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**AN ANALYTIC MODEL OF CYLINDRICAL METAL
LINER COMPRESSION OF DT PLASMA**

James W. Shearer

June, 1974

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Prepared for U.S. Atomic Energy Commission under contract no. W-7405-Eng-48

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AN ANALYTIC MODEL OF CYLINDRICAL METAL LINER COMPRESSION OF DT PLASMA

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ABSTRACT

An analytical model is formulated to give an approximate description of metal liner compression of DT-plasma (and magnetic field) in cylindrical geometry. The model includes a correction for liner compression. Calculations are carried out for a copper liner. "Breakeven" conditions are found for a pure fusion system and for a hybrid system with a fission blanket. The results show that a pure fusion system requires gigajoules of energy per explosion, but the hybrid system needs only megajoules of energy per explosion.

I. INTRODUCTION

A well-known^(1,2) fusion energy proposal is the compression of deuterium-tritium plasma to thermonuclear temperatures by means of an imploding metallic liner. Experimental research programs based on this concept are in progress at Krasnaya Pakhra, USSR,⁽³⁾ and at the Naval Research Laboratory.⁽⁴⁾

In addition to computer studies,^(5,6) there has been some analytical work done on the concept.^(7,8,9,10) The purpose of this report is to construct an approximate analytical model of cylindrical liner compression of DT plasma which will permit evaluation of sizes and efficiencies for practical systems. We will concentrate our attention on the metal liner (including its compressional energy), and on the DT plasma (including the magnetic field needed to insulate it from the walls).

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II. PLASMA-FIELD MODEL

The "plasma beta" β is defined:

$$(1) \quad \beta \equiv \frac{P_p}{P_m} = \frac{16\pi n k T}{B_p^2}$$

where P_p = plasma pressure, P_m = magnetic pressure inside the plasma and B_p = corresponding axial magnetic field. T is the temperature of both electrons and ions (assumed equal); n is the density of either electrons or ions ($Z = 1$ in a fully ionized DT plasma).

The "fill factor" f_f is defined:

$$(2) \quad f_f \equiv \frac{A_p}{A_t} = \left(\frac{r_p}{r_t} \right)^2$$

where A_p is the area and r_p is the radius of the plasma. A_t and r_t are the area and radius defined by the inner surface of the liner at turnaround.

Two possible plasma-liner configurations (at turnaround) are sketched in Fig. 1. In the theta pinch case (Fig. 1a), one expects that the fill factor f_f will be low because the B_z magnetic field acts like a zero-density fluid with $\gamma = 2$:

$$(3) \quad P_m \sim B_z^2 \sim \left(\frac{\phi}{r^2} \right)^2 \sim \frac{1}{v^2} \sim \frac{1}{v^\gamma}$$

where ϕ = flux, and v = specific volume. Adiabatic compression of the plasma ($\gamma = 5/3$) leads to the proportionality:

$$(4) \quad P_p \sim \frac{1}{v^\gamma} \sim \left(\frac{1}{v} \right)^{5/3}$$

Since $P_m = P_p$ throughout the liner compression, comparison of Eqs. (3) and (4) shows that the plasma volume will shrink relative to the field volume. Therefore, the fill factor f_f will be low in this case, as stated.

The stabilized Z-pinch case (Fig. 1b) is different because the azimuthal B_θ field varies more slowly with radius than either the axial

field or the plasma. In the final, low radius limit, we have neglected it entirely, and filled the entire cavity with a low β plasma-axial magnetic field mixture.

We now write down the plasma energy per unit length E_p and magnetic field energy per unit length E_m at turnaround:

$$(5) \quad E_p = (3nkT) f_f \pi r_t^2$$

$$(6) \quad E_m = (B_p^2 f_f + B_m^2 (1 - f_f)) \frac{r_t^2}{8}$$

where B_m is the axial magnetic field outside the plasma. The total pressure at turnaround, P_t , is:

$$(7) \quad P_t = (2nkT) \left(1 + \frac{1}{\beta}\right) = B_m^2/8\pi$$

Expressing the energies in terms of P_t :

$$(8) \quad E_p = \left(\frac{3}{2} \frac{\beta}{\beta + 1} f_f\right) (\pi r_t^2) P_t$$

$$(9) \quad E_m = \left(1 - \frac{\beta}{\beta + 1} f_f\right) (\pi r_t^2) P_t$$

Finally, we remark that the pressure of the axial magnetic field requires that either the plasma β or the fill factor f_f be low; therefore, the magnetic energy E_m at turnaround will dominate the plasma energy E_p . The axial magnetic field is important because it suppresses instabilities, thermal conduction, and diffusion losses,⁽¹¹⁾ and it cannot easily be reduced. This is an inherent energy inefficiency of the liner compression system that must be taken into account.

III. TURNAROUND DYNAMICS

We consider the motion of the liner up to the time of turnaround, as shown in Fig. 2. We ignore the subsequent expansion, on the assumption

that Rayleigh-Taylor instabilities on the inner surface of the liner will destroy the symmetry at later times.⁽¹²⁾ Assuming a thin, slow-moving liner:

$$(10) \quad \ddot{r} \approx \frac{P}{\sigma} = \frac{P}{(\rho)(\Delta r)}$$

where r is the inner radius, P is the pressure and σ is the mass per unit area of the liner.

In cylindrical geometry the thin liner σ will vary with the radius r :

$$(11) \quad \sigma = \sigma_t r_t / r$$

where σ_t , r_t are the respective values at turnaround time ($t = 0$, as in Fig. 2).

Adiabatic compression in cylindrical geometry will vary as:

$$(12) \quad P = P_t \left(\frac{v_t}{v} \right)^\gamma = P_t \left(\frac{r_t}{r} \right)^{2\gamma} \approx P_t \left(\frac{r_t}{r} \right)^4$$

where we have chosen $\gamma = 2$ as the best approximation for γ near turnaround. The reason for this choice is the dominance of the axial magnetic field B_z , as described in the last section.

Combining Eqs. (10), (11), and (12):

$$(13) \quad \ddot{r} = \frac{P_t r_t^3}{\sigma_t} \frac{1}{r^3}$$

The first integral of Eq. (13) is obtained from:

$$(14) \quad \int_u^0 u du = \frac{P_t r_t^3}{\sigma_t} \int_r^{r_t} \frac{dr}{r^3}$$

where $u \equiv \dot{r}$, and $\ddot{r} = u(du/dr)$. The result for the velocity u is:

$$(15) \quad u = \frac{dr}{dt} = - \left(\frac{P_t r_t^3}{\sigma_t} \right)^{1/2} \left(1 - \left(\frac{r_t}{r} \right)^2 \right)^{1/2}$$

The integration of Eq. (15) is accomplished with the substitution:

$$(16) \quad x = \left(\frac{r}{r_t} \right)^2 - 1 \quad dx = \frac{2r dr}{r_t^2}$$

$$(17) \quad - \int_t^0 \frac{dt}{\tau_t} = \int_{(r/r_t)^2 - 1}^0 \frac{dx}{2x^{1/2}}$$

where the characteristic time τ_t is defined by:

$$(18) \quad \tau_t \equiv \sqrt{\frac{\sigma_t r_t}{P_t}}$$

The final result is:

$$(19) \quad \left(\frac{r}{r_t} \right)^2 = 1 + \left(\frac{t}{\tau_t} \right)^2$$

Equation (19) is the equation of a hyperbola in the r - t plane. For large values of r and t , the velocity u of the liner approaches (r_t/τ_t) . This derivation is not exact because it ignores compressibility and thickness effects in the liner.^(7,9) However, in a later section we will make an approximate correction for liner compressibility.

IV. DT NUCLEAR REACTIONS AT TURNAROUND

The DT reaction energy yield Y (per unit length) is written, similar to Eqs. (8) and (9):

$$(20) \quad Y = E_{DT} \left(\int_{-\infty}^0 R_{DT} dt \right) \pi r_t^2 f_f$$

where E_{DT} is the useful energy release per reaction, and where R_{DT} is the reaction rate in the plasma per unit volume. Constant density is assumed, as in the configurations of Fig. 1. The reaction rate is written:

$$(21) \quad R_{DT} = n_D n_T \overline{\sigma v}(T)$$

where n_D = deuterium ion density, n_T = tritium ion density, and $\overline{\sigma v}(T)$ is the DT reaction cross section averaged over the ion velocity distribution, which is assumed to be Maxwellian at temperature T (keV).

We will approximate $\overline{\sigma v}(T)$ by a square law fit which is accurate to about 20% in the temperature range $7 \text{ keV} < T < 20 \text{ keV}$:

$$(22) \quad \overline{\sigma v}(T) \approx 10^{-18} T^2$$

where $\overline{\sigma v}$ is in cm^3/sec . Then, in a 50:50 DT mixture, the reaction rate is written:

$$(23) \quad R_{DT} = 2.5 \times 10^{-19} (nT)^2 = \frac{P_p^2}{1.6 \times 10^{19} k^2}$$

where $n = n_D + n_T$ and where k is Boltzmann's constant. But from Eqs. (7) and (12), we can substitute for the plasma pressure P_p , and obtain

$$(24) \quad R_{DT} = \frac{1}{40.96} \left(\frac{\beta}{\beta + 1} P_t \left(\frac{r_t}{r} \right)^4 \right)^2 \equiv R_t \left(\frac{r_t}{r} \right)^8$$

where we have defined the maximum reaction rate R_t , and have substituted Boltzmann's constant $k = 1.6 \times 10^{-9} \text{ esu/keV}$. In this approximation we have neglected the difference in the adiabatic γ between the plasma and the field, but this neglect is consistent with Eq. (12).

Next we integrate the reaction rate over time by substituting Eq. (19) into Eq. (24)

$$(25) \quad R_t \tau_R \equiv \int_{-\infty}^0 R_{DT} dt = R_t \int_0^{\infty} \frac{dt}{(1 + (t/\tau_t)^2)^4}$$

where we have defined the effective reaction time τ_R , and where the limits have been changed owing to the symmetry of the integral. If we make the substitution:

$$(26) \quad u \equiv \left(\frac{t}{\tau_t} \right)^2 \quad du = \frac{2t dt}{\tau_t^2}$$

We obtain the beta function $B\left(\frac{1}{2}, \frac{7}{2}\right)$: ⁽¹³⁾

$$(27) \quad \begin{aligned} \tau_R &= \frac{\tau_t}{2} \int_0^\infty \frac{u^{-1/2} du}{(1+u)^4} = \frac{\tau_t}{2} B\left(\frac{1}{2}, \frac{7}{2}\right) \\ &= \frac{\tau_t}{2} \frac{\Gamma\left(\frac{1}{2}\right) \Gamma\left(\frac{7}{2}\right)}{\Gamma(4)} = .491 \tau_t \end{aligned}$$

For convenience we will approximate:

$$(28) \quad \tau_R \approx \frac{1}{2} \tau_t = \frac{1}{2} \left(\frac{\sigma_t r_t}{P_t} \right)^{1/2}$$

Thus we have found that the effective reaction time constant τ_R is shorter than the dynamic time constant τ_t because of its steeper dependence on the radius r .

Now we substitute Eqs. (24), (25) and (28) back into Eq. (20), and we find:

$$(29) \quad \begin{aligned} Y &= E_{DT} R_t \frac{\tau_t}{2} \pi r_t^2 f_f \\ &\approx \frac{E_{DT}}{82} \left(\frac{\beta}{\beta+1} \right)^2 f_f (\pi r_t^2) P_t^2 \tau_t \end{aligned}$$

This equation for the DT reaction energy yield Y can be directly compared to the input energies E_p and E_n (Eqs. (8)-(9)). For example, consider the "scientific breakeven" condition, that $Y = E_p$; in that case we find, using $E_{DT} = 17.6$ MeV/reaction:

$$(30) \quad P(\text{MB}) \tau_R (\mu\text{sec}) \approx 2.2 \frac{\beta+1}{\beta}$$

where we have also used Eq. (28) for τ_R . In the plasma temperature range where we fit the cross section (Eq. (22)), this corresponds to the Lawson criterion. ⁽¹⁴⁾

$$(31) \quad 4 \times 10^{13} \lesssim n\tau \frac{\beta}{\beta+1} \lesssim 10^{14}$$

Similar comparison can be made with the magnetic energy E_n (Eq.(9)), if desired.

These estimates assume that the percent burn is low compared to unity, and they ignore heating of the plasma by alpha particles and neutrons.

V. COMPRESSIONAL ENERGY OF METAL LINER

In addition to the adiabatic compressional energy imparted to the plasma and to the magnetic field, the liner also does work on itself:

$$(32) \quad W = - \int_{v_0}^v P dv$$

where W is the work done per unit mass, P is the pressure, v is the specific volume (cc/gm), and v_0 is the STP value of the specific volume for the metal of the liner.

Pressure versus volume curves for metals are well-known from both static and dynamic (high-explosive) experiments.⁽¹⁵⁾ We shall not consider them in detail; rather, we shall make an algebraic fit to the data which is easily integrable in Eq. (32). By trial and error, we have found that the following exponential fit to the data gives simple analytical results:

$$(33) \quad \frac{v}{v_0} = \sum_i a_i e^{-P/P_i}$$

where v/v_0 is the specific volume ratio, and where the coefficients a_i , P_i are fitted to the data. For example, the compressibility of copper^(15,16) in the pressure range 0-4.5 MB is closely fitted by the equation:

$$(34) \quad \left(\frac{v}{v_0} \right)_{Cu} = 0.3 e^{-P/.65} + 0.7 e^{-P/15}$$

where P is the pressure in megabars. This fit is illustrated in Fig. 3.

Using this simple fit, we first find the derivative:

$$(35) \quad \frac{dv}{dP} = - \frac{1}{\rho_0} \sum_i \frac{a_i}{P_i} e^{-P/P_i}$$

Then we integrate Eq. (32):

$$(36) \quad W = - \int_{v_0}^v P \frac{dv}{dP} dP = + \frac{1}{\rho_0} \sum_i \frac{a_i}{P_i} \int_0^P P e^{-P/P_i} dP$$

$$W = \frac{1}{\rho_0} \sum_i a_i P_i \left[1 - \left(1 + \frac{P}{P_i} \right) e^{-P/P_i} \right]$$

Thus, we now have an approximate result for the liner compression energy per gram.

In order to obtain an approximate pressure and compressional energy distribution for a thin, slow-moving liner (as previously assumed for Eq. (10)), it is most convenient to use the thickness parameter η in units of mass per unit area:

$$(37) \quad d\eta \equiv \rho dr$$

At the outside radius of the liner, we define $\eta \equiv 0$. Then at the inside radius at turnaround time, we have:

$$(38) \quad \sigma_t = \int_0^{\sigma_t} d\eta = \int_{r_{t+\Delta}}^{r_t} \rho dr$$

where Δ is the liner thickness (See Fig. 2), and where σ_t is the liner mass per unit area introduced in Eqs. (10)-(11). The pressure distribution is then obtained by means of the impulse-momentum theorem:

$$(39) \quad \int P dt = P \tau_t = \eta u_0$$

where u_0 is the velocity at an early time, long before turnaround. Remembering that $P = P_t$ when $\eta = \sigma_t$, we write:

$$(40) \quad P = P_t \frac{\eta}{\sigma}$$

See Fig. 4. This formula for the pressure distribution P neglects the inertia of the plasma, convergence effects in thick liners, and liner heating. However, it should be a good first approximation for the purpose of estimating the compressional energy of the liner.

The liner compression energy E_L per unit length at turnaround time is now found by integration:

$$(41) \quad E_L = \int_0^{\sigma_t} 2\pi r_t W dn$$

Substitution of Eq. (36) leads to:

$$(42) \quad E_L = 2\pi \frac{r_t}{\rho_0} \sum_i a_i P_i \int_0^{\sigma_t} \left(1 - (1 + \alpha_i n) e^{-\alpha_i n} \right) dn$$

where the parameter α_i is defined by:

$$(43) \quad \alpha_i \equiv P_t / \sigma_t P_i$$

After carrying out the integration, one obtains:

$$(44) \quad E_L = 2\pi \frac{r_t \sigma_t}{\rho_0} Y(P_t)$$

where the function $Y(P)$ is defined:

$$(45) \quad Y(P) \equiv \sum_i a_i P_i \left\{ \left(e^{-P/P_i} + 1 \right) + 2 \frac{P_i}{P} \left(e^{-P/P_i} - 1 \right) \right\}$$

This is the expression for the liner energy which we sought.

In order to compare the liner energy E_L (Eq. (44)) with the other energies in the system (Eqs. (8), (9), and (29)), it is convenient to rewrite it in the following way:

$$(46) \quad E_L = (\pi r_t^2) P_t \left(\frac{2\sigma}{\rho_0 r_t P_t} \right) Y(P_t)$$

It is also convenient to express the liner thickness Λ at turnaround as

a multiple of the turnaround radius r_t (See Fig. 2).

$$(47) \quad \Delta \equiv f_L r_t = \int_{r_t}^{r_t + \Delta} dr$$

We wish to know how Δ and f_L vary with the pressure P_t ; to find this we must again integrate through the layer:

$$(48) \quad \frac{dr}{d\eta} = v = \frac{1}{\rho_0} \sum_i a_i e^{-\alpha_i \eta}$$

where we have used Eqs. (33), (37), and (40). Combining Eqs. (47) and (48), and carrying out the integration, one obtains:

$$(49) \quad \Delta = f_L r_t = \frac{\sigma_t}{\rho_0} Z(P_t)$$

where the function $Z(P)$ is defined:

$$(50) \quad Z(P) \equiv \sum_i a_i \frac{P_i}{P_t} \left(1 - e^{-P_t/P_i} \right)$$

Substitution back into Eq. (46) gives us the following form for liner energy:

$$(51) \quad E_L = (\pi r_t^2 P_t) (f_L) (S(P_t))$$

where the function $S(P)$ is defined:

$$(52) \quad S(P) \equiv 2Y(P)/PZ(P)$$

The function $S(P)$ can be calculated for any particular liner metal. In particular for the copper fit (Eq. (34)), one calculates the results shown in Fig. 5.

VI. FORMULATION OF RADIAL SYSTEM SCALING RELATIONS

We are now ready to consider the total system of liner, plasma, magnetic field, and DT reactions. We define the overall energy multiplication:

$$(53) \quad M \equiv \frac{Y}{E_P + E_M + E_L} = \frac{\frac{E_{DT}}{82} \left(\frac{\beta}{\beta + 1} \right)^2 f_f P_t \tau_t}{1 + \frac{1}{2} \frac{\beta}{\beta + 1} f_f + f_L S(P_t)}$$

where we have used Eqs. (8), (9), (29), and (51). This equation can be solved for the impulse product P_t (MB) τ_t (μ sec):

$$(54) \quad P_t \tau_t = 2.91 \left(\frac{\beta + 1}{\beta} \right)^2 \left(\frac{M}{f_f} \right) \left(1 + \frac{1}{2} \frac{\beta}{\beta + 1} f_f + f_L S(P_t) \right)$$

where we regard the turnaround pressure P_t (MB) as the independent variable, and β , f_f , f_L , and M as parameters of the model.

Equation (54) is consistent with Eq. (30), which was solved for the special case $\beta \rightarrow \infty$, $f_f = 1$, $M = 1$, and negligible compressibility. Two other incompressible liner cases of interest are models (a) and (b) of Fig. 1. In model (a), where the plasma is not mixed with the field, we obtain

$$(55) \quad (P_t \tau_t)_a \approx 13M \text{ Mb-}\mu\text{sec}$$

where the liner is incompressible (E_L negligible). With the same assumptions, the mixed plasma and field case (model (b)), gives the result

$$(56) \quad (P_t \tau_t)_b \approx 52M \text{ Mb-}\mu\text{sec}$$

which is a factor of four higher. Note that in these two cases the last factor in Eq. (54) was the same number. These cases show that for pulsed systems it is better to try to keep the plasma from mixing with the field, when one wishes to achieve high values of energy multiplication M . The reason for this is the quadratic density dependence of the nuclear reaction (Eq. (21)).

In order to apply Eq. (54) to specific examples, one uses the solution for $P_t \tau_t$ to solve for the radius r_t :

$$(57) \quad r_t = P_t \tau_t \left(\frac{Z(P_t)}{P_t f_L} \right)^{1/2}$$

which can be derived from Eqs. (18) and (49). One proceeds from there to find the energies (per unit length) E_P (Eq. (8)), E_M (Eq. (9)), and E_L (Eq. (51)). The yield Y can be found either from Eq. (29), or by multiplying M into the sum $E_P + E_M + E_L$.

Another parameter of interest is the liner mass per unit length M_L :

$$(58) \quad M_L = 2\pi r_t \sigma_t = 2\pi \frac{(P_t \tau_t)^2}{P_t}$$

where we have used Eq. (18). Let us assume that the sum of plasma, magnetic, and liner energies was originally entirely kinetic energy of the liner - this assumption ignores plasma preheat and initial magnetic field energy. In that case, we can find the initial liner velocity u_L :

$$(59) \quad u_L = \left\{ \frac{2}{M_L} (E_P + E_M + E_L) \right\}^{1/2}$$

This velocity is important when considering liner acceleration methods.

VII. AN APPROXIMATE AXIAL SYSTEM SCALING RELATION

The necessary L_p of the long hot plasma column depends on the time it takes for the plasma to escape out the ends. This time must be greater than the reaction time τ_R ; thus we have:

$$(60) \quad L_p \geq v_a \tau_R$$

where v_a is the acoustic velocity of the plasma. A number of analyses of this end loss problem have been made in connection with liner theta pinch studies; three of these are cited in the references. (17,18,19)

A number of proposals for shortening this length have been made, such as multiple mirrors, (17) or cusp geometry terminations. (20)

For the purposes of this report, we will adopt the arbitrary criterion that the length L_p can be simply scaled according to the equation:

$$(61) \quad L_p \text{ (meters)} \approx \frac{10^{20}}{n} M \approx \frac{3.2}{P_t \text{ (MB)}} M$$

where we have put $T = 10$ KeV, and where the reaction time τ_R varies linearly with M , and inversely with P_t . In the density range $10^{16} < n < 10^{18}$, this criterion agrees with the estimates of the references^(17,18,19) within a factor of 2. In the lower density range the ~ 1 km length has thus far daunted any serious attempt to build a reactor-size theta pinch. However, at the densities considered here, the length is much shorter.

VIII. "LINER BREAKEVEN" DT FUSION CALCULATIONS FOR COPPER LINER

We define "liner breakeven" as the condition that $M = 1$ (see Eq. (53)); this is a more stringent condition than "scientific breakeven" (Eq. (30)). We shall choose model (a) of Fig. 1 as our plasma configuration, and we shall use a copper liner, as modeled by Eq. (34). Under these conditions the impulse product (Eq. (54)) reduces to:

$$(62) \quad P_t \text{ (MB)} \tau_t \text{ (}\mu\text{sec)} = 13.1 \left(1 + \frac{8}{9} f_L S(P_t) \right)$$

where $S(P_t)$ is given in Eq. (52) and Fig. 5.

In Figs. 6-8 we present various parameters for the breakeven system, using Eqs. (57)-(62). One finds that at low pressures the inner liner radius r_t is large, implying a large bulky system. On the other hand, at high pressures the initial velocity u_L is large; in fact, it is greater than the velocities one would expect even from a liner driven by TNT.⁽¹⁶⁾

In Figs. 7 and 8 we find that the initial liner kinetic energy is very large over the entire pressure range. Using a rough figure of 5 megajoules per kilogram for the energy release from TNT, we see that the total energy is equivalent to ~ 400 kg of TNT at $P = 1$ MB, and ~ 25 kg of TNT at $P = 20$ MB. Thus, the low pressure systems require very large

containment vessels. The high pressure systems, as remarked before, require very high velocity u_L (Fig. 6).

It is possible to conceive of systems at lower pressure ($P < 1$ MB), that use thicker liners ($f_L > 2$). However, this analytical model is based on the assumption of comparatively thin liners, and thick liners might better be analyzed by the method of Robson.⁽⁷⁾ In addition, such lower pressure systems would have to be many meters long in order to prevent prohibitive end losses (see Eq. (61)).

It would be possible to obtain almost nine times lower total energy results for a plasma model in which the fill factor f_f is raised from $1/4$ to $3/4$ (see Eq. (54)). However, it would be much harder to obtain such a fill factor in practice, particularly at the end of the liner compression. Thus, it is felt that the chosen example is more realistic.

The effect of the liner compressional energy is most prominent at high pressure. In fact, this is the reason that the thicker liner curve ($f_L = 2$) crosses over the thinner liner curve ($f_L = 1$) near 17 MB in Figs. 7-8.

Finally, it should be emphasized that even these large, high energy systems will not be large enough for a working fusion energy extraction system. Inefficiencies in the liner propulsion system,⁽¹⁰⁾ and the necessity to produce a surplus of useful power dictate the requirement that the overall liner multiplication M should be at least 5-10. But inspection of Eqs. (8), (9), (29), (51), (54), (57), and (61) shows that: r_t and L_p scale as M ; E_p , E_M , and E_L scale as M^2 ; and Y scales as M^3 . The total energy input scales as M^3 , and the total yield scales as M^4 .

These total energy scaling factors are apparently true for any three-dimensional inertial system - they have already been shown to be true for

spherical deuterium pellets.⁽²¹⁾ Future attempts to create a more practical system for fusion power by the liner method will have to find some way to overcome this difficulty that low pressure systems require enormous sizes and energies.

IX. FUSION LINER COMBINED WITH FISSIONABLE BLANKET

In computing the energy yield of a pure fusion D-T liner contained explosion, we used the energy value $E_{DT} = 17.6$ MeV/reaction (see Eqs. (29)-(30)). It was assumed that most of this energy reached a neutron-absorbing blanket outside of the containment vessel (see Fig. 9). Although the alpha particles would not reach the blanket, there is additional compensatory energy released in the blanket by the $n\gamma$ capture reaction. Let us now consider the energy situation where the blanket contains a considerable amount of fissionable material.

We assume that the fissionable material is nowhere near the amount needed for a critical assembly, but that there is sufficient neutron multiplication (by n_2n and nf reactions) to have, on the average, one fission reaction in the blanket for every 14 MeV neutron produced by the liner-compressed DT plasma. However, remember that the energy release per fission is more than ten times the energy release per DT reaction. Therefore, for this example, overall system energy breakeven is reached when the fusion multiplication parameter $M \approx 0.1$, or one tenth its value for the previous example.

But from the remarks on scaling just given in the previous section, we see that changing M from 1.0 to 0.1 will have profound effects on the system size. To show this we have replotted the parameters of the previous example for the case $M = 0.1$; see Fig. 10. We find that the total energy per pulse in this system is less than 1 kilogram of TNT equivalent - an energy which is much more accessible to common engineering containment practice.

Hybrid fusion-fission systems have previously been discussed,⁽²²⁾ but not in combination with an inertially contained DT fusion source.

From the point of view of liner applications, these results suggest that such a hybrid system is a much more achievable goal than a pure fusion reactor.

X. COMPARISONS WITH OTHER WORK

This analysis is in qualitative agreement with some recent computer calculations, which have been done on the LASNEX code.⁽⁶⁾ The comparison is not exact because most of the computer calculations used thicker liners and different fill factors. For these conditions the computer runs gave a DT fusion breakeven level for thick copper liners of 90 megajoules/meter. The analytical results for thinner liners at breakeven were from 500-800 megajoules/meter (see Fig. 7). Some computer runs using thinner liners at 90 megajoules/meter did not reach breakeven, in agreement with the results of the analysis.

More exact comparisons could be made by means of Eq. (53) - choosing β , f_f , f_L , P_t , and τ_t from a computer run and comparing the calculated value of M (or Y) with the computed one. This has not yet been done.

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Figure 1

Two extreme models of possible plasma-liner configurations: (a) A theta pinch with no magnetic field diffused into the plasma, (b) A stabilized Z-pinch with axial magnetic field and plasma mixed. In terms of Eqs. (1) and (2) we have:

<u>Model</u>	<u>β</u>	<u>f_f</u>
(a)	∞	$1/4$
(b)	$1/3$	1
Field Only	0	1

Figure 2

Radius r versus time t plot of the "turn-around" geometry of liner and plasma.

Figure 4

Linear pressure distribution as a function
the parameter η , where $\eta = \int_{r_t + \delta}^r \rho dr$

Figure 5

Compression functions for copper (Eqs. 34, 45, 50, and 52). Dotted lines are an extrapolation beyond the pressures tabulated in Ref. 15.

Figure 6

"Liner breakeven" DT fusion conditions - final liner radius and initial liner velocity vs pressure for copper liner - ($M = 1$, $\beta = \infty$, $f_f = 1/4$)

Figure 7

"Liner breakeven" DT fusion conditions - initial kinetic energy and mass of copper liner vs pressure
($M = 1$, $\beta = \infty$, $f_f = 1/4$)

Figure 8

"Liner Breakeven" DT fusion conditions - length and total energy vs pressure for copper liner ($M = 1$, $\beta = \infty$, $f_f = 1/4$)

Figure 9

Overall Liner Reactor Schematic

lithium, which acts as both a heat exchanger and a tritium regenerator - for a hybrid system this would be a mixture of lithium and fissionable material.

Figure 10

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