PERTURBATION THEORY AND APPLICATIONS

I THEORETICAL

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SUMMARY

The bases for the two group perturbation theory for reflected reactors are presented. Working equations for calculating the effects of arbitrary perturbations are derived.

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I. INTRODUCTION

Two group perturbation methods have been used extensively in Phillips' Atomic Energy Division to calculate reactivity changes in the Materials Testing Reactor and in other reactors now operated by Phillips. High speed computing devices have rendered this method obsolete for some purposes, but it is still a useful tool where computing machines are not available. Also, there are some problems, such as those arising from unsymmetrically distributed perturbations, which can be handled readily by perturbation methods but would tax the capacities of any but the larger computing machines, unless approximation methods were used with them too.

A general outline of the perturbation method is available in the unclassified Reactor Handbook ⁽¹⁾. The present work is based on formulations of Nordheim ⁽²⁾ and Wigner ⁽³⁾ which are available only in early classified reports. The two group theory is developed and the equations which have been used in most applications at the MTR are derived. A report describing specific applications is planned for the future. The theoretical development reported here is essentially a revision of work reported in 1952 ⁽⁶⁾.

II. THEORY

A. The Two Group Flux Equations

The differential equations satisfied by the neutron fluxes and the sources of delayed neutrons are:⁽²⁾

$$\nabla \cdot D_f \nabla \phi_f - \frac{D_f \phi_f}{\mathcal{T}} + \frac{k_{\infty}(1-\beta)}{p} \Sigma_a \phi_s + \sum_I \lambda_I n_I = \frac{1}{v_f} \frac{\partial \phi_f}{\partial t} \quad (1a)$$

$$\nabla \cdot D_s \nabla \phi_s - \Sigma_a \phi_s + \frac{p D_f \phi_f}{\mathcal{T}} = \frac{1}{v_s} \frac{\partial \phi_s}{\partial t} \quad (1b)$$

$$\frac{\partial n_I}{\partial t} = \beta_I \frac{k_{\infty} \Sigma_a \phi_s}{p} - \lambda_I n_I \quad (1c)$$

where

D_f, D_s = fast and slow diffusion constants, respectively (cm).
These are one-third the transport mean free paths.

$\mathcal{T}$ = age of fission neutrons at thermal energy (cm²)

Σ_a = total macroscopic absorption cross section (cm²/cm³)

k_{∞} = local multiplication constant i.e. the number of neutrons per thermal capture which would return to thermal energy in an infinite medium of the local composition.

p = resonance escape probability (always taken to be 1 in MTR applications)

$\phi_f(\underline{r}, t), \phi_s(\underline{r}, t)$ = flux of above thermal neutrons and thermal neutrons, respectively, at position $\underline{r}$ and time t . (neutrons/cm²,sec)

$n_I(\underline{r}, t)$ = concentration of fission products at $(\underline{r}, t)$ which decay with a mean life $1/\lambda_I$ and produce one neutron in the process.

λ_I = decay constant for delayed neutron emitters n_I

β_I = fraction of fissions which produce delayed neutron emitters with mean life $1/\lambda_I$

$\beta = \sum_I \beta_I$ = fraction of neutrons which are produced by delayed emission from any given number of fissions

v_f, v_s = average speed of above thermal neutrons and thermal neutrons, respectively (cm/sec)

The terms in Equation 1a for the fast neutron density have the following significances:

$\nabla \cdot D_f \nabla \phi_f$ is the leakage

$-\frac{D_f \phi_f}{\tau}$ is the loss by degradation into the thermal group and capture during slowing down

$\frac{k_{\infty}(1-\beta)\Sigma_a \phi_s}{p}$ is the prompt neutron production. The p appears in the denominator because k_{∞} is defined as the number of neutrons which would reach thermal energy in an infinite medium of the local composition, and this number is actually p times the number produced.

$\sum_I \lambda_{I n_I}$ = neutrons produced by delayed emission.

The terms in Equation 1b are as follows:

$\nabla \cdot D_s \nabla \phi_s$ is the leakage

$-\Sigma_a \phi_s$ is the loss by thermal capture

$\frac{p D_f \phi_f}{\tau}$ is the production by degradation of fast neutrons which escape capture during slowing down.

The terms in Equation 1c are as follows:

$\beta_I \frac{k_{\infty} \Sigma_a \phi_s}{p}$ is the production of delayed emitters of type I

$-\lambda_{I n_I}$ is the loss of delayed emitters by decay (with the release of a neutron for each decay).

It can be shown that solutions to equations 1 exist in which the spatial and time dependences of ϕ_f , ϕ_s and n_I are separated (4). Such solutions can have the forms:

$$\phi_f(\underline{r}, t) = S_f(\underline{r}) e^{\lambda t} \quad (2a)$$

$$\phi_s(\underline{r}, t) = S_s(\underline{r}) e^{\lambda t} \quad (2b)$$

$$n_I(\underline{r}, t) = C_I(\underline{r}) e^{\lambda t} \quad (2c)$$

This separation is possible provided the chosen solutions have the properties:

$$\frac{S_f(\underline{r})}{S_s(\underline{r})} = \text{const} \quad (3a)$$

$$\frac{S_s(\underline{r})}{C_I(\underline{r})} = \text{const} \quad (3b)$$

If separable solutions of the forms in equations 2 (which must also satisfy the conditions in equations 3) are assumed (2) equations 1a and 1b take the forms:

$$\nabla \cdot D_f \nabla S_f - \frac{D_f S_f}{\mathcal{J}} + \frac{k_{\infty} \Sigma_a S_s}{p} = \frac{\lambda S_f}{v_f} + \frac{\lambda k_{\infty} \Sigma_a S_s}{p} \left[\frac{\Sigma}{I} \frac{\beta_I}{\lambda + \lambda_I} \right] \quad (4a)$$

$$\nabla \cdot D_s \nabla S_s - \Sigma_a S_s + \frac{p D_f S_f}{\mathcal{J}} = \frac{\lambda S_s}{v_s} \quad (4b)$$

Equations 4a and 4b must be solved subject to the usual boundary conditions of continuity of fluxes and currents at all interfaces and vanishing of fluxes at the extrapolated outer boundaries.

In the following it will be convenient to employ a matrix operator to express equations 4a and 4b by the single equation.

$$MS = \lambda \mathcal{A}(\lambda) S \quad (5)$$

where

$$M = \begin{pmatrix} \nabla \cdot D_f \nabla - \frac{D_f}{\mathcal{J}} & \frac{k_{\infty} \Sigma_a}{p} \\ \frac{p D_f}{\mathcal{J}} & \nabla \cdot D_s \nabla - \Sigma_a \end{pmatrix} \quad (6a)$$

$$\mathcal{A}(\lambda) = \begin{pmatrix} \frac{1}{v_f} & \frac{k_{\infty} \Sigma_a}{p} \left[\frac{\Sigma}{I} \frac{\beta_I}{\lambda + \lambda_I} \right] \\ 0 & \frac{1}{v_s} \end{pmatrix} \quad (6b)$$

$$S = \begin{pmatrix} S_f \\ S_s \end{pmatrix} \quad (6c)$$

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B. The Adjoint Equations

The first order perturbation theory is concerned only with solutions to equation 5 which are positive everywhere in the reactor, vanish at the extrapolated outer boundaries, and satisfy the usual interfacial conditions of continuity of fluxes and currents. Such solutions have the following properties.

1. They have characteristic values λ . This is true for systems with non-uniform compositions and irregular shapes as well as for systems which are symmetrical and have uniform compositions in each region.
2. For reactors with calculable geometries solutions exist for but one size (when $\lambda = 0$ this is the critical size). Calculable geometries include spheres reflected in but one direction with concentric interfaces, cylinders reflected in but one direction with coaxial interfaces, and slabs reflected in but one direction with interfaces normal to this direction.

The first property is proved by considering solutions to the "adjoint" equations which are formed from equation 5 by transposing M and $\mathcal{A}$. This equation is

$$M^t S^+ = \lambda^+ \mathcal{A}^t(\lambda^+) S^+ \quad (7)$$

where M^t is the transpose of equation 6a and $\mathcal{A}^t$ is the transpose of equation 6b with λ^+ in place of λ . The adjoint solutions are denoted by:

$$S^+ = \begin{pmatrix} S_f^+ \\ S_s^+ \end{pmatrix} \quad (8)$$

If solutions S^+ are obtained which satisfy the same boundary conditions as the solution S (positive everywhere inside the reactor, vanishing at the extrapolated outer boundaries, and with S_f^+ , S_s^+ , $D_f \nabla S_f^+$ and $D_s \nabla S_s^+$ continuous at all interfaces) the following identity is valid

$$\int_V (S^+)' M S dV - \int_V S' M^+ S^+ dV = 0 = [\lambda - \lambda^+] \left[\int_V \frac{S_f^+ S_f dV}{v_f} + \int_V \frac{S_s^+ S_s dV}{v_s} + \left\{ \frac{\sum \beta_I \lambda_I}{(\lambda + \lambda_I)(\lambda^+ + \lambda_I)} \right\} \int_V \frac{k_a \sum S_f^+ S_s dV}{p} \right] \quad (9)$$

the subscripts V denote integration over the entire reactor volume.

If the β_I were equal to zero it would follow immediately that $\lambda = \lambda^+$. Furthermore, no other solution S with a different λ could then exist because the integrals on the right side must be positive always when solutions S^+ and S which are everywhere positive are considered.

When the β_I are not zero the right side could be made zero by a suitable choice of λ and λ^+ having opposite signs. However, it is reasonable on physical grounds that if the β_I were small the solutions S^+ and S would be very close to those with $\beta_I = 0$. This implies that $\lambda = \lambda^+$ is true also in the general case when the β_I are not zero.

The proof of property 2 is quite simple. Suppose that two solutions S_1 and S_2 having identical values of λ existed for different sizes of the reactor. Consider first the case where the size of the second reactor is larger than for the first.

Let S_1^+ denote the adjoint solution for the first reactor.

If the integrations in equation 9 are over the boundaries of the first (smaller) reactor the left side becomes

$$\int_A S_{f2} D_f \nabla S_{f1}^+ \cdot dA + \int_A S_{s2} D_s \nabla S_{s1}^+ \cdot dA \neq 0 \quad (10)$$

where the subscript A denotes integration over the outer surface of the first reactor. The fact that S_{f1}^+ , S_{s1}^+ , S_{f2}^+ and S_{s2}^+ are positive inside the reactor means that both integrals in equation 10 have the same sign. The inequality in equation 10 is not allowed because equation 9 shows that with $\lambda = \lambda^+$ this surface integral must be zero. Hence it is proved that the second reactor cannot have a larger size than the first one if λ does not change.

When the second reactor is assumed to be smaller than the first the same procedure can be followed by integrating over the boundary of the second reactor and noting that S_{f1}^+ and S_{s1}^+ are finite along this boundary. The same reasoning used to prove that the second reactor cannot be larger than the first now shows that it cannot be smaller. There must, therefore, be but one particular size for each λ . For $\lambda = 0$ this is the "critical" size.

C. The Perturbation Equations

Let M_0 and M_0' denote operators for an unperturbed system and S_0 and S_0^+ the corresponding unperturbed solutions which are positive everywhere inside the reactor. If this system is perturbed in any way new operators M and M' will be required. It is convenient to express M by the relation

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$$M = M_0 + P \quad (11)$$

where

$$P = \begin{pmatrix} \nabla \cdot (\delta D_f \nabla) - \delta \left(\frac{D_f}{\mathcal{J}} \right) & \frac{\delta(k_\infty \Sigma_a)}{p} \\ \delta \left(\frac{p D_f}{\mathcal{J}} \right) & \nabla \cdot (\delta D_s \nabla) - \delta \Sigma_a \end{pmatrix} \quad (12)$$

the operator P is obtained by assuming D_s , D_f , $\mathcal{J}$, Σ_u , and Σ_a can change in arbitrary ways. Any change δD_f , $\delta \Sigma_a$ etc. at a position $\underline{r}$ is defined as the difference between the perturbed and unperturbed values there. For example, if $D_{f1}(\underline{r})$ and $D_{f0}(\underline{r})$ are the values of D_f at $\underline{r}$ after and before the perturbation, respectively, then $\delta D_f(\underline{r})$ is simply

$$\delta D_f(\underline{r}) = D_{f1}(\underline{r}) - D_{f0}(\underline{r})$$

For generality equation 12 includes the resonance escape probability p . However, in all applications at the MTR $p = 1$ and $\delta p = 0$ have been assumed. For many purposes the term $\frac{\delta(k_\infty \Sigma_a)}{p}$ is conveniently replaced by its equivalent $2.1 \delta \Sigma_u$.

Using the operator M equation 5 becomes

$$MS = (M_0 + P)S = \lambda S \quad (13)$$

where S is the perturbed flux function and λ the characteristic value for the perturbed problem. Noting that $\lambda_0 = \lambda_0^+$ for the unperturbed problem the basic perturbation theory formulas follow from the relation

$$\int_V (S_0^+)^\dagger M S dV - \int_V S_0^+ M_0^\dagger S_0^+ dV = (\lambda - \lambda_0) \left\{ \int_V \frac{S_{f0}^\dagger S_f dV}{v_f} + \int_V \frac{S_{s0}^\dagger S_s dV}{v_s} + \int_V \frac{k_\infty \Sigma_a S_{f0}^\dagger S_s dV}{p} \left[\frac{\Sigma_I \beta_I \lambda_I}{I (\lambda + \lambda_I)(\lambda_0 + \lambda_I)} \right] \right\} \quad (14)$$

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The subscripts V denote integration over the entire reactor volume. In most problems k_{∞} is finite in only one region, the core volume.

The left side of equation 14 may be expanded as follows:

$$\int_V (S_o^+)' M S dV - \int_V S' M_o' S_o^+ dV = \int_V S_o^+ M_o S dV - \int_V S' M_o' S_o^+ dV + \int_V S_o^+ P S dV \quad (15)$$

those terms arising from the constant elements of M_o and M_o' cancel in equation 15 leaving the following expression

$$\begin{aligned} & \int_V S_{fo}^+ \nabla \cdot D_{fo} \nabla S_f dV - \int_V S_f \nabla \cdot D_{fo} \nabla S_{fo}^+ dV + \int_V S_{so}^+ \nabla \cdot D_{so} \nabla S_s dV \\ & - \int_V S_s \nabla \cdot D_{so} \nabla S_{so}^+ dV + \int_V S_{fo}^+ \nabla \cdot \delta D_f \nabla S_f dV + \int_V S_{so}^+ \nabla \cdot \delta D_s \nabla S_s dV + \int_V S_o^+ P_1 S dV \end{aligned} \quad (16)$$

where P_1 includes all terms in P (equation 12) except those involving differentials.

Each of the terms in equation 16 involving differentials can be expanded by the formula

$$A \nabla \cdot \vec{B} = \nabla \cdot A \vec{B} - \nabla A \cdot \vec{B}$$

where the scalar A is to be interpreted as S_{fo}^+ , S_f , S_{so}^+ or S_s as the case requires, and the vector $\vec{B}$ is $D_{fo} \nabla S_f$, $D_{fo} \nabla S_{fo}^+$, $D_{so} \nabla S_s$, $D_{so} \nabla S_{so}^+$, $\delta D_f \nabla S_f$, or $\delta D_s \nabla S_s$ as the case requires. This substitution yields, after cancellations, the expression:

$$\begin{aligned}
 & \int_V \nabla \cdot S_{fo}^+ D_{fo} \nabla S_f dV - \int_V \nabla \cdot S_f D_{fo} \nabla S_{fo}^+ dV + \int_V \nabla \cdot S_{so}^+ D_{so} \nabla S_s dV \\
 & - \int_V \nabla \cdot S_s D_{so} \nabla S_{so}^+ dV + \int_V \nabla \cdot S_{fo}^+ \delta D_f \nabla S_f dV + \int_V \nabla \cdot S_{so}^+ \delta D_s \nabla S_s dV \\
 & - \int_V \delta D_f \nabla S_{fo}^+ \cdot \nabla S_f dV - \int_V \delta D_s \nabla S_{so}^+ \cdot \nabla S_s dV + \int_V S_o^+ P_1 S dV \quad (17)
 \end{aligned}$$

The quantities $S_f D_{fo} \nabla S_{fo}^+$ and $S_s D_{so} \nabla S_{so}^+$ are continuous across all boundaries as are $S_{fo}^+ D_{f1} \nabla S_f$ and $S_{so}^+ D_{s1} \nabla S_s$ where D_{f1} and D_{s1} are the perturbed values. If the substitutions $D_{fo} = D_{f1} - \delta D_f$ and $D_{so} = D_{s1} - \delta D_s$ are made in the terms involving $D_{fo} \nabla S_f$ and $D_{so} \nabla S_s$, and Green's theorem is used to convert the volume integrals to surface integrals, almost all the terms in equation 17 disappear because surface integrals over one side of any boundary cancel those over the other side, and the integrals over the outer boundary are zero because all fluxes and adjoints vanish there. Remembering that equation 17 is just the left side of equation 14 the net result is that equation 14 becomes

$$\begin{aligned}
 & - \int_V \delta D_f \nabla S_{fo}^+ \cdot \nabla S_f dV - \int_V \delta D_s \nabla S_{so}^+ \cdot \nabla S_s dV + \int_V S_o^+ P_1 S dV = \\
 & (\lambda - \lambda_o) \left\{ \int_V \frac{S_{fo}^+ S_f dV}{v_f} + \int_V \frac{S_{so}^+ S_s dV}{v_s} + \int_V \frac{k_\infty Z_a S_{fo}^+ S_s dV}{p} \left[\sum_I \frac{\beta_I \lambda_I}{(\lambda + \lambda_I)(\lambda_o + \lambda_I)} \right] \right\} \quad (18)
 \end{aligned}$$

In more compact form equation 18 may be written

$$\int_V S_o^+ F_o S dV = [\lambda - \lambda_o] \left\{ \int_V \frac{S_{fo}^+ S_f dV}{v_f} + \int_V \frac{S_{so}^+ S_s dV}{v_s} + \int_V \frac{k_\infty Z_a S_{fo}^+ S_s dV}{p} \left[\sum_I \frac{\beta_I \lambda_I}{(\lambda + \lambda_I)(\lambda_o + \lambda_I)} \right] \right\} \quad (19)$$

$$P_0 = \left(\begin{array}{l} -\delta D_f \nabla S_{f0}^+ \cdot \nabla - \frac{\delta D_f}{J_1} + \frac{D_{f0}}{J_0} \frac{\delta J}{J_1} \quad \delta \left(\frac{k_{\infty} \Sigma_a}{p} \right) \\ \frac{D_{f0} \delta p}{J_1} + \frac{P_0 \delta D_f}{J_1} - \frac{D_{f0} P_0}{J_0} \frac{\delta J}{J_1} \quad -\delta D_s \nabla S_{s0}^+ \cdot \nabla - \delta \Sigma_u - \delta \Sigma_e \end{array} \right) \quad (20)$$

In equation 20 subscripts 0 and 1 refer to quantities before and after the perturbation, respectively. The term $\delta \left(\frac{k_{\infty} \Sigma_a}{p} \right)$ has not been expanded for reasons which will become apparent presently. It is often convenient to replace it by its equivalent $2.1 \delta \Sigma_u$. The quantity $-\delta \Sigma_u - \delta \Sigma_e$ is the change in total macroscopic cross section, $-\delta \Sigma_a$, expressed as the sum of parts due to U^{235} ($-\delta \Sigma_u$) and to materials other than U^{235} ($-\delta \Sigma_e$).

Equation 19 is the basic perturbation theory formula.

It is strictly valid only when the perturbed fluxes S_f and S_s are used. In most applications the integrals on the right side will not be sensibly changed when S_f and S_s are replaced by the unperturbed values S_{f0} and S_{s0} . This may also be true in some cases for the terms on the left side. However, for perturbations which affect the flux significantly, in particular for large localized perturbations, the perturbed fluxes will be needed.

D. Basic Relations

1. Expression For The Inverse Period λ

If the reactor is initially in the steady state $\lambda_0 = 0$ and equation 19 yields for λ , the inverse of the period of the perturbed reactor, the relation

$$\frac{1}{T} = \lambda = \frac{\int (S_0^+)' P_0 S_0 dV}{V} \left\{ \frac{\int S_{f0}^+ S_{f0} dV}{V v_f} + \frac{\int S_{s0}^+ S_{s0} dV}{V v_s} + \frac{\int \frac{k_{\infty} \Sigma_a S_{f0}^+ S_{s0} dV}{p}}{V} \left[\frac{\Sigma}{I} \frac{\beta_I}{\lambda + \lambda_I} \right] \right\} \quad (21)$$

In the denominator of equation 21 S_{f0} and S_{s0} have been used in place of the perturbed fluxes S_f and S_s .

2. Expression For The Change in k_{∞} Alone Which is Equivalent To a Perturbation

It is often convenient to evaluate a perturbation in terms of the uniform change in k_{∞} alone throughout the core which would produce the same period $1/\lambda$ as does the actual perturbation. This change in k_{∞} is a fictitious thing, but it is useful in comparing perturbations from different sources. (Sometimes a fictitious change in ν , the number of neutrons per fission is used for the same purpose). If δk_{∞} denotes this change in k_{∞} it must be true that

$$\int_V S_0^+ P_0 S_0 dV \approx \int_C S_0^+ \begin{pmatrix} 0 & \delta k_{\infty} & \frac{\Sigma_a}{P} \\ 0 & 0 & \end{pmatrix} S_0 dV \quad (22)$$

or

$$\delta k_{\infty} \approx \frac{\int_V (S_0^+)^+ P_0 S_0 dV}{\int_C \frac{\Sigma_{a0}}{P} S_{f0} S_{s0} dV} \quad (23)$$

In equations 22 and 23 the subscript C denotes integration over the core region where k_{∞} differs from zero.

3. The Generalized Inhour Relation ⁽²⁾

Equations 21 and 23 yield the relation

$$\frac{\delta k_{\infty}}{k_{\infty}} = \frac{\lambda l}{k_{\infty}} + \lambda \frac{\sum_I \beta_I}{\lambda + \lambda_I} \quad (24)$$

where

$$\overline{k_{\infty}} = \frac{\int_C \frac{k_{\infty}}{p} \sum_a S_{fo}^+ S_{so} dV}{\int_C \frac{\sum_{ao}}{p} S_{fo}^+ S_{so} dV} \quad (25)$$

$$\beta = \frac{\int_V \frac{S_{fo}^+ S_{fd} dV}{v_f} + \int_V \frac{S_{so}^+ S_{sd} dV}{v_s}}{\int_C \frac{\sum_{ao}}{p} S_{fo}^+ S_{so} dV} \quad (26)$$

Equation 24 has the same form as the inhour equation for the bare reactor⁽⁵⁾. Equation 25 tells how the average value of k_{∞} is to be computed for a core with variable composition, and equation 26 tells how the neutron lifetime is to be computed for a reflected reactor.

4. The δk_{∞} Due To An Arbitrary Perturbation

Using P_0 from equation 20 the expansion of equation 23 yields the following relation for a general perturbation

$$\begin{aligned} \delta \overline{k_{\infty}} \int_C \frac{\sum_{ao}}{p} S_{fo}^+ S_{so} dV = & - \int_V \delta D_f \nabla S_{fo}^+ \cdot \nabla S_{fd} dV - \int_V \frac{\delta D_f}{J_1} S_{fo}^+ S_{fd} dV \\ & + \int_V \frac{p_0}{J_1} \delta D_f S_{so}^+ S_{fd} dV + \int_V \frac{D_{fo}}{J_0} \frac{\delta J}{J_1} S_{fo}^+ S_{fd} dV - \int_V \frac{D_{fo} p_0}{J_0} \frac{\delta J}{J_1} S_{so}^+ S_{fd} dV \\ & + \int_V \frac{D_{fo}}{J_1} \delta p S_{so}^+ S_{fd} dV - \int_V \delta D_s \nabla S_{so}^+ \cdot \nabla S_{sd} dV + 2.1 \int_V \delta \sum_u S_{fo}^+ S_{sd} dV \\ & - \int_V \delta \sum_u S_{so}^+ S_{sd} dV - \int_V \delta \sum_e S_{so}^+ S_{sd} dV \end{aligned} \quad (27)$$

subscripts C and V denotes integration over the core volume and over the volume of the perturbation, respectively.

When the composition in a region is uniform such that D_{fo} and D_{so} are constants, and the perturbation is such that D_{fl} and D_{sl} are also uniform in the region, the terms involving $\nabla S_{fo}^+ \cdot \nabla S_f$ and $\nabla S_{so}^+ \cdot \nabla S_s$ may sometimes be replaced by approximate expressions obtained from equations 4a and 5a with $\lambda = 0$. These expressions are:

$$\nabla S_{fo}^+ \cdot \nabla S_f \approx \nabla \cdot S_{fo}^+ \nabla S_f - \frac{S_{fo}^+ S_f}{J_1} + \frac{k_{\infty} \Sigma_a S_{fo}^+ S_s}{p D_{fl}} \quad (28)$$

$$\nabla S_{so}^+ \cdot \nabla S_s \approx \nabla \cdot S_{so}^+ \nabla S_s + \frac{p_1 D_{fl} S_{so}^+ S_f}{J_1} - \frac{\Sigma_a S_{so}^+ S_s}{D_{sl}} \quad (29)$$

If the approximations in equations 28 and 29 are assumed equation 27 becomes:

$$\begin{aligned} \delta k_{\infty} \int_V \frac{\Sigma_a}{c p} S_{fo}^+ S_s dV = & - \delta D_f \int_A S_{fo}^+ \nabla S_f \cdot d\vec{A} - \frac{2.1 \delta D_f}{D_{fl}} \int_V \Sigma_u S_{fo}^+ S_s dV \\ & + \delta D_f \int_V \frac{p_0}{J_1} S_{so}^+ S_f dV + \int_V \frac{D_{fo}}{J_0} \frac{\delta J}{J_1} S_{fo}^+ S_f dV - \int_V \frac{D_{fo} p_0}{J_0} \frac{\delta J}{J_1} S_{so}^+ S_f dV \\ & + \int_V \frac{D_{fo}}{J_1} \delta p S_{so}^+ S_f dV - \delta D_s \int_A S_{so}^+ \nabla S_s \cdot d\vec{A} - \delta D_s \int_V \frac{p_1 D_{fl}}{J_1} S_{so}^+ S_f dV \\ & + \frac{\delta D_s}{D_{sl}} \int_V \Sigma_a S_{so}^+ S_s dV + 2.1 \int_V \Sigma_u S_{fo}^+ S_s dV - \int_V \Sigma_u S_{so}^+ S_s dV \\ & - \int_V \Sigma_a S_{so}^+ S_s dV \quad (30) \end{aligned}$$

Equation 30 is approximately valid if the diffusion constants D_f and D_s are uniform in the unperturbed and perturbed condition, and it is justifiable to use the approximations giving equations 28 and 29 (i.e. $\lambda = 0$)

in equations 4a and 4b). The subscripts C, V, and A denote integration over the core volume, the perturbed volume, and the surface of the perturbed volume with the direction of dA being outward. Subscripts 0 and 1 denote values before and after the perturbation, respectively. Equation 30 assumes that all constants except D_f and D_s may vary from point to point. Obvious simplifications are possible when they are independent of position.

It is planned to give applications of equations 26, 27 and 30 in a subsequent report.

The writer is indebted to J. W. Webster for helpful consultations.

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