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**CALCULATED INTENSITY THRESHOLD FOR THE
MAINTENANCE OF LASER-SUPPORTED DETONATION WAVES
WITH VARIOUS ELECTRON DENSITIES**

N. M. Ferriter

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June 10, 1974

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CALCULATED INTENSITY THRESHOLD FOR THE MAINTENANCE OF LASER-SUPPORTED DETONATION WAVES WITH VARIOUS ELECTRON DENSITIES

Abstract

A one-dimensional intensity threshold for the maintenance of laser-supported detonation (LSD) waves is calculated based on the free electron density at the Chapman-Jouguet point. A free electron density equal to 7% of the critical electron density correlates well with available experimental threshold intensity measurements in air and with LASNEX calculations. Thresholds are calculated in air as a function of density. A minimum at about

2% of normal density is found for $\lambda = 10.6 \mu\text{m}$, due to the effect of molecular binding energy, but no minimum is found for $\lambda = 1.06 \mu\text{m}$. For a given wavelength, a maximum height is found above which LSD waves cannot be maintained. Results are also given for argon.

For intensities sufficiently high above threshold, we predict the existence of supercritical electron density at the Chapman-Jouguet point.

Introduction

Laser absorption waves in gases have been observed which are ignited at a solid target and move up the laser beam at high velocities, decoupling the laser energy from the target. These waves have been produced in various atmospheres, including air, with densities corresponding to altitudes from sea level to 30 km. For laser intensities greater than a certain threshold, these waves propagate by shock ionization and are then called laser-supported detonation (LSD) waves. It is of interest to predict this threshold, which varies with laser wavelength, the nature of the gas, and the gas density.

The one-dimensional theory of Raizer¹ accounts for many of the observed properties of LSD waves but sheds no light on the question of the threshold intensity for maintaining them. One theory for the maintenance threshold is based on lateral hydrodynamic relaxation in the detonation front.^{1,2} It has been shown, however, that a threshold is determined by one-dimensional phenomena,³ and our purpose is to explore a one-dimensional threshold model.

The simplest absorption wave to consider, if one is willing to ignore the complexities of absorption, is a wave

which propagates with the critical electron density, defined as that density where the plasma frequency is equal to the laser frequency. Near this density the laser energy will be strongly absorbed and the absorption length may be taken as zero. The LSD wave structure will be determined by the electron cascade, and energy will be deposited nonlinearly between the wave front and the point of critical density. It will be shown that a threshold for propagation of this class of waves is given by the one-dimensional Raizer theory.

However, LSD waves have been observed at densities and intensities below that predicted for critically dense wave propagation. The threshold intensity for these waves can also be related, by means of Raizer's theory, to the electron density

at the Chapman-Jouguet point, which we find to be approximately 7% of the critical density. This electron density predicts the available experimental thresholds and also agrees with threshold calculations made with the computer program LASNEX.³

Laser intensities which generate lower electron densities than this produce absorption waves, but these fail to satisfy the Chapman-Jouguet conditions and propagate more slowly than the LSD waves of Raizer's theory. We have developed no explanation for the absorption of all of the laser energy in the detonation front, by 7% of the critical electron density, while perhaps 3% waves are not strongly enough absorbing to drive a detonation.

Development

The currently accepted theory for LSD waves, due to Raizer,¹ is derived from the hydrodynamic equations for the conservation of mass, momentum, and energy, and results in the following expression for the absorbed light intensity W required to propagate a wave of a given specific energy E :

$$W = \frac{(\gamma^2 - 1)^{1/2}}{2} \left[\left(\frac{\gamma + 1}{\gamma} \right) E_J \right]^{3/2} \rho_0 \quad (1)$$

where γ is the exponent of the adiabat defined by $\gamma \equiv 1 + (p/\rho E)$, p is the pressure, E_J is the specific energy at the Chapman-Jouguet point J , and ρ_0 is the ambient gas density.

The absorption of the light energy takes place between S , the detonation

front, and J . Strong energy absorption implies a substantial free electron density n_e . For n_e equal to, or constituting an appreciable fraction of, the critical electron density n_c , the absorption length will be short and the intensity used in Eq. (1) will be the total laser intensity. In view of this, it seems natural to relate E_J to n_c . Equation (1) then gives the absorbed intensity required to maintain an LSD wave in which n_e is a given fraction f of n_c ; the threshold intensity is that value of W below which f is too small to support LSD waves. This intensity threshold is based on one-dimensional considerations; in this respect it differs from current two-dimensional threshold theories.^{1,2}

For intensities above threshold, Eq. (1) can also be used to predict the

maximum value of f to be expected in an LSD wave driven by a given intensity. For sufficiently large W , we find that we may have $f > 1$, i.e., supercritical electron density.

In order to determine E_J from n_c , we use a two-temperature model for the gas with electron and heavy-particle temperatures T_e and T_i , respectively, which LASNEX calculations have shown to be not necessarily equal, and corresponding specific energies E_e and E_i . At the shock front we assume equilibrium, with $T_e = T_i = T_s$, the shock temperature. We find the electron temperature T_{eJ} at J by assuming that it satisfies the equation

$$n_{eJ} \approx n_e [T_{eJ}, \rho_J(T_{eJ})] = f n_c, \quad (2)$$

where n_c is the critical electron density for laser wavelength λ given by

$$n_c = \frac{\pi m c^2}{2 \lambda^2} = 1.115 \times 10^{21} \lambda^{-2} \cdot \text{cm}^{-3}, \quad (3)$$

where λ is in μm , n_{eJ} is the electron density at J , $f > 0$, and

$$\rho_J = \left(\frac{\gamma + 1}{\gamma} \right) \rho_0, \quad (4)$$

according to Raizer.¹ Assuming that at J the electrons are in Saha equilibrium at the electron temperature, we solve Eq. (2) for T_{eJ} by an iterative method using a fast Saha equation subroutine with tabulated values of the ionization potentials.³ Since ρ_J depends on T_{eJ} through γ , we also iterate on γ , using equation-of-state tables for air,⁴ which take into account molecular dissociation, electron attachment, and ionization.

To complete the calculation of E_J we need to find T_{iJ} . In the absence of information about the LSD wave structure, we have assumed both equilibrium and nonequilibrium models, as follows.

Model 1: Equilibrium at J

Here we assume that $T_{ij} = T_{eJ}$, so that the desired E_J is given simply by

$$E_J = E(T_{eJ}, \rho_J), \quad (5)$$

where T_{eJ} is obtained from Eq. (2) and ρ_J from Eq. (4). To evaluate Eq. (5), we use our equation-of-state tables for air.⁴ Alternatively, we can approximate E_J in the form:

$$E_J = E_{eJ} + E_{iJ}, \quad (6)$$

where we assume ideal gases with potential energies

$$E_{eJ} = n_e \left(\frac{3}{2} T_{eJ} + \frac{1}{J} \right), \quad (7a)$$

$$E_{iJ} = n_a \left(\frac{3}{2} T_{eJ} + B \right) + n_m \frac{5}{2} T_{eJ}, \quad (7b)$$

and n_a and n_m are the specific number of atoms and molecules, respectively, B is the molecular dissociation and excitation energy per atom, and I_J is the average ionization energy per electron. For our calculations in air, T_{eJ} usually exceeds 1 eV, whereas air is completely dissociated at $T \approx 0.9$ eV. Hence we set $n_m = 0$, assign to B its maximum value $B = 4.4$ eV, and neglect excitation energy. Values of E_J obtained by this approximation agree with the values obtained from Eq. (5) to within 15% when applied to air at heights from sea level to 30 km.

Nonequilibrium Models at J

The equilibrium model gives us an upper limit for E_J at the specified electron density n_c . To obtain lower limits we set $T_{iJ} = T_s$. Using Eq. (6) and assuming an ideal-gas model for the heavy particles, we get two different nonequilibrium models, depending on how we treat molecular dissociation and excitation.

Model 2

We set

$$E_{eJ} = E(T_{eJ}, \rho_J) - \frac{3}{2} n_a T_{eJ} \quad (8a)$$

$$E_{iJ} = \frac{3}{2} n_a T_s. \quad (8b)$$

In Eq. (8a) we are effectively letting the molecular dissociation and excitation be determined by T_{eJ} , which is obtained from Eq. (2) as before. To obtain T_s we use the relations¹

$$\rho_s = \left(\frac{\gamma + 1}{\gamma - 1} \right) \rho_0$$

and

$$E_s = 2 \left(\frac{\gamma - 1}{\gamma} \right) E_J \quad (9)$$

to get an implicit equation for T_s

$$E_s(T_s, \rho_s) = 2 \left(\frac{\gamma - 1}{\gamma} \right) \left(E_{eJ} + \frac{3}{2} n_a T_s \right). \quad (10)$$

where E_{eJ} is given by Eq. (8a). We solve Eq. (10) for T_s by iteration and substitute Eqs. (8a) and (8b) into Eq. (6). Since γ is slowly varying as a function of temperature, we have used the equilibrium value of γ evaluated at J in Eqs. (9) and (10), as well as in Eq. (12), below.

Model 3

If we assume that heavy-particle collisions determine molecular dissociation and excitation, we then treat the electrons as an ideal gas with potential energy

$$E_{eJ} = n_{eJ} \left(\frac{3}{2} T_{eJ} + I_J \right) \quad (11a)$$

and obtain the energy of the ions from the equation-of-state tables for air⁴:

$$E_{iJ} = E(T_s, \rho_J) - n_{eJ}(T_s, \rho_J) \times \left[\frac{3}{2} T_s + I_J(T_s, \rho_J) \right]. \quad (11b)$$

To obtain T_s we again use Eq. (9), so that we have another implicit equation for T_s :

$$E(T_s, \rho_s) = 2 \left(\frac{\gamma - 1}{\gamma} \right) \left\{ n_{eJ} \left(\frac{3}{2} T_{eJ} + I_J \right) + E(T_s, \rho_J) - n_{eJ}(T_s, \rho_J) \times \left[\frac{3}{2} T_s + I_J(T_s, \rho_J) \right] \right\}. \quad (12)$$

We solve Eq. (12) for T_s by iteration in the table, taking into account the dependence of n_{eJ} and I_J in the last two terms on T_s through the Saha equation, and then substitute Eqs. (11a) and (11b) into Eq. (6).

Thus, by assuming equilibrium at S and either equilibrium ($T_{ij} = T_{eJ}$) or nonequilibrium ($T_{ij} = T_s$) at J, we have determined three models for E_J as a function of λ , f , γ , and ρ_0 , which give upper and lower limits for W . LASNEX calculations indicate that coupling between ions and electrons occurs during the absorption process, leading to significantly

more molecular dissociation than occurs with model 3, but not leading to the equilibration assumed in model 1. Additionally, since in all three models ionization is determined by the electron temperature, it might be argued that dissociation should be determined in the same way since the energies and

cross sections are similar. In view of this, we believe model 2 to be the most accurate.

In the next section we discuss results obtained when this theory is applied to LSD propagation in air, and in the fourth section we give results for LSD waves in argon.

Results for Air

Critically Dense LSD Waves ($f = 1$)

In Fig. 1 we show T_{eJ} and T_s from the different models described above, and in Figs. 2 and 3 the threshold intensity W vs altitude h for $\lambda = 10.6 \mu\text{m}$ and $3.5 \mu\text{m}$, respectively, for $f = 1$, i.e., for critically dense LSD waves. The partial ionization calculation used air with volume fractions 0.7848 N_2 , 0.2105 O_2 , and 0.0047 A. We convert dry air density to altitude using Ref. 4, as shown in Fig. 4. As expected, the equilibrium model requires a higher intensity in all cases since it assumes that $T_{iJ} = T_{eJ}$, whereas $T_{ij} < T_{eJ}$ for the nonequilibrium models. The values of γ obtained at J , corresponding to Fig. 1, are shown in Fig. 5.

The general trend of the equilibrium curves can be understood from Eqs. (5)

Fig. 1. (a) The temperature required for electrons in Saha equilibrium to be at the critical electron density at the Chapman-Jouguet point vs altitude. (b) The temperature of the shock front for molecular binding determined by the electrons and ions vs altitude.

and (6), which we write (setting $n_m = 0$) in the form:

$$E_J = n_a \left[\frac{3}{2} (1 + Z) T + ZI + B \right]_J, \quad (13)$$

where $Z \equiv n_e/n_a$ and ZI is the average ionization energy per atom. These three components of E_J are plotted vs h in Fig. 6. For $h < 10$ km, the dominant term

Fig. 2. The intensity required at $\lambda = 10.6 \mu\text{m}$ to drive critically dense waves vs altitude for equilibrium (Model 1), binding energy determined by electrons (Model 2), and binding energy determined by ions (Model 3).

Fig. 3. The intensity required at $\lambda = 3.5 \mu\text{m}$ to drive critically dense waves vs altitude for equilibrium (Model 1), binding energy determined by electrons (Model 2), and binding energy determined by ions (Model 3).

Fig. 4. Air density vs altitude for the earth's atmosphere.

Fig. 5. The specific heat ratio, γ , for waves with various values of the electron density vs altitude.

is B , assumed constant, and so the initial decrease of W with h is due entirely to the decrease of ρ_0 in Eq. (1). This decrease of air density requires an increase of temperature in order to produce the critical electron density, and so W begins to increase, with the ionization energy ZI becoming the dominant term for $h \geq 15$ km.

On the other hand, for $\lambda = 3.5 \mu\text{m}$ we have complete dissociation even at sea level because the temperatures are higher

Fig. 6. Various components of the energy density associated with critically dense waves at $\lambda = 10.6 \mu\text{m}$ vs altitude.

Fig. 7. The shortest wavelength possible for LSD waves with various values of f vs altitude.

than for $\lambda = 10.6 \mu\text{m}$ ($T_{eJ} = 3.5 \text{ eV}$ instead of 1.5 eV). The first two terms in Eq. (13), which are increasing, dominate the third term so that there is no minimum and the curves in Fig. 3 resemble the upper portion of Fig. 2.

The minimum air density at which the electron density n_c can be reached by full ionization is given by

$$\rho_{\min} = \left(\frac{\gamma}{\gamma + 1} \right) \frac{n_c}{N_{av}} \frac{A}{Z}.$$

Using Eq. (3) for n_c and converting air density to altitude, we obtain Fig. 7, showing the maximum altitude at which LSD waves for a given wavelength can propagate. At sea level, the minimum wavelength (λ) for which critical LSD

waves can exist is $1.28 \mu\text{m}$. Hence, critical-density LSD waves cannot exist at sea level for $\lambda = 1.06 \mu\text{m}$.

The experimental threshold for $\lambda = 10.6 \mu\text{m}$ in normal-density air as determined by Hall⁵ is shown on Fig. 1. Our results for a critically dense wave from models 1 and 2 are seen to be approximately twice Hall's results, while model 3 is at his lower limit. Using our model 2 calculations, we interpret these results as indicating that Hall's threshold measurements involve subcritical LSD waves, with $f < 1$.

Subcritical LSD Waves ($f < 1$)

For $\lambda = 1.06 \mu\text{m}$, Rudder⁶ has observed totally absorbing LSD waves in normal-density air which must be subcritical since at this wavelength full ionization yields

Fig. 8. The threshold intensity necessary to propagate an LSD wave with a specified value of the electron density vs electron density for various wavelengths.

only $f = 0.68$. To determine the correct value of f , we show in Fig. 8 the threshold W from our theory for various wavelengths as a function of f , together with the measurements of Hall and Rudder for normal-density air. For $\lambda = 1.06 \mu\text{m}$ we require $0.03 \leq f \leq 0.07$, and for $\lambda = 10.6 \mu\text{m}$ we must have $f \lesssim 0.2$ for agreement with the experiments.

The inverse bremsstrahlung absorption length l for subcritical waves may be readily calculated for given values of λ and f , as shown in Fig. 9. In order that subcritical LSD waves be totally absorbing, their thicknesses should be greater than these absorption lengths. From Hall's measurements we estimate this thickness for his experiments to be somewhat less than 0.1 cm. The value of l for

$\lambda = 10.6 \mu\text{m}$ and $f = 0.07$ is 0.08 cm, as shown in Fig. 9, which is compatible with Hall's experiments, whereas a value of f smaller than 0.05 would lead to too large a value of l .

In Figs. 10 and 11 we show W vs h for various values of f , using model 2. For $\lambda = 10.6 \mu\text{m}$, a value of $f = 0.05$ will bring the threshold measurements at normal density into better agreement with Hall's measurements, but the minimum in the curve of W vs h is intensified (see Fig. 10). Further experiments at low density are needed to verify this effect.

One-dimensional calculations made with LASNEX at $\rho_0 = 2 \times 10^{-5} \text{ g/cm}^3$ and $W = 10^6 \text{ watts/cm}^2$ give an LSD wave with $f = 0.07$; however, when W is reduced to $5 \times 10^5 \text{ watts/cm}^2$, the absorption wave

Fig. 9. The electron density vs inverse bremsstrahlung absorption lengths for various wavelengths at sea level.

Fig. 10. Intensity for propagation of various fractions of the critical electron density at $\lambda = 10.6 \mu\text{m}$ vs altitude.

fails to reach maximum compression at the Chapman-Jouguet point [Eq. (9)], and the wave velocity is less than the Raizer value [Eq. (14)]. Similar results are obtained at sea level density

Choosing $f = 0.07$ for all densities, we show T_{ej} vs h in Fig. 12 and the corresponding values of γ in Fig. 5. There is a maximum in γ at $h = 45 \text{ km}$, corresponding to $Z \sim 5$; a constant γ approximation would be inaccurate in this region.

Supercritical LSD Waves ($f > 1$)

Walters⁷ has measured the speed of LSD waves in air at $\lambda = 10.6 \mu\text{m}$ at intensities up to $W = 3 \times 10^8 \text{ watts/cm}^2$

for $\rho/\rho_0 = 0.33$ and 1. Using $\gamma \approx 1.15$, he finds agreement to within about 18% with the theoretical values

$$v = \left[2(\gamma^2 - 1) \frac{W}{\rho_0} \right]^{1/3} \quad (14)$$

Fig. 11. Intensity for propagation of various fractions of the critical electron density at $\lambda = 1.06 \mu\text{m}$ vs altitude.

predicted by Raizer's theory. If f could not exceed unity, v would cease to increase for $W \geq 4 \times 10^7$ watts/cm² at $\rho = \rho_0$. Therefore, we interpret Walters' results as indicating the existence of supercritical LSD waves; i.e., some portion of these waves must have values of $f > 1$, and in fact $f \approx 10$ for $W = 3 \times 10^8$ watts/cm² (cf. Fig. 8). As the intensity increases, f gradually approaches an upper limit characteristics of the gas and wavelength; e.g., for $\lambda = 10.6 \mu\text{m}$ in air, this limiting value is 67.

Two-Dimensional Effects

Two-dimensional effects depend on the ratio of the LSD wave thickness L to the radius R of the laser beam, where L is a combination of shock thickness, absorption length, and electron cascade length. In the cascade length the growth of electron density takes place which, although essential, does not require an appreciable fraction of the laser energy; the latter is primarily deposited near J , not uniformly as assumed in Refs. (1) and (2).

For a laser beam of finite radius, the intensity threshold will be raised because radial expansion, by reducing ρ_J , will require a higher degree of ionization for the same value of f . Thus two-dimensional thresholds will be higher than the one-dimensional thresholds we have discussed. In both the experiments mentioned above, the beam radius was 0.1 cm, which is somewhat greater than the detonation thickness in air at sea level. Hence, at sea level correction for this effect is marginal. However as ρ_0 is reduced, L increases; for example, the one-dimensional LASNEX calculation with $\rho_0 = 2 \times 10^{-5}$ g/cm³ and $W = 10^6$ watts/cm²

Fig. 12. Saha equilibrium temperatures predicted for an electron density of 0.07 critical vs altitude for various wavelengths.

gives $L = 3$ cm. Thus for this beam radius, two-dimensional effects at low densities may raise the threshold significantly. This probably explains why Hall did not observe a decrease in threshold intensity as ρ_0 was decreased.

Consideration of two-dimensional effects must thus include the breakdown of the gas. The time of interest is that required for the electrons to cascade from their density at the shock front to the Chapman-Jouguet point. This is true whether or not an electron avalanche⁸ exists ahead of the shock, because the electron pressure in front of the shock will not be sufficient to cause any appreciable hydrodynamic effect.

Results for Argon

We have repeated the above calculations and for argon, with an equation of state of the form of Eqs. (6) and (7), with $B = n_m = 0$ since argon is monatomic. Again assuming Saha equilibrium at the electron temperature, we obtain

$$E = n_a \left[\frac{3}{2} (1 + \bar{Z}) T + Z I \right],$$

$$p = n_a (1 + \bar{Z}) T \rho.$$

Fig. 13. Threshold intensity vs density ratio to propagate various fractions of the critical density in argon.

$$\gamma = 1 + \frac{(1 + \bar{Z}) T}{\frac{3}{2} (1 + \bar{Z}) T + \bar{Z} I}.$$

where $Z(T, \rho)$ and $I(T, \rho)$ are obtained from the Saha equation.

A graph of W vs ρ/ρ_0 is shown in Fig. 13. Here $\rho_0 = 0.00170 \text{ g/cm}^3$, the density at which the pressure is 1 atm for $T = 20^\circ\text{C}$. The two nonequilibrium results are now identical and differ from the equilibrium result less than do results for air. At 1 atm, using $f = 0.07$, we predict a threshold value of $W \approx 10^6 \text{ watts/cm}^2$. A shallow minimum at $\rho/\rho_0 \approx 0.04$ is also predicted. Values of γ vs ρ/ρ_0 are shown in Fig. 14.

Fig. 14. The ratio of specific heats, γ , for various fractions of the critical density in argon vs the density of argon.

Conclusions

We have calculated a one-dimensional threshold for the maintenance of LSD waves based solely on the ability of the laser beam to produce a sufficient density of free electrons. Using Raizer's theory we relate the required beam intensity to the electron density at the Chapman-Jouguet point by making use of a two-temperature equation of state and assuming Saha equilibrium. The single parameter in our theory, which we call f , is the ratio of this electron density to the critical electron density for the given laser wavelength. Using LASNEX calculations and experimental threshold intensity measurements, we find that the threshold may be satisfactorily explained as that intensity which will produce $f \approx 0.97$.

Our theory predicts that, for $\lambda = 10.6 \mu\text{m}$, the maintenance threshold will fall at higher altitudes, reach a minimum at about 30 km, and then rise. This minimum will be detectable, however, only with beam sizes that are at least as large as the detonation thickness, which LASNEX calculations show reaches 3 cm at 30 km and even larger values at higher altitudes. With only millimeter beam sizes, two-

dimensional effects will raise the threshold and may obscure this minimum.

The theory can also be applied to lasers of different wavelengths and to different gases. For $\lambda = 1.06 \mu\text{m}$ in air, for example, we find that the one-dimensional threshold intensity shows no minimum but increases monotonically with altitude. This difference is due to the effect of the molecular dissociation energy. We have also calculated thresholds for LSD waves in argon.

For a given value of f , our theory yields a maximum height in the atmosphere above which LSD waves cannot be maintained. For $f = 0.07$, this height is found to be 18 km for $\lambda = 1.06 \mu\text{m}$, 33 km for $\lambda = 3.5 \mu\text{m}$, and 49 km for $\lambda = 10.6 \mu\text{m}$. Two-dimensional effects would tend to lower these heights.

As a by-product of our theory we can calculate the maximum electron density to be found in an LSD wave for beam intensities greater than threshold. We predict that, for sufficiently high intensities, supercritical electron densities will exist. As far as we know, these have not been observed experimentally.

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