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STATISTICAL ANALYSIS OF
MTR FUEL ELEMENT NON-DESTRUCTIVE ASSAY DATA

F. H. Tingey
F. P. Vance

October 8, 1954

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U. S. ATOMIC ENERGY COMMISSION

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REACTORS—RESEARCH AND TESTING

UNITED STATES ATOMIC ENERGY COMMISSION

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By
F. H. Tingey
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Atomic Energy Division
Phillips Petroleum Company
Idaho Falls, Idaho

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SUMMARY

By means of an analysis of variance of the the two-fold, nested, unequal-number type, data reported by Forbes (1) have been studied in order to derive a procedure for routine use of the fuel element assay method developed by Forbes (2). The parameters reflecting the reproducibility of total counts for a given instrument setting, from setting to setting, as well as variation in fuel length and precision of nominal fuel assay as reported by ORNL have been evaluated. This allows expression of empirical equations for measurement precision under a variety of conditions.

When apparent fuel core length measurements are not made,

$$(C.V.)^2 \left(\sum_{i=1}^m F_i \right) = 0.07 + 1.97 \left(\frac{1}{n} + \frac{1}{mk} \right) + 0.63 \left(1 + \frac{1}{m} \right)$$

and when core length measurements are made,

$$(C.V.)^2 \left(\sum_{i=1}^m F_i \right) = 0.07 + 1.97 \left(\frac{1}{n} + \frac{1}{mk} \right)$$

where $(C.V.) \left(\sum_{i=1}^m F_i \right)$ is defined as the standard deviation of the measurement precision, expressed as percentage of the total of "m" elements, $\sum_{i=1}^m F_i$. Ninety-five per cent confidence limits are obtained by multiplying this by two.

n = number of measurements performed on the reference standard, and

k = number of measurements performed on each individual fuel element.

For a given amount of time, the better precision is provided by performing core-length measurements, than duplicate measurements on individual assemblies. This increased precision, however, is difficult to justify, as it halves the measurement rate.

The overall precision of the method can be improved slightly by procurement of a fuel element assembled by taking one plate from each of the nineteen different alloy melts. The greatest improvement can be realized, however, by changed conditions resulting in better reproducibility of total counts, such as higher frequency of measurements with reference standard, i.e., reduction in the length of a series of fuel elements.

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I. INTRODUCTION

During the period May 1953 to June 1954, in the course of routinely assaying each assembly (1) repeated determinations were made on an arbitrarily selected standard (I-403) under various operating conditions and on different days. Statistical analysis of these data provides an estimate of the reproducibility of the method under various operating conditions and for various methods of calculation.

II. STATISTICAL ANALYSIS OF DATA

The mathematical-statistical model which was assumed to reasonably explain each observation on total counts in terms of the primary variables involved was as follows:

Define

$X_{k(ij)}$ = the kth determination on fuel element I-403 for the jth day (setting) and the ith operating condition,

μ = hypothetical average value over all possible operating conditions,

α_i = deviation from the hypothetical value due to the ith operating condition,

$\beta_j(i)$ = multiplicative deviation from $\mu + \alpha_i$ due to the jth day (setting) within the ith operating condition,

$\gamma_{k(ij)}$ = multiplicative deviation associated with the kth observation for the jth day (setting) under operating condition i,

then

$$X_{k(ij)} = (\mu + \alpha_i)(1 + \beta_j(i))(1 + \gamma_{k(ij)})$$

$$\cong \mu + \alpha_i + \beta_j(i)(\mu + \alpha_i) + \gamma_{k(ij)}(\mu + \alpha_i) \dots \dots \dots (1)$$

This model was assumed over other possible models primarily because a preliminary examination of the data indicated that the variation of the observations within a given setting and given operating condition was approximately proportional to the square of the average total counts indicating a constant coefficient of variation (constant percentage error).

If we further define

$\sigma_{X_{k(ij)}}^2$ = theoretical variation in observed total counts for given i and j (operating condition and setting),

σ_{α}^2 = theoretical variation in the α_i 's,

σ_{β}^2 = theoretical variation in the $\beta_j(i)$ (assumed to be the same for each operating condition).

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σ_f^2 = theoretical variation in the $X_{k(ij)}$ (assumed to be the same for each operating condition and setting,

then it follows from the model that for a given i and j,

$$\sigma_f^2 \approx \frac{\sigma_{X_{k(ij)}}^2}{(\mu + \alpha_i)^2}.$$

An estimate of this parameter will provide an estimate for the reproducibility in total counts for a given operating condition and setting.

Similarly the same sort of relationship between the mean and the variation between days within operating conditions exists so that the relative variation in the observed total counts for a given operating condition is given by

or

$$\sigma_{X_{k(ij)}}^2 = \sigma_\beta^2 (\mu + \alpha_i)^2 + \sigma_f^2 (\mu + \alpha_i)^2$$
$$\sigma_\beta^2 + \sigma_f^2 = \frac{\sigma_{X_{k(ij)}}^2}{(\mu + \alpha_i)^2}.$$

An estimate of σ_β^2 along with an estimate of σ_f^2 will provide an estimate for the precision or reproducibility of total counts on a given fuel assembly for a given operating condition. A comparison of the magnitudes of σ_β^2 and σ_f^2 will provide a basis for determining the assaying method which yields the most precision at the least cost.

To facilitate the estimation of these parameters we note that by a logarithmic transformation on the $X_{k(ij)}$ the original model (1) is transformed to

$$\ln X_{k(ij)} = \ln(\mu + \alpha_i) + \ln(1 + \beta_j(i)) + \ln(1 + \gamma_{k(ij)}) \dots\dots\dots(2)$$

the effects now being additive and thus the transformed data are amenable to a variance analysis treatment.

If we let

$$\left. \begin{aligned} \ln X_{k(ij)} &= X_{k(ij)}^* \\ \ln(\mu + \alpha_i) &= \alpha_i^* \\ \ln(1 + \gamma_{k(ij)}) &= \gamma_{k(ij)}^* \\ \ln(1 + \beta_j(i)) &= \beta_j^*(i) \end{aligned} \right\} \dots\dots\dots(3)$$

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then we can write (2) as,

$$X_{k(ij)}^* = \alpha_i^* + \beta_{ji}^* + \gamma_{k(ij)}^* \dots\dots\dots(4)$$

and an analysis of variance of the two-fold, "nested", unequal-number type(2) on the $X_{k(ij)}^*$ will provide estimates for the variation in the α_i^* 's (σ_{α}^2), variation in the β_{ji}^* 's (σ_{β}^2), and the variation in the $\gamma_{k(ij)}^*$'s, (σ_{γ}^2). By applying the propagation of errors formula (3) to the equations (3) we finally observe that

$$\sigma_{\beta}^2 \approx \sigma_{\beta}^2$$

and

$$\sigma_{\gamma}^2 \approx \sigma_{\gamma}^2$$

The analysis of variance is summarized in Table I.

TABLE I

<u>Source of Variation</u>	<u>Degrees of Freedom</u>	<u>Sum of Squares</u>	<u>Mean Squares</u>	<u>Expectation of Mean Square</u>
Between Operating Conditions	10	14.6075	1.46075	$\sigma_{\gamma}^2 + 4.13\sigma_{\beta}^2 + 7.90\sigma_{\alpha}^2$
Between Settings Within Operating Conditions	14	.0363	.002593	$\sigma_{\gamma}^2 + 3.15\sigma_{\beta}^2$
Within Settings	65	.0128	.000197	σ_{γ}^2
Total	89			

From the mean squares and expectation of mean squares we obtain the estimate

$$\sigma_{\gamma}^2 \approx 0.000197 \dots\dots\dots(5)$$

and

$$\sigma_{\beta}^2 \approx \frac{0.002593 - 0.000197}{3.15} = 0.000761 \dots\dots\dots(6)$$

It is interesting to note that the hypothesis $\sigma_{\beta}^2 = 0$

(i.e., that the variation from setting to setting for a given operating condition is explainable by the variation from determination to determination within a setting and operating condition)

may be tested by comparing the ratio

$\frac{.002593}{.000197} = 13.16$ with critical values of the F distribution with 14 and 65 degrees of freedom.

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The 1 per cent point of this distribution (value exceeded by pure chance 1 per cent of the time) is 2.38 and hence we conclude that the variation from setting to setting is real. An examination of the magnitude of σ_x and σ_p indicates that the reproducibility of total counts for a given operating condition for a given setting is fairly good (approximately $\pm 3\%$ * 95% confidence limits) however the reproducibility of total counts for a given operating condition for different settings is approximately $\pm 6\%$ **.

To convert from total counts to fuel element uranium content we assume the following calibration equation:

Define

F_i = calculated fuel content of the i th fuel assembly in a given series.
(All fuel elements measured under a given operating condition and given setting constitute a series).

$T_{Ei}^{(r)}$ = r th measurement of total counts for the E_{i th fuel assembly in the series.

$T_s^{(j)}$ = j th measurement of total counts for the arbitrarily selected "Standard" S.

$\bar{T}_{Ei} = \sum_{r=1}^k \frac{T_{Ei}^{(r)}}{k}$ (we assume k measurements on each fuel element assembly).

$\bar{T}_s = \sum_{j=1}^n \frac{T_s^{(j)}}{n}$ (we assume n measurements on the "standard" in each series).

W_s = nominal content of the arbitrarily selected "standard".

L_{Ei} = apparent fuel length of the i th fuel element in the series as determined by a single measurement by the established technique (1).

L_s = apparent fuel length of the "standard"; single determination by the same method as above

$$F_i = \left[\frac{\frac{1}{k} \sum_{r=1}^k T_{Ei}^{(r)}}{\frac{1}{n} \sum_{j=1}^n T_s^{(j)}} \right] W_s \left[\frac{L_{Ei}}{L_s} \right] = \left[\frac{\bar{T}_{Ei}}{\bar{T}_s} \right] W_s \left[\frac{L_{Ei}}{L_s} \right] \dots\dots\dots(7)$$

$$* 2 \sqrt{.000197} \times 100, \quad ** 2 \sqrt{.000197 + .000761} \times 100$$

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If we denote $\frac{\sigma_x}{x}$ by C.V.(x) (coefficient of variation or relative precision of(x)) then it can be shown by applying the propagation of errors formula to (7) that

$$C.V.^2(F_i) \cong C.V.^2(\bar{T}_{Ei}) + C.V.^2(\bar{T}_s) + C.V.^2(W_s) + C.V.^2(L_{Ei}) + C.V.^2(L_s) - \frac{2 \text{Cov.}(\bar{T}_{Ei}, \bar{T}_s)}{\bar{T}_{Ei} \bar{T}_s} \dots\dots\dots(8)$$

where Cov. (xy) denotes the covariance of x,y.

We note that under our assumed model (1) for observed total counts, $\bar{T}_{Ei}$ and $\bar{T}_s$ have the $\beta_j(\mu + \alpha_i)$ component in common and are hence correlated; in fact it can be derived from the model that $\text{Cov.}(\bar{T}_{Ei}, \bar{T}_s) = \sigma_\beta^2(\mu + \alpha_i)^2 \alpha$ being the deviation due to the particular operating condition under which the series measurements are made. Since both $\bar{T}_{Ei}$ and $\bar{T}_s$ estimate $\mu + \alpha_i$, it follows that $-2 \frac{\text{Cov.}(\bar{T}_{Ei}, \bar{T}_s)}{\bar{T}_{Ei} \bar{T}_s}$ estimates $-2\sigma_\beta^2$.

Similarly from equation (1) it follows that

$$C.V.^2(\bar{T}_{Ei}) \cong \sigma_\beta^2 + \frac{\sigma_f^2}{k}$$

and

$$C.V.^2(\bar{T}_s) \cong \sigma_\beta^2 + \frac{\sigma_f^2}{n}$$

so that

$$C.V.^2(F_i) \cong \sigma_\beta^2 + \frac{\sigma_f^2}{k} + \sigma_\beta^2 + \frac{\sigma_f^2}{n} + C.V.^2(W_s) + C.V.^2(L_{Ei}) + C.V.^2(L_s) - 2\sigma_\beta^2$$

$$\cong \sigma_f^2 \left[\frac{1}{k} + \frac{1}{n} \right] + 2C.V.^2(L_s) + C.V.^2(W_s) \dots\dots\dots(9)$$

It is important to note that equation (9) can be interpreted in two ways depending upon the way we define $C.V.^2(L_{Ei}) \cong C.V.^2(L_s)$. If $C.V.^2(L_s)$ reflects relative precision with which apparent fuel length measurements can be made then (9) gives the square of the relative precision of F_i if both the standard and the assembly in question are measured for apparent length. However, if we assume that the actual measurement as to apparent fuel length is very precise, i.e., having essentially zero measurement error, then we can define C.V.(L_s) or C.V.(L_{Ei}) to be the precision with which the nominal value represents the true length and equation (9) would then yield the square of the relative precision of the F_i when no apparent fuel length measurements are made.

Clearly under this second assumption the equation for estimating $C.V.^2(F_i)$ if apparent fuel length measurements were made would be

$$C.V.^2(F_i) \cong \sigma_f^2 \left[\frac{1}{k} + \frac{1}{n} \right] + C.V.^2(W_s) \dots\dots\dots(10)$$

It is also very interesting to note that σ_β^2 , i.e. the relative variation between settings within operating conditions does not appear in either (9) or (10).

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To determine the precision to be associated with the cumulative content of a series consisting of "m" fuel elements, excluding the standard, it is not valid to sum the uncertainties associated with each by the root mean square method since the F_i 's have the component $\beta_j(\mu + \alpha)$ in common and are therefore correlated. To evaluate the precision we note by summing (7) on i, that

$$\sum_{i=1}^m F_i = \left[\frac{W_s}{\bar{T}_s} \right] \frac{\sum_{i=1}^m \bar{T}_{E_i} L_{E_i}}{L_s} \quad \dots\dots\dots(11)$$

Applying the propagation of errors formula to (11) gives

$$C.V.^2\left(\sum_{i=1}^m F_i\right) \cong C.V.^2(W_s) + C.V.^2(\bar{T}_s) + C.V.^2(L_s) + C.V.^2\left(\sum_{i=1}^m \bar{T}_{E_i} L_{E_i}\right) - \frac{2 \text{Cov.}(\bar{T}_s \cdot \sum_{i=1}^m \bar{T}_{E_i} L_{E_i})}{\bar{T}_s \sum_{i=1}^m \bar{T}_{E_i} L_{E_i}} \dots(12)$$

Again it can be derived from the model (1) that

$$C.V.^2(\bar{T}_s) \cong \sigma_\beta^2 + \frac{\sigma_f^2}{n}$$

$$C.V.^2\left(\sum_{i=1}^m \bar{T}_{E_i} L_{E_i}\right) \cong \frac{1}{m} \left[\sigma_\beta^2 + \frac{\sigma_f^2}{k} + C.V.^2(L_s) + (m-1) \sigma_\beta^2 \right]$$

and

$$\frac{\text{Cov.} \bar{T}_s \sum_{i=1}^m \bar{T}_{E_i} L_{E_i}}{\bar{T}_s \sum_{i=1}^m \bar{T}_{E_i} L_{E_i}} \cong \sigma_\beta^2$$

so that (12) reduces to

$$\begin{aligned} C.V.^2\left(\sum_{i=1}^m F_i\right) &\cong C.V.^2(W_s) + \sigma_\beta^2 + \frac{\sigma_f^2}{n} + C.V.^2(L_s) + \frac{1}{m} \left[\sigma_\beta^2 + \frac{\sigma_f^2}{k} + C.V.^2(L_s) + (m-1) \sigma_\beta^2 \right] - 2\sigma_\beta^2 \\ &\cong C.V.^2(W_s) + \sigma_\beta^2 \left(\frac{1}{n} + \frac{1}{mk} \right) + C.V.^2(L_s) \left(1 + \frac{1}{m} \right) \dots\dots\dots(13) \end{aligned}$$

The interpretation of (12) depends on the interpretation of $C.V.(L_s)$ as outlined above. Under the second assumption (12) gives the square of the relative precision of $\sum_{i=1}^m F_i$ if apparent fuel length measurements are not made. The counterpart to equation (10) becomes

$$C.V.^2\left(\sum_{i=1}^m F_i\right) \cong C.V.^2(W_s) + \sigma_f^2 \left(\frac{1}{n} + \frac{1}{mk} \right) \quad \dots\dots\dots(14)$$

if apparent fuel length measurements are made.

To evaluate (12) and (13) as a function of k, m, and n it is necessary that the parameters $C.V.^2(W_s)$, σ_f^2 and $C.V.^2(L_s)$ be estimated. The analysis of variance given in Table I provides an estimate for σ_f^2 . An estimate for $C.V.^2(L_s)$ is provided by an analysis of variance of the reported data (1). The data are summarized in Appendix B; the analysis of variance is given in Appendix E. It will be observed that the model is based upon the assumption that measurement error in the length measurement is nil, and that $C.V.(L_s)$ is the relative precision with which the nominal length represents the true length.

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An analysis of these data yields as an estimate for $C.V.^2(L_s)$

$$C.V.^2(L_s) \cong 0.63 \times 10^{-4} \quad \dots\dots\dots(15)$$

It is interesting to note also that on the basis of the data in Appendix B a significant difference exists between the average fuel length for nominally 168 gram elements and the 200 gram elements.

To evaluate the precision of the assay of the arbitrarily selected standard as reported by ORNL the data given in Appendices C and D were obtained from ORNL and analyzed. The analysis (Appendix F) gave an estimate for $C.V.^2(W_s)$ of

$$C.V.^2(W_s) \cong 0.07 \times 10^{-4} \quad \dots\dots\dots(16)$$

Hence if these estimates,

$$\sigma_y^2 \cong 1.97 \times 10^{-4}$$

$$C.V.^2(L_s) \cong .63 \times 10^{-4}$$

$$C.V.^2(W_s) \cong .07 \times 10^{-4}$$

are substituted into equation (12) and (13) we have

$$C.V.^2\left(\sum_{i=1}^m F_i\right) \cong 0.07 \times 10^{-4} + 1.97 \times 10^{-4} \left[\frac{1}{n} + \frac{1}{mk}\right] + 0.63 \times 10^{-4} \left[1 + \frac{1}{m}\right] \quad \dots\dots(17)$$

if apparent fuel length measurements are not made and

$$C.V.^2\left(\sum_{i=1}^m F_i\right) \cong 0.07 \times 10^{-4} + 1.97 \times 10^{-4} \left[\frac{1}{n} + \frac{1}{mk}\right] \quad \dots\dots(18)$$

if apparent fuel length measurements are made.

The equations (17) and (18) provide the basis for determining the most efficient combination of n, the number of measurements on the standard, k, the number of measurements on the element and m, the number of elements in a series as well as the increase in precision resulting from apparent fuel length measurements.

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III. DISCUSSION OF RESULTS

General-case equations have been derived from the available data for several conditions:

- (a) number of fuel elements in a group,
- (b) single and duplicate measurements on each fuel element,
- (c) number of measurements performed on the reference standard,
- (d) fuel core-length measurements.

Core-length measurements require a more elaborate procedure than ordinary scanning; about twice as much time is required when these are made (5). Thus, it is interesting to note that the improvement in precision afforded by duplicate measurements is less than for core-length measurements. Consequently, for a given amount of time, the better precision will be afforded by performing length measurements, rather than duplicate measurements on each element.

Overall precision of the method can be improved by reducing the coefficients $(C.V.)^2(W_S)$, and σ_f^2 in the equations. As described in Appendix F, if a fuel element could be assembled consisting of nineteen plates, each from a different alloy melt, the factor $(C.V.)^2(W_S)$ could be reduced from 0.07 to 0.01. Since the actual case is such that the value is less than .07, there is little advantage in attempting to improve precision by this means.

The factor σ_f^2 represents the variation in total counts within a series of fuel element measurements. Under the procedure in use when these measurements (1) were made, usually two and not more than three measurements were made on the reference standard. The effect of variation in beam strength and background level is built into this factor; consequently, increasing the frequency of reference standard measurements, i.e., reducing the length of a series of "unknowns" can be expected to reduce the factor σ_f^2 , appreciably below 1.97. In these data this accounted for most of the imprecision.

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Operating Conditions: New Atomic Instrument Company decade scalers were placed in permanent operation in the assaying facility at this time. Preset count = 100,000.

<u>Date</u>	<u>Total Counts</u>		
<u>4-22-54</u>	<u>4-26-54</u>	<u>4-27-54</u>	
233,762	214,620	238,575	
239,985	240,378	239,415	
240,318	242,577		
226,312	242,136		
227,514			
239,190			

Operating Conditions: Preset time = 3 minutes. Some improvement in rejection of power line disturbances were made in the amplifiers.

<u>Date</u>	<u>Total Counts</u>			
<u>5-10-54</u>	<u>5-11-54</u>	<u>5-12-54</u>	<u>5-17-54</u>	<u>5-18-54</u>
298,233	299,214	302,517	272,853	271,989
297,108	301,383	298,602	272,232	273,384
297,828			272,709	
296,596			273,456	
295,749				

Operating Conditions: Preset time = 5 minutes per point.

<u>Date</u>	<u>Total Counts</u>
	<u>6-4-54</u>
	502,232

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APPENDIX B

Apparent Fuel Length Measurements (Centimeters)

<u>Date</u>	<u>168 gm U-235 (Nominal)</u>	<u>Date</u>	<u>200 gm U-235(Nominal)</u>
4-26-54	58.60	5-11-54	59.05
	58.35		58.25
	57.90		58.10
	58.15		57.45
	58.30		58.70
	58.50		
4-27-54	58.30	5-12-54	58.60
			58.50
			59.05
5-10-54	57.65		58.75
	58.35		58.25
	57.90		58.95
			59.25
		5-17-54	58.50
			58.80
			58.75
			58.05
			58.25
		5-18-54	58.85
			59.90
			59.35
			59.40
			59.05
		6-4-54	58.60
			59.15
		6-7-54	58.90
			59.05
			59.00
			59.25
			59.15
			59.10
			59.25
			59.30
		6-8-54	59.50
			59.45
			59.50

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APPENDIX C

Per Cent Uranium From Duplicate Samples From Fuel Element Heats

(ORNL Data)

<u>Heat No.</u>	<u>Sample A</u>	<u>Sample B</u>
D-251	18.70	18.65
252	18.71	18.62
253	18.69	18.60
254	14.35	14.37
255	21.05	21.20
256	15.92	15.95
257	17.05	16.95
258	16.96	16.88
259	17.13	17.01
260	16.92	16.90
261	16.96	16.89
262	16.87	16.88
263	16.95	16.96
264	17.09	17.07
265	17.10	17.12
266	17.35	17.25
267	27.09	26.90
268	14.36	14.37
269	14.40	14.37
270	14.56	14.44
271	14.53	14.41
272	14.69	14.59
273	14.53	14.45
274	14.48	14.52
275	18.63	18.64
276	18.63	18.71
277	18.48	18.44
278	14.37	14.28
279	18.49	18.43
280	18.57	18.48
281	18.48	18.42
282	18.38	18.44
283	18.29	18.30
284	18.39	18.41
285	18.41	18.56
286	18.52	18.55
287	18.46	18.51
288	18.45	18.53
289	18.38	18.49
290	18.39	18.52
291	18.44	18.50

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APPENDIX C (continued)

<u>Heat No.</u>	<u>Sample A</u>	<u>Sample B</u>
292	18.53	18.46
293	17.01	16.98
294	17.16	17.12
295	16.94	17.04
296	16.92	17.07
297	17.01	17.04
298	16.85	17.09
299	16.87	16.97
300	16.91	16.95

APPENDIX D

Weight of Uranium Fuel Plate Punchings (ORNL Data)
Grams

<u>Punching No.</u>	<u>Ingot A</u>	<u>Punching No.</u>	<u>Ingot B</u>
1	58.323	1	58.496
2	58.605	2	58.395
3	58.708	3	58.369
4	58.645	4	58.284
5	58.623	5	58.386
6	58.484	6	58.395
7	58.510	7	58.373
8	58.382	8	58.404
9	58.564	9	58.424
10	58.555	10	58.489
11	58.417	11	58.410
12	58.467	12	58.486
13	58.561	13	58.538
14	58.522	14	58.469
15	58.507	15	58.486
16	58.598	16	58.492
17	58.532	17	58.601
18	58.474	18	58.567
19	58.270	19	58.336
20	58.344	20	58.355
21	58.333	21	58.324
22	58.326	22	58.367
23	58.260	23	58.366
24	58.272	24	58.419
25	58.358	25	58.472
26	58.362	26	58.423
27	58.312	27	58.501
28	58.400	28	58.492
29	58.414	29	58.440
30	58.382	30	58.567
31	58.240	31	58.585

APPENDIX D (continued)

<u>Punching No.</u>	<u>Ingot A</u>	<u>Punching No.</u>	<u>Ingot B</u>
32	58.226	32	58.552
33	58.325	33	58.494
34	58.324	34	58.460
35	58.321	35	58.356
36	58.352	36	58.309

APPENDIX E

Estimation of C.V.²(L_g)

An analysis of variance was performed on the data of Appendix B. The mathematical model assumed was

$$L_{ij} = \xi_i + \eta_{ij} \quad \begin{matrix} i = 1, 2 \\ j = 1, 2, \dots, n_i \end{matrix}$$

where the subscript i refers to the group as determined by the nominal weight and j refers to the jth element measured in that group. n_i denotes the number of elements in the ith group. We define

ξ_i = hypothetical mean length of element in the ith group

η_{ij} = random deviation from ξ_i peculiar to the jth element in the ith group,

$$\bar{\xi} = \frac{\xi_1 + \xi_2}{2}$$

Furthermore, it was assumed that

$\sigma_{\eta_{ij}}^2 = \sigma_{\eta}^2$ that is, the variation within groups is the same for both groups.

Analysis of Variance

<u>Source</u>	<u>Degree of Freedom</u>	<u>Sum of Squares</u>	<u>Mean Square</u>	<u>Expected Mean Square</u>
Between Groups	1	3.6571	3.6571	$\sigma_{\eta}^2 + \sum_{i=1}^2 n_i (\xi_i - \bar{\xi})^2$
Within Groups	43	9.3929	.2184	σ_{η}^2

The estimate .2184 for σ_{η}^2 divided by the square of the mean of all the apparent fuel length measurements $(58.74)^2$ gives $.63 \times 10^{-4}$ as an estimate for C.V.²(L_g).

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APPENDIX F

Estimate of $C.V.^2(W_s)$

The procedure, as explained by Mr. J. E. Cunningham at ORNL, used to assign nominal weights to fabricated fuel elements generally consists of the following steps.

- (1). Two samples are taken from each melt of Uranium-Aluminum alloy, and each sample individually analyzed for per cent uranium. A sample of such data is given in Appendix C. The average of the two samples is computed and is then used as the per cent uranium in all plates made from that heat.
- (2). Two ingots are made from each heat and after rolling into plates about 0.25 inch thick, punchings which are later rolled into fuel assembly cores are taken from each plate. Usually 36 punchings are obtained from each rolled ingot (72 from a heat).
- (3). All punchings are then weighed simultaneously with essentially no weighing error and the average weight of a punching from the heat computed. A sample of these data is given in Appendix D.
- (4). The uranium content of a given punching fabricated into a fuel plate is found by multiplying the average weight of a punching from the heat by the average concentration of uranium in that heat as found from the analysis of the two samples.

Two models will be assumed. First we will assume that all "p" plates of a given fuel assembly are from the same heat. Secondly we will assume that each of the p plates in a given fuel assembly come from different heats. The actual situation is usually somewhere between these two models; however, the specific cases would be too numerous to evaluate. An analysis under each of these models will serve to evaluate the range of precision for the intermediate cases.

In the first case the total calculated assay of a fuel assembly consisting of "p" plates can be symbolically represented by the equation

$$W_s = p \left(\frac{s_1 + s_2}{2} \right) \left(\sum_{i=1}^{72} \frac{W_i}{72} \right)$$

where p = number of plates in the assembly, $s_i (i = 1, 2)$ is the per cent uranium in the melt as estimated by the i th sample from the heat

W_i = weight of the i th punching in the heat and we assume 72 punchings from a heat.

The actual assay can be symbolically represented by

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APPENDIX F (continued)

$$T = \sum_{j=1}^P W_j^* C_j$$

where

W_j^* is the weight of the j th punching used in the assembly

and

C_j = uranium percentage of the j th punching. Under very reasonable assumptions as to the mathematical models explaining the s_i 's, C_j 's and W_j 's it can be shown that on the average $W_S - T$ will be zero and that the variance of $W_S - T$ from which $C.V.^2(W_S)$ can be derived is given by the relationship

$$\text{Var.}(W_S - T) \cong p \frac{(P-p)}{P-1} (w^2 \sigma_c^2 + c^2 \sigma_w^2) + \frac{p^2 w^2 \sigma_c^2}{2}$$

where

w = average weight of a punching,

c = average uranium concentration in the punchings,

σ_c^2 = variation in concentration from punching to punching,

σ_w^2 = variation in weight from punching to punching.

Since $C.V.^2(W_S)$ is estimated by

$$\frac{\text{Variance}(W_S - T)}{(p w c)^2} = \frac{(P-p)}{p(P-1)} [C.V.^2(c) + C.V.^2(w)] + \frac{1}{2} C.V.^2(c), \dots\dots\dots(19)$$

estimates for $C.V.^2(c)$ and $C.V.^2(w)$ in (19) provide an estimate for $C.V.^2(W_S)$.

$C.V.^2(w)$ was estimated directly from the data of Appendix D.

$C.V.^2(c)$ was assumed to be the same as that for sample-to-sample in the same heat and was hence estimated from the data of Appendix C.

Hence

$$C.V.^2(w) \cong (.183)^2 \times 10^{-4}$$

$$C.V.^2(c) \cong (.356)^2 \times 10^{-4}$$

P was assumed as 472

(continued on next page)

and

$p = 19$ so that finally

$$C.V.^2(W_s) \approx .07 \times 10^{-4}$$

which is equivalent to ± 0.26 per cent.

Under the second model, that is each plate in the assembly comes from a different heat, it can be shown that

$$C.V.^2(W_s) = \frac{1}{p} \left[C.V.^2(c) \left(1 + \frac{1}{2}\right) + C.V.^2(w) \right]$$

which when the estimates of $C.V.^2(c)$ and $C.V.^2(w)$ are substituted yields

$$\begin{aligned} C.V.^2(W_s) &\approx \frac{1}{19} \left[(.356)^2 \left(1 + \frac{1}{2}\right) + (.183)^2 \right] \\ &= \frac{1}{19} \left[.190 + .033 \right] 10^{-4} \approx .01 \times 10^{-4} \end{aligned}$$

which is equivalent to ± 0.1 per cent.

The larger value, $(C.V.)^2(W_s) \approx 0.07 \times 10^{-4}$ was used in the general case equations, Nos. (17) and (18).

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