutronic and thermohydraulic characteristics of the reactor system and the interdependence of various inherent reactivity-feedback networks. EROS solves the space-independent reactor-kinetics equation and computes the temperature-induced reactivity feedback in the system by solving a set of equations representing the spatial heat- and mass-transfer model for representative fuel, axial-reflector, and radial-reflector channels in the EBR-II irradiation cores. The EROS digital program is used to maintain a surveillance of the dynamic-response characteristics of EBR-II irradiation core loadings.

The component malfunctions considered in this continuing safety surveillance include abnormal operations of (a) control and safety rods, (b) fuel-handling gripper mechanism, and (c) primary coolant pumps. The various malfunctions are analyzed in the unprotected mode--i.e., hypothetical case of no action by the PPS--and are followed to the onset of fuel melting in either a driver-fuel or an experimental subassembly.

C. Program Logic and Structure

It is helpful in using a program to have a description of the programming logic. The flow chart shown in Fig. 1 does not correspond to any subroutine in EROS, and subroutine names are not used. It is intended for the user rather than for the programmer, and as such depicts the overall logic of the program in an attempt to clarify its operation. References made to various sections of this report are in parentheses.

Figure 2 describes the overlay structure of EROS. Calls from a subroutine in one level of the hierarchy will bring a lower-level segment, consisting of subroutines, into the Central Processing Unit. Following are a list of the EROS subroutines as they appear in the overlay structure and a brief description of their principal functions. The order of appearance of a subroutine in its segment does not indicate the sequence in which it is called. Numbers in parentheses indicate reference sections.

SEGMENT 1 (Root segment)

MAIN	Main driver initialization; reads title.
SUMRY	Determines maximum, minimum, and the times of occurrence of the principal variables, e.g., power, reactivity, and flow.
YLAG	LaGrangian interpolation.

Fig. 1

EROS Flow Chart. ANL Neg. No. 103-R5069 Rev. 1.

Fig. 2. EROS Overlay Structure. ANL Neg. No. 103-R5070.

<u>SEGMENT 2</u>	(Calculational control)
MPRI	Logical control, data initialization.
DECRD	Card-input routine, floating point.
<u>SEGMENT 3</u>	(Input generator)
INPGEN	Input generator driver, iteration for initial temperature (II.A).
FORMUA	Overall heat-transfer coefficients (II.A.5).
SCHEME	Generates indices for nodes and connectors; calculates coefficients.
READ	Reads and prints channel input data (II.A.6).
DECRD2	Card-input routine for region composition.
READ2	Reads channel data from disk, and modifies data as required.
FILM	Calculates film coefficient (II.A.3).
CHKPRT	Debugging subroutine.
CKSET	Checks for some input errors; sets control items from users' data.
ODENS	Normalized power fraction, geometrical parameters and miscellaneous constants (II.A.9).
DENOM	Intermediate heat-transfer calculations.
PROPTS	Material-property tables and interpolation, entry point CHPROP for changes and additions (II.A.3).
TCALC	Calculates steady-state temperatures (II.A.8).
NODOUT	Calculates indices of feedbacks to be printed and plotted.
<u>SEGMENT 4</u>	(Reactivity and flow calculation and intermediate storage)
HTEST	Primary interval switching (Appendix B).
FLOW	Flow rates (II.B.4).
SETBAK	Returns value of reactivity due to exceeding an upper or lower setpoint (II.B.7).
RT	Time-dependent reactivity insertion (II.B.3).
Q	Computes total power, decay heat (II.B.2,5).
DF	Integration integrals, $D_n(x)$ (Appendix A).
EOPT	Tests for end of problem (III.E).
STORE	Stores pointwise data on external devices.
QTEST	Error test (Appendix B).

SAVE	Stores data of current time point for interval testing (Appendix B).
AXRODB	Calculates displacement of control rod for Sec. L_1 (II.B.10).
REAC	Returns system reactivity from Eqs. 35 and 41 (II.B.3).
<u>SEGMENT 5</u>	(Dynamics data input, preliminary kinetics calculations, and zero-power kinetics)
INPUT	Dynamics data input.
IREAD	Card input routine, integer data.
ICJCC	Input connectors if INPGEN not used (II.B.2).
PRELIM	Dynamics initialization.
AXROD	Determines indices of plotted and printed feedbacks.
KINIKS	Zero-power solution (III.A.3).
READ 3	Reads dynamics data from disk, and modifies data as required.
PRDP	Prints steady-state feedback reactivity.
<u>SEGMENT 6</u>	(Integration of equations)
AIREK	Primary solution (Cohn's method) (Appendix A).
TEMP	Evaluates feedback equation by using feedback triplets (II.B.2).
ALGEBA	Evaluates algebraically determined feedbacks (II.B.2).
TEST	Change-of-phase tests (II.C).
AJUSTH	Adjusts time interval when phase condition changes.
VARINT	Varies inlet coolant temperature of selected channels during transient.
<u>SEGMENT 7</u>	(Print and display output)
TAPOUT	Retrieves data from external devices and is the output driver routine.
PRINT	Prints output.
ANLCRT	Graphics driver.
ROUND	Numerical truncation for graphics.
<u>SEGMENT 8</u>	(Transfer-function analysis)
TRNFCN	Transfer-function driver (II.B.9).
FILON	Filon quadrature.
FUNC	Data interpolation.

II. MODELS

The models used in EROS are for input generator, reactor dynamics, and change of phase. Input-generation and change-of-phase models are separated from the reactor-dynamics models, not only because of the historical development but also because of their optional use.

A. Input Generator

The input-generation routines reduce the labor of preparing data for the EROS program. Use of these routines is optional, but is highly recommended for any time that heat transfer is to be simulated. Input is in the form of geometrical description of the fuel element and coolant channel, compositions, total power, and options that determine which of several forms of the various equations are to be used. The outputs of these routines are the heat-transfer coefficients, connectors, and other coefficients of the differential (or algebraic) equations to be solved in the reactor-dynamics simulation.

1. Geometry

a. Nodes, Regions, and Zones. Fuel elements are represented in two dimensions by cylindrical or Cartesian coordinates in the horizontal plane and by Cartesian coordinates in the vertical plane, i.e., R-Z or X-Z geometry. Fuel elements may be of the plate, annular, rod, or pin type. The extent of the coolant may be artificial, as with fuel pins, for which one uses the volume of coolant associated with each pin to compute an effective coolant boundary.

The model in Appendix C illustrates the terminology and preparation of input data. The terminology is designed to be independent of the geometry of the element. A nodal representation of the geometry is constructed by dividing the element axially into NZ zones and radially into NR regions. The region may be an artificial division of a physical region or may correspond to a physical region. For example, one might divide the fuel into five regions to obtain a temperature profile. The nodes are the volumes bounded by the regions and zones.

The above terms and use of input data may be illustrated by means of two examples based on the information in Appendix C.

b. Examples

(1) Example 1. To obtain a temperature profile in the fuel, the following regions are used:

		<u>Fuel Pin</u>					<u>Gap</u>	<u>Cladding</u>	<u>Coolant</u>	
<u>Zones</u>	3	3	6	9	12	15	18	21	24	27
	2	2	5	8	11	14	17	20	23	26
	1	1	4	7	10	13	16	19	22	25
		<u>Regions</u>								
		1	2	3	4	5	6	7	8	9

The fuel is divided into five regions; the gap and coolant form one region each, and the cladding is divided into two regions. The 28 nodes are numbered as shown. Node 28 is added to represent the coolant inlet temperature.

For the type of data in Sec. III.B of Volume II, the arrays NCOOL and TYPE assume the following values:

$[NCOOL(I), I = 1, NR] = 0, 0, 0, 0, 0, 0, 0, 0, 0, 1;$

$[TYPE(I), I = 1, NR] = 1, 1, 1, 1, 1, 4, 2, 2, 3;$ and

$NR = 9, NZ = 3.$

The NCOOL array, which differentiates coolant from noncoolant regions, indicates that mass-transport effects are to be taken into account in generating the coefficients and connectors. No two coolant regions may be adjacent. Coolant material can be in a region without the region being designated as a "coolant region."

The TYPE array is input by region, but is not used by the input generator. This array is used in the change-of-phase models to determine the type of node--i.e., fuel, cladding, coolant, or "other."

(2) Example 2. In this example, detail is avoided and computer time is minimized:

		<u>Fuel Pin</u>	<u>Gap</u>	<u>Cladding</u>	<u>Coolant</u>
<u>Zones</u>	3	3	6	9	12
	2	2	5	8	11
	1	1	4	7	10
					13
		<u>Regions</u>			
		1	2	3	4

The fuel, gap, and cladding are each represented by one region, resulting in 13 nodes, as shown. Node 13 is added to represent the coolant inlet temperature.

The geometric information of the above data from Volume II have the following values:

$$[\text{NCOOL}(I), I = 1, \text{NR}] = 0, 0, 0, 1;$$

$$[\text{TYPE}(I), I = 1, \text{NR}] = 1, 4, 2, 3; \text{ and}$$

$$\text{NR} = 4, \text{NZ} = 3.$$

2. Pseudochannel

The basic structure of the input generator does not allow a geometry in which there is a material change in the direction of coolant flow. Additional flexibility in modeling may be obtained by the concept of a pseudochannel, an independently defined channel "connected" to a previously defined "source" channel. It is independent in its material composition and in the number of regions and zones. Connection to the source channel is by the coolant inlet. Thus, a pseudochannel connector to the model in example 1 above would have node 27 for its coolant inlet source.

Fig. 3. Pseudochannel Stacking Model.
ANL Neg. No. 103-R5068.

outlet-coolant node 18 to inlet-coolant node 36. Pseudochannel 3 is connected to its source channel 2 by outlet-coolant node 35 to inlet-coolant node 57. Such a model provides a method for modeling the thermal and reactivity feedback effects over the entire length of a subassembly.

Figure 3 shows the pseudochannel capability in which three channels with different geometrics and material compositions are connected vertically by the pseudochannel connectors. Pseudochannel 2 is connected to its source channel 1 by

3. Region Properties

The temperature-dependent physical properties are obtained from an iteration procedure that calculates the initial temperature distribution and selects the physical properties from the material table by using a second-order LaGrangian interpolation scheme.

Properties⁵ for several materials are built into the code in the form of a table (Table I), which can be added to when the code is executed.

TABLE I. Temperature-dependent Physical Properties for Common Fuels, Structural Materials, and Coolant

Material	Temperature, °F	Density, lb/ft ³	Conductivity, 10 ³ Btu/sec/ft-°F	Specific Heat, 10 ² Btu/lb-°F	Viscosity, 10 ⁴ lb/sec-ft
U5FS ^a	700	1120	4.21	3.64	0
	1000	1120	5.0	4.5	0
	1200	1120	5.75	4.8	0
	1400	1120	7.0	4.8	0
	1600	1120	8.25	4.9	0
	1800	1120	9.25	5.9	0
Oxide	700	614	1.307	7.4	0
	1000	614	0.955	7.9	0
	2000	614	0.560	8.3	0
	3000	614	0.430	8.7	0
	5000	581	0.273	9.1	0
	7000	581	0.226	9.9	0
Carbide	500	864	2.55	5.5	0
	1000	864	2.94	5.85	0
	2000	864	3.6	6.25	0
	3000	864	4.26	6.65	0
	5000	864	5.58	7.45	0
	6000	864	6.0	7.85	0
Stainless Steel	700	488	2.9	13.3	0
	900	484	3.15	13.8	0
	1200	480	3.5	14.4	0
	1500	475	3.9	15.0	0
	1800	470	4.2	15.5	0
	2500	456	5.03	16.5	0
Sodium	700	53.8	11.7	30.8	1.90
	900	52.2	10.8	30.2	1.50
	1100	50.5	10.0	30.0	1.30
	1300	48.8	9.3	30.0	1.20
	1500	47.2	8.6	30.2	1.15
	1700	45.6	7.9	30.7	1.10

^aUranium-5-wt % fissium alloy.

The overall physical properties from different materials in a single region are obtained by volume-weighting the properties of up to five materials specified by the user for that region. For each region, the following formula is used:

$$\bar{X} = \frac{\sum_{i=1}^m V_f^i X^i}{\sum_{i=1}^m V_f^i} \quad \text{for } m \leq 5, \quad (1)$$

where

X^i = one of the above properties for the i th material,

V_f^i = the volume fraction of the i th material,

and

$\bar{X}$ = the average value of the property.

The volume fractions must add to 1.000 ± 0.001 , or an error indication is printed, and the job will be terminated after input testing is complete.

4. Film Coefficient

Four methods for obtaining the film coefficient are available:

a. The user may supply a value that will be used in all coolant regions.

b. The film coefficient representative of LMFBR's⁶ can be computed by the following equation:

$$h = \frac{k}{D} (5.0 + 0.025 Pe^{0.8}). \quad (2)$$

c. The remaining two methods for calculating the film coefficient were originally in AIROS²⁻⁴ and remain in EROS, but are not used in simulations of EBR-II transients.

Lyon's equation with Dwyer's modification:

$$h = \frac{k}{D} \left\{ 7.0 + 0.025 \left[\left(1 - \frac{1.82}{Pr(\epsilon_m/\nu)_{\max}^{1.4}} \right) Pe^{0.8} \right] \right\}; \quad (3a)$$

Dittus-Boelter equation:

$$h = 0.0243 \frac{k}{D} Re^{0.8} Pr^{0.4}. \quad (3b)$$

In the above equations,

- h = film coefficient, (Btu/sec)/ft²-°F,
 k = conductivity, (Btu/sec)/ft-°F,
 D = equivalent flow diameter, ft,
 Pe = Peclet number, $PrRe$,
 Pr = Prandtl number, $C_p\mu/k$,
 Re = Reynolds number, $\rho vD/\mu$,
 ρ = density, lb/ft³,
 v = flow velocity, ft/sec = $\omega/\rho A$,
 μ = viscosity, lb/sec-ft,
 C_p = specific heat, Btu/lb-°F,
 ω = mass flow rate as supplied by the user, lb/sec,
 A = geometric area computed from the dimensions of the region, ft²,
 ϵ_m = eddy diffusivity for mass transfer, ft²/hr,

and

ν = kinematic viscosity, ft²/hr.

The value for $(\epsilon_m/\nu)_{\max}$ is obtained from a least-squares fit to the curve for circular pipes given in Fig. 2.5 of Ref. 7.

All the above properties are average values obtained by volume-weighting as described in Sec. II.A.3 above.

5. Power Distribution

The power distribution is taken to be the product of a user-supplied radial distribution by region and a computed or user-supplied axial distribution.

a. Computed Distribution. The computed axial distribution, Z_i , is formed by integrating a sine distribution over the length of the fuel, as

$$\begin{aligned}
 Z_i &= \frac{\int_{\ell_{ex}+(i-1)\ell}^{\ell_{ex}+i\ell} \sin \frac{\pi Z}{L} dZ}{\int_{\ell_{ex}}^{\ell_{ex}+H} \sin \frac{\pi Z}{L} dZ} \\
 &= \frac{\cos \left\{ \frac{\pi}{L} [\ell_{ex} + (i-1)\ell] \right\} - \cos \left[\frac{\pi}{L} (\ell_{ex} + i\ell) \right]}{\cos \left(\frac{\pi}{L} \ell_{ex} \right) - \cos \left[\frac{\pi}{L} (H + \ell_{ex}) \right]}; \quad i = 1, NZ,
 \end{aligned} \tag{4a}$$

where

H = fuel length,

$\ell_{ex} = H + 2\ell_{ex}$, the extrapolated length,

$\ell = H/NZ$, the length of a node,

and

Z_i = axial distribution of power by node.

Appendix D contains a curve of ℓ_{ex}/H versus peak/average flux.

b. User-supplied Distribution. Input for axial power distribution supplied by the user consists of NFPZ ($NFPZ \leq 25$) points in an array AXPOW corresponding to values at heights given in an array AXH.

The normalized power fraction is then computed as

$$Z_i = \frac{\int_{(i-1)\ell}^{i\ell} AXPOW \, dx}{\sum_{i=1}^{NZ} \int_{(i-1)\ell}^{i\ell} AXPOW \, dx}. \quad (4b)$$

Integrations over each interval are obtained by Simpson's rule, where the required points are obtained by three-point LaGrangian interpolation of the input data. Then

$$P_{ij} = Z_i FPR_j \quad (5)$$

where

P_{ij} = fraction of the power in axial zone i in region j , i.e., node ij ,

and

FPR_j = radial distribution of power per region as supplied by the user.

The power in node N is

$$P_N = PP_{ij}, \quad (6)$$

where

ij = coordinate of node n ,

P_N = power generation rate of node N , Btu/sec,

and

P = total power generation rate of the channel, Btu/sec.

6. Overall Heat-transfer Coefficient

The overall heat-transfer coefficient, UA , is formed for the outer boundary of each region by adding the "resistances" between the points in the regions at which the temperature is to be calculated. Because each zone may be at a different temperature, UA is calculated for each node.

The following illustration will help in the discussion:

The subscript J on UA_J indicates that the heat-transfer coefficient is for the outer (right) boundary of region J .

a. Slab Geometry

(1) No Coolant in Regions J or $J + 1$

$$UA_J = \frac{l}{\frac{W_J}{2K_J} + \frac{W_{J+1}}{2K_{J+1}} + RC_J}, \quad (7)$$

where

l = height of node (H/NZ),

W_J = width of region J ,

K_J = conductivity of the node evaluated at the initial temperature,

R_J = outer boundary of region J ,

$\bar{R}_J$ = midpoint of region J ,

and

RC_J = contact resistance per unit length of surface at the J th boundary.

(2) Region $J + 1$ Is Coolant; Region J Is Not

$$UA_J = \frac{l}{\frac{W_J}{2K_J} + \frac{1}{h_{J+1}} + RC_J}, \quad (8)$$

where

h = film coefficient (either a given input value, or evaluated at each node as a function of the initial conditions).

(3) Region J Is Coolant; Region J + 1 Is Not

$$UA_J = \frac{\ell}{\frac{1}{h_J} + \frac{W_{J+1}}{2K_{J+1}} + RC_J} \quad (9)$$

b. Cylindrical Geometry. Define the average radius in each region to correspond to equal volumes; i.e.,

$$\bar{R}_J = \left(\frac{R_J^2 + R_{J-1}^2}{2} \right)^{1/2} \quad (10)$$

(1) No Coolant in Region 1 or Region 2

$$UA_1 = \frac{2\pi\ell}{\frac{R_1^2 - \bar{R}_1^2\sigma}{2K_1R_1^2} + \frac{P_2(\bar{R}_2^2 - R_1^2)}{P_1(2K_2)R_1^2} + \frac{(P_1 - P_2)\ell\ln(\bar{R}_2/R_1)}{P_1K_2} + \frac{RC_1}{R_1}} \quad (11)$$

where

$$\sigma = \begin{cases} 0 & \text{if peak temperatures are to be calculated for region 1} \\ 1 & \text{if average temperatures are to be calculated for region 1,} \end{cases}$$

and

P_J = power density, Btu/sec-ft³ in region J.

(2) No Coolant in Region 1; Coolant in Region 2

$$UA_1 = \frac{2\pi\ell}{\frac{R_1^2 - \bar{R}_1^2\sigma}{2K_1R_1^2} + \frac{1}{R_1h_2} + \frac{RC_1}{R_1}} \quad (12)$$

(3) Coolant in Region J; No Coolant in Region J + 1, J ≥ 1

$$UA_J = \frac{2\pi\ell}{\frac{1}{R_Jh_J} + \frac{\ell\ln(\bar{R}_{J+1}/R_J)}{K_{J+1}} + \frac{RC_J}{R_J}} \quad (13)$$

(4) No Coolant in Regions J or J + 1, J > 1

$$UA_J = 2\pi\ell \left[\frac{P_J(R_J^2 - \bar{R}_J^2)}{2K_JS_J} + \frac{S_{J-1} - R_{J-1}^2P_J}{S_J} \frac{\ell\ln(R_J/\bar{R}_J)}{K_J} \right. \\ \left. + \frac{P_{J+1}(\bar{R}_{J+1}^2 - R_J^2)}{2K_{J+1}S_J} + \frac{S_J - R_J^2P_{J+1}}{S_J} \frac{\ell\ln(\bar{R}_{J+1}/R_J)}{K_{J+1}} + \frac{RC_J}{R_J} \right]^{-1} \quad (14)$$

where

$$S_J = \sum_{L=1}^J (R_L^2 - R_{L-1}^2) P_L$$

and

$$S_0 = R_0 = 0.$$

(5) No Coolant in Region J, Coolant in Region J + 1, J > 1

$$UA_J = 2\pi\ell \left[\frac{P_J (R_J^2 - \bar{R}_J^2)}{S_J 2K_J} + \frac{S_{J-1} - R_{J-1}^2 P_J}{S_J} \frac{\ln(R_J/\bar{R}_J)}{K_J} + \frac{1}{R_J h_{J+1}} + \frac{RC_J}{R_J} \right]^{-1}. \quad (15)$$

7. Coefficients for Nodal Heat Transfer

The equations for calculating nodal heat transfer can be cast into the form of the generalized feedback equation discussed in Sec. II.B.2 (see Appendix C). The coefficients and connectors are generated in the following manner.

Let the triplet $I(\ell)$, $J(\ell)$, $C(\ell)$ have the following meaning:

$I(\ell)$ = node number for the ℓ th triplet,

$J(\ell)$ = node to which node $I(\ell)$ is "connected,"

and

$C(\ell)$ = coupling coefficient $C_{I,J}$,

where ℓ counts the number of triplets.

The node numbers I and J are determined by assigning to the node at the lower left corner of channel 1 the number 1 and increasing the node number as shown in Sec. II.A.1. The node in the lower left corner of channel 2 is numbered one higher than the coolant-inlet node of channel 1, etc. Thus, each channel has $(NR \cdot NZ) + 1$ nodes.

a. Noncoolant Nodes. Noncoolant nodes are "connected to" both the node on the left and the node on the right to represent radial conduction. Axial conduction is ignored.

For node N, the coefficients and connectors generated are:

- (1) $I(\ell) = N$, $J(\ell) = N - NZ$, $C(\ell) = UA_{\text{left}}/(\rho C_p V)_N$;
- (2) $I(\ell + 1) = N$, $J(\ell + 1) = N + NZ$, $C(\ell + 1) = UA_{\text{right}}/(\rho C_p V)_N$;
- (3) $Y(N) = -[C(\ell) + C(\ell + 1)]$;
- (4) $U(N) = 0$;
- (5) $P(N) = P_N/(\rho C_p V)_N$ (see Sec. II.A.4).

If no node exists to the left or right, only one triplet is generated and $Y(N) = -C(\ell + 1)$, and if no node exists to the right only, one triplet is generated and $Y(N) = C(\ell)$. The subscripts left and right indicate that the boundary value is to be used.

b. Coolant Nodes. Two formulations can be used: The first accounts for transport delay by "connecting" to all lower coolant nodes; the second connects to one lower node and assumes that outlet (rather than average) temperatures are being calculated.

The first several connectors and coefficients are the same as for noncoolant nodes; i.e.,

- (1) $I(\ell) = N$, $J(\ell) = N - NZ$, $C(\ell) = UA_{\text{left}}/(\rho C_p V)_N$;
- (2) $I(\ell + 1) = N$, $J(\ell) = N + NZ$, $C(\ell + 1) = UA_{\text{right}}/(\rho C_p V)_N$;
- (3) $Y(N) = -[C(\ell) + C(\ell + 1)]$;
- (4) $U(N) = 0$;
- (5) $P(N) = P_N/(\rho C_p V)_N$.

For the transport-delay model, the additional triplets are generated as follows:

For a given node, let K_{max} represent the axial position of that node with respect to the coolant-inlet node. Thus, for this node, $K_{\text{max}} = 1$, and K_{max} for the top (coolant-outlet) node = $NZ + 1$.

Then the connectors and coefficients representing mass transport are generated as follows:

- (6) $I(\ell + 2) = N$, $J(\ell + 2) = N$, $C(\ell) = -2\dot{w}/(\rho V)_N$;
- (7) $C(\ell + 2 + K) = \begin{cases} -[4\dot{w}/(\rho V)_N](-1)^K, & K \neq K_{\text{max}}, \\ 2\dot{w}/(\rho V)_N, & K = K_{\text{max}}, \end{cases}$

where, for each node $K = 1, K_{\max}$, and

$$J(\ell + 2 + K) = N - K;$$

$$I(\ell + 2 + K) = N.$$

For the coolant-outlet model, the additional triplets are as follows:

$$(8) \quad I(\ell + 2) = N, \quad J(\ell + 2) = N, \quad C(\ell + 2) = -\dot{w}/(\rho V)_N;$$

$$(9) \quad I(\ell + 3) = N, \quad J(\ell + 3) = N - 1, \quad C(\ell + 3) = \dot{w}/(\rho V)_N.$$

To indicate what flow equation is to be used, the J part of the triplet is modified:

If $IFLOW = 0$ (normal flow equation), a negative sign is attached to the J part of the triplet.

If $0 < IFLOW \leq 20$ ($IFLOW$ indicates an equation number), the quantity $1000*IFLOW$ is added to J . See Secs. II.B.2 and II.B.4 for more detail.

No connectors are generated for the coolant-inlet node of each channel, and Y , U , and P are set equal to zero.

8. Initial Temperature Calculation

The initial temperature calculation is done by solving the nodal heat-balance equations (as given in Appendix C) for the temperature of the particular node. This calculation is performed only if the coolant region is the rightmost region. Here we can use the fact that the heat flux into each node is equal to the total power generated in all nodes to the left; i.e.,

$$UA_{I,J-1}(T_{I,J-1} - T_{I,J}) = \sum_{K=1}^{J-1} P_{I,K}. \quad (16)$$

By substituting Eq. 16 into Eqs. C.2 and C.4, setting the left sides equal to zero, we obtain:

a. Noncoolant Node

$$T_{I,J} = \frac{\sum_{K=1}^J P_{I,K}}{UA_{I,J}} + T_{I,J+1}. \quad (17)$$

b. Coolant Node: Transport-delay Model

$$T_{I,J} = \frac{\sum_{K=1}^J P_{I,K}}{2\dot{\omega}C_p} - \left[2 \sum_{K=1}^{I-1} (-1)^K T_{I-K,J} + (-1)^I T_{inlet} \right]. \quad (18)$$

c. Coolant Node: Outlet-temperature Model

$$T_{I,J} = \frac{\sum_{K=1}^J P_{I,K}}{\dot{\omega}C_p} + T_{I-1,J}. \quad (19)$$

Equation 18 or 19 is solved first for $I = 1, NZ$. Then Eq. 16 is solved for $J = NR - 1, NR - 2, \dots, 1$; each time, I is varied from 1 to NZ .

If the rightmost region does not contain coolant, initial temperature estimates must be supplied as input. See NODET and the TAVG or T arrays in the data in Sec. III of Volume II.

The initial temperatures are implicit functions of the material properties at each node. These properties--density, conductivity, specific heat, viscosity and film coefficients--are in turn functions of temperature. Current estimates of temperatures $T_{I,J}$ are used to recalculate the values of these parameters and the overall heat-transfer coefficients. The iterative process is continued until either the difference between successive estimates is less than a given input value or a maximum number of iterations are performed.

9. Axial-importance Distribution

The axial-importance distribution may be taken to be the square of a sine distribution, integrated over the length of the node.

$$I_i = \frac{\int_{\ell_{ex} + (i-1)\ell}^{\ell_{ex} + i\ell} \sin^2 \frac{\pi Z}{L} dZ}{\int_{\ell_{ex}}^{\ell_{ex} + H} \sin^2 \frac{\pi Z}{L} dZ}$$

$$= \frac{\frac{\ell}{2} - \frac{L}{4\pi} \left(\sin \left[\frac{2\pi}{L} (\ell_{ex} + i\ell) \right] - \sin \left\{ \frac{2\pi}{L} [\ell_{ex} + (i-1)\ell] \right\} \right)}{\frac{H}{2} - \frac{L}{4\pi} \left\{ \sin \left[\frac{2\pi}{L} (H + \ell_{ex}) \right] - \sin \left[\frac{2\pi}{L} (\ell_{ex}) \right] \right\}}, \quad (20)$$

where I_i is the axial-importance distribution by node, and the other symbols are the same as in Sec. II.A.5.

Alternatively, the axial-importance distribution may be supplied by the user as input values to the array $EIM(I)$, $I = 1, NZ$. An independent user-supplied distribution for the coolant nodes may be supplied as input values to the array $EIMCL(I)$, $I = 1, NZ$.

10. Reactivity Coefficients

The coefficient $A(N)$ to be used in the reactivity calculation (see Sec. II.B.3) is calculated as follows:

$$A(N) = A_{i,j} = AFR_j I_i, \quad (21)$$

where, for each channel, node N corresponds to region j and zone i ; the radial value AFR_j is the total worth of region j represented by the channel; and I_i is either calculated by Eq. 20 or supplied by the user $EIM(I)$, $I = 1, NZ$ and/or $EIMCL(I)$, $I = 1, NZ$. For example, if only one channel is used to represent the reactivity feedback, then in the fuel, AFR_j is set equal to the temperature coefficient of reactivity for all the fuel in the core.

If multiple channels are to contribute to the reactivity feedback, the various coefficients are weighted according to the radial importance represented by each channel.

The exponent of the Doppler coefficient XN (Sec. II.B.3) is determined for each node by the $TYPE$ array and the input values XNF , XNC , and XNO .

$A(N)$ and $XN(N)$ are zero for the inlet node of each channel.

B. Reactor Dynamics

1. Neutron Behavior

The space-independent equations for prompt and delayed precursor neutrons² are:

$$\dot{N}(t) = \frac{\rho(t) - \beta}{\ell} N(t) + \sum_i \lambda_i C_i(t) + S_0; \quad (22)$$

$$\dot{C}_i(t) = \frac{\beta f_i}{\ell} N(t) - \lambda_i C_i(t), \quad (23)$$

where the reactivity, $\rho(t)$, is a known function of time, power level, and the feedback variables.

In these equations, the prompt lifetime, ℓ , and the precursor fractions, f_i and β , are assumed to be constant. In general, these parameters are affected by changes in temperature, spectrum, control-rod position, and other variables of the system. Fortunately, for most applications, the consequences of postulated accidents are not changed by accounting for these variations. Often the uncertainty in the parameters is greater than their variation during a transient.

Equations 22 and 23 are transformed into

$$\dot{N}(t) = \frac{\beta}{\ell} \left\{ [r(t) - 1]N(t) + \sum_i f_i W_i(t) + S \right\} \quad (24)$$

and

$$\dot{W}_i(t) = \lambda_i [N(t) - W_i(t)] \quad (25)$$

by the substitutions

$$r(t) = \frac{\rho(t)}{\beta}, \quad (26)$$

$$W_i(t) = \frac{\ell \lambda_i C_i(t)}{\beta f_i}, \quad (27)$$

and

$$S = \frac{\ell S_0}{\beta}, \quad (28)$$

so that $W_i(t)$ is proportional to the ratio of delayed-neutron precursors at time t to time zero, and $r(t)$ is the reactivity in dollars. Equations 24 and 25 are integrated by a numerical method, as discussed in Appendix A.

2. Feedback Equations

Generalized feedback equations are used to provide flexibility in the models that represent multichannel heat transfer, including conduction and convection, energy, pressure, and other phenomena. (The term "feedback" arises because, in general, these quantities affect reactivity, and hence, in control language, these variables are feedbacks.) The equations used in EROS are of the form

$$\begin{Bmatrix} \dot{F}_i(t) \\ F_i(t) \end{Bmatrix} = P_i Q(t) + U_i N(0) + Y_i F_i(t) + \sum_{\ell=1}^{LMAX} [\alpha_{\ell} \delta_{i,K(\ell)}] F_{J(\ell)}(t), \quad (29)$$

where

$$Q(t) = q(t) + [1 - q(0)]N(t); \quad (30)$$

$$\alpha_{\ell} = \begin{cases} C(\ell); & 0 < J(\ell) < 1000, \\ \dot{\omega}_0(t)C(\ell); & J(\ell) < 0, \\ \dot{\omega}_{\text{IFLOW}}(t)C(\ell); & J(\ell) > 1000 \text{ and of the form } 1000 \text{ IFLOW} + J, \end{cases} \quad (31)$$

and $\delta_{i,I(\ell)}$ is the Kröner delta function. $[I(\ell), J(\ell), C(\ell)]$, the triplets discussed in Sec. II.A.7, have the following meaning:

$I(\ell)$ = node number for the ℓ th triplet,

$J(\ell)$ = node to which node $I(\ell)$ is "connected,"

and

$C(\ell)$ = coupling coefficient, $C_{I,J}$.

The numerical method used in EROS presupposes diagonal dominance; i.e., Y_i in Eq. 29 is large, and thus $\exp(Y_i t)$ may be used as an integrating factor. Accuracy and stability can be increased by taking the integrating factor as a function of $\partial \dot{F}_i / \partial F_i$. For all nodes except the coolant nodes, we have $\partial \dot{F}_i / \partial F_i \equiv Y_i$. For coolant nodes, $|J(\ell)| = i$ for some values of ℓ , and this term should be included into the integrating factor.

Ideally, the integrating factors for the coolant nodes should change at each time step if the coolant flow changes. The extra calculational effort does not seem to warrant this change. Instead, the initial value of $(\partial \dot{F}_i / \partial F_i)|_0$ has been used. Thus, for coolant nodes,

$$\begin{aligned} \dot{F}_i = & P_i Q(t) + U_i N(0) + \hat{Y}_i F_i + \sum_{\substack{\ell=1 \\ \ell \neq \ell^*}}^{\text{LMAX}} [\alpha_{\ell} \delta_{i,I(\ell)}] F_{J(\ell)}(t) \\ & + C(\ell^*) [\dot{\omega}(t) - \dot{\omega}(0)] F_i, \end{aligned} \quad (31a)$$

where

$$\hat{Y}_i = Y_i + C(\ell^*) \dot{\omega}(0),$$

and

$$\ell^* \text{ such that } |J(\ell)| = i.$$

Use of the (I, J, C) triplets reduces a sparse two-dimensional array, $C(I, J)$, to three compact arrays: $I(\ell)$, $J(\ell)$, and $C(\ell)$. This technique was first used by Blaine and Berland in the 500-THGC heat-transfer code.⁸ The sum in Eq. 29 is formed for all feedbacks as follows:

- a. Set the sum for each node equal to zero.
- b. Starting with $\ell = 1$, add $\alpha_{\ell} F_{J(\ell)}$ to the sum for feedback $I(\ell)$.
- c. Repeat step 2 for each triplet $\ell = 1, LMAX$.

Feedback equations may be differential or algebraic. In general, numbering of the feedback equations will be in the following order:

- a. First there are heat-transfer differential equations, which correspond to the node-numbering scheme in Sec. II.A.1. If the input generator is used,

$$NHT = \sum_{L=1}^{NC} [NR(L) * NZ(L) + 1].$$

If the input generator is not used, the user will provide NHT as input ($NHT \geq 0$). These feedbacks are calculated as solutions to the differential equation form of Eq. 29.

- b. If the change-of-phase models are to be used, the next group of feedbacks is used internally by the code to calculate the heat flux into the coolant in each axial zone, once nucleate boiling begins. The feedback numbers used for this purpose are

$$NHT + 1, NHT + \sum NZ(L),$$

where the sum includes only those channels to be tested for change of phase. The user must not enter any array in the corresponding positions.

- c. The next set of feedback equations are the general differential equations. The user indicates the number of these (NGE). The starting number for these is

$$NHT + \sum NZ(L) + 1.$$

These feedback equations are used for instrument response, calculations for coolant-flow decay, additional heat-transfer equations, and for any other purpose the user may have, e.g., calculation of total energy. Note that $NGE \geq 0$.

- d. If thermal expansion of the EBR-II control rods and control-rod suspension system is to be calculated, six additional feedback equations will be set up automatically when this option is selected. The starting number will be

$$NHT + \sum NZ(L) + NGE + 1.$$

e. The last set of equations are algebraic (NA). The starting number for these is

$$NHT + \sum NZ(L) + NGE + 6 + 1.$$

For the general differential and algebraic equations, the user must provide the $[I(\ell), J(\ell), C(\ell)]$ triplets and, in general, $0 < J(\ell) < 1000$. The coefficients of Eq. 29 can be manipulated (1) to obtain the magnitude of the flow, various integrals, and other such quantities for display or (2) to trigger instrument scrams, or both.

Because $N(t)$ and $q(t)$, and consequently $Q(t)$, are usually expressed as fractions of full power, the coefficients U_j and P_j have units of $^{\circ}\text{F}/\text{sec}$, whereas Y_j and α_{ℓ} are relaxation times with units of sec^{-1} .

a. Simple Feedback Equations. Many kinds of feedback can be represented by Eq. 29. By setting $U_j = Y_j = \alpha = q = 0$ for some value of j , Eq. 29 becomes

$$\dot{F}_j(t) = P_j N(t). \quad (32)$$

If P_j represents the initial reactor power, $F_j(t)$ is the energy produced to time t .

b. Heat Transfer. Equations for multichannel heat transfer are represented by Eq. 29. Appendix C shows a simplified model consisting of one channel containing four radial nodes (fuel, gap, cladding, and coolant), each divided into three axial nodes, and the manner in which the coefficients for Eq. 29 are obtained from the basic heat-balance equations.

c. Instrumentation Response. Any feedback variable can initiate a trip (see Sec. II.B.3). In a real reactor system, abnormal behavior, such as high temperature, power, or inverse period, is sensed by a fission plate, thermocouple, or period meter, and generally by some instrument having its own response to the signal. This signal is finally relayed to a servomechanism, which in turn initiates the trip. Therefore, using an additional feedback equation to represent the response from each instrument that may initiate the trip signal is convenient. Let $F_k(t)$ represent the response of some instrument. Then, from Eq. 29, if

$$Y_k = -\frac{1}{\tau}, \quad \alpha_{\ell} = \frac{1}{\tau}, \quad P_k = U_k = 0, \quad F_k(0) = \text{initial value},$$

then

$$\dot{F}_k(t) = \frac{1}{\tau} [F_j(t) - F_k(t)], \quad (33)$$

which is the desired equation for the instrument response to one of the F_j 's. Response to the power can be found if

$$P_k = \frac{1}{\tau}, \quad Y_k = -\frac{1}{\tau}, \quad U_k = 0, \quad F_k(0) = \text{initial value};$$

viz.,

$$\dot{F}_k(t) = \frac{1}{\tau} [Q(t) - F_k(t)], \quad (34)$$

where τ is the instrument-response time. Similarly, other instrument-response functions can be simulated.

3. Reactivity

The reactivity subroutine is flexible in that any equation can be programmed and many special equations have been coded for special application. The form chosen for the present version of EROS is as follows:

a. Normal Reactivity

$$r(t) = r(0) + R(t) + RS(t) + R_{CTR}(t) + R_{set}(t) + \sum_{j=1}^{NF} A_j \delta F_j(t);$$

$$t \leq (t_s + \tau_D); \quad (35)$$

where

$r(0)$ = initial reactivity given in the input or determined by the solution of the inhour equation for constant inverse period (\$) (see Sec. III.A.2),

$R(t)$ = time-dependent reactivity insertion, \$,

$RS(t)$ = structural feedback reactivity, \$,

$R_{CTR}(t)$ = feedback reactivity due to axial thermal expansion of the EBR-II control-rod subassemblies and control-rod shaft (Sec. II.B.10),

$R_{set}(t)$ = reactivity from setback and controller (Sec. II.D),

NF = number of feedback equations,

t_s = time at which the trip signal is sensed as discussed below,

and

τ_D = delay between the time the signal is sensed and rod motion starts.

This is found from τ_{j_s} by using the DTAU data array, where j_s is the number of the feedback causing the scram (see Eq. 36).

The time at which the trip occurs, t_s , is determined by comparing each feedback variable that can initiate a trip against a trip level FM_j specified in the input. Thus t_s is the earliest time at which the condition

$$F_j(t) \geq |FM_j|; \quad j = j_s \quad (36)$$

is satisfied. The absolute value of FM_j is taken because the program determines those feedbacks, j , which can initiate the trip by the condition

$$FM_j < 0. \quad (37)$$

The terms of Eq. 35 represent:

(1) Step insertion at $t = 0$.

(2) Time-dependent reactivity insertions (can be terminated after a trip occurs or continue the insertion after trip occurs) are generated by the following methods:

(a) Linear reactivity insertion:

$$R(t) = \begin{cases} \min(\gamma_1 t, \gamma_4), & t \leq (t_s + \tau_D), \\ \min[\gamma_1(t_s + \tau_D), \gamma_4], & t \geq (t_s + \tau_D), \end{cases} \quad (37a)$$

where γ_1 = linear reactivity-insertion rate, \$/sec, and γ_4 = maximum reactivity insertion, \$.

(b) LaGrangian N-point interpolation from a user-supplied table. This table may consist of as many as 500 arbitrarily spaced points.

(c) Sine variation:

$$R(t) = A \sin(\omega t + \varphi).$$

(d) Linear combination of sine and tabular values.

(3) Structural feedback reactivity is the equilibrium change in reactivity with coolant-temperature rise over the coolant region of a selected channel and is obtained by a LaGrangian N-point interpolation from a user-supplied table. This table consists of 100 arbitrarily spaced points of structural reactivity as a function of coolant-temperature rise.

(4) Reactivity feedback, determined by the last term in Eq. 25, is dependent on results from the generalized feedback equation, Eq. 29. Use of the reactivity values for coefficient A_j are determined from the following considerations: The Doppler feedback coefficient can be expressed as

$$F_j^{n_j/2} \frac{\partial \rho}{\partial F_j} = A_j, \quad (38)$$

where n_j is 0, 1, or 2, depending on the temperature range. Thus, the reactivity feedback is found by integrating; viz.,

$$\begin{aligned} (\delta \rho)_j &= \int_{F_j(0)}^{F_j(t)} \frac{\partial \rho}{\partial F_j} dF_j = \int_{F_j(0)}^{F_j(t)} A_j F_j^{-n_j/2} dF_j \\ &= \begin{cases} \frac{A_j}{1 - n_j/2} \left[F_j^{1-(n_j/2)}(t) - F_j^{1-(n_j/2)}(0) \right]; & n_j \neq 2, \\ A_j \ln \frac{F_j(t)}{F_j(0)}; & n_j = 2, \end{cases} \end{aligned} \quad (39)$$

so that the symbol, $\delta F_j(t)$, used in Eq. 35 is defined to be:

$$\delta F_j(t) = \begin{cases} [F_j(t) - F_j(0)]; & n_j = 0, \\ 2 \left\{ [F_j(t) + 460]^{1/2} - [F_j(0) + 460]^{1/2} \right\}; & n_j = 1, \\ \ln \frac{F_j(t) + 460}{F_j(0) + 460}; & n_j = 2. \end{cases} \quad (40)$$

b. Reactivity after a Trip. Reactivity after trip is assumed to be due to several scram rods dropping from the full-inserted position downward and out of the core owing to gravity and friction with various delay times. Further assumed is that the axial flux distribution is a sine, so that the importance (relative worth) is given by a sine squared. Thus the reactivity after a trip is given by

$$\begin{aligned} r(t) &= r(0) + R(t) + RS(t) + R_{set}(t) + \sum_{j=1}^{NF} A_j \delta F_j(t) \\ &\quad + \gamma_2 \sin^2[\gamma_3 x_0(t)]; \quad t \geq t_s + \tau_{D_0}, \\ &\quad + \gamma_5 \sin^2[\gamma_6 x_1(t)]; \quad t \geq t_s + \tau_{D_1}, \\ &\quad + \gamma_7 \sin^2[\gamma_8 x_2(t)]; \quad t \geq t_s + \tau_{D_2}, \\ &\quad + \gamma_9 \sin^2[\gamma_{10} x_3(t)]; \quad t \geq t_s + \tau_{D_3}, \end{aligned} \quad (41)$$

where

$\gamma_2, \gamma_5, \gamma_7$, and γ_9 = the maximum rod worths, \$,

and

x_0, x_1, x_2 , and x_3 = the distances from the top of core at time t , as calculated below.

$$x_0(t) = \begin{cases} 0; & t \leq t_s + \tau_D, \\ [t - (t_s + \tau_D)]^2; & 0 \leq x_0(t) \leq L_0, \quad t > t_s + \tau_D, \\ \frac{2}{a_0} L_0; & t_s + \tau_D < t \rightarrow \infty, \end{cases} \quad (42)$$

$$x_1(t) = \begin{cases} 0; & t \leq t_s + \tau_{D1}, \\ [t - (t_s + \tau_{D1})]^2; & 0 \leq x_1(t) \leq L_1, \quad t > t_s + \tau_{D1}, \\ \frac{2}{a_1} L_1; & t_s + \tau_{D1} < t \rightarrow \infty, \end{cases} \quad (43)$$

$$x_2(t) = \begin{cases} 0; & t \leq t_s + \tau_{D2}, \\ [t - (t_s + \tau_{D2})]^2; & 0 \leq x_2(t) \leq L_2, \quad t > t_s + \tau_{D2}, \\ \frac{2}{a_2} L_2; & t_s + \tau_{D2} < t \rightarrow \infty, \end{cases} \quad (44)$$

$$x_3(t) = \begin{cases} 0; & t \leq t_s + \tau_{D3}, \\ [t - (t_s + \tau_{D3})]; & 0 \leq x_3(t) \leq L_3, \quad t > t_s + \tau_{D3}, \\ \frac{1}{a_3} L_3; & t_s + \tau_{D3} < t \rightarrow \infty, \end{cases} \quad (45)$$

and where

L_0, L_1, L_2 , and L_3 = distances of fall, ft,

a_0, a_1 , and a_2 = acceleration constants, ft/sec²,

a_3 = constant, ft/sec,

and

γ_3 , γ_6 , γ_8 , and γ_{10} are defined as follows:

$$\gamma_3 = \frac{1}{2} a_0 \frac{\pi}{2L_0}, \quad (46)$$

$$\gamma_6 = \frac{1}{2} a_1 \frac{\pi}{2L_1}, \quad (47)$$

$$\gamma_8 = \frac{1}{2} a_2 \frac{\pi}{2L_2}, \quad (48)$$

and

$$\gamma_{10} = a_3 \frac{\pi}{2L_3}. \quad (49)$$

The rod simulation may use any combination of the four available rods.

4. Equations for Coolant Mass Flow

The following total-flow conditions can be represented now: constant flow; tabular data describing either measured or assumed flow conditions not adequately described by Eqs. 50 and 51; sinusoidal flow variation about 1.0; flow decay from full power; and flow decay after a trip.

Individual channel-flow conditions can be represented by Eq. 53, 54, or 55.

a. Measured or Assumed Flow Conditions from Tabular Data.

Equations 50 and 51a presented below, may be modified by replacing the exponential term and the constant term (C) with user-supplied tabular values. As many as 100 tabular values of flow at arbitrary time intervals may be used. The final values are computed by using N-point LaGrangian interpolation.

b. Sinusoidal Flow Variation. External loop equations are planned. In anticipation of this, transfer-function analyses of the effect of flow changes have been made. For these studies, the flow deviations may be expressed as the sum of as many as five sinusoids. Each sinusoid may have its own amplitude and phase angle.

c. Decay of Coolant Flow from Full Power. The reactor is assumed to be operating at some power level at $t = 0$, and an accident such as pump failure or pipe rupture is assumed to occur that causes the flow to decay from its normal value according to the relation

$$\dot{\omega}_0(t) = A \exp(Bt + B_1 t^2) + C + E(F - t); \quad t \geq 0. \quad (50)$$

The last term is set equal to zero if $F < t$. This is the normal flow equation and is used if $D = 0$. This option can also be used to simulate pump startup for increasing power levels, although no provision (other than end of problem) is made to terminate the increase.

d. Flow Decay after Scram. In this case, the reactor operates according to Eqs. 24, 25, 29, and 35 and at constant flow up to the time of a scram; i.e.,

$$\dot{\omega}_0 = A + C; \quad t \leq (t_s + \tau_{\dot{\omega}}), \quad (51)$$

where $\tau_{\dot{\omega}}$ is a delay time, and thereafter according to

$$\dot{\omega}_0(t) = A \exp(Bt_m + B_1 t_m^2) + C + E(F - t_m), \quad t_m > 0, \quad (51a)$$

where

$$t_m = t - (t_s + \tau_{\dot{\omega}}). \quad (52)$$

The last term is set equal to zero, unless $0 < t_m < F$.

e. Equations for Channel Flow. The flow equation expressed by $\dot{\omega}_l$ is used in addition to Eqs. 51 and 51a if

$$D = -NW,$$

where NW is the number of flow equations.

For each value of l , the flow equation may take any of three forms, depending on the choice of the flow coefficients ϵ_{il} . If the subscript l is dropped, the forms are determined as follows:

The sign of ϵ_1 will determine which major form is desired, and $|\epsilon_1|$ is the number of the feedback equation to be used. Let this be feedback number k , and let Z be the value of feedback variable k ; i.e., let

$$k = \epsilon_1$$

and

$$Z = F|_k|(t).$$

(1) For $k < 0.0$,

$$\dot{\omega} = \epsilon_2 Z + \epsilon_3 Z^2 + \frac{\epsilon_4}{Z} + \frac{\epsilon_5}{Z^2}. \quad (53)$$

(2) For $k \geq 0.0$, the two remaining forms of $\dot{\omega}$ are determined by ϵ_{10} . Let

$$H = \begin{cases} 0.0 & \text{for } k = 0.0 \\ Z & \text{for } k > 0.0 \end{cases}$$

and

$$R(t) = \begin{cases} 0.0 & \text{for } t \leq 0.0 \\ \epsilon_3 t + \epsilon_4 t^2 & \text{for } t > 0.0. \end{cases}$$

(a) For $\epsilon_{10} = 0.0$,

$$\dot{\omega} = H + R(t^*) + \epsilon_2 + \epsilon_5 \exp(\epsilon_6 t + \epsilon_7 t^2), \quad (54)$$

where $t^* = \epsilon_9 - t$.

(b) For $\epsilon_{10} > 0.0$,

$$\dot{\omega} = \begin{cases} H + \epsilon_{10} & \text{before scram or } t_m \leq 0.0 \\ H + R(t_m^*) + \epsilon_2 + \epsilon_5 \exp(\epsilon_6 t_m + \epsilon_7 t_m^2) & \text{after scram and } t_m > 0.0, \end{cases} \quad (55)$$

where

$$t_m = t - (t_s + \epsilon_8)$$

and

$$t_m^* = \epsilon_9 - t_m.$$

These forms of flow equations should be adequate to represent most possible types of flow decay.

5. Afterglow-heating Level

Equation 30 defines normalized reactor power as the sum of neutron level and afterglow heating level. One can simulate the afterglow heating for a startup after a previous trip by

$$q(0) \neq 0.$$

Afterglow heating level after a trip is determined by either (1) tabular data (GLWDAT) with an N-point LaGrangian interpolation of the input data, which may consist of as many as 100 nonequidistant points; or (2) a two-term exponential based on Ref. 9.

The following equation describes afterglow heating level before and after a trip:

$$q(t) = \begin{cases} q(0); & t \leq t_s; \\ \text{GLWDAT}(t - t_s); & t > t_s, \text{ GLOW} \neq 0; \\ \delta_1 \exp[\delta_3(t - t_s)] + \delta_2 \exp[\delta_4(t - t_s)]; & t > t_s, \text{ GLOW} = 0. \end{cases} \quad (56)$$

6. Change of Phase

Models used in EROS to simulate change of phase are phenomenological rather than highly detailed microscopic representations of the complex processes of fuel melting and of coolant boiling and vaporization. There are several reasons for the use of the simpler models: (1) Other existing codes treat the above processes in great detail;¹⁰⁻¹² (2) using simplified models minimizes running time of EROS; and (3) the models are easy to extend or modify for comparison with experiment.

Because of the nature of these models, much intuition, experience, and work are required to prepare the data used in describing the change-of-phase processes. The models are simple, but adequately represent the physical processes, their reactivity feedback, and the results of postulated accidents.

The routines that simulate change of phase are entered at the end of each time interval. For each channel to be tested for change of phase, a loop is performed, one column of nodes at a time. The TYPE array (Sec. II.A.1) determines whether that column is fuel, cladding, coolant, or other, and, depending on the type, one of the following sections describes the equations used.

a. Fuel and Cladding. If the column of nodes is fuel or cladding, the temperature of each axial node is tested against an input value, TMF1 (TMC1),* which determines at what temperature melting begins. When any node in the column reaches that temperature, all coefficients $[P(N), Y(N), \text{ and } C(I, J)]$ for the node are modified to account for a change in specific heat. The specific heat is assumed to increase by a factor FCPF (FCPC). When the temperature reaches TMF2 (TMC2), melting is complete and the specific heat is returned to its initial value. The assumed change in specific heat is an artifice to account for the noncongruent manner in which most fuels melt; i.e., melting occurs over a temperature range that accounts properly for the latent heat of vaporization of the fuel or cladding. The fractional change in specific heat can be set at one if desired.

If the temperature drops (with resolidification of material) the process is reversed; i.e., at TMF2 (TMC2) the specific heat is returned to its

*These terms, and following ones, are in the User's Manual.

initial value. Reactivity feedback is accounted for through the normal reactivity, Eq. 35, as discussed in Sec. II.B.3, by means of the coefficient array $A(I)$.

If the temperature reaches TFD (TCD), the fuel or cladding is assumed to disassemble. When this condition occurs, no further heat is transferred out of the node, the temperature is held at TFD (TCD), and all coefficients for this node are set to zero. This condition is not reversible, and no additional reactivity is introduced in the present model.

If the heat flux from any axial node into the coolant exceeds an input value, QBO, creation of a film that blocks any further flow of heat into the coolant is assumed. The formulation is as follows.

The heat current is given by

$$q_N = UA_N[T_{N-NZ}(t) - T_N(t)], \quad (57)$$

where

q_N = the heat current into coolant node N from node N - NZ,

and

UA_N = the overall heat-transfer coefficient at the boundary of the coolant.

Then, if $q_N \geq QBO$, burnout occurs. This representation is reasonable for steady-state or slow transients, but should not be used for fast transients.

b. Coolant. Nucleate boiling, bulk boiling, and vaporization of the coolant are represented in the following manner.

If the temperature of any coolant node reaches TNB, nucleate boiling occurs and the overall heat-transfer coefficient is modified to reflect the film-boiling coefficient.

If the temperature of any coolant node reaches TCB, bulk boiling occurs and the flow in that channel is modified by the fraction FWF. The temperature is held fixed at the boiling temperature TCB, and the heat flux is integrated until an amount of heat QVL corresponding to the latent heat of vaporization has passed into the coolant. Once coolant in any node has vaporized, the temperatures of all coolant nodes in that channel are held fixed.

Void reactivity is inserted by means of feedbacks $NHT + 1$ to $NHT + \sum [NZ(t)]$, where the sum is over all channels to be tested for change of phase (see Sec. II.B.2). Each of these feedbacks is the integral of the heat flux into the coolant, beginning at the time when bulk boiling starts, and has, as a

maximum value, QVL. The numbering is continuous over channels to be tested for change of phase. $NZ(l)$ is the number of axial nodes in the l th such channel.

The value of the reactivity coefficients, A , for these feedbacks is not supplied by the input generator; the user must input a value A such that

$$A(I) = \frac{A'(I)}{QVL}, \quad (58)$$

where $A'(I)$ is the void worth of the corresponding coolant node.

Thus void reactivity is accounted for by the expression (see Eqs. 35 and 40):

$$A(I)[F(I) - F_0(I)] = A'(I) \min \left[1, \frac{\int U A_N (T_{N-NZ} - T_N) dt}{QVL} \right]. \quad (59)$$

When the integral in Eq. 59 is equal to QVL (i.e., an amount of heat equal to the latent heat of vaporization has passed into the coolant), the coolant is assumed to have vaporized, and the temperatures of all coolant nodes and the integrals of heat flux for the channel are held fixed.

7. Setback and Controller

EROS can simulate setback mode or controller action of a single control rod or a bank of control rods. This process can be described as:

One to NF feedbacks can be monitored with the following data arrays:

FSPH(I), $I = 1, NF$ --upper set point of I th variable;

FSPL(I), $I = 1, NF$ --lower set point of I th variable;

TAUSET(I), $I = 1, NF$ --delay of I th variable.

In addition,

γ_{UP} = the rate at which the bank of rods is raised;

γ_{DN} = the rate at which the bank of rods is lowered;

X_0 = the initial rod position;

Worth = the reactivity worth of the bank of rods when fully inserted (negative), \$;

L = distance of travel;

and

$X(t)$ = position of rod bank at time t , $0 \leq X(t) \leq L$.

Then the reactivity contribution due to setback or controller action, R_{set} , is determined by

$$X(t) = \begin{cases} X_{end}; & \text{FSPL(I)} < F(I) < \text{FSPH(I)}, \\ & \text{for all I or } t < t_s + \xi, \\ \text{Min}\{X_{end} + \gamma_{DN}[t - (t_s + \xi)], L\}; & \text{F(I)} > \text{FSPH(I)} \text{ for any I,} \\ & \text{or } t < t_{end} + \xi, \\ \text{Max}\{X_{end} - \gamma_{UP}[t - (t_s + \xi)], 0\}; & \text{FSPL(I)} > F(I) \text{ for any I,} \\ & \text{or } t < t_{end} + \xi, \end{cases} \quad (60)$$

where

X_{end} is the position at which the bank stops (initiated to X_0);

t_s is the time at which a variable, I , first crosses its setpoint;

t_{end} is the time at which the same variable recrosses its setpoint;

and

ξ is the delay associated with that variable;

and

$$R_{set}(t) = \text{Worth} \left(\frac{X(t) - X_0}{L} - \frac{1}{2\pi} \left\{ \sin \left[\frac{2\pi}{L} X(t) \right] - \sin \frac{2\pi X_0}{L} \right\} \right). \quad (61)$$

Then Eqs. 35 and 41 are modified by the addition of the above reactivity unless WORTH is zero, when the subroutine is not used. If any other variable has crossed one of its setpoints when a rod is to stop moving, the following actions are taken:

a. New variable crossed same setpoint: Continue bank motion in the same direction without delay.

b. New variable crossed opposite setpoint: Stop bank motion until appropriate delay has passed.

These equations are best understood by the example shown in Fig. 4.

8. Reactor Trips

Four parameters are available to start a reactor trip:

a. Power trips:

(1) Reactor power: $Q(t) = q(t) + [1 - q(0)]N(t)$.

(2) Neutron power: $N(t)$.

- b. Relative coolant-flow trip.
- c. Inverse period trip.
- d. Any number of the 1000 feedbacks.

Fig. 4. Example of Setback and Controller Action

Associated with each available reactor trip is a time delay to simulate the time at which the trip was sensed and the time at which the trip rod or rods begin to move. See Sec. II.B.3 for reactivity after a trip.

9. Transfer-function Analysis

Stability characteristics of power reactors may be investigated by analysis of transfer functions. Computational abilities for transfer functions have been added to EROS for investigating the stability characteristics of various models.

A systems transfer function is the complex-plane ratio of the system response function to its driving function:

$$TF = \frac{L\{R\}_{i\omega}}{L\{D\}_{i\omega}} = \frac{\int_0^{\infty} e^{-i\omega t} R(t) dt}{\int_0^{\infty} e^{-i\omega t} D(t) dt}, \quad (62)$$

where

$L\{ \}$ = LaPlace transform,

$R(t)$ = response function,

and

$D(t)$ = driving function.

Transforms of wide variety may be calculated. Examples of response functions are reactor power and coolant outlet temperatures. Examples of driving functions are coolant inlet temperatures, coolant inlet flow, and reactivity.

Numerical estimates of the transforms are obtained by Filon¹³ quadrature of a given function between user-supplied limits corresponding to the time limit or portion of the interval of the current EROS problem. The user must also supply a step interval for the quadrature.

Filon Quadrature may be considered to be an extension of Simpson's rule, which evaluates the integral

$$I = \int_a^b f(x) dx$$

by approximating $f(x)$ with a string of parabolas. Filon's method evaluates the integral

$$I = \int_a^b \sin(\omega x) f(x) dx$$

or

$$I = \int_a^b \cos(\omega x) f(x) dx$$

by approximating $f(x)$ with a string of parabolas. The cosine integral reduces to Simpson's rule in the limiting case of $\omega = 0$.

Transfer-function analysis may be studied by using arbitrary driving functions. However, if the driving function is a sinusoid, computation time may be shortened by using built-in analytic evaluations of the transforms:

$$\int_0^{t_m} e^{-i\omega t} \sin kt \, dt = \frac{1}{k^2 - \omega^2} \left\{ k - e^{-i\omega t_m} [i\omega \sin(kt_m) + k \cos(kt_m)] \right\},$$

$$k \neq \omega,$$

(63)

and

$$\int_0^{t_m} e^{-i\omega t} \sin kt \, dt = \frac{\sin^2 \omega t_m}{2\omega} + i \left(-\frac{t_m}{2} + \frac{\sin^2 \omega t_m}{4\omega} \right), \quad k = \omega. \quad (64)$$

10. Feedback Reactivity due to Axial Thermal Expansion of the EBR-II Control-rod Subassembly and Control-rod Shaft

A reactor with control-rod subassemblies suspended from above and immersed in the outlet coolant can generate feedback reactivity due to axial thermal expansion of the control rods and drive shaft as the coolant temperature changes with power and flow. In some systems, including EBR-II, depending on how far the control rods are inserted into the core, the amount of such reactivity feedback is significant in the total reactivity feedback.

The EBR-II control rod and drive shaft are divided into four sections with distinctly different dynamic heat-transfer characteristics.¹⁴ The sections are shown as L_1 - L_4 in Fig. 5. Reactivity feedback resulting from thermal expansion of these four sections is described by

$$R_{CTR}(t) = \sum_{\ell=1}^4 d_{\ell}(t) W_{CTR}. \quad (65)$$

Fig. 5. EBR-II Control-rod Suspension System. ANL Neg. No. 103-R5067.

where $d_l(t)$ is the calculated displacement of each of the four sections shown in Fig. 5, and W_{CTR} is the reactivity coefficient in dollars per foot of fuel movement for a particular inserted position, summed over all control rods.

The first section (L_1) of the control-rod hexagonal can will expand or contract in a way governed by its coolant temperature and by the transport time required for a change in coolant temperature to travel the length of the section. Section L_1 is modeled by using the pseudochannel option shown in Fig. 3. The channel stacking results in an axial temperature distribution that is used to calculate the displacement $d_1(t)$ for section (L_1) by using the following equation:

$$d_1(t) = \sum_{i=1}^{NAXRZ} [T_i(0) - T_i(t)] \alpha H_i, \quad (66)$$

where

$NAXRZ$ = total number of coolant nodes used in section L_1 , Fig. 3;

α = linear-expansion coefficient for stainless steel,
 1.0×10^{-5} ft/ft-°F;

H_i = height of each node (Fig. 3);

$T_i(0)$ = steady-state coolant temperature of node i ;

and

$T_i(t)$ = coolant temperature of node i .

All remaining sections (L_2 , L_3 , and L_4) respond to the plenum coolant temperature, which is assumed in the model to be the outlet coolant temperature of section L_1 . The general form of the equation to calculate the displacement of these remaining sections to a change in plenum temperature¹⁴ is

$$\frac{D_l(s)}{T(s)} = \frac{k}{1 + \tau_p s} \frac{C_l}{1 + \tau_l s}, \quad (67)$$

where

τ_p = time constant for coolant holdup time in plenum;

τ_l = time constant for section L_2 , L_3 , or L_4 ;

C_l = expansion of section L_2 , L_3 , or L_4 in feet per degree change in temperature;

and

k = fraction of change in temperature not lost to the surrounding reactor-vessel cover.

Table II lists the constants used in Eq. 67 that have been programmed in subroutine AXROD.

TABLE II. Constants Used in Eq. 67

Control-rod Section L_l	τ_p^a sec	τ_l^a sec	k	C_l , 10^5 ft/ $^{\circ}$ F
2	1.49	0.093	1.0	1.5092
3	1.49	21.0	1.0	1.666
4	1.49	8.1	0.9	1.6666

^aReference 14.

Equation 67 is solved by using the generalized feedback equation,
Eq. 29.

III. SPECIAL FEATURES

EROS has several special features that can be used to great advantage; they either reduce running time or make the program easier to use because fewer input data are required.

A. Initial Conditions

Because the program solves a system of first-order differential equations, the initial conditions for the levels of prompt and delayed precursor neutrons, reactivity, flow, etc., must be provided. To make input easy, the user can set certain of the initial conditions by a combination of three methods.

1. Built-in Conditions

At the beginning of the first run of a set of cases, the initial levels of neutrons and delayed precursors are set equal to one; i.e.,

$$N(0) = W_i(0) = 1; \quad i = 1, \dots, 15. \quad (65)$$

These values are used as the initial conditions for all cases until changed by one of the two following methods. The initial reactivity $[r(0)]$, flow, and $\dot{w}(0)$ are not initialized by the program. The $F_j(0)$ are initialized by the input generator for $j = 1, \text{NHT}$.

2. Conditions Specified by Input

The initial conditions for coolant flow, decay heat, and any feedback levels not calculated by the input generator must be specified in the input data, and the initial neutron level must be specified if different from one. The initial reactivity and levels of delayed precursors may be specified in the input data or can be determined by a third method, which follows.

3. Inhour Equation

The third option for specifying the levels of delayed-neutron precursors and initial reactivity is to obtain them from the inhour equation, which is solved for the case of constant period; that is, the levels of prompt- and delayed-neutron precursors are assumed to be increasing exponentially with inverse period, ω_0 . Then the levels of delayed-neutron precursors are given in terms of the initial prompt-neutron level by

$$W_i(0) = \frac{\lambda_i N(0)}{\lambda_i + \omega_0}; \quad i = 1, \text{ND}, \quad (68)$$

and the initial reactivity by

$$r(0) = \omega_0 \left(\sum_i \frac{f_i}{\lambda_i + \omega_0} + \frac{\ell}{\beta} \right) - \frac{S}{N(0)}. \quad (69)$$

B. Present Data

For convenience, portions of the data are set before the first case and are used in subsequent cases, unless changed by input data. First, all addressable data arrays for Type III and IV data* are set to zero. Levels of prompt and delayed neutrons are set to one, and the following additional values are preset.

1. Decay Constants and Fractions of Delayed-neutron Precursors

The data on delayed-neutron precursors for EBR-II¹⁵ are presented in Table III. These values will be used unless modified.

TABLE III. Data on Delayed-neutron Precursors

Group	f_i	λ_i	Group	f_i	λ_i
1	0.0346	0.0127	4	0.4041	0.311
2	0.2034	0.0317	5	0.1407	1.40
3	0.1847	0.1150	6	0.0325	3.87

2. Maximum Values of Feedbacks

To preclude problem termination because of exceeding the maximum value of some feedback not used in the problem, all feedback maxima are set equal to 10^{35} ; i.e.,

$$FM_j = 10^{35}; \quad j = 1, \dots, 1000. \quad (70)$$

Feedbacks used to terminate the problem or to initiate a trip must have maximum values provided in the input data.

3. Change-of-phase Levels

All data that could initiate change of phase are set to 10^{35} --i.e., the following arrays (20 values for each):

<u>Array</u>	<u>Starting Address**</u>	<u>Array</u>	<u>Starting Address**</u>
TNB	111	TMC1	271
TCB	151	TMC2	291
TMF1	191	TCD	331
TMF2	211	QVL	351
TFD	251	QB	371

*Data types in Volume II.

**For data in Sec. IV.A, Volume II.

If the change-of-phase models are to be used, the user must provide the levels at which the various phenomena are to be initiated.

C. Interval Switching

The interval size, h , is automatically adjusted by EROS so that a given accuracy results. Because performing an exact analysis of the truncation error or of the propagation of error is extremely difficult, an approximate method was developed that has proven satisfactory in many test cases. This method is discussed in Appendix B and is implemented in the following way.

Let Q be the maximum relative truncation error in the neutron or feedback equations as derived in Appendix B, and AL and AU be specified by the user. Then:

1. If $Q \leq AL$, the interval is doubled.
2. If $AL < Q < AU$, the interval is unchanged.
3. If $AU \leq Q$, the interval is halved and the calculation is repeated for that interval.

Thus a value of h is known for the next step.

Further restrictions exist, in that h must satisfy the condition $HL \leq h \leq HU$, with HL and HU specified in the input data. HU will be the maximum value used for h throughout the calculation. To preclude wasted machine time, if the error test requires a step size, h , that is less than the minimum, HL , the problem will be terminated, an edit will occur.

The nature of various problems, e.g., flow variation and startup, requires different values of the control parameters. See Volume II for appropriate values of AL , AU , HL , and HU .

D. Startup Problems

One of the classic accidents investigated in every hazards report is the startup accident. In this accident, a control rod is inserted to achieve full power, and, because of some malfunction, continues to be inserted, causing the reactor to go highly supercritical. The initial power (that of the source) is small enough (such as 1 W) that negligible heat is generated during a large part of the transient. Consequently, no useful information is gained in solving Eq. 29 for the temperatures, as they are constant; however, much computer time is wasted because the computer may be solving several hundred feedback equations.

Provision is made in EROS to solve only Eqs. 24, 25, and 35 for the prompt-neutron level, delayed-neutron-precursor levels, and reactivity, respectively, until $N(t) \geq N_{\min}$, where $N_{\min}$ is provided as input data; then Eq. 29 is solved. If desired, the user can bypass this option by setting $N_{\min} = 0$; then the feedback equations will be solved at all time points. Normally, however, $N_{\min}$ should be such that the feedback equations are solved when the prompt power exceeds 0.1% of full power.

E. Conditions for Problem Termination

A problem normally terminates when one of the following three conditions is met:

1. $t = \text{TIMEM}$, the maximum problem time specified in the input;
2. $N(t) \geq N_{\max}$, the maximum neutron level specified in the input; or
3. $F_j(t) \geq FM_j$, the maximum value of one of the feedbacks.

Abnormal termination will occur if:

1. The time interval, h , is less than HL ; or
2. An error is detected by the input generator.

IV. CONCLUSION

The EROS code provides the necessary dynamic simulation of normal and off-normal EBR-II reactor operating conditions to permit (1) a continuing plant-kinetics and safety surveillance of the dynamic behavior of the reactor system and (2) a continuing evaluation of the EBR-II plant protective system.

Future programmatic emphasis in the EBR-II dynamic-simulation effort will be in three additional areas:

A. Dynamic Simulation of the EBR-II Primary and Secondary Coolant Systems

Dynamic modeling of the EBR-II primary and secondary coolant systems is required to properly compare expected system responses with actual ones. Steady-state and transient conditions will be simulated to allow a more complete description of the EBR-II primary coolant system and primary-tank characteristics. Dynamic modeling of the secondary coolant system will provide a meaningful basis for improving process system control.

B. Transient Behavior of Irradiated Fuels

A considerable challenge is represented in dynamic modeling of transient behavior of irradiated fuels. Present fuel-modeling codes such as BEMOD,^{17,18} for metallic fuels, solve for the steady-state behavior of irradiated fuel. Initially, a quasi-steady-state interface can be made between fuel-modeling codes and EROS. Long-range objectives in this area include a completely integrated dynamic fuel-modeling program.

C. Real-time Dynamic Plant Simulations

During operation, the EBR-II power plant has associated with it a digital computer monitoring system, an XDS Sigma 5 system. Evolution of this system requires dynamic plant simulations of all subsystems. Present efforts on the EROS program will provide a strong technical basis for future real-time EBR-II dynamic plant simulations.

APPENDIX A

Numerical Method

Of fundamental importance in reactor dynamics is the ability to follow the time behavior of a reactor, including the physical phenomena that affect the behavior. Although one can solve the space-independent equations (including those for delayed neutrons) analytically, doing so is generally not possible if feedbacks are included; hence, a numerical method is required. As recently as 1957, the best* numerical method available was the Runge-Kutta method. This method is severely limited in the size of the time step because of the existence of a solution of the form $\exp(-\beta t/\ell)$, where β/ℓ is of the order 10^2 - 10^6 . Thus, thousands of calculations were required to follow reactor transients.

Cohen developed a numerical method that was not so limited.² This method, a generalization of the Runge-Kutta method, accounted explicitly for the exponential nature of the solutions and for the first time provided a fast reliable method for reactor-kinetics calculations. As a result, the first AIREK code was developed. This code, used in one form or another for several years, was reprogrammed in 1962 (AIREK 3)¹ to take advantage of new techniques for programming and graphical display.

AIREK 3 became the standard engineering tool for space-independent reactor kinetics (in addition to analog computation) at Atomics International, throughout the country, and in other parts of the world. The equations solved by AIREK 3 are the usual space-independent equations.

The numerical method used for the solution of Eqs. 24, 25, and 29 is described below.

Simplify Eqs. 22 and 23 by the following substitutions:

$$r(t) = \frac{\rho(t)}{\beta},$$

$$W_i(t) = \frac{\ell \lambda_i C_i(t)}{\beta f_i},$$

and

$$S = \frac{\ell S_0}{\beta}.$$

*Fastest method for a given accuracy.

Obtaining Eqs. 24, 25, and 29:

$$\left. \begin{aligned} \dot{N}(t) &= \frac{\beta}{\ell} \left\{ [r(t) - 1]N(t) + \sum_i f_i W_i(t) + S \right\}; \\ \dot{W}_i(t) &= \lambda_i [N(t) - W_i(t)]; \\ \dot{F}_j(t) &= P_j Q(t) + U_j N(0) + Y_j F_j(t) + \sum_{\ell=1}^{LMAX} [\alpha_{\ell} \delta_{i,I(\ell)}] F_j(\ell), \end{aligned} \right\} \quad (A.1)$$

where

$r(t)$ = some known function of t , $N(t)$, A_j , and $F_j(t)$.

To facilitate numerical solution, Eqs. A.1 are further transformed as follows:

$$\left. \begin{aligned} \dot{N}(t) &= \alpha_N N(t) + R_N; \\ \dot{W}_i(t) &= \alpha_{W_i} W_i(t) + R_{W_i}; \\ \dot{F}_j(t) &= \alpha_{F_j} F_j(t) + R_{F_j}, \end{aligned} \right\} \quad (A.2)$$

where

$$\alpha_N = \frac{\beta}{\ell} [r(t) - 1]; \quad \alpha_{W_i} = -\lambda_i; \quad \alpha_{F_j} = Y_j;$$

$$R_N = \frac{\beta}{\ell} \left\{ [r(t + \xi) - r(t)] N(t + \xi) + \sum f_i W_i(t + \xi) + S \right\};$$

$$R_{W_i} = \lambda_i N(t + \xi); \quad R_{F_j} = P_j Q(t + \xi) + U_j N(0) + \text{sum}(t + \xi);$$

and

$$\text{sum}(t) = \sum_{\ell=1}^{LMAX} [\alpha_{\ell}(t) \delta_{i,I(\ell)}] F_j(\ell)(t),$$

and $0 \leq \xi \leq h$ over the time interval h .

Now define the function $D_k(x)$ as follows:

For $|x| < 1$,

$$D_3(x) = 2 \sum_{n=0}^{10} \frac{x^n}{(n+3)!}. \quad (A.3)$$

Chebyshev telescoping of the power series for $D_3(x)$ reduces the required number of terms for single precursor on the IBM-360/75 from ten to five terms:

$$\begin{aligned} D_3(x) = & (((((5.0578E-5 * X + 4.0549E-4) * X \\ & + 2.777288E-3) * X + 1.6663524E-2) * X \\ & + 8.3333394E-2) * X + 0.33333308; \end{aligned} \quad (A.3.1)$$

$$D_2(x) = \frac{1}{2}[xD_3(x) + 1]; \quad (A.3.2)$$

$$D_1(x) = xD_2(x) + 1. \quad (A.3.3)$$

For $|x| \geq 1$,

$$D_1(x) = \frac{e^x - 1}{x}. \quad (A.3.4)$$

Computation of e^x may be eliminated for large negative values of x . For $x < -17$, e^x is equal to zero relative to 1. Thus,

$$D_1(x) \simeq \frac{1}{x}; \quad x < -17; \quad (A.3.5)$$

$$D_2(x) = \frac{D_1(x) - 1}{x}; \quad (A.3.6)$$

$$D_3(x) = \frac{2D_2(x) - 1}{x}. \quad (A.3.7)$$

Interval sizes are usually increased by doubling. For these occurrences, the doubling formulas

$$D_1(2x) = D_1(x)[1 + xD_1(x)/2], \quad (A.3.8)$$

$$D_2(2x) = [D_1^2(x) + 2D_2(x)]/4, \quad (A.3.9)$$

and

$$D_3(2x) = [D_1(x)D_2(x) + D_2(x) + D_3(x)]/4 \quad (A.3.10)$$

are used for all values of x .

To show the numerical procedure used to go from time t to time $t + h$, the process will be written first for the single equation

$$\dot{N}(t) = \alpha N + R$$

(given initial conditions at time t), and then expanded for Eq. A.2.

For the equation above, we have:

$$\left. \begin{aligned} N_0 &= N(t); \quad R_0 = R(N_0, t); \\ \delta_1 &= \frac{1}{2} h D_1 \left(\alpha \frac{h}{2} \right) (R_0 + \alpha N_0). \end{aligned} \right\} \quad (\text{A.4.0})$$

$$\left. \begin{aligned} N_1 &= N_0 + \delta_1; \quad R_1 = R \left(N_1, t + \frac{h}{2} \right); \\ \delta_2 &= \frac{1}{2} h D_2 \left(\alpha \frac{h}{2} \right) (R_1 - R_0). \end{aligned} \right\} \quad (\text{A.4.1})$$

$$\left. \begin{aligned} N_2 &= N_1 + \delta_2; \quad R_2 = R \left(N_2, t + \frac{h}{2} \right); \\ \delta_3 &= h D_1 (\alpha h) (R_0 + \alpha N_0) + 2h D_2 (\alpha h) (R_2 - R_0). \end{aligned} \right\} \quad (\text{A.4.2})$$

$$\left. \begin{aligned} N_3 &= N_0 + \delta_3; \quad R_3 = R(N_3, t + h); \\ \delta_4 &= h [2D_3(\alpha h) - D_2(\alpha h)] (R_0 - 2R_2 + R_3). \end{aligned} \right\} \quad (\text{A.4.3})$$

$$\left. \begin{aligned} N_4 &= N_3 + \delta_4; \quad R_4 = R(N_4, t + h); \\ \delta_5 &= h [2D_3(\alpha h) - D_2(\alpha h)] (R_4 - R_3). \end{aligned} \right\} \quad (\text{A.4.4})$$

$$N(t + h) = N(t) + \delta_3 + \delta_4 + \delta_5. \quad (\text{A.4.5})$$

Given $N(t)$, $W_i(t)$, and $F_j(t)$, the equations defining the functional values $N(t + h)$, $W_i(t + h)$, and $F_j(t + h)$ are

$$\left. \begin{aligned} N_0 &= N(t); \quad W_{i0} = W_i(t); \quad F_{j0} = F_j(t); \quad Q_0 = Q(t); \\ R_{N0} &= \frac{\beta}{\ell} \left(\sum_i f_i W_{i0} + S \right); \quad R W_{i0} = \lambda_i N_0; \\ R_{F_{j0}} &= P_j Q_0 + U_j N(0) + \text{sum}^0; \\ \delta_{N1} &= \frac{1}{2} h D_1 \left(\alpha_N \frac{h}{2} \right) (R_{N0} + \alpha_N N_0); \\ \delta_{W_{i1}} &= \frac{1}{2} h D_1 \left(\alpha_{W_i} \frac{h}{2} \right) (R W_{i0} + \alpha_{W_i} W_{i0}); \\ \delta_{F_{j1}} &= \frac{1}{2} h D_1 \left(\alpha_{F_j} \frac{h}{2} \right) (R_{F_{j0}} + \alpha_{F_j} F_{j0}). \end{aligned} \right\} \quad (\text{A.5.0})$$

$$\begin{aligned}
N_1 &= N_0 + \delta N_1; \quad W_{i1} = W_{i0} + \delta W_{i1}; \quad F_{j1} = F_{j0} + \delta F_{j1}; \quad Q_1 = Q\left(t + \frac{h}{2}\right); \\
R_{N1} &= \frac{\beta}{\ell} \left\{ \left[r\left(t + \frac{h}{2}\right) - r(t) \right] N_1 + \sum_i f_i W_{i1} + S \right\}; \quad R_{W_{i1}} = \lambda_i N_1; \\
R_{F_{j1}} &= P_j Q_1 + U_j N(0) + \text{sum}^1; \\
\delta N_2 &= \frac{1}{2} h D_2 \left(\alpha_N \frac{h}{2} \right) (R_{N1} - R_{N0}); \\
\delta W_{i2} &= \frac{1}{2} h D_2 \left(\alpha_{W_i} \frac{h}{2} \right) (R_{W_{i1}} - R_{W_{i0}}); \\
\delta F_{j2} &= \frac{1}{2} \left[h D_2 \left(\alpha_{F_j} \frac{h}{2} \right) \right] (R_{F_{j1}} - R_{F_{j0}}).
\end{aligned}
\tag{A.5.1}$$

$$\begin{aligned}
N_2 &= N_1 + \delta N_2; \quad W_{i2} = W_{i1} + \delta W_{i2}; \quad F_{j2} = F_{j1} + \delta F_{j2}; \quad Q_2 = Q\left(t + \frac{h}{2}\right); \\
R_{N2} &= \frac{\beta}{\ell} \left\{ \left[r\left(t + \frac{h}{2}\right) - r(t) \right] N_2 + \sum_i f_i W_{i2} + S \right\}; \quad R_{W_{i2}} = \lambda_i N_2; \\
R_{F_{j2}} &= P_j Q_2 + U_j N(0) + \text{sum}^2; \\
\delta N_3 &= h D_1 (\alpha_N h) (R_{N0} + \alpha_N N_0) + 2h D_2 (\alpha_N h) (R_{N2} - R_{N0}); \\
\delta W_{i3} &= h D_1 (\alpha_{W_i} h) (R_{W_{i0}} + \alpha_{W_i} W_{i0}) + 2h D_2 (\alpha_{W_i} h) (R_{W_{i2}} - R_{W_{i0}}); \\
\delta F_{j3} &= h D_1 (\alpha_{F_j} h) (R_{F_{j0}} + \alpha_{F_j} F_{j0}) + 2h D_2 (\alpha_{F_j} h) (R_{F_{j2}} - R_{F_{j0}}).
\end{aligned}
\tag{A.5.2}$$

$$\begin{aligned}
N_3 &= N_0 + \delta N_3; \quad W_{i3} = W_{i0} + \delta W_{i3}; \quad F_{j3} = F_{j0} + \delta F_{j3}; \quad Q_3 = Q(t + h); \\
R_{N3} &= \frac{\beta}{\ell} \left\{ \left[r(t + h) - r(t) \right] N_3 + \sum_i f_i W_{i3} + S \right\}; \quad R_{W_{i3}} = \lambda_i N_3; \\
R_{F_{j3}} &= P_j Q_3 + U_j N(0) + \text{sum}^3; \\
\delta N_4 &= h [2D_3 (\alpha_N h) - D_2 (\alpha_N h)] [R_{N0} - 2R_{N2} + R_{N3}]; \\
\delta W_{i4} &= h [2D_3 (\alpha_{W_i} h) - D_2 (\alpha_{W_i} h)] [R_{W_{i0}} - 2R_{W_{i2}} + R_{W_{i3}}]; \\
\delta F_{j4} &= h [2D_3 (\alpha_{F_j} h) - D_2 (\alpha_{F_j} h)] [R_{F_{j0}} - 2R_{F_{j2}} + R_{F_{j3}}].
\end{aligned}
\tag{A.5.3}$$

$$\left. \begin{aligned}
 N_4 &= N_3 + \delta N_4; \quad W_{i4} = W_{i3} + \delta W_{i4}; \quad F_{j4} = F_{j3} + \delta F_{j4}; \quad Q_4 = Q(t+h); \\
 R_{N4} &= \frac{\beta}{\lambda} \left\{ [r(t+h) - r(t)] N_4 + \sum_i f_i W_{i4} + S \right\}; \quad R W_{i4} = \lambda_i N_4; \\
 R_{F_{j4}} &= P_j Q_4 + U_j N(0) + \text{sum}^4; \\
 \delta N_5 &= h[2D_3(\alpha_{Nh}) - D_2(\alpha_{Nh})](R_{N4} - R_{N3}); \\
 \delta W_{i5} &= h[2D_3(\alpha_{W_i h}) - D_2(\alpha_{W_i h})](R W_{i4} - R W_{i3}); \\
 \delta F_{j5} &= h[2D_3(\alpha_{F_j h}) - D_2(\alpha_{F_j h})](R_{F_{j4}} - R_{F_{j3}}).
 \end{aligned} \right\} \quad (\text{A.5.4})$$

$$\left. \begin{aligned}
 N(t+h) &= N(t) + \delta N_3 + \delta N_4 + \delta N_5; \\
 W_i(t+h) &= W_i(t) + \delta W_{i3} + \delta W_{i4} + \delta W_{i5}; \\
 F_j(t+h) &= F_j(t) + \delta F_{j3} + \delta F_{j4} + \delta F_{j5}.
 \end{aligned} \right\} \quad (\text{A.5.5})$$

Equation A.5.5 completes the numerical integration across the time step h from t to $t+h$ to yield values of all variables at $t+h$. The process may be repeated for the next time interval, h , with the value of the variables from above as initial conditions.

APPENDIX B

Interval Switching

An important feature in the AIREK and AIROS codes is the automatic interval-switching facility; i.e., the time interval, h , is adjusted to satisfy some prescribed condition related to numerical accuracy. The condition used in AIREK 3 is lacking, in that only experimentation could determine the error bounds AL and AU as a function of the accuracy desired. Thus how to apply this condition to the feedbacks is not clear, and there is no guarantee that the bounds giving desired accuracy for one problem would result in the same accuracy for another. Consequently, an attempt was made to relate the error bounds AL and AU to the truncation error on a given step. The procedure, used previously in AIREK 3A and 3B, can be described as follows.

A measure of the truncation error at time $t + h$ is given by E , where

$$E_N = \left| \frac{[N^*(t + h) - N(t + h)]}{N(t + h)} \right|, \quad (B.1)$$

where

t = time,

h = present interval,

h' = previous interval,

$N(t + h)$ = the power as determined by the normal integration scheme based on $N(t)$,

and $N^*(t + h)$ is based on $N(t - h')$ and is a prediction of the value that would have been obtained had the interval been $h + h'$.

Because of the strong stability inherent in the method, E is the assumed error at time $t + h$. Although not highly refined, this method gives physical meaning to the error bounds AL and AU.

Calculation of $N^*(t + h)$ is, of course, the crux of the error test. Although several formulations were attempted, only the most successful is reported here. The quantities $N(t - h')$, $N(t)$, and $N(t + h)$ are obtained by the numerical procedure described in Eq. A.4, and we will define $N^*(t + h)$ as an estimate of the value of $N(t + h)$ based on $N(t - h')$ using an interval $h' + h$.

The numerical procedure defined by Eq. A.4 can be thought of in the following way: Find a function $f(t + \xi)$ expressed as a second-order polynomial, which represents the deviation of $N(t)$ from exponential behavior over the interval $t, t + h$; thus,

$$\dot{N}(t) = \alpha N(t) + R[N(t), F_j(t), W_i(t), t], \quad (B.2)$$

which can be solved formally by quadrature:

$$N(t+h) = N(t)e^{\alpha h} + \int_0^h R[N(t+\xi), F_j(t+\xi), W_i(t+\xi), t+\xi] e^{\alpha(h-\xi)} d\xi. \quad (B.3)$$

Thus the numerical procedure described by Eq. A.4 determines the coefficients of such a polynomial by an iterative procedure, and as a result,

$$f(t+\xi) \simeq R[N(t+\xi), W_i(t+\xi), t+\xi], \quad (B.4)$$

where

$$f(t+\xi) = R_0 + \frac{\xi}{h}(4R_2 - 3R_0 - R_4) + 2\left(\frac{\xi}{h}\right)^2(R_4 - 2R_2 + R_0). \quad (B.5)$$

Substituting Eq. B.5 into Eq. B.3 gives Eq. A.4.5, which determines $N(t+h)$.

Thus, using Eq. B.5 for $f(t+\xi)$, we can predict $N(t+h)$ based on $N(t-h')$ by extending the integration over two intervals, $h'^{\dagger}$ and h :

$$\begin{aligned} N^*(t+h) = & N(t-h') + (h+h') D_1[\alpha'(h+h')][R_0' + \alpha'N(t-h')] \\ & + \frac{(h+h')^2}{h'} D_2[\alpha'(h+h')][4R_2' - 3R_0' - R_4'] \\ & + 2 \frac{(h+h')^3}{h'^2} D_3[\alpha'(h+h')][R_0' - 2R_2' + R_4']. \end{aligned} \quad (B.6)$$

A similar equation can be derived for $F_j^*(t+h)$, defining the relative error in the value of the feedback by

$$E_{F_j} = \left| \left[\frac{F_j^*(t+h) - F_j(t+h)}{F_j(t+h)} \right] \right|. \quad (B.7)$$

The interval size determined in AIROS II-A is based on the following.

Let

$$Q = \begin{cases} EN; & N(t) \leq N_{\min}, \\ \max\{E_N, E_{F_j}\}_j; & N(t) \geq N_{\min}. \end{cases} \quad (B.8)$$

Then, if $Q \leq AL$, the interval is doubled; if $AL < Q < AU$, the interval is unchanged; and if $AU \leq Q$, the interval is halved, and the calculation is repeated for that interval.

[†]All primed quantities refer to the previous interval.

APPENDIX C

Sample Heat-transfer Model

Consider the simple model for nodal heat transfer consisting of four radial and three axial nodes as shown in Fig. 6.

Fig. 6
Sample Heat-transfer Model

The geometry is as described in Sec. II.A.1. The index I is used in the axial direction, and the index J in the radial direction. The node number, N, is assigned as shown above. A given node has two possible indexes: N, the node number; and (I, J), the coordinate position. Both of these indexing schemes are used in the text.

Let us write the nodal heat-balance equations as used in AIROS II-A (neglecting axial conduction).

1. Noncoolant Nodes

$$(\rho C_p V)_{I,J} \dot{T}_{I,J}(t) = H_{I,J} N(t) + UA_{I,J-1} [T_{I,J-1}(t) - T_{I,J}(t)] + UA_{I,J} [T_{I,J+1}(t) - T_{I,J}(t)]. \quad (C.1)$$

Dividing Eq. C.1 by $(\rho C_p V)_{I,J}$ and rearranging terms gives

$$\dot{T}_{I,J}(t) = \frac{H_{I,J}}{(\rho C_p V)_{I,J}} N(t) + \left[-\frac{UA_{I,J-1}}{(\rho C_p V)_{I,J}} - \frac{UA_{I,J}}{(\rho C_p V)_{I,J}} \right] T_{I,J}(t) + \frac{UA_{I,J-1}}{(\rho C_p V)_{I,J}} T_{I,J-1}(t) + \frac{UA_{I,J}}{(\rho C_p V)_{I,J}} T_{I,J+1}(t). \quad (C.2)$$

2. Coolant Nodes

$$(\rho C_p V)_{I,J} \dot{T}_{I,J}(t) = H_{I,J} N(t) + UA_{I,J-1} [T_{I,J-1}(t) - T_{I,J}(t)] + UA_{I,J} [T_{I,J+1}(t) - T_{I,J}(t)] - (\dot{w} C_p)_{I,J} [\theta_I(t) - \theta_{I-1}(t)]. \quad (C.3)$$

Dividing Eq. C.3 by $(\rho C_p V)_{I,J}$ and rearranging terms gives

$$\begin{aligned} \dot{T}_{I,J}(t) = & \frac{H_{I,J}}{(\rho C_p V)_{I,J}} N(t) + \left[-\frac{UA_{I,J-1}}{(\rho C_p V)_{I,J}} - \frac{UA_{I,J}}{(\rho C_p V)_{I,J}} \right] T_{I,J}(t) \\ & + \frac{UA_{I,J-1}}{(\rho C_p V)_{I,J}} T_{I,J-1} + \frac{UA_{I,J-1}}{(\rho C_p V)_{I,J}} T_{I,J}(t) \\ & - \frac{(\dot{w}C_p)_{I,J}}{(\rho C_p V)_{I,J}} [\theta_I(t) - \theta_{I-1}(t)], \end{aligned} \quad (C.4)$$

where

$$T_{I,J}(t) = \frac{\theta_I(t) + \theta_{I-1}(t)}{2} \text{ for coolant nodes:} \quad (C.5)$$

$$\theta_0 = T_{\text{inlet}};$$

$H_{I,J}$ = the volumetric heat generation rate, Btu/sec;

$UA_{I,J}$ = the overall heat-transfer coefficient across the right boundary of node (I, J);

and

$\rho C_p V$ = the density, specific heat, and volume of node (I, J).

Density, specific heat, and the overall heat-transfer coefficients are taken as functions of the initial steady-state temperature at each node, $T_{I,J}(0)$.

a. Transport-delay Model

Solving Eq. C.5 for $\theta_I(t)$ gives

$$\theta_I(t) = 2T_{I,J}(t) - \theta_{I-1}(t). \quad (C.6)$$

Substituting Eq. C.6 into Eq. C.4 and solving recursively transforms the last term of Eq. C.4 into

$$-2 \frac{(\dot{w}C_p)_{I,J}}{(\rho C_p V)_{I,J}} \left[T_{I,J}(t) + 2 \sum_{k=1}^{I-1} (-1)^k T_{I-k,J}(t) + (-1)^I T_{\text{inlet}} \right], \quad (C.7)$$

which is of the proper form.

b. Outlet-temperature Model

Equation C.5 is replaced by

$$T_{I,J}(t) = \theta_I(t); \quad (C.8)$$

i.e., the outlet temperatures rather than the average temperatures of the coolant nodes are calculated.

Except for the last term (mass transport) in Eq. C.4, there is a one-to-one correspondence between the terms of Eq. 29 and those of Eqs. C.8 and C.2.

Substituting Eq. C.8 into the last term of Eq. C.4 yields

$$- \frac{(\dot{w}C_p)_{I,J}}{(\rho C_p V)_{I,J}} [T_{I,J}(t) - T_{I-1,J}(t)], \quad (C.9)$$

which is again of the form required.

That the generalized feedback Eq. 29 will handle other models must be emphasized; however, Eqs. C.2, C.4, C.5, and C.8 represent the model used in the input generator.

APPENDIX D

Extrapolation Length versus Peak-to-Average Flux

The user is frequently given the peak-to-average flux ratio and would like to know the extrapolation distance.

For a sine distribution, this distance is found as follows:

$$\frac{\varphi_{pk}}{\varphi_{av}} = \frac{1}{\int_{l_{ex}}^{l_{ex}+H} \sin \frac{\pi z}{L} dz} \quad (D.1)$$

$$= \frac{\pi}{1 + 2l_{ex}/H} \left[\cos \frac{\pi l_{ex}/H}{1 + 2l_{ex}/H} - \cos \pi \frac{1 + l_{ex}/H}{1 + 2l_{ex}/H} \right]^{-1}, \quad (D.2)$$

where

H = the fuel length,

l_{ex} = the extrapolation distance,

$L = H + 2l_{ex}$,

$\varphi_{pk}/\varphi_{av}$ = the peak-to-average flux ratio,

and $\varphi_{pk}/\varphi_{av}$ is plotted as a function of l_{ex}/H in Fig. 7.

Fig. 7. Normalized Extrapolation Length as a Function of Peak-to-Average Flux Ratio

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