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Richard E. Siemon

Los Alamos
scientific laboratory
of the University of California
LOS ALAMOS, NEW MEXICO 87544

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A TRANSFORMER FOR INJECTION EXPERIMENTS

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ABSTRACT

This report describes an air-core transformer designed to provide a slow rising field in a single-turn coil. The primary, driven by available 10-kV capacitors, carries a relatively small current (~ 200 kA), while the secondary carries a large current (over 4 MA). Although the efficiency is modest (about 20%), the transformer gives the proper rise time and the small primary current requires few switches and cables in the capacitor bank.

I. INTRODUCTION

For injection experiments, one desires a magnetic field, varying slowly in time, with the same magnitude as the rapidly pulsed fields in adjacent coil sections.^{1,2} The conventional theta-pinch single-turn coil is convenient to fabricate, and helical fields can be created when its inner surface is shaped. However, the low inductance of a single-turn coil is not convenient for use with the low voltage Zeus capacitors available to power the injected region. Therefore, a transformer has been designed to permit a better impedance match.

II. TRANSFORMER DESIGN

A. Geometry Parameters. Figure 1 shows the various geometrical parameters needed in the design. The coil is shown in Figure 2.

Figure 1. Transformer with single-turn secondary.

$$D_3 = D_1 + 2\Delta + 2T.$$

Figure 2. Primary coil with n turns. $W = L/(n + 1)$.

B. Maxwell's Equations.

As an approximation, we assume the B fields are everywhere straight and uniform with $B_2 = B_1$ given by the current in the coil. B_2 is the same outside the coil and inside D_2 .

$$\text{From } \nabla \times E = - \frac{\partial B}{\partial t}$$

$$\text{or } V = \oint$$

$$\text{we conclude } I_2 R_2 + \dot{B}_1 A_1 + \dot{B}_2 A_2 = 0 \quad (1)$$

$$V_{\text{coil}} = I_1 R_1 + n \dot{B}_1 A_1 \quad (2)$$

where I_2 = current in secondary (collector plate)

R_2 = resistance in secondary

$$A_1 = \pi D_1^2 / 4$$

$$A_2 = \pi D_3^2 / 4 + \pi D_2^2 / 4 + 4t - \pi (D_1 + 2T)^2 / 4$$

A_2 = area outside coil but inside secondary.

From $\nabla \times B = \mu_0 j$

$$B_1 - B_2 = \frac{\mu_0 I_1 n}{L} \quad (3)$$

$$B_2 = \frac{\mu_0 I_2}{L} \quad (4)$$

These can be rearranged as follows:

$$\dot{B}_2 = \frac{\mu_0 \dot{I}_2}{L}$$

$$\dot{B}_1 = \frac{\mu_0}{L} (n \dot{I}_1 + \dot{I}_2)$$

$$I_2 R_2 + \frac{\mu_0 n}{L} \dot{I}_1 A_1 + \frac{\mu_0 \dot{I}_2}{L} A_1 + \frac{\mu_0 A_2}{L} \dot{I}_2 = 0$$

$$V_{\text{coil}} = I_1 R_1 + \dot{I}_1 \left(\frac{\mu_0 n^2 A_1}{L} \right) + \dot{I}_2 \left(\frac{n A_1 \mu_0}{L} \right) \quad (5)$$

$$I_2 R_2 + \dot{I}_1 \left(\frac{\mu_0 n A_1}{L} \right) + \dot{I}_2 \frac{\mu_0}{L} (A_1 + A_2) = 0 \quad (6)$$

In the low resistance limit, $I_2 R_2$ can be considered small compared to the other terms in equation (6).

In that case

$$\frac{I_1}{I_2} = - \frac{A_1 + A_2}{n A_1}$$

We have an effective turns ratio m

$$m = \frac{n A_1}{A_1 + A_2}$$

$$I_2 = -m I_1$$

From equation (6)

$$\dot{I}_2 \frac{\mu_0 n A_1}{L} = - \dot{I}_1 \frac{\mu_0 n m A_1}{L} + m^2 R_2 I_1$$

From equation (5)

$$V_{\text{coil}} = \frac{\mu_0 n^2 A_1}{L} \left(\frac{A_2}{A_1 + A_2} \right) \dot{I}_1 + I_1 (R_1 + m^2 R_2) \quad (7)$$

C. Circuit Parameters.

In this approximation, we see that the coil has an effective inductance and resistance:

$$L_{\text{eff}} = \frac{\mu_0 n^2 A_1}{L} \frac{A_2}{A_1 + A_2} = \frac{\mu_0 n m A_2}{L} \quad (8)$$

$$R_{\text{eff}} = R_1 + m^2 R_2$$

We now imagine connecting the effective coil to a capacitor bank with initial charge V_0 , capacitance C , inductance L_b , resistance R_b , and ask for the efficiency of transfer from $1/2 C V_0^2$ to magnetic energy in the bore D_2

$$\epsilon \equiv \frac{1/2 L_c I_2^2 (\text{max})}{1/2 C V_0^2}$$

$$\text{where } L_c = \frac{\mu_0 \pi D_2^2}{4 L}$$

This simple R-L-C circuit has a current given by

$$I_1 = \frac{V_0}{\omega_n L} e^{-\alpha t} \sin \omega_n t \quad (9)$$

$$\text{where } \omega_n = \sqrt{\omega_o^2 - \alpha^2} \quad (10)$$

$$\omega_o^2 = 1/LC \quad (11)$$

$$\alpha = R/2L \quad (12)$$

$$R = R_{\text{eff}} + R_b \quad (13)$$

$$L = L_{\text{eff}} + L_b \quad (14)$$

The current maximum comes at t_m where

$$t_m = \frac{1}{\omega_n} \tan^{-1} (\omega_n / \alpha)$$

When $\alpha^2 > \omega_o^2$ we rewrite these as

$$I_1 = \frac{V_o}{\omega_n L} e^{-\alpha t} \sinh(\omega_n t)$$

$$t_m = \frac{1}{2\omega_n} \ln \left(\frac{\alpha + \omega_n}{\alpha - \omega_n} \right)$$

To evaluate these formulas, we need expressions for R_1 and R_2 which we obtain by assuming all the current flows in $\delta/2$ where δ is the classical skin depth and the frequency is ω_o .

$$\delta = \sqrt{2 \epsilon_o c^2 / \sigma \omega_o} = \sqrt{\frac{2}{\mu \sigma \omega_o}} \quad (15)$$

$$\epsilon_o c^2 = \frac{10^7}{4\pi} = \frac{1}{\mu_o}$$

$$\sigma_{\text{alum}} = 3.5 \times 10^7 \text{ mho/meter}$$

$$\sigma_{\text{copper}} = 5.7 \times 10^7$$

We will take R_1 to be the bank resistance plus the resistance of the coil plus one $m\Omega$ to allow for lead resistance, and we will assume that a copper strap is used to fabricate a coil filling the available space. Then

$$R_{\text{coil}} = \frac{(\text{length})}{\sigma(\text{Area})}$$

$$\text{Area} = (\delta/2) W = (\delta/2) \left(\frac{L}{n+1} \right)$$

$$\text{Length} = n\pi(D_1 + T)$$

$$R_1 = R_{\text{coil}} + R_b + .001$$

$$R_1 = \frac{2n(n+1)\pi(D_1 + T)}{\sigma_{\text{Cu}} \delta_{\text{Cu}} L} + R_b + .001 \quad (16)$$

For R_2 we have

$$R_2 = \frac{2L + \pi D_3 + \pi D_2}{\sigma_{\text{Al}} L (\delta_{\text{Al}}/2)} \quad (17)$$

D. Numerical Results.

With equations (1) through (17), we take certain parameters as given and calculate the remaining parameters as follows:

Given	FORTRAN Symbol	Calculate	FORTRAN Symbol
D_1 (m)	D1	c	EPS
D_2 (m)	D2	$I_1(t_m)$ (kA)	XI1
Δ (mm)	DD	$I_2(t_m)$ (kA)	XI2
		R_1 (kG)	B1
L (m)	XL1	R_2 (kG)	B2
L (m)	XL2	R_1 (m Ω)	R1
n	XN	$n^2 R_2$ (m Ω)	XR2
T (mm)	TH1	t_m (μ s)	TM
t (mm)	TH2		
C (μ F)	C	α	ALP
V_o (kV)	V	δ_{Al} (mm)	DEL1
R_b (m Ω)	RB	L_{eff} (μ h)	EFFL
L_b (nh)	XLB	δ_{Al} (ω_n) (mm)	XY

A FORTRAN program was written to carry out this calculation. For a given n, D_1 was varied to determine the maximum efficiency. The capacitance and V_o , available in two racks of Zeus capacitors (192 capacitors), were chosen for the bank. We assume that for the Zeus capacitors, a "shelf" of 48 capacitors has the following characteristics:

$$C = 48 \times 62 = 2976 \mu\text{F}$$

$$V_{\text{max}} = 10.0 \text{ kV}$$

$$L_b = 26 \text{ nH}$$

$$R_b = 0.9 \text{ m } \Omega$$

We estimate a bank of 192 capacitors to have

$$C = 11,904 \mu\text{F}$$

$$L_b = 6.5 \text{ nH}$$

$$R_b = .225 \text{ m } \Omega$$

Results for a typical case are given in Figures 3-5. Input parameters were the following (see Appendix):

Input

D_2 .11 m (large bore) or .0635 m (small bore)
 Δ 4 mm
 l .3 m
 L 1.0 m
 T 6.0 mm
 t 3.0 mm
 C 11,904 μ f
 V_o 9.7 kV
 R_b .225 m Ω
 L_b 6.5 nH

Figure 3. Coil diameter for maximum efficiency as a function of n assuming $D_2 = .11$ m.

Figure 4. Primary current required to produce 50 kG (on left) and efficiency (on right) as functions of n for $D_2 = .11$ m (large bore) and $D_2 = .0635$ m (small bore).

Figure 5. The time of field maximum (on left) and the skin depth in aluminum (on right) as a function of n and large or small bore.

The coil diameter shown in Figure 3 was calculated to give maximum efficiency as a function on n in the large bore case. The same coil diameter gives a lower efficiency in the small bore but it is adequate for the smaller energy needed.

Figure 4 gives the primary current needed to obtain 50 kG, and the transfer efficiency as defined above. Figure 5 gives the time to field maximum, t_m , and the skin depth in aluminum.

The rather low efficiencies (20% or so) are due to: 1) significant resistance which absorbs about 60% of the initial energy, and 2) area effects (useless flux) which waste about 20% of the initial energy.

The main advantages to the use of a transformer, rather than direct coupling from the capacitor bank coil are the following: 1) the rise time becomes long, 2) the current from the capacitors (primary current) is small, 3) not many switches are needed since the current is small, and 4) not many cables are needed since the current is small.

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APPENDIX
MODIFIED DESIGN

Figure 6. Modified design needed to fit transformer and connections between adjacent theta-pinch collector plates.

For the final design, a shape like that shown in Figure 6 was chosen. In that case, A_2 must be calculated as the sum of areas which are weighted according to the local B field. B_2 is the field outside the winding and $B_3 = f B_2$ is the field inside bore D_2 .

The following formulas must be modified:

$$A_2 = \pi D_3^2 / 4 - \pi (D_1 + 2T)^2 / 4 + L_1 + L_2 t \left(\frac{1+f}{2} \right) + f L_3 t + f \pi D_2^2 / 4$$

$$R_2 = \left(2 / \sigma_{A1} L \delta_{A1} \right) \left[2L_1 + \pi D_3 + \frac{4L_2 f}{f+1} + 2 f L_3 + f \pi D_2 \right]$$

$$L_c = \frac{f \mu_0 \pi (D_2)^2}{4 L}$$

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