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ONE-COMPONENT TWO-PHASE CRITICAL FLOW:
AN EXPERIMENTAL STUDY USING FREON-11
AT SUBATMOSPHERIC PRESSURES

by

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MASTER

TABLE OF CONTENTS

	<u>Page</u>
NOMENCLATURE.....	8
ABSTRACT.....	9
I. INTRODUCTION.....	10
Two-phase Critical Flow.....	10
Purpose of this Study.....	13
II. MATHEMATICAL MODELS.....	14
Homogeneous Equilibrium Model.....	17
Slip Equilibrium Model.....	18
Homogeneous Nonequilibrium Model.....	19
III. EXPERIMENTAL APPARATUS AND PROCEDURE.....	23
APPARATUS SECTION.....	23
Refrigeration Unit and Low-pressure Reservoir.....	23
Freon Reservoir.....	24
Glass Test Sections.....	26
Stainless Steel Test Section.....	27
Instruments and Photography.....	28
Pressure Control System.....	28
PROCEDURE SECTION.....	30
Procedure with Glass Test Section.....	31
Procedure with Stainless Steel Test Section.....	31
IV. EXPERIMENTAL RESULTS.....	32
Entrance Subcoolings.....	32
ID and L/D Effects.....	35
Effects of Entrance Configuration.....	37
Flow Regimes at the Entrance.....	38
Observations of Flow Regimes.....	39
Critical Pressure Ratio.....	41
Temperature and Pressure Profiles.....	42

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TABLE OF CONTENTS

	<u>Page</u>
V. DISCUSSION OF RESULTS AND CONCLUSIONS.....	45
Entrance Subcoolings.....	45
Comparison with Models.....	46
Critical Pressure Ratio.....	48
Conclusions.....	51
APPENDICES.....	53
A. Sample Calculations of All-liquid Test Runs.....	53
B. Steady-state Data for Glass Test Section.....	56
C. Exit-pressure Accuracy.....	64
D. Critical Flow Data from Stainless Steel Test Section.....	65
E. Calculations of Pressure Drop for the Stainless- steel Test Section.....	67
F. 0.121-in.-ID Exit-pressure Error.....	68
G. Estimation of the Empirical Constant N of the Homogeneous Nonequilibrium Model.....	72
ACKNOWLEDGMENTS.....	74
REFERENCES.....	75

LIST OF FIGURES

<u>No.</u>	<u>Title</u>	<u>Page</u>
1	Analogy between Single-phase and Two-phase Critical Flows.....	11
2	Experimental Apparatus.....	23
3	Schematic of Refrigeration Unit and Low-pressure Reservoir.....	24
4	0.0995-in.-ID Glass Test Section.....	25
5	Stainless Steel Test Section.....	27
6	Pressure Profiles of Typical (a) Steady-state and (b) Quasi-steady Test Runs.....	29
7	Schematic of Pressure Control System and Entrance Configurations.....	29
8	Critical Mass Flux vs Entrance Subcooling.....	33
9	Critical Exit Pressure vs Entrance Subcooling.....	33
10	Entrance Flow Regimes.....	34
11	Density Ratio at Exit Plane vs Entrance Subcooling.....	35
12	Results of Quasi-steady Tests.....	36
13	Critical Mass Flux vs Entrance Subcoolings for Various IDs.....	36
14	Effects of Entrance Configuration for 0.25-in.-ID Tubes and Freon-11 Initially at 40°F.....	38
15	Mass Flux during Cyclic Flashing.....	39
16	Flow Regimes in Long Tubes.....	40
17	Critical Pressure Ratio Variation with Increasing Stagnation Pressure for Constant Initial Freon-11 Temperature of $75 \pm 2^\circ\text{F}$	42
18	Experimental Pressure Profiles.....	43
19	Experimental Temperature Profiles.....	43
20	Correlation between Mass Flux and Entrance Subcooling for Freon-11 Temperature of $75 \pm 2^\circ\text{F}$	45

LIST OF FIGURES

<u>No.</u>	<u>Title</u>	<u>Page</u>
21	Interpretation of Data in Terms of the Slip and Homogeneous Equilibrium Models for 0.162-in.-ID Test Section.....	47
22	Interpretation of Data in Terms of the Homogeneous Equilibrium and Homogeneous Nonequilibrium Models for 0.250-in.-ID Test Section.....	47
23	0.121-in.-ID Glass Test Section.....	69
24	0.121-in.-ID Critical Exit Pressure Data.....	71
25	Calculation of the Upper Bound of the Actual Exit Quality.....	73

LIST OF TABLES

<u>No.</u>	<u>Title</u>	<u>Page</u>
I	Experimental Test Data.....	32

NOMENCLATURE

NOMENCLATURE

<u>Symbol</u>		<u>Units</u>
A	Cross-sectional area	sq ft
C_c	Contraction coefficient	
C_p	Specific heat	Btu/lb _m -°F
D	Diameter	ft
f	Friction factor	
G	Mass flux	lb _m /ft ² -sec
g_c	Unit conversion factor	32.17 lb _m -ft/sec ² -lb _f
h	Enthalpy	Btu/lb _m
k	Slip ratio	($k = u_g/u_l$)
L	Length	ft
$\dot{m}$	Mass flow rate	lb _m /sec
N	Experimental value	
P	Pressure	lb _f /in. ²
$\dot{q}$	Energy in flux	Btu/hr
s	Entropy	Btu/lb _m -°F
T	Temperature	°F
u	Velocity	ft/sec
v	Specific volume	ft ³ /lb _m
w	Work	ft-lb _f
x	Quality	
y	Height	ft
z	Length	ft
α	Void fraction	
ρ	Density	lb _m /ft ³
τ	Shear stress	lb _f /ft ²

Actu

Cris

Calc

At e

Fric

Vapo

Liqu

Mom

At e

Rece

At e

Sub

Tota

Wall

Sing

Two-

BLANK PAGE

TURE

Units

Symbol

Subscripts

act	Actual
c	Critical
cal	Calculated
e	At exit
f	Friction
g	Vapor phase
l	Liquid phase
m	Momentum
o	At entrance
R	Receiver
sat	At saturation
sub	Subcooling
t	Total
w	Wall
1ϕ	Single-phase
2ϕ	Two-phase

2

ONE-COMPONENT TWO-PHASE CRITICAL FLOW:
AN EXPERIMENTAL STUDY USING FREON-11 AT SUBATMOSPHERIC PRESSURES.

by

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ABSTRACT

Low-pressure, one-component, two-phase critical-flow studies in long cylindrical tubes were undertaken using Freon-11. A stainless steel test section, equipped with numerous pressure transducers and thermocouples, was used to obtain experimental pressure and temperature profiles. Critical discharge pressures from 3.0 to 13.5 psia were observed with measured mass fluxes from 250 to 2100 lb/ft²-sec in tubes in which the length-to-diameter (L/D)^m ratios ranged from 25 to 300 for varying amounts of entrance subcooling. The effects of four different entrance configurations on critical mass flux and liquid superheat were also investigated. Critical pressure ratios (ratio of critical exit pressure to entrance stagnation pressure) for various L/Ds and entrance subcoolings are presented and discussed. Steady-state critical flow was compared to "blowdown" critical flow, and it was found that a "blowdown" can be approximated by a succession of steady states. Visual observations of flow regimes are related to entrance stagnation conditions, and photographs of entrance flow regimes are also presented. The data are compared to four well-known critical-flow models.

I. INTRODUCTION

Critical flow in a constant-area duct is that flow phenomenon in which the mass flux is independent of the pressure downstream of the exit plane. During all other types of fluid flow, the mass flux increases with increasing pressure differences across the duct. If the fluid is a gas, one-phase critical flow occurs when a sonic condition exists at the exit plane, thus prohibiting any transmission of pressure reduction into the duct and altering the flow conditions. If the fluid is a liquid, two-phase critical flow might be observed if the exit pressure is below the saturation pressure. This flow condition occurs in cascade drain pipes of turbines, cryogenic coolant flow, and rocket-nozzle coolant systems. A complete literature survey through 1968 of two-phase critical flow can be found in Refs. 2 and 9.

Critical-flow studies have been prompted by safety aspects of both water-cooled^{6,10} and sodium-cooled reactors.⁷ The literature at present covers two-phase critical flow in the high-pressure ranges typical of water-cooled reactors quite well. However, current developments in sodium-cooled reactors have contributed to renewed interest in low-pressure ranges.

Two-phase Critical Flow

One method of describing one-component, two-phase critical flow is to describe single-phase critical flow and draw analogies between the two flows. The left half of Fig. 1 shows two large adiabatic reservoirs of gas connected by a thermally insulated constant-area duct. Initially, both are at the same pressure. If one reservoir is lowered in pressure while the other remains constant, flow will be initiated. Steady state would be reached when the acceleration and frictional pressure drops along the tube were equal to the pressure difference between the reservoirs. A pressure gradient would develop within the duct, causing the density of the gas to decrease and the velocity to increase as it moves along the length. Pressure profile "b" of Fig. 1 shows this condition. Further pressure reductions in the downstream reservoir pressure would cause still higher mass fluxes and higher velocities until the gas reaches its sonic velocity at the exit plane (pressure profile "c"). But now further pressure reductions in the downstream reservoir will not affect the mass flux since the lower pressure cannot be transmitted through the exit plane to change the pressure gradient. Flow at the exit plane is now commonly referred to as critical or choked flow. The lower two sketches in the left half of Fig. 1 show the choked condition. This is represented mathematically as

$$\left. \frac{\partial G}{\partial h} \right|_s = 0 . \quad (1)$$

In Fanno flow, as is being considered here, the properties at the exit plane can be evaluated isentropically because the entropy has reached a maximum value.¹⁴ Since the receiver pressure cannot be transmitted upstream through the exit plane, it is the compressibility of the gas which determines the critical mass flux. A general form for the critical mass flux of a perfect gas is¹⁸

Fig. 1. Analogy between Single-phase and Two-phase Critical Flows.

$$G_c = \sqrt{\frac{-g_c}{\left(\frac{\partial v}{\partial P}\right)_s}} \quad (2)$$

Now the analogy between single-phase critical flow and one-component two-phase critical flow will be made. The right half of Fig. 1 shows the upstream reservoir containing a nearly saturated liquid. Holding the pressure and temperature in this reservoir constant and lowering the downstream reservoir pressure would initiate flow. Consider the flow to be such that departure from thermodynamic equilibrium is negligible. The dashed lines of Fig. 1 show the actual pressure, and the solid lines indicate the saturation pressure of the liquid. When the downstream pressure is lowered below saturation pressure, vaporization takes place, causing the average density of the mixture to decrease. If the densities of the phases differ by orders of magnitude, as is typical for pressures far removed from the thermodynamic critical point, the effect of even small quantities would greatly increase the average specific volume of the mixture. As in the single-phase case, the velocity and the specific volume of the mixture increase as it passes along to lower pressures, and a pressure gradient would exist along the length of the duct. In this case, however, the pressure gradient must include components for the phase change as well as for friction. Continuing the pressure reduction would bring the system to a choked condition, $(\partial G/\partial h)_s = 0$ (profile c in Fig. 1). As with the single-phase case, reductions in the receiver pressure beyond this point would not affect the mass flux.

The two-phase compressible system is more complex than the corresponding single-phase system because of the extra degree of freedom associated with the additional phase. This allows for the possibility of differences in velocities between the phases and, for a one-component system, differences in the relative amounts of the phases present. These differences are strongly influenced by flow regime, that is, the geometric configuration of liquid and vapor in the flow channel. The three most general flow regimes encountered in two-phase flow are the bubbly, slug, and annular flow regimes.

The bubbly-flow pattern is characterized by a suspension of discrete bubbles in a continuous liquid.

The slug-flow pattern is characterized by a series of individual large bubbles which almost fill the available flow cross section, separated axially by elements of the continuous liquid phase.

The annular-flow pattern is characterized by a continuous film flowing along the sides of the pipe while the vapor flows continuously in a central core.¹⁹ These flow patterns can be seen in Fig. 16.

The ideal system discussed above to compare single-phase and one-component two-phase critical flow is similar to the experimental apparatus used in this study. Freon-11 was the fluid, and precision-bore glass tubing was the constant-area duct.

Purpose of this Study

Data for one-component two-phase critical flow have been obtained at low pressures. The data include entrance and exit pressures as well as critical mass flux. These data are compared with well-known critical-flow models. Some of the data have been obtained in glass test sections where visual flow observations were possible; the remainder of the data were obtained in a stainless steel test section where pressure and temperature profiles were recorded. The length/diameter (L/D) parameter is not found in any of the critical-flow models discussed here. Several values of this parameter have been tested and the data compared with calculations from these models. Four different entrance configurations were also tested. Observations were made as to the effects of changes in this parameter upon the critical mass flux and the extent to which the fluid departed from thermodynamic equilibrium. Data were also obtained by different testing procedures. Some data were recorded from steady-state conditions, and other data were obtained from tests with varying entrance pressure (blowdown test). By comparing both sets of data, one can then determine if a "blowdown" test can be approximated by a sequence of steady-state conditions. The experimental facility used in this investigation is described in Ch. III.

II. MATHEMATICAL MODELS

Several models for two-phase critical flow in a constant-area duct have been developed,^{2,8,9,13,15} and three well-known models will be discussed here. All are consistent with the critical-flow criterion, Eq. 1, for steady-state flow and lead to an expression analogous to Eq. 2 for the critical mass flux. Explicit evaluation of Eq. 2 depends upon the particular assumptions that one chooses.

Steady, one-dimensional, two-phase, compressible flow in a horizontal constant-area duct can be described by the following differential equations, which are valid without considerations of flow patterns. After a general form of the equilibrium critical-flow equation has been developed, assumptions will be made to reduce this general form to a more applicable form.

The mass flow rate of the gas and liquid phases are defined in terms of the one-dimensional average velocity as

$$\dot{m}_g = \rho_g \alpha A u_g \quad (3)$$

and

$$\dot{m}_l = \rho_l (1 - \alpha) A u_l \quad (4)$$

The void fraction α is that fraction of the duct cross-sectional area A occupied by the vapor phase. The mass balance for both phases is then written as

$$\frac{d}{dz} \left\{ \rho_g \alpha u_g + \rho_l (1 - \alpha) u_l \right\} = 0 \quad (5)$$

Momentum and energy balances for both phases can also be written:

$$\frac{d}{dz} \left[\rho_g \alpha u_g^2 + \rho_l (1 - \alpha) u_l^2 \right] + \frac{4}{D} \tau_w + \frac{dP}{dz} = 0 \quad (6)$$

and

$$\begin{aligned} \frac{d}{dz} \left[\rho_g \alpha u_g h_g + \rho_l (1 - \alpha) u_l h_l \right] + \frac{d}{dz} \left[\rho_g \alpha \frac{u_g^3}{2} + \rho_l (1 - \alpha) \frac{u_l^3}{2} \right] \\ - \frac{d\dot{q}}{dz} = 0 \end{aligned} \quad (7)$$

If the total mass flux G is defined as

$$G = \frac{\dot{m}_t}{A} = \frac{\dot{m}_g + \dot{m}_l}{A} , \quad (1)$$

and the velocity of each phase is related to G by

$$u_g = \frac{x}{\alpha \rho_g} G \quad (9)$$

and

$$u_l = \frac{(1-x)}{(1-\alpha)\rho_l} G , \quad (10)$$

where x is the vapor mass fraction, the momentum equation (6) can then be written in the convenient general form

$$G^2 \frac{d}{dz} \left[\frac{x^2}{\alpha \rho_g} + \frac{(1-x)^2}{(1-\alpha)\rho_l} \right] + \frac{4\tau_w}{D} + \frac{dP}{dz} = 0 . \quad (11)$$

Further, if the terms in the brackets of Eq. 11 are defined as the two-phase specific volume:

$$v_{2\phi} = \frac{x^2}{\alpha \rho_g} + \frac{(1-x)^2}{(1-\alpha)\rho_l} , \quad (12)$$

then the momentum equation can be rewritten in a form that appears analogous to single-phase compressible flow:

$$G^2 \frac{dv_{2\phi}}{dz} + \frac{4}{D} \tau_w + \frac{dP}{dz} = 0 . \quad (13)$$

Further similarities with single-phase compressible flow can be illustrated by rearranging Eq. 13 to obtain a relation for the pressure gradient and noting these facts about Eq. 12. The two-phase specific volume is clearly some function of α , x , ρ_l , and ρ_g :

$$v_{2\phi} = \text{Func}_1(\alpha, x, \rho_l, \rho_g) ;$$

since ρ_l and ρ_g are evaluated at the saturation pressure, we may write

$$v_{2\phi} = \text{Func}_2(\alpha, x, P) .$$

If there is no change of flow regime in the region of interest, the void fraction and the quality will at most be a function of pressure only. Therefore, the derivative of $v_{2\phi}$ with respect to position is

$$\frac{dv_{2\phi}}{dz} = \text{Func}_3(P) .$$

Now, noting that

$$\frac{dv_{2\phi}}{dz} = \left(\frac{dv_{2\phi}}{dP} \right) \left(\frac{dP}{dz} \right) ,$$

Eq. 13 can be rewritten in the form

$$\frac{dP}{dz} = \frac{-4\tau_w/D}{\left[1 - G^2 \left(\frac{-dv_{2\phi}}{dP} \right) \right]} . \quad (14)$$

The $v_{2\phi}$ of Eq. 14 can be considered to be a function of two independent intensive properties. However, in the Fanno flow considered here, the entropy has reached a maximum value at the exit plane. Therefore, the derivative of Eq. 14 can be evaluated at constant entropy. Equation 14 clearly has a singularity when the flow rate has a value of

$$G_c = \left[\frac{-1}{\left[\frac{dv_{2\phi}}{dP} \right]} \right]^{\frac{1}{2}} . \quad (15)$$

The above relation is consistent with Eq. 2 and can serve as a general definition of two-phase critical flow rate:

$$G_c = \left\{ - \frac{d}{dP} \left[\frac{x^2}{\alpha \rho_g} + \frac{(1-x)^2}{(1-\alpha)\rho_l} \right] \right\}^{-\frac{1}{2}} . \quad (16)$$

Differences between various models are found specifically in the physical reasoning which establishes: 1) a relationship between the local quality x and the void fraction α ; 2) the extent of thermodynamic equilibrium or local variation of quality x with pressure; 3) the local variation of phase velocity with pressure. Often the latter variation is presented in terms of the ratio of phase velocities, that is, the slip ratio k .

Homogeneous Equilibrium Model

The classic critical-flow model is the homogeneous equilibrium model which assumes:

- 1) the vapor and liquid phases always remain in thermodynamic equilibrium, and
- 2) the vapor and liquid phases travel at the same velocities.

Under the second assumption above, the ratio of Eqs. 3 and 4 yields

$$\frac{x}{1-x} = \frac{\alpha}{1-\alpha} \frac{\rho_g u_g}{\rho_l u_l} = \frac{\alpha}{1-\alpha} \frac{\rho_g}{\rho_l} \quad (17)$$

or simply

$$\alpha = \frac{xv_g}{xv_g + (1-x)v_l} \quad (18)$$

By use of Eq. 18, the two-phase specific volume of Eq. 12 becomes

$$v_{2\phi} = xv_g + (1-x)v_l \quad (19)$$

The expression for the critical flow rate is derived directly from Eqs. 15 and 19:

$$G_c^2 = \frac{-1}{\left[x \frac{dv_g}{dP} + (v_g - v_l) \frac{dx}{dP} + (1-x) \frac{dv_l}{dP} \right]_s} \quad (20)$$

The terms $\frac{dv_g}{dP}$ and $\frac{dv_l}{dP}$ must be evaluated along the saturation line, whereas the $\frac{dx}{dP}$ term can be evaluated isentropically or from the thermodynamic relations^{dp}

$$Td\epsilon = dh - vdP \quad (21)$$

For a process of constant entropy

$$dh = vdP$$

or

$$dh_f + xdh_{fg} + h_{fg}dx = vdP \quad (22)$$

Far from the thermodynamic critical point, $dh_{fg} = 0$, so that

$$C_p dT + h_{fg} dx = v dP \quad (23)$$

or

$$\left. \frac{dx}{dP} \right|_s = \frac{v}{h_{fg}} - \frac{C_p}{h_{fg}} \frac{dT}{dP} \quad (24)$$

From the Clapeyron equation one obtains

$$\frac{dT}{dP} = \frac{v_{fg} T}{h_{fg}} \quad (25)$$

Combination of Eqs. 24 and 25 gives

$$\left. \frac{dx}{dP} \right|_s = \frac{v_f + x v_{fg}}{h_{fg}} - \frac{C_p v_{fg} T}{h_{fg}^2} \quad (26)$$

Then one can readily evaluate Eq. 20 by use of tabulated thermodynamic properties.¹

Slip Equilibrium Model

High-pressure water data were obtained by Fauske² for exit qualities from 0.01 to 0.70. These data showed that the homogeneous equilibrium model worked well at high pressures and high qualities. However, the accuracy decreased with decreasing exit quality. At an exit quality of 30%, the homogeneous equilibrium model predicted a flow rate of 50% the actual mass flux. Fauske developed the slip equilibrium model to correlate his data.

The basic assumptions of the slip equilibrium model are:

- 1) The phases are considered to be in thermodynamic equilibrium, and
- 2) the phases are considered to travel at different velocities.

The first assumption is generally valid in long tubes where the fluid resides within the tube long enough to approach equilibrium. As a result of the second assumption, the term "slip" is introduced, which is defined as

$$k = u_g / u_l \quad (27)$$

Again, the relation between void fraction and quality x is written from Eqs. 17 and 27:

$$\alpha = \frac{xv_g}{xv_g + (1-x)v_l k} \quad (28)$$

Fauske showed that minimization of the specific volume, Eq. 12 (which leads to a maximum in the expression for critical-flow rate Eq. 16), with respect to slip ratio occurs when

$$k = \sqrt{\frac{\rho_l}{\rho_g}} \quad (29)$$

By use of Eq. 28, Eq. 12 becomes

$$v_{2\phi} = \left(xv_g + (1-x)v_l k \right) \left(\frac{1}{k} + \frac{k-1}{k} x \right) \quad (30)$$

Combining Eqs. 29 and 30 with Eq. 16 gives the following expression for the critical flow rate in the slip equilibrium model:

$$\begin{aligned} G_{crit}^2 = -k \left[\left\{ (1-x+kx)x \right\} (dv_g/dP) + \left\{ v_g(1+2kx-2x) \right. \right. \\ \left. \left. + v_l(2xk-2k-2xk^2+k^2) \right\} (dx/dP) + \left\{ k \left[1+x(k-2) \right. \right. \right. \\ \left. \left. \left. - x^2(k-1) \right] \right\} (dv_l/dP) \right]^{-1} \quad (31) \end{aligned}$$

The question might arise as to whether one should evaluate the terms of Eq. 31 isentropically or isenthalpically. The phase change is assumed to be an equilibrium process and the system adiabatic. However, the system has different phase velocities and a nonzero friction factor. Irreversibilities will be present, which rules out a constant entropy process. Fauske chose to consider the process as isenthalpic because of this reason. In practice, the difference between the two calculations is small.

Homogeneous Nonequilibrium Model

Bubbly-flow regimes are usually associated with low-quality critical flow. Such a flow regime would have a slip close to unity. However, the slip equilibrium model assumes a slip equal to the square root of the density ratio at all qualities. Therefore, at low qualities, the assumed

process and the actual physical process would differ and, hence, shortcomings of the model should be expected at such conditions.

High-pressure data obtained by Henry⁹ showed the slip equilibrium model to predict too low a mass flux at qualities <0.01. A homogeneous bubbly mixture was assumed by Henry, and a mathematical model was developed to describe the steam-water data he obtained for exit qualities less than 2%.

The basic assumptions of this model are:

- 1) the vapor and liquid phases travel at nearly equal velocities,
 $u_g \doteq u_l$;
- 2) the quality of the mixture is not the thermodynamic equilibrium as determined from Eq. 7, but is related empirically to the equilibrium quality according to

$$x_{act} = kNx_{equilibrium} \quad (32)$$

In Eq. 32, k is the slip ratio and N is an empirically determined constant. The general expression for critical flow, Eq. 16, and Eq. 28, can be combined to obtain

$$\begin{aligned} G_c^2 = & - \left\{ k \left[\left[1 + x(k-1) \right] x \right] \frac{\partial v_g}{\partial P} \right|_{h_o} \\ & + \left\{ v_g \left[1 + 2x(k-1) \right] + kv_l \left[2(x-1) + k(1-2x) \right] \right\} \frac{\partial x}{\partial P} \Big|_{h_o} \\ & + \left\{ k \left[1 + x(k-2) - x^2(k-1) \right] \right\} \frac{\partial v_l}{\partial P} \Big|_{h_o} \\ & + x(1-x) \left(kv_l - \frac{v_g}{k} \right) \frac{\partial k}{\partial P} \Big|_{h_o} \right\}^{-1}. \end{aligned} \quad (33)$$

The process is considered isenthalpic, and the differentials are evaluated as such.

An estimate of the orders of magnitude involved can reduce Eq. 33. At moderate pressures:

$$\frac{v_l}{v_g} < 10^{-2}$$

and

$$\frac{\frac{\partial v_g}{\partial P}}{\frac{\partial v_g}{\partial P}} < 10^{-3}$$

Hence, Eq. 33 becomes

$$G_c^2 = -1/k \left[\left[1 + x(k-1) \right] x \frac{\partial v_g}{\partial P} \Big|_{h_o} + v_g \left[1 + 2x(k-1) \right] \frac{\partial x}{\partial P} \Big|_{h_o} - x(1-x) \frac{v_g}{k} \frac{\partial k}{\partial P} \Big|_{h_o} \right] \quad (34)$$

Now, due to Eq. 32, the term $\partial x / \partial P$ becomes

$$\frac{\partial x}{\partial P} = kN \frac{\partial x_{eq}}{\partial P} \Big|_{h_o} + kx_{eq} \frac{\partial N}{\partial P} \Big|_{h_o} + Nx_{eq} \frac{\partial k}{\partial P} \Big|_{h_o}.$$

Henry showed that order-of-magnitude estimates can eliminate the dk/dP term. He invoked experimental data to show that for flow in long tubes $dN/dP \neq 0$ at the exit or choked plane.

Under these conditions the expression for critical flow of the homogeneous nonequilibrium model becomes

$$G_c^2 = -1/k \left\{ \left[1 + Nkx_{eq}(k-1) \right] Nkx_{eq} \frac{\partial v_g}{\partial P} \Big|_{h_o} + v_g \left[1 + 2Nkx_{eq}(k-1) \right] \frac{\partial x_{eq}}{\partial P} \Big|_{h_o} \right\} \quad (35)$$

The term $\partial x_{eq} / \partial P \Big|_{h_o}$ is evaluated isentropically since, as before, the entropy has $\partial v_g / \partial P \Big|_{h_o}$ reached a maximum and is constant. The quantity $\partial v_g / \partial P \Big|_{h_o}$ is not an equilibrium term and therefore is not evaluated in this manner. If one considers the process to be polytropic with a polytropic exponent of unity, the following relationships can be applied:

$$Pv = \text{constant}; Pdv + vdp = 0; \frac{dv}{dp} = -\frac{v}{p} \quad (36)$$

During homogeneous flow, the slip ratio is very close to unity and this value is assumed. By combining Eqs. 35 and 36, the critical mass flux becomes

$$G_c^2 = \frac{+1}{N \left\{ \frac{x_{eq} v_g}{P} - v_g \left. \frac{\partial x_{eq}}{\partial P} \right|_s \right\}_e} \quad (37)$$

The only term of Eq. 37 yet to be discussed is N , which was introduced in Eq. 32. Henry determined this term by measuring the void fraction during critical flow. The value which he obtained was $N = 20x_{eq}$ for his water system. One would expect different values of N for different fluids; $N = 4x_{eq}$ was used with the Freon-11 data. Justification of this choice of N is given in Appendix G.

III. EXPERIMENTAL APPARATUS AND PROCEDURE

The test facility shown in Fig. 2 consisted of four basic units. The limitations and functions of each unit will be individually discussed. This chapter also describes the test procedure for both the glass and the stainless steel test sections.

Fig. 2. Experimental Apparatus. ANL Neg. No. 900-1321 (Rev. 1).

APPARATUS SECTION

Refrigeration Unit and Low-pressure Reservoir

A reservoir capable of maintaining a low pressure was the largest component of this test facility. A schematic of this vessel and the attached refrigeration unit are shown in Fig. 3. The refrigeration unit was necessary in order to maintain a temperature lower than the saturation temperature of the Freon which would enter the vessel. Since the saturation temperature of Freon-11 vaporizing at 2.5 psia, the lower limit of exit pressures in this study, is 0°F, the vessel had to be kept at least this cold. This was accomplished with a closed-circuit cooling system and

*Fig. 3.
Schematic of Refrigeration
Unit and Low-pressure
Reservoir.*

refrigeration coils wrapped around the low-pressure reservoir. Ethylene glycol solution was cooled in the refrigeration unit, pumped through the coils which were wrapped around the low-pressure vessel, and returned to the refrigeration unit. The low-pressure vessel itself was constructed of stainless steel and was cylindrical in shape, approximately 4 ft wide and 5 ft long. The low-pressure reservoir was separated from the test section by a 2 in. ball valve, which can be seen in Fig. 4.

Freon Reservoir

The upstream reservoir, shown in Fig. 2, stored the Freon just prior to its passage through the test section. Also shown in Fig. 2 is the movable rack which supported the reservoir in a vertical position. This reservoir was a 6-in.-ID by 60-in.-long glass pipe with a reducing glass tee attached to the bottom. The entire vessel was wrapped in fiberglass insulation to reduce heat transfer. One branch of the glass tee contained the filler valve, base pressure transducer, and base thermocouple. The temperature of the Freon, measured by this thermocouple, was controlled in the following manner. If the temperature was too low, it could be increased by means of heating tapes wrapped around the vessel. These heating tapes were positioned between the vessel wall, and the insulation and the power to the tapes was controlled by a Variac. If the temperature was too high, it could be reduced by lowering the pressure within the vessel, causing adiabatic vaporization of the Freon within the vessel. When the desired temperature was obtained, the dome pressure was increased to a value greater than the saturation pressure of the Freon to prevent further vaporization. The other branch of the glass tee held the test-section mounting plate (see Fig. 4). The schematic in Fig. 7 below also shows the Freon reservoir with the apparatus just described.

Freon-11 was used exclusively in the tests for the following reasons:

Fig. 4. 0.0995-in.-ID Glass Test Section.
ANL Neg. No. 900-1320 (Rev. 1).

- 1) Freon-11 is nontoxic and nonflammable.
- 2) Its thermodynamic properties are tabulated and readily available.
- 3) A near-room-temperature atmospheric boiling point reduces the amount of insulation required to prevent significant heat transfer.
- 4) The photographic qualities of the two-phase mixture are good.
- 5) Freon-11 has been used before for critical flow studies, and the data are available for comparison.

Glass Test Sections

Four glass test sections with inner diameters ranging from 0.0995 to 0.250 in. and L/D ratios from 25 to 300 were used to obtain critical mass flux, critical exit pressure, and flow-regime data. The entrances of all the test sections, including the stainless steel test section, protruded into the Freon liquid to form a reentrant geometry, but since the tube walls were thick, a sharp-edged entrance can be assumed. A fifth glass test section of 0.094-in.-ID was used for flow-regime observations. The glass test sections were similar in construction, their bodies being composed of precision-bore glass tubing. The upstream end was connected to the Freon reservoir by a modified Conax fitting; the downstream end diverged with a 22° total included angle to form a 15-mm glass joint. Each test section had a pressure tap located at the junction of the test-section body and diffuser. Figure 4 shows the 0.0995-in.-ID test section mounted in position. The receiver pressure transducer, which was located 4 in. downstream of the exit plane, can be seen in that figure. Both exit pressure and receiver pressure transducers were kept at 200°F to prevent Freon liquid from entering by capillary action.

Pressure measurements at the exit plane were part of the data to be taken in this study. As mentioned above, four glass test sections were equipped with pressure taps at the exit plane. Although every precaution was taken to minimize the error in pressure measurement, there were certain problems which could not be avoided:

- 1) Difficulty was encountered in boring the hole in the glass tubing. The exit tap hole was ultrasonically drilled in each test section. A small area increase caused by chipping around the inside edges of the tap hole as the drill bit broke through the tube wall would result in too high a pressure reading. An area increase of 10% is possible, which would cause an error of 0.5 psi.* The actual pressure is a function of mass flux squared (refer to Appendices C and F).

* By use of the Bernoulli theorem, an area increase would decrease the velocity and increase the pressure.

- 2) A steep pressure gradient existed at the exit plane during critical flow (refer to "Pressure Profile Section" of Ch. IV). The pressure variation across the tap itself was estimated at 0.5 psi. Although the tap was placed as close as possible to the actual exit plane, a mispositioning of one tap diameter results in an error of ± 0.5 psi.
- 3) Kelly¹¹ showed that the two-dimensional effects at the exit plane during critical flow could be minimized by means of a small angle (7°) diffuser. Each test section with an exit-plane pressure tap was equipped with a 22° diffuser. The amount of residual error should be considered to be -1.0 psi, that is, the actual centerline pressure may have been as much as 1.0 psi greater than the recorded value. This error is a function of the compressibility of the mixture.¹¹

When one considers the above possible errors a tolerance of -1.5 to $+1.0$ psi should be assumed. Calculations in Appendix C show the exit-pressure error to be within these limits. However, if one views the exit-pressure data in a different manner, more confidence can be placed in them. Errors due to chipping and mispositioning of the pressure tap hole would be common to all data taken from that test section. It is for this reason that data from a particular test section are grouped together, thus revealing the trends in critical exit pressure more readily.

Stainless Steel Test Section

A fully instrumented stainless steel test section was used to obtain simultaneous pressure and temperature data for critical flow. Experimental values from 10 pressure transducers and 15 thermocouples were recorded on strip charts. These values included stagnation pressure and temperature as well as 9 pressures and 14 temperatures along the length of the test section. Figure 5 shows the test section with the pressure-tap and

Fig. 5. Stainless Steel Test Section.

thermocouple locations, indicated respectively by integers and capital letters. All the transducers were mounted at the same height as the test section to eliminate any hydrostatic head errors. The entire apparatus was covered with insulation to minimize heat transfer. The total included angle of the test-section diffuser was 14° . Freon was able to enter the test section directly from the reservoir without passing through any valves. The test section is identical to that of Fauske and Grolmes³ for critical-flow studies of

sodium. Freon data from this test section is tabulated in Appendix D. The accuracy of pressure measurement was better with this test section than with the glass test sections because of the machineability of stainless steel, which was not possible with glass.

Instruments and Photography

All pressure, temperature, and mass-flux data were taken with two-pen strip-chart recorders. The pressures at the entrance, exit plane, downstream reservoir, and dome (gas pressure above the Freon liquid level) were measured with pressure transducers and recorded directly onto strip charts. All pressure transducers were calibrated each day with a 0-25-psia Heise gauge. The entrance temperature was measured with a chromel-alumel-sheathed thermocouple, the tip of which was positioned approximately 2 in. from the test-section entrance. The mass fluxes were measured by direct visual observations of Freon liquid level in the Freon reservoir. Timing marks were made on each of the recorders when the Freon liquid level passed inch-mark graduations on the side of the Freon reservoir. The distance between timing marks represented the time to pass a known volume of Freon through the test section.

A strobe light was synchronized with the shutter of a Polaroid camera to permit the photographing of the flow regimes at the entrance (see Fig. 10 below). For these photographs, the entrance of the test section was placed in a specially constructed glass fixture. Two opposite branches of a 2-in.-ID glass cross were shortened and capped with optically flat glass plates. During photographing, the test-section entrance was positioned between the glass caps, and the strobe light was mounted below while the camera viewed the entrance through the flat plates. In all tests conducted, the Freon was able to enter the test section without passing through any valves or other restrictions.

Pressure Control System

Test-section stagnation pressure may be thought of as having two components:

- 1) the gas pressure above the Freon liquid, also called dome pressure, and
- 2) the hydrostatic head pressure of the Freon liquid itself (a maximum height of 50 in. or 2.7 psi).

In steady-state test runs, the entrance pressure was held constant by increasing the dome pressure an amount equal to the decrease in the hydrostatic head pressure. Figure 6 shows how the dome pressure changed during a typical test run to maintain a constant entrance pressure. This was accomplished in the following manner.

A schematic of the major components of the pressure control system is shown in Fig. 7. A reference pressure chamber was pressurized to a desired entrance stagnation pressure. A Foxburo differential pressure transducer (D/P cell) compared the reference pressure and the actual entrance pressure. If the pressures were equal, a null signal was sent to the Bailey Controller and no further action was taken. If, on the other hand, the reference pressure was higher than the actual entrance pressure, a high signal was sent to the Bailey Controller. Upon receiving this signal, the controller increased the dome pressure. The entrance pressure

Fig. 6. Pressure Profiles of Typical (a) Steady-state and (b) Quasi-steady Test Runs.

Fig. 7. Schematic of Pressure Control System and Entrance Configurations.

would then increase until it equaled the reference pressure, at which point the D/P cell sent a null signal. Similar action was taken in lowering the dome pressure. If the D/P cell sensed the entrance pressure as too high, it was reduced by the Bailey Controller. After control instabilities were corrected, the above system worked well with a maximum variation of entrance pressure of 0.05 psi.

For the test run shown in Fig. 6, the hydrostatic head pressure decreased by 0.5 psi (10-in. liquid level drop in the Freon reservoir) while the dome pressure increased from 16.5 to 17.0 psia, with a resulting subcooling of 11.5°F. The only necessary requirement to make a run with low subcooling was to have the liquid level in the Freon reservoir as low as possible. For example, if a test with 2°F subcooling were desired (from Appendix B, Table II: 25 L/D, 0.250-in. ID, $P_0 = 15.2$ psia, $T_0 = 75^\circ\text{F}$), the dome pressure was initially at 14.7 psia and the Freon liquid level 10 in. above the test-section entrance. During the test, as the liquid dropped, the dome pressure was slowly increased from 14.7 to 15.2 psia, thus maintaining the constant entrance pressure.

PROCEDURE SECTION

The testing procedure used in this study insured that all the data taken were critical-flow data. At the end of each test run, the receiver pressure (pressure downstream of the exit) was raised, and no noticeable change in exit pressure was observed. The independence of the exit pressure on variation of the receiver pressure was taken as experimental evidence of choked two-phase flow. When this was verified, the data were recorded. As mentioned in Ch. I, the effects of density ratio on mass flux were to be investigated. By choosing two different stagnation temperatures, data could be obtained in such a manner as to maximize the experimental range of this parameter. The temperatures were chosen with the limitations of the Freon reservoir as the controlling factor. Freon-11 at 40°F has a saturation pressure of 7.0 psia and a density ratio of 520. Therefore, in adiabatic two-phase critical flow, the exit-plane pressure would have to be less than 7.0 psia and the density ratio greater than 520. The other entrance temperature chosen was 75°F with a saturation pressure of 14.7 psia. Similarly, the exit pressure would have to be less than P_{sat} and the density ratio greater than 250. These two entrance temperatures were used in most of the tests conducted here.

The testing method for all the tests can be separated into two groups: 1) steady state, and 2) quasi-steady state. In steady-state tests, the entrance pressure and temperature were held constant throughout the test. The upstream Freon reservoir was insulated to reduce temperature changes. The mass flux was measured by recording the amount of time required for a known amount of Freon to pass through the test section. All the data listed in the appendices are steady-state data.

In quasi-steady tests, the stagnation temperature was held constant, but the entrance pressure was varied slowly by maintaining a constant dome pressure and letting the head pressure decrease. Figure 6 shows the pressure profile for a typical quasi-steady test. Instantaneous mass fluxes were obtained by observing the time required for the Freon liquid level to

dropped by 1 in. In this manner, the mass flux was monitored continuously. Data from such runs are given as graphs such as in Fig. 12 below, rather than as individual points.

Procedure with Glass Test Section

The testing procedure for each glass test section was as follows:

The test section was initially tested with four different entrance configurations at the largest value of L/D . After these tests are conducted, the test section was cut in half and tested at this new value of L/D . This procedure was repeated for all values of L/D of interest.

The four different entrance configurations shown in Fig. 7, were the same for all glass test sections. Initially, a sharp entrance was tested, obtained by sawing the glass tube with a carbide saw. Next, the entrance was fire polished to remove the sharp edges but retain the abrupt nature of the entrance. Then, the entrance was flared to provide as smooth and gradual an opening as possible. Finally, this flared or bell mouth was mechanically chipped to provide possible nucleation sites. The chipping was done in a glass blaster using 60- μ glass beads.

Procedure with Stainless Steel Test Section

Only steady-state tests were conducted in a stainless steel test section. A predetermined entrance pressure was set and flow started. Data were not recorded until the system had reached steady state. Sufficient recorders were not available to record simultaneously all thermocouples and transducers. A switching arrangement was developed to record temperatures and pressures at selected positions on the test section. After steady state was reached, all the readings were recorded in sequence. Readings were then recorded again; if the second readings were the same as the first, steady state was assured and the measurements considered valid.

IV. EXPERIMENTAL RESULTS

The effects of entrance subcooling, L/D , and entrance configuration on critical flow are given in this chapter. The experimental data for the glass and stainless steel test sections are presented in Table I. A complete listing of steady-state tests in which exit pressures were measured is given in Appendices B and D.

TABLE I. Experimental Test Data

Test-section L/D	Temp, (°F)	Exit Pressure (psia)	Critical Mass Flux (lb _m /ft ² -sec)	Equilibrium Exit Quality (Z)	Density Ratio at Exit (ρ_l/ρ_g)
25	40	5.3 - 6.7	432 - 1500	3.0 - 0.4	680 - 540
	75	10.7 - 12.8	701 - 1573	4.2 - 1.9	330 - 290
50	40	4.5 - 7.1	330 - 1500	4.7 - 0.1	795 - 510
	75	9.4 - 13.6	499 - 1666	5.6 - 1.2	390 - 270
100	40	3.7 - 7.3	300 - 1140	6.6 - 0.0	990 - 490
	75	7.6 - 13.3	405 - 1360	7.4 - 1.1	480 - 280
200	40	3.1 - 5.3	281 - 888	7.3 - 4.7	1140 - 680
	75	5.7 - 12.1	346 - 1198	10.4 - 2.6	630 - 390
300	40	-----	-----	-----	-----
	75	4.5 - 9.0	246 - 495	12.7 - 6.6	795 - 400
Stainless	40	5.8 - 6.7	635 - 1388	2.2 - 1.0	620 - 540
Steel(44)	75	11.8 - 12.7	931 - 2101	2.7 - 2.1	315 - 290

Entrance Subcooling

Different entrance subcoolings were obtained by using a constant entrance temperature and different entrance pressures. The subcooling, with units of °F, were measured along the saturation curve, that is, $\Delta T_{sub} = T_{sat} - T_o$. The following effects of entrance subcooling on critical flow were observed in glass test sections with sharp entrances; the effect of subcooling on critical mass flux with other entrance configurations is given in the "Effects of Entrance Configuration" section.

Regardless of the value of L/D , critical mass flux increased with increasing entrance subcooling (see Fig. 8). Also shown in that figure is a dashed line connecting some of the data for $L/D = 200$. It was in the subcooling range depicted by that line that two-phase flow could not be initiated. With the 200- L/D test section, two-phase flow ceased when the entrance subcooling was increased above 8°F or decreased below 18°F.

Consider critical flow with various L/D s and entrance pressures but with a constant entrance temperature. Since two-phase critical mass flux

Fig. 8. Critical Mass Flux vs Entrance Subcooling.

Transitions in the entrance flow regime can be seen in Fig. 8. At entrance subcoolings less than the transition subcooling, the test section had a vapor annulus at the entrance; a photograph of such a flow regime can be seen in Fig. 10-A. This was the only entrance flow regime observed in the 300-L/D test section. The vapor annulus at the entrance had an all-liquid core surrounded by vapor or two-phase mixture. When the subcooling was zero, the vapor annulus was longest, approximately five or six diameters; increasing the entrance pressure shortened the length of the annulus. At entrance subcooling greater than the transition subcooling as indicated by Fig. 8, the entrance was of the bubbly type (see Figs. 10-B, -C, and -D). This was the only entrance flow regime observed in the 25-L/D test section.

is determined at the exit plane, one would expect similar flow conditions at the exit plane at equal mass fluxes. Changes in the entrance pressure will strongly affect the critical mass flux with a short tube. However, when the L/D becomes very long, changes in entrance pressure do not affect the exit-plane pressure as much. Extrapolation to the extreme case, shows that the exit conditions would be independent of the entrance pressure in an infinitely long duct. This can be seen to some degree in Figs. 8 and 9. The effect of entrance subcooling (entrance pressure) on the critical mass flux and critical exit pressure are seen to be less with the 300-L/D tubes than with the others.

Fig. 9. Critical Exit Pressure vs Entrance Subcooling.

CRITICAL
MASS FLUX:
480 $\text{lb}_m/\text{ft}^2\text{-sec}$
PRESSURE:
 $P_0 - P_{\text{sat}} = 0.3 \text{ psi}$

599 $\text{lb}_m/\text{ft}^2\text{-sec}$
 $P_0 - P_{\text{sat}} = 1.5 \text{ psi}$

859 $\text{lb}_m/\text{ft}^2\text{-sec}$
 $P_0 - P_{\text{sat}} = 3.5 \text{ psi}$

1215 $\text{lb}_m/\text{ft}^2\text{-sec}$
 $P_0 - P_{\text{sat}} = 7.5 \text{ psi}$

← FLOW
DIRECTION

$L/D = 120$ $ID = 0.250 \text{ in.}$

Fig. 10. Entrance Flow Regimes. ANL-Neg. No. 900-1322.

The density ratio $\rho_e/\rho_v|_e$ at the exit plane for test sections of various values of L/D can be seen in Fig. 11. This ratio was determined at

Fig. 11. Density Ratio at Exit Plane vs Entrance Subcooling.

mately 1/2-in. in diameter, the flow returned to single-phase flow, no matter where the heat was applied.

For large entrance subcoolings and for tubes shorter than $L/D = 300$, bubbles were formed at the inner edges of the test section. Figures 10-B, -C, and -D show photographs of such a condition; it can also be seen that the diameter of the bubbles decreased with increasing entrance subcooling. If a two-phase flow regime, either bubbly or annular, was not found at the entrance, then two-phase critical flow could not be initiated.

Figure 12 compares the steady-state data with quasi-steady data from the same test section. The critical mass fluxes from both sets of data agree with each other within experimental accuracy. Comparison of other quasi-steady and steady-state data show similar results. This indicates that these "blowdown" critical-flow tests can be approximated by a succession of steady states.

ID and L/D Effects

The graph in Fig. 13 shows the effects of ID on critical mass flux in tubes for which $L/D = 100$. For small subcoolings at the entrance, the two smaller-ID test sections have lower critical mass fluxes than the two larger ones. As the entrance subcooling increased, however, the differences become smaller than experimental error. If one were to compare similar data for the test sections with $L/D = 25$, no difference in the critical mass flux

the measured exit-plane pressure for the same data shown in Fig. 9. For a given quality the average two-phase specific volume increased with the density ratio. The higher specific volume was indicative of annular or slug flow, which was observed for $L/D = 300$ and at low subcoolings for $L/D = 200$.

Self-sustained critical flow in tubes with $L/D = 300$ and greater could only be obtained with entrance subcoolings $< 15^\circ\text{F}$.

With entrance subcoolings greater than 15°F , all-liquid flow was observed with superheats as high as 80°F (see Appendix A). (The liquid superheat was obtained because of the gradual depressurization of the Freon in the absence of nucleation sites.) In an effort to obtain critical flow during such a test, heat was applied to the test section, causing two-phase flow downstream of that point. Upon removal of the heat, which was supplied by a flame approxi-

Fig. 12. Results of Quasi-steady Tests.

Fig. 13. Critical Mass Flux vs. Entrance Subcoolings for Various IDs.

would be found. One possible explanation of this would be the existence of different flow regimes in tubes of different ID due to the relative importance of surface tension and viscosity. In the tube with $L/D = 25$, the observed flow regime was always bubbly flow while in tubes with $L/D \geq 100$, slug flow was observed for small subcooling at the entrance. In long tubes, the major portion of the pressure drop in the test section pressure, changes from frictional losses to entrance losses with increasing critical mass flux. It is proposed that at low entrance subcoolings (low critical mass flux) surface tension caused a slug-flow regime to exist in the smaller-ID tubes, whereas a bubbly-flow regime existed in the larger-ID test sections. Since the frictional pressure drop is important at low critical mass fluxes, the total test-section pressure drop would be significantly changed if the friction factor were changed. However, as the mass flux increases, the influence of friction becomes less and the differences become smaller.

The ID effect exhibited in Fig. 13 is for tubes in the capillary range; there is no evidence for any ID effect in larger tubes. This is because the importance of surface tension can only be realized in very small ID tubes. The upper limit of the ID for the capillary range for Freon is less than approximately 0.1 in. The modeling parameter of L/D should still be considered valid in tubes above the capillary range.

In order to study the effect of variation of L/D , a

particular test section was tested, then shortened and the tests repeated. The following effects of L/D were observed:

- 1) Figure 8 shows the effect of L/D upon the critical mass flux for the data reported here and the data of Fauske and Min.⁴ It can be seen that increasing the tube length (that is, increasing the value of L/D), decreases the mass flux for equal entrance conditions. Since entrance pressure drops are proportional to the square of the mass flux, these losses would be smaller for the longer tube, in which the mass flux is smaller. Upon examining Fig. 9, however, one can see that the total test-section pressure drop is greater for the longer tube. This indicates that the pressure drops due to friction and changes in specific volume are larger.
- 2) The effects of L/D on critical exit pressure are most pronounced with entrance subcoolings $<10^\circ\text{F}$. Decreasing the entrance subcooling increases the void fraction and, consequently, the friction factor. Since the frictional pressure drop is proportional to the length, the greatest effect will be seen when the friction factor is at its highest level, that is, at low entrance subcoolings. Conversely, at high entrance subcoolings and low void fractions, the effects of friction are small and the critical exit pressures tend to group together.
- 3) Shorter tubes sustained a vapor annulus at the entrance at higher entrance subcoolings than longer tubes.

Effects of Entrance Configuration

Each of the glass test sections was tested with four different entrance shapes (see Fig. 7). All the test sections were of the reentrant-type geometry, that is, all the test-section entrances protruded into the Freon reservoir. Sharp and fire-polished entrances did not affect the type of flow regime at the entrance. The flow regime at the entrance appears to be more dependent upon stagnation conditions and L/D than on geometry. It should be noted that the fire-polished entrance produced a slightly higher exit pressure and critical mass flux than the sharp-edged entrance.

Pasqua^{15,16} studied the effects of entrance configuration upon critical flow in short tubes. He observed that the slightest rounding at the entrance increased the mass flux for equal entrance conditions. This seems reasonable since in short tubes the flow is effectively determined at the entrance. To a small extent this phenomena was observed here in tubes with $L/D = 25$ (see Fig. 14). The effect of fire polishing is negligible at low entrance subcoolings with a tube for which $L/D = 25$, but increases with entrance subcooling. There does not appear to be any effect in tubes with values of L/D greater than 50. Figure 14 shows the data for sharp and fire-polished entrances when Freon-11 was initially at 40°F ; however, the effect at 75°F is more pronounced.

The bell-mouthed entrance, formed by flaring the fire-polished entrance into a gradual opening, and the glass-blasted entrance, which had the same shape as the bell mouth but with a roughened surface to provide nucleation sites, never produced two-phase critical flow for all the L/D

Fig. 14.
Effects of Entrance Configuration
for 0.25-in.-ID Tubes and Freon-11
Initially at 40°F.

and ID combinations used here. By using a bell mouth with $L/D = 300$ and greater, considerable amounts of liquid superheat was obtained (see Appendix A).

Flow Regimes at the Entrance

All the critical-flow data obtained here were associated with initial vapor production at the entrance. This vapor could have been Freon or dissolved nitrogen gas coming out of solution. The entrance flow regime during critical flow was either bubbly or annular.

An annular flow regime at the entrance was observed during critical flow with low entrance subcooling. The length of the vapor annulus surrounding an all-liquid core decreased with increasing entrance subcooling. A flow-regime transition from annular to bubbly was caused by increasing the entrance subcooling in tubes with $L/D > 50$. The amount of subcooling needed to cause the transition varied with L/D (see Fig. 8). Photographs of these entrance flow regimes are shown in Fig. 10.

An oscillating flow regime at the entrance was observed in the fire-polished test section with $L/D = 200$ and with Freon at 42°F. Figure 12 shows how the exit pressure varied with entrance subcooling during the quasi-steady test run. The measured frequency of 1.7 Hz, a period of 590 msec, can be related to the transit time of the single-phase and two-phase mixtures. Figure 15 shows the probable shape of the mass flux and the exit pressure for one period. For $0 \leq \text{time} \leq t_1$, the flow at the exit plane was all liquid, the mass flux was very high, and vapor was produced at the entrance. At time t_1 , the first bubble produced at the entrance reached the exit. At this instant the mass flux decreased due to the step

Fig. 15. Mass Flux during Cyclic Flashing.

change in the density of the fluid reaching the exit, and vapor production ceased at the entrance because of the lower entrance losses. At time t_2 , the all-liquid flow reached the exit, causing the mass flux to increase and vapor production to start at the entrance again.

The value of $G_{1\phi}$ was found from the single-phase pressure-drop equation and a knowledge of the total pressure drop. The two-phase mass flux was calculated with the Slip Equilibrium Model for the measured exit pressure. The time during which there was all liquid at the exit plane was found from the average velocity ($u = G_{1\phi} / \rho_l$) and the test-section length. The integral of the proposed mass flux over one period is approximately equal to the measured mass flux. It should be noted that this phenomenon was not observed in the same test section

with a sharp entrance; however, the phenomena was reproducible.

Observations of Flow Regimes

Observations of the flow regime were made in a glass tube with $L/D = 413$ and of 0.094-in. inner diameter. Figure 16 depicts observations made with this tube, which had a sharp entrance. During critical flow in long tubes, the flow-regime transition always appeared in the sequence bubbly to slug to annular. These transitions only occurred at low mass fluxes and could be suppressed by decreasing L/D or increasing the entrance subcooling. The bubbly-slug interface was considered to be that point where two bubbles were no longer seen abreast of one another; bubble size at that point was greater than half of the diameter of the test section. The slug-annular interface was considered to be that point where most of the slugs broke up. A strobe light was used to help find the interfaces but precise locations were not possible. In Fig. 16 the interface locations should be considered to be $\pm 20 L/D$ and the mass fluxes to be $\pm 10\%$.

Further observations of the flow regime made with this test section were:

- 1) Immediately downstream of the entrance, numerous small bubbles grew in size. These small growing bubbles grouped together to form a fewer number of larger growing bubbles.
- 2) Slugs were initially paraboloid in shape.

Fig. 16. Flow Regimes in Long Tubes.

- 3) Slugs grew from one diameter in length to 10 diameters.
- 4) Visual observations were made concerning the length of the vapor slug just upstream of the slug-annular interface. The vapor-slug length was found to be slightly longer than the liquid-slug length.
- 5) The slug finally developed a long-leading tapered tip, expanding to nearly full diameter and a slightly rounded tail.
- 6) Increasing the entrance subcooling caused the bubble-slug and the slug-annular interface to move toward the exit.
- 7) When the slug-annular interface reached the exit, due to increased entrance subcooling, all-liquid flow immediately followed.

The last two observations deserve further comment. As was mentioned earlier in the "Entrance Flow Regime" section, increasing the entrance subcooling caused the vapor annulus at the entrance, if one existed, to decrease in length. In tubes with $L/D \geq 200$, increasing the subcooling not only decreased its length but actually collapsed it. The increased entrance subcooling was also directly related to the observed flow regimes. With increased subcooling, the initially smaller bubbles caused the bubbly-flow regime to exist further downstream. This then moved the bubbly-slug interface further downstream, leaving less tube length for the slug and annular flows, which have higher friction factors. The mass flux was then correspondingly increased. Continuing to increase the entrance subcooling eventually eliminated the annular flow regime entirely. In fact, when the slug-annular interface approached to within 30-50 L/D of the exit, it accelerated to the exit, resulting in a flow-regime transition to single-phase flow. One possible reason for the flow changing to all liquid is that the decreased frictional pressure drop increased the mass flux just enough to cause the vapor annulus at the entrance to be drawn into the tube. Another possible explanation is that the increased entrance pressure was too great for the existence of vapor at the entrance at these low mass fluxes, and the flow returned to all liquid. It is not known which phenomena caused the other, that is, if the change of entrance flow regime caused the total flow to become all liquid or if the decreased frictional pressure drop increased the mass flux and drew the entrance vapor into the test section.

Critical Pressure Ratio

The critical pressure ratio (CPR), which is the ratio of the critical exit pressure to the entrance stagnation pressure, has been determined from the data obtained here. Although the CPR itself was not experimentally measured, it will be presented here along with the other experimental results; a discussion of the results is in Ch. V.

CPR data derived from results with two different test sections are shown in Fig. 17. The following can be deduced from these data:

Fig. 17.
Critical Pressure Ratio Variation with
Increasing Stagnation Pressure for
Constant Initial Freon-11 Temperature
of $75 \pm 2^\circ\text{F}$.

- 1) The CPR exhibited a clear maximum with respect to entrance subcooling for all test-section L/Ds except that with 25.
- 2) The peak in the CPR curve appears to shift to higher entrance subcooling with increasing L/D.
- 3) The CPR is lower with a larger L/D at equal entrance conditions than with a smaller L/D. The effect appears to be greater on the left side of the peak.

Temperature and Pressure Profiles

A stainless steel test section used in this study was instrumented with pressure transducers and thermocouples both upstream and downstream of the exit plane. A diagram of this test section is shown in Fig. 5. The data tabulated in Appendix D are representative of data obtained with the stainless steel test section. Other tests were performed, but their results were redundant. The data are, therefore, considered reproducible and accurate.

Pressure profiles (see Fig. 18) from the stainless steel test section show evidence of the presence of some vapor upstream of the point where the fluid passes below its saturation pressure. Calculations of the single-phase pressure drop, similar to those in Appendix E, show too high a pressure drop for the fluid to be all liquid. If thermodynamic equilibrium is assumed to exist between the phases, any Freon vapor formed at the entrance would have been condensed. An alternative possibility is that nitrogen gas comes out of solution due to depressurization.

Figure 19 shows the results of a test in which the entrance pressure P_0 was much greater than P_{sat} . It is seen that the saturation temperature

Fig. 18. Experimental Pressure Profiles.

Fig. 19. Experimental Temperature Profiles.

remained above the actual temperature until the vicinity of the exit plane. In this adiabatic system, a temperature drop would indicate vapor production. Substantial vapor production is not indicated by Fig. 19 until the fluid pressure is below saturation.

Figure 19 also shows the results of a test in which the entrance pressure was close to saturation. In this case, vapor production occurred earlier, but not until the pressure was below saturation.

V. DISCUSSION OF RESULTS AND CONCLUSIONS

Entrance Subcooling

Figure 8 indicates the existence of a seemingly linear relationship between the entrance subcooling and the critical mass flux. Griebel⁸ proposed a relationship between the mass flux and the entrance stagnation conditions of the form

$$G_c = C_c \sqrt{2\rho(P_0 - P_{sat})} g_c, \quad (38)$$

where C_c is the contraction coefficient at the entrance. In the development of this model, a vapor annulus was assumed to exist at the entrance of the test section, and the pressure within the annulus was assumed to be equal to the vapor pressure of the liquid. It was postulated that the all-liquid core was accelerated by the pressure difference $P_0 - P_{sat}$. This model was developed from data obtained in test sections ranging in L/D from 6 to 48. The observed entrance flow regime was of the annular type. In Fig. 20, calculated results from this model are compared with data obtained

Fig. 20.
Correlation between Mass Flux
and Entrance Subcooling for
Freon-11 Temperature of
75 ± 2°F.

here in test sections with L/D s of 25 and 200. For $L/D = 25$, a vapor annulus was the observed entrance flow regime, and this simple model worked well. It should also be remembered that at these low L/D s the flow is effectively determined at the entrance, that is, the entrance pressure drop is greater than the frictional pressure drop. The contraction coefficient of 0.612 was the one proposed by Griebe.

For comparison, results obtained by use of this model are also shown with data obtained for a test section with $L/D = 200$. In such a long test section, the frictional pressure drop will be greater than the entrance pressure drop; hence, the mass flux cannot be determined by examining the entrance conditions only. It should also be noted that the assumed entrance flow regime did not always exist in this test section. If the contraction coefficient of Eq. 38 were considered to have the form

$$C_c = \left(1 + \frac{1}{K_c} + \frac{4fL}{D} \right)^{-1}, \quad (39)$$

a different value of C_c can be assigned to tubes with different L/D s. The first, second, and third terms of Eq. 39 are respectively entrance momentum, entrance loss, and friction. If the friction is negligible as was the case for Greibe, the original value of 0.612 is obtained. If, however, the tube is long and friction is not negligible, the value of C_c will be smaller. This very simple approach was applied to the data from the 200- L/D test section; the result is shown in Fig. 20

Comparison with Models

The three critical-flow models of Ch. II will now be compared with the data obtained here (see Figs. 21 and 22). Since these models do not use L/D as a parameter, the data have been plotted as mass flux vs exit pressure at constant entrance enthalpy. Calculations from the Homogeneous Equilibrium Model have been plotted in both figures for the sake of comparison. The difference between the actual mass flux and the value predicted by the Homogeneous Equilibrium Model is seen to decrease with decreasing exit pressures.

Figure 21 shows that predictions of the Slip Equilibrium Model are more accurate at lower than at higher exit pressures, that is, the Slip Equilibrium Model appears to yield more accurate predictions at lower entrance subcoolings and for longer tubes. These conditions are more conducive to higher exit qualities and, hence, a more annular type of flow. As the exit pressure approaches P_{sat} , however, the quality decreases and the Slip Equilibrium Model begins to predict too low a mass flux. The data compared with the calculations based on the Slip Equilibrium Model were obtained from test sections with L/D values of 50, 100, and 200.

In the Homogeneous Nonequilibrium Model equal phase velocities are assumed. For a bubbly-flow regime differences of phase velocities would be small or negligible. Flow-regime observations in this study showed bubbly-flow regimes near the exit when the exit pressure was near saturation

Fig. 21.
Interpretation of Data in Terms of the
Slip and Homogeneous Equilibrium
Models for 0.162-in.-ID Test Section.

Fig. 22.
Interpretation of Data in Terms of the
Homogeneous Equilibrium and Homogeneous
Nonequilibrium Models for 0.250-in.-ID
Test Section.

pressure. Such low-quality bubbly-flow regimes were the result of high entrance subcooling.

The data shown in Fig. 22 were obtained from test sections with L/D values of 25, 50, and 100. These test sections were shorter than the test sections used to obtain the data shown in Fig. 21. In addition, a stainless steel test section with L/D = 44 was used.

The shorter test sections would not have as large a frictional pressure drop and, hence, would have lower qualities. The Homogeneous Nonequilibrium Model was initially proposed for exit qualities of less than 2% for water, but is being compared here to data of 6% quality for Freon-11. It is indicated in Fig. 22 that N has been set equal to $4X_e$. Henry experimentally determined the value of this parameter for his water data to be $N = 20X_e$, using the relationship

$$N = \frac{\alpha V_1}{X_e (1 - \alpha) V_g} \quad , \quad (40)$$

and measuring the void fraction and pressure at the exit. Since the void fraction was not measured here, the value of $N = 4X_e$ was chosen on the basis of calculations similar to those in Appendix G.

Critical Pressure Ratio

We now discuss the effects of entrance subcoolings and L/D on the CPR. Referring to Fig. 17, first consider the effects of entrance subcooling. For the 25-L/D test section the CPR remains constant and then decreases with entrance subcooling. The test sections with L/D = 50 and greater show a definite maximum.

For the sake of this discussion, let us consider the 25 L/D test section to be "short" and those with L/D = 50 and larger as "long." The CPR can be written as

$$CPR = \frac{P_e}{P_o} = \frac{P_e}{P_e + \Delta P_{total}} \quad (41)$$

where

$$\begin{aligned}
\Delta P_{\text{total}} &= \Delta P_{\text{entrance acceleration}} + \Delta P_{\text{entrance loss}} \\
&+ \Delta P_{\text{specific volume change}} + \Delta P_{\text{two-phase friction}} \\
&= \frac{G^2}{2\rho} \left[1 + K_c + 2\rho \Delta v_{\ell} + \frac{4fL}{D} \right] \\
&= \frac{G^2}{2\rho} \left[1 + K_c + 2\rho v_{\ell} \left[\frac{1}{1-\alpha} - 1 \right] + \frac{4fL}{D} \left(\frac{1}{1-\alpha} \right) \right]. \quad (42)
\end{aligned}$$

The change in specific volume has been approximated as $\{[1/(1-\alpha)] - 1\}v_{\ell}$, which implies homogeneous flow.¹² Clearly, the two latter terms of Eq. 42 are a function of α , whereas the first two are constants. The magnitude of α was found experimentally to be a function of both entrance subcooling and L/D . The value of α increased with increasing L/D and decreasing entrance subcooling. Therefore, the two latter terms will be at their maximum value for long tubes at low entrance subcoolings.

Consider a long tube at low entrance subcooling. Figure 9 shows the critical exit pressure increases with increasing entrance subcooling. In order for the CPR to increase with entrance subcooling, it can be seen from Eq. 41 that ΔP_{total} must remain constant while the exit pressure or entrance subcooling is increased. However, Eq. 42 shows that ΔP_{total} increases as the mass flux squared. Since increasing the entrance subcooling also increases the mass flux, one would expect ΔP_{total} to increase. Since ΔP_{total} was found not to increase under the conditions of long tubes with low entrance subcooling, the sum of all the terms in the brackets of Eq. 42 must have decreased at least as fast as $1/G^2$, that is, if

$$\frac{1}{1-\alpha} \approx G^{-2},$$

ΔP_{total} would remain constant even though the mass flux were increasing. Clearly this can only be the case for flow with a high void coefficient. A large value of α would result in a large value for $1/(1-\alpha)$. As α becomes small, $1/(1-\alpha)$ cannot change very much and ΔP_{total} would be almost directly proportional to G^2 . This would cause the CPR to decrease.

The above argument can be seen graphically in Fig. 9. The total test-section pressure drop can be found from that figure by subtracting the exit pressure from the entrance pressure. In short tubes ΔP_{total} increases continuously whereas in the long tubes, ΔP_{total} initially decreases and then, as entrance subcooling is farther increased, ΔP_{total} increases. Since the CPR is defined as P/P_o , and both P and P_o vary with subcooling, the trend of CPR with subcooling is not obvious. If we denote CPR as η , the relationship between the trends of CPR of Fig. 17 and the entrance and exit pressures of Fig. 9 is

$$\frac{\partial \eta}{\partial \Delta T_{\text{sub}}} = \frac{\partial}{\partial \Delta T_{\text{sub}}} \left\{ \frac{P_e}{P_o} \right\}$$

If T_o is a constant, P_e can be approximated as

$$P_e = P_{\text{sat}} - \Delta P_{\text{flow}} - \Delta P_e e^{-\lambda \Delta T_{\text{sub}}},$$

where ΔP_{flow} is the saturation pressure minus the exit pressure as the subcooling increases indefinitely. It can be seen from Fig. 9 that ΔP_{flow} increases with L/D and that ΔP_e , which is the change in critical exit pressure with subcooling, is also a function of L/D . The parameter λ has dimensions of $^{\circ}\text{F}^{-1}$; since the data tend to converge together, λ can at most be only a weak function of L/D . Over a small pressure range, the saturation pressure can be adequately approximated as

$$P_{\text{sat}}|_{T_2} = P_{\text{sat}}|_{T_1} + C(T_2 - T_1);$$

hence,

$$P_o = P_{\text{sat}} + C \Delta T_{\text{sub}},$$

where C is determined by the physical properties of the fluid. By using these two approximations and differentiating η with respect to entrance subcooling, we obtain

$$\begin{aligned} \frac{\partial \eta}{\partial \Delta T_{\text{sub}}} &= \frac{\partial}{\partial \Delta T_{\text{sub}}} \frac{P_e}{P_o} \\ &= \frac{\partial}{\partial \Delta T_{\text{sub}}} \frac{P_{\text{sat}} - \Delta P_{\text{flow}} - \Delta P_e e^{-\lambda \Delta T_{\text{sub}}}}{P_{\text{sat}} + C \Delta T_{\text{sub}}} \\ &= \frac{1}{P_{\text{sat}} + C \Delta T_{\text{sub}}} \left\{ \lambda \Delta P_e e^{-\lambda \Delta T_{\text{sub}}} \right. \\ &\quad \left. - \frac{C (P_{\text{sat}} - \Delta P_{\text{flow}} - \Delta P_e e^{-\lambda \Delta T_{\text{sub}}})}{P_{\text{sat}} + C \Delta T_{\text{sub}}} \right\}. \end{aligned}$$

In order for η to increase with subcooling, the difference of the terms in the braces must be positive; hence, for a given subcooling

$$\frac{\partial \Delta P_e}{\partial T_{sub}} e^{-\lambda \Delta T_{sub}} > \frac{P_{sat} - \Delta P_{flow} - \Delta P_e e^{-\lambda \Delta T_{sub}}}{P_{sat} + C \Delta T_{sub}} = \frac{P_e}{P_o} = \eta .$$

The term $(1/C) \Delta P_e e^{-\lambda \Delta T_{sub}}$ can be considered as the ratio of the exit to the entrance pressure gradient, where differentiation is with respect to subcooling. This is the ratio of the slope of the exit and entrance pressures of Fig. 9. For example, the use of this criterion and the 200-L/D data from Fig. 9 at zero entrance subcooling gives the following

$$\Delta P_{flow} = 2.5 \text{ psi}$$

$$\Delta P_e = 6 \text{ psi}$$

$$\lambda = 0.1/^\circ\text{F}$$

$$c = 0.33 \text{ psi}/^\circ\text{F}$$

$$\frac{\lambda \Delta P_e}{C} \Big|_{\Delta T_{sub} = 0} = \frac{(0.1)(6)}{0.33} = 1.8 > \frac{P_e}{P_o} = \frac{6.1}{14.7}$$

Figure 17 shows η to be increasing at this entrance subcooling. Now consider the same test section but at 20°F entrance subcooling:

$$\frac{\lambda \Delta P_e}{C} \Big|_{\Delta T_{sub} = 20} = \frac{(0.1)(0.1)}{0.35} = 0.03 \nless \eta .$$

Figure 17 shows that at this entrance subcooling η is decreasing.

Conclusions

- 1) No differences between test data from steady-state and quasi-steady state were observed.
- 2) The two smaller-ID test sections appeared to have lower mass fluxes at low entrance subcoolings than the two larger-ID test sections. It is speculated that this is a surface-tension effect.

- 3) A streamlined entrance was conducive to all-liquid flow. Considerable amounts of liquid superheat were observed with smooth entrances.
- 4) If initial vapor was not generated at the entrance, all-liquid flow resulted.
- 5) The Slip Equilibrium Model was accurate at low entrance subcoolings. The range of subcooling where accuracy was good increased with increasing values of L/D .
- 6) The Homogeneous Nonequilibrium Model accurately predicted critical mass flux at high entrance subcoolings. The range in which accuracy was good increased with decreasing L/D .
- 7) Measurements of the critical pressure at the exit plane showed a clear maximum of the CPR for tubes with $L/D \geq 50$.

APPENDIX A

Sample Calculations of All-liquid Test Runs

A series of tests were conducted in order to find an accurate friction factor for the glass test sections. The following experimental data were obtained:

Test fluid: Freon-11 at 57°F

Test section L/D = 300

Test section ID = 0.0995 in.

$\mu = 1.11 \text{ lb}_m/\text{hr-ft}$

Exit pressure = 14.34 psia (vented to atmosphere)

P_o (psia)	G ($\text{lb}_m/\text{sec-ft}^2$)	ΔP_{act} (psi)
17.88	562	3.45
20.31	751	5.97
23.30	955	8.96

The single-phase pressure-drop equation is¹⁷

$$\Delta P_{1\phi} = \frac{G^2}{2\rho} \left[1 + K_c + \frac{fL}{D} \right]. \quad (\text{A.1})$$

In evaluating Eq. A.1, the entrance loss coefficient K_c was set equal to 0.5. A thick-walled reentrant duct is equal to a sharp entrance. Since this flow is in the turbulent-flow regime, the von Karman friction-factor correlation seems appropriate:

$$\frac{1}{\sqrt{f}} = 1.74 - 2. \log \frac{18.6}{R_e \sqrt{f}}. \quad (\text{A.2})$$

The following table shows the calculated results using these two equations:

R_e	$\left[1 + K_c + \frac{fL}{D}\right]$	ΔP_{cal} (psi)
15043	4.785	3.54
20301	4.476	5.95
25619	4.252	8.99

The above calculations show Eqs. A.1 and A.2 to work well in this system. During later tests when the exit pressure was not experimentally measured, this equation could be used to calculate it. Such calculations can be used to find the amount of superheat that this system could produce. The experimental data of such a run are:

$$L/D = 362$$

$$ID = 0.094 \text{ in.}$$

$$F-11 \text{ temperature} = 80^\circ F$$

$$\mu = 0.992 \text{ lb}_m/\text{ft-hr}$$

P_o (psia)	G $\text{lb}_m/\text{ft}^2\text{-sec}$	R_e	$\left[1 + K_c + \frac{fL}{D}\right]$	P_{cal} (psi)	P_e (psia)
18.36	1277	36604	4.14	15.84	2.52

The above calculation shows the liquid Freon at $80^\circ F$ to be depressurized to 2.52 psia, corresponding to a superheat of $80^\circ F$. Superheats of this magnitude were common with test sections of at least $L/D = 300$.

III. 25 L/D; 0.250-in. ID; Fire-polished Entrance

P_o (psia) (± 0.1 psi)	T_o ($^{\circ}$ F) (± 0.1 $^{\circ}$ F)	P_e (psia) (± 1.0 psi)	G_c (lb _m /ft ² -sec) ($\pm 10\%$)
8.0	41.5	5.4	432
9.0	43.0	6.1	684
10.0	41.0	6.45	930
11.0	39.0	6.4	1115
12.0	40.0	6.6	1370
13.0	40.0	6.6	1470
14.0	40.0	6.7	1500
15.1	75.0	10.9	761
18.12	75.0	12.00	956
19.1	75.0	12.50	1231
21.3	75.0	12.75	1573

The 25-L/D, 0.250-in.-ID data for both the sharp and fire-polished entrances were observed with a vapor annulus at the entrance and a bubbly-flow regime in the tube.

IV. 50 L/D; 0.0995-in. ID; Sharp Entrance

P_o (psia) (± 0.1 psi)	T_o ($^{\circ}$ F) (± 0.1 $^{\circ}$ F)	P_e (psia) (± 1.0 psi)	G_c (lb _m /ft ² -sec) ($\pm 10\%$)
8.15	40.0	5.0	394 Bubbly Flow
9.0	40.0	5.8	648 Bubbly Flow
10.55	40.0	5.9	1000 Bubbly Flow
12.20	43.5	6.6	1135
13.33	41.5	6.45	1350
15.35	74.5	9.4	499
16.10	76.3	10.3	540
17.00	76.7	11.4	714
17.95	76.7	12.0	832
18.7	76.7	12.4	958 Vapor Annulus
19.95	76.7	12.7	1156
20.9	76.7	12.8	1287
21.8	76.7	12.8	1412
22.65	76.7	12.8	1510 Bubbly Flow
23.35	77.0	12.85	1575 Bubbly Flow

APPENDIX B

Steady-state Data for Glass Test SectionI. 25 L/D; 0.121-in. ID; Sharp Entrance

P_o (psia) (± 0.1 psi)	T_o ($^{\circ}$ F) (± 0.1 $^{\circ}$ F)	P_e (psia) (± 1.0 psi)	G_c (lb/ft ² -sec) _m ($\pm 10\%$)
8.65	41.0	6.3	577
9.85	40.0	6.7	880
11.0	39.0	6.9	1072
12.35	41.0	7.55	1220
13.6	40.0	7.75	1423
15.55	72.5	12.0	729
16.5	75.2	12.75	820
18.2	76.0	13.9	969
19.6	76.0	14.4	1155
20.8	75.5	14.7	1313
22.0	75.5	15.0	1454
23.2	75.5	15.3	1558

II. 25 L/D; 0.250-in. ID; Sharp Entrance

P_o (psia) (± 0.1 psi)	P_o ($^{\circ}$ F) (± 0.1 $^{\circ}$ F)	P_e (psia) (± 1.0 psi)	G_c (lb/ft ² -sec) _m ($\pm 10\%$)
7.6	40.0	5.3	460
8.5	40.0	5.5	653
9.0	40.0	5.95	780
10.0	39.3	6.2	920
11.0	38.5	6.3	974
11.5	40.0	6.5	1145
12.5	39.5	6.6	1280
13.8	39.3	6.7	1450
15.2	75.0	10.7	701
16.6	75.0	11.7	863
18.35	75.0	12.2	1099
20.0	75.0	12.58	1319
21.8	75.0	12.82	1519

V. 50 L/D; 0.162-in. ID; Sharp Entrance

P_o (psia) (± 0.1 psi)	T_o (°F) (± 0.1 °F)	P_e (psia) (± 1.0 psi)	G_c (lb _m /ft ² -sec) ($\pm 10\%$)
7.55	40.0	4.65	368
8.7	41.4	5.75	618
9.3	41.0	5.95	748
9.9	41.0	6.2	852
11.8	50.0	7.6	858
12.5	48.0	7.5	1008
13.0	46.0	7.35	1173
13.5	45.0	7.1	1253
14.0	43.5	6.9	1404
14.7	73.0	9.5	559
15.5	74.0	10.5	683
16.0	74.0	10.8	715
16.5	75.5	11.2	757
17.35	75.5	11.75	804
17.3	75.9	11.55	770
18.0	75.7	12.05	874
18.5	75.6	12.35	910
19.0	75.5	12.55	1000
19.75	75.0	12.8	1112
20.6	74.3	12.7	1234
21.05	74.5	12.8	1325 Bubbly Flow
21.5	74.5	12.85	1325 Bubbly Flow
22.0	75.0	12.9	1430 Bubbly Flow
22.5	75.0	13.0	1453 Bubbly Flow
23.0	75.5	13.1	1528 Bubbly Flow

VI. 50 L/D; 0.250-in. ID; Sharp Entrance

P_o (psia) (± 0.1 psi)	T_o ($^{\circ}$ F) (± 0.1 $^{\circ}$ F)	P_e (psia) (± 1.0 psi)	G_c (lb _m /ft ² -sec) ($\pm 10\%$)
7.45	39.0	5.0	577
8.45	39.0	5.35	701
9.45	40.2	5.9	820
10.45	39.8	6.05	967
11.45	39.8	6.15	1114
12.45	40.0	6.25	1254
13.45	40.0	6.3	1345
17.95	74.7	11.85	952
18.45	76.0	12.25	1009
18.95	74.6	12.35	1055
19.45	75.6	12.37	1153
19.95	74.7	12.37	1217
20.45	75.2	12.37	1324

VII. 50 L/D; 0.250-in. ID; Fire-polished Entrance

P_o (psia) (± 0.1 psi)	T_o ($^{\circ}$ F) (± 0.1 $^{\circ}$ F)	P_e (psia) (± 1.0 psi)	G_c (lb _m /ft ² -sec) ($\pm 10\%$)
8.0	40.9	5.15	561 Vapor Annulus Entrance
9.0	40.6	5.9	682 Entrance Regime Transition
10.0	40.6	6.2	848 Bubbly Entrance
11.0	40.6	6.25	1087 Bubbly Entrance
12.0	40.0	6.4	1200 Bubbly Entrance
13.0	39.5	6.4	1290 Bubbly Entrance
14.0	38.8	6.4	1498 Bubbly Entrance
15.4	74.2	10.3	631 Vapor Annulus Entrance
16.0	74.9	10.8	703 Vapor Annulus Entrance
16.5	75.0	11.3	770 Vapor Annulus Entrance
17.0	75.0	11.7	857 Vapor Annulus Entrance
17.5	75.0	11.98	928 Vapor Annulus Entrance
17.95	74.5	12.15	995 Bubbly Entrance
19.2	74.5	12.33	1173 Bubbly Entrance
20.5	76.0	12.54	1451 Bubbly Entrance
21.0	76.0	12.65	1499 Bubbly Entrance
21.4	74.5	12.43	1483 Bubbly Entrance
21.5	76.1	12.58	1519 Bubbly Entrance
22.0	76.2	12.62	1551 Bubbly Entrance
22.5	76.3	12.7	1666 Bubbly Entrance

VIII. 100 L/D; 0.0995-in. ID; Sharp Entrance

P_o (psia) (± 0.1 psi)	T_o ($^{\circ}$ F) (± 0.1 $^{\circ}$ F)	P_e (psia) (± 1.0 psi)	G_c (lb/ft ² -sec) _m ($\pm 10\%$)
15.15	72.3	7.6	405 Almost Slug Flow
17.90	75.7	11.0	646 Bubbly Flow
19.45	75.5	12.0	832 Bubbly Flow
20.90	75.5	12.5	1112 Bubbly Flow
21.65	75.5	12.6	1141 Bubbly Flow
22.45	75.0	12.5	1240 Bubbly Entrance
23.40	75.0	12.5	1337 Bubbly Flow

IX. 100 L/D; 0.121-in. ID; Sharp Entrance

P_o (psia) (± 0.1 psi)	T_o ($^{\circ}$ F) (± 0.1 $^{\circ}$ F)	P_e (psia) (± 1.0 psi)	G_c (lb/ft ² -sec) _m ($\pm 10\%$)
8.15	39.0	4.7	325 Annular Entrance
8.4	39.0	5.6	403 Annular Entrance

Critical flow could not be obtained above 9.0 psia at 40°F.

15.35	72.5	8.65	411 Annular Entrance
18.7	76.2	12.85	722 Annular Entrance
20.0	76.2	13.75	925 Bubbly Entrance
21.1	76.0	14.05	1050 Bubbly Entrance

Critical flow could not be obtained between 21.9 and 23.3 psia at 76°F.

A transition from slug flow to annular flow was observed in the 100 L/D 0.121 in. ID test section. This transition took place at 41 ± 3 L/D upstream of the exit with an entrance pressure of 8.15 psia and at 16 ± 3 L/D with an entrance pressure of 8.4 psia.

X. 100 L/D; 0.162-in. ID; Sharp Entrance

P_o (psia) (± 0.1 psi)	T_o ($^{\circ}$ F) (± 0.5 $^{\circ}$ F)	P_e (psia) (± 1.0 psi)	G_c (lb _m /ft ² -sec) ($\pm 10\%$)
7.5	40.0	3.7	300
8.5	43.0	5.2	400
9.5	39.5	5.7	700
10.0	39.5	5.8	940
14.5	72.0	7.8	460
15.5	73.5	9.4	520
16.8	75.5	10.4	598
17.6	75.5	11.1	718
18.5	76.0	12.15	790
19.3	75.5	12.35	870
21.2	75.5	13.05	1084
22.35	75.5	13.25	1195 Bubbly Flow
23.5	75.5	13.2	1340

XI. 100 L/D; 0.250-in. ID; Sharp Entrance

P_o (psia) (± 0.1 psi)	T_o ($^{\circ}$ F) (± 0.5 $^{\circ}$ F)	P_e (psia) (± 1.0 psi)	G_c (lb _m /ft ² -sec) ($\pm 10\%$)
8.43	40.5	5.13	515
9.16	40.0	5.43	646
10.0	41.0	5.8	750
11.0	41.0	6.0	850
12.0	41.0	6.1	950
13.0	41.0	6.3	1030
14.0	41.0	6.35	1140
15.85	75.0	9.5	562
16.3	75.3	9.8	600
17.2	75.5	10.6	700
18.3	75.5	11.65	830
19.5	75.0	11.95	962
20.3	75.5	12.05	1020
21.7	75.5	12.25	1180
22.3	75.5	12.45	1300
23.3	75.5	12.45	1360

XII. 100 L/D; 0.250-in. ID; Fire-polished Entrance

P_o (psia) (± 0.1 psi)	T_o (°F) (± 0.1 °F)	P_e (psia) (± 1.0 psi)	G_c (lb _m /ft ² -sec) ($\pm 10\%$)
8.0	39.0	5.1	450 Vapor Annulus Entrance
9.0	41.0	5.2	550 Vapor Annulus Entrance
10.0	40.0	6.0	580 Vapor Annulus Entrance
11.0	40.0	6.5	770 Bubbly Entrance
12.0	41.5	7.0	800 Bubbly Entrance
13.0	41.0	7.2	930 Bubbly Entrance
14.0	40.5	7.3	1000 Bubbly Entrance
15.86	75.0	9.5	575
16.40	75.0	10.1	624
17.66	73.2	10.95	633
19.12	73.6	12.4	778
20.77	74.0	13.3	960

XIII. 200 L/D; 0.162-in. ID; Sharp Entrance

P_o (psia) (± 0.1 psi)	T_o (°F) (± 0.1 °F)	P_e (psia) (± 1.0 psi)	G_c (lb _m /ft ² -sec) ($\pm 10\%$)
7.7	41.6	3.1	418
8.0	41.6	3.7	598
8.1 - 14.0 psia; only single-phase flow could be obtained.			
14.2	72.8	6.15	346
15.0	73.4	7.5	419
16.15	73.9	9.1	516
16.9	74.6	9.37	569
17.2 - 20.5 psia; only single-phase flow could be obtained.			
21.0	75.3	12.07	983
21.5	75.1	12.02	1038
21.5	74.0	11.96	1090
22.0	74.0	11.98	1119
22.5	74.0	12.0	1141
23.0	74.2	12.1	1158
23.5	74.5	12.1	1208

XIV. 200 L/D; 0.162-in. ID; Fire-polished Entrance

P_o (psia) (± 0.1 psi)	T_o ($^{\circ}F$) (± 0.1 $^{\circ}F$)	P_e (psia) (± 1.0 psi)	G_c (lb _m /ft ² -sec) ($\pm 10\%$)
7.9	41.1	3.5	281
8.7	41.3	4.9	430
10.5	43.9	5.45	598
11.0	43.4	5.3	688

For entrance pressures between 11.25 to 11.95 psia, cyclic pulses of vapor were observed in the test section. While the entrance pressure changed slowly from 11.25 to 11.95 psia, the exit pressure varied from 4.05 to 4.10 psia when the flow at the exit plane was all-liquid and from 5.5 to 5.7 when the flow was two-phase. During this time, the mass flux increased from 753 to 824 lb_m/ft²-sec. With the entrance pressure below 11.25 psia, the flow was continuously two-phase, and when the entrance pressure was above 11.95 psia, the flow was all-liquid; between these two limits was the cyclic flashing at a rate of 1.7 Hz. It should be noted that these same entrance stagnation conditions were tested with this test section when it had a sharp entrance and the cyclic flashing was not observed. Refer to Fig. 12.

XV. 200 L/D; 0.162-in. ID; Fire-polished Entrance

P_o (psia) (± 0.1 psi)	T_o ($^{\circ}F$) (± 0.1 $^{\circ}F$)	P_e (psia) (± 1.0 psi)	G_c (lb _m /ft ² -sec) ($\pm 10\%$)
15.3	74.8	6.7	354
15.9	75.1	7.3	414
16.05	75.3	7.55	429
16.5	75.3	8.35	467
17.2	75.7	9.1	512
17.8	76.0	9.7	542

XVI. 200 L/D; 0.0995-in. ID; Sharp Entrance

P_o (psia) (± 0.1 psi)	T_o ($^{\circ}$ F) (± 0.1 $^{\circ}$ F)	P_e (psia) (± 1.0 psi)	G_c (lb _m /ft ² -sec) ($\pm 10\%$)
14.7	72.0	5.7	
16.25	76.0	6.7	
16.9	76.0	7.8	
17.0*	75.6	8.0	428
17.45	76.0	9.0	
17.53	75.5	9.08	500 Annular Entrance
17.95†	76.0	9.8	567
18.2	76.0	10.0	

* This test had a transition from slug to annular flow.

† A bubble flow regime was observed the entire length.

XVII. 300 L/D; 0.0955-in. ID; Sharp Entrance

P_o (psia) (± 0.1 psi)	T_o ($^{\circ}$ F) (± 0.1 $^{\circ}$ F)	P_e (psia) (± 1.0 psi)	G_c (lb _m /ft ² -sec) ($\pm 10\%$)
15.2	73.0	5.2	274
15.45	74.5	5.35	
15.50	76.0	4.85	246
16.5	75.0	5.4	
17.3	76.0	5.9	
17.7	74.0	6.8	371
18.9	75.0	8.45	444
19.4	74.0	9.0	496

At 16.0 psia the slug-annular interface was 140 L/D from the exit.

At 17.7 psia the slug-annular interface was 70 L/D from the exit.

At 18.9 psia the bubbly-slug interface was 70 L/D from the exit.

At 19.4 psia the flow regime was bubbly the entire length.

APPENDIX C

Exit-pressure Accuracy

Calculations similar to those in Appendix A were carried out to show the accuracy of the exit-pressure measurement. The single-phase pressure-drop equation has already been verified as accurate for tests conducted here (see Appendix A). The following data are single-phase flow.

Test information:

200 L/D; 0.0995 in. ID; Bell Entrance

P_o (psia) (± 0.1 psi)	T_o (°F) ($\pm 1.0^\circ\text{F}$)	P_e (psia) (± 1.0 psi)	G_c (lb _m /ft-sec) ($\pm 10\%$)	ΔP calculated (psi)	ΔP actual (psi)	Exit Super- heat (°F)
16.7	74.5	6.8	1271	10.42	9.9	36
18.0	74.5	7.2	1332	11.42	11.3	34
19.6	74.5	7.3	1450	14.19	12.3	33
22.7	74.5	7.9	1572	15.12	14.8	30
23.9	74.5	8.0	1621	16.26	15.9	29

APPENDIX D

Critical Flow Data from Stainless Steel Test SectionI. Runs

Run No.	Flow Rate, lb _m /ft ² -sec (±10%)	Stagnation Pressure, psia (±0.2 psi)	Stagnation Temperature, °F (±1°F)
1	635	8.1	40.5
2	955	9.2	40.0
3	1180	10.3	40.0
4	1388	12.20	42.0
5	931	16.0	74.5
6	1283	18.1	74.5
7	1623	20.15	75.5
8	1869	22.0	75.5
9	2101	24.1	75.5

II. Pressure Data

Pressure Transducer (see Fig. 5)	Run No.								
	1	2	3	4	5	Exit	7	8	9
#1	7.6	8.1	8.45	9.70	14.8	15.7	16.50	17.25	18.1
#2	7.15	7.3	7.50	8.70	14.45	15.15	15.55	15.90	16.4
#3	6.85	7.0	7.15	8.0	13.75	14.50	14.85	15.10	15.6
#4	6.40	6.7	6.8	7.6	13.20	13.85	14.15	14.65	14.9
#5	6.30	6.4	6.45	7.3	13.00	13.60	13.95	14.20	14.45
Exit	5.80	5.9	6.0	6.7	11.80	12.30	12.4	12.60	12.7
#7	3.5	4.4	4.9	5.5	5.95	8.60	9.5	10.0	9.85
#8	2.1	3.4	4.1	4.8	4.40	6.1	7.4	8.6	9.6
#9	2.7	3.3	3.7	4.4	3.95	6.8	9.4	8.3	8.3

III. Temperature Data

Thermocouple Location (see Fig. 5)	Run No.								
	1	2	3	4	5	Exit	7	8	9
A	41.0	40.5	40.0	42.0	73.0	73.0	73.5	74.0	74.5
B	40.0	40.0	40.0	42.0	72.3	72.7	73.0	73.5	73.5
C	41.0	40.0	40.5	42.5	72.7	73.0	74.0	74.0	74.0
D	40.0	40.0	40.0	41.5	72.3	72.3	73.0	73.5	72.5
E	40.0	40.0	40.0	41.5	72.3	72.3	73.0	73.5	72.5
F	40.0	40.0	40.5	42.5	72.7	73.0	73.5	74.0	74.0
G	40.0	40.0	39.7	42.0	71.5	72.0	72.7	73.0	73.0
H	39.3	39.3	39.7	42.0	71.5	72.0	72.7	73.5	73.5
I	38.0	39.3	39.7	42.0	71.0	72.0	72.7	73.0	74.0
J	36.0	37.0	38.5	40.5	65.0	67.0	68.0	68.0	68.5
K	24.0	28.5	31.5	34.0	47.0	55.0	58.0	58.7	59.0
L	11.5	2.1	24.5	28.5	31.5	45.0	51.5	58.0	54.3
M	0.0	9.5	17.5	22.7	15.7	29.3	39.0	44.7	49.5
N	0.0	8.0	14.0	20.0	15.0	26.3	36.0	42.0	47.5

APPENDIX E

Calculations of Pressure Drop for the Stainless-steel Test Section

The single-phase pressure-drop equation, Eq. A.1 of Appendix A, was used to calculate the pressure drop for data obtained stainless-steel test number 4, for which

$$G = 1.388 \text{ lb}_m/\text{ft}^2\text{-sec};$$

$$D = 0.138 \text{ in.};$$

$$\mu = 1.215 \text{ lb}_m/\text{hr-ft};$$

$$R_e = GD/\mu = 47020;$$

$$f = 0.0791 R_e^{-0.25} = 0.0053;$$

$$\Delta P = G^2/2\rho g_c = \text{minimum all-liquid acceleration pressure drop} = 2.18 \text{ psi};$$

$$\frac{\Delta P}{L/D} = \frac{2G^2f}{\rho g_c} = 0.0469 \frac{\text{psi}}{L/D}.$$

The pertinent values obtained were:

Pressure (psia)	Measured Pressure Change (psi)	L/D	Calculated All-liquid Pressure Drop (psi)	$\frac{P_{act}}{P_{cal}}$
12.2				
9.7	2.5	5.43	2.18 + 0.255	1.01
8.7	1.0	18.1	0.850	1.17
8.0	0.7	10.8	0.507	1.38
7.6	0.4	5.43	0.255	1.56
7.3	0.3	1.81	0.084	3.57
6.7	0.6	1.81	0.084	7.14

APPENDIX F

0.121-in.-ID Exit-pressure Error

After the data had been compiled, the accuracy of the 0.121-in.-ID exit-pressure data became questionable. The experimental results, tabulated in Appendix B, show these data to be consistently higher than similar data from other test sections. It was for this reason that the test section was cut in such a manner as to expose the interior of the tube wall at the exit-pressure tap junction.

In the description in Ch. III of the glass test section, it was stated that difficulty was encountered in drilling a hole in the side of the glass tube wall. Photographs in Fig. 23 show how the inside edges of this tube wall chipped away during the drilling process. This chipping resulted in an area increase at the exit plane causing the error in measurement of the exit pressure. By applying the Bernoulli equation, F.1, and the continuity equation, F.2, one can see how the high exit-pressure data can be brought within the range of the other data. Now

$$P_1 + \frac{1}{2} \rho_1 u_1^2 = P_2 + \frac{1}{2} \rho_2 u_2^2, \quad (F.1)$$

where P_1 , ρ_1 , and u_1 are evaluated just upstream of the area increase, and P_2 , ρ_2 , and u_2 are evaluated at the tap junction. If there is negligible mixture expansion at this area increase, $\rho_1 = \rho_2$. By continuity

$$\rho_1 A_1 u_1 = \rho_2 A_2 u_2 \quad (F.2)$$

or

$$u_1 = \frac{A_2}{A_1} u_2 ;$$

$$P_1 + \frac{1}{2} \rho \left[\frac{A_2}{A_1} u_2 \right]^2 = P_2 + \frac{1}{2} \rho u_2^2 ;$$

$$\frac{1}{2} \rho u_2^2 \left[\frac{A_2^2}{A_1^2} - 1 \right] = P_2 - P_1 ;$$

$$\frac{G^2}{2\rho} \left[\left(\frac{A_2}{A_1} \right)^2 - 1 \right] = \Delta P. \quad (F.3)$$

Fig. 23. 0.121-in. ID Glass Test Section. ANL-Neg. No. 900-2225.

In Eq. F.3 ρ was determined in the following manner. Since the fluid in the test section was observed to be a two-phase mixture, the density would be less than the all-liquid density. The estimated exit-plane density was

$$\rho_{\text{exit}} = \frac{1}{v_{\text{exit}}} = \frac{1}{v_l + \Delta v} = \frac{1}{v_l \left[1 + \frac{1}{1 - \alpha_{\text{exit}}} \right]} \quad (\text{F.4})$$

The value of α_{exit} was approximated as a function of entrance subcooling and L/D :

$$\alpha_{\text{exit}} = C_1 \exp(-C_2 T_{\text{sub}}) , \quad (\text{F.5})$$

where

$$C_1 = 1.0 - [0.691 \exp(-0.01 L/D)]$$

and

$$C_2 = [1.0 - \exp(-0.01(L/D)^2)] [2.0/(L/D + 1)] .$$

This expression has little physics in it, but does show the proper trends in the extreme cases. The constant C_1 represents the maximum value of α possible for zero entrance subcooling. As the duct length becomes very small, the situation approaches flow through an orifice. As L/D increases, the maximum value of α increases, and in the extreme case for an infinitely long duct, the fluid would be all vapor and $\alpha = 1$.

The second constant, C_2 , controls the rate of change of α as a function of L/D . There are two terms for this constant because of different trends for short and long tubes. With short tubes, the flow is determined almost entirely at the entrance; the fluid resides in the tube only a very short time, and little vapor is formed regardless of subcooling. This is determined by the first term. As the duct length increases, however, the effects of subcooling become more noticeable, and then the second term of C_2 becomes effective. By using Eqs. F.5 to estimate the exit-plane void fraction and F.4 to obtain exit-plane density, Eq. F.3 was used to estimate the pressure-measurement error.

In adiabatic two-phase critical flow, as was studied here, the critical exit pressure must be lower than the saturation pressure of the fluid. The chipping at the exit tap junction caused the high-pressure measurements of the 0.121-in. ID tube; some of the data was above the saturation pressure, a physical impossibility. Figure 24 shows the data

Fig. 24. 0.121-in. ID Critical Exit Pressure Data.

compared to the data from the 0.250-in. tube data. An area increase of 20%, twice the experimental tolerance, assumed to exist, was chosen for these calculations.

APPENDIX G

Estimation of the Empirical Constant N
of the Homogeneous Nonequilibrium Model

One possible way to estimate the value of N of the Homogeneous Nonequilibrium Model is to find an upper and lower bound of the actual quality. The value of N is an indicator of how close the actual quality is to the equilibrium quality, thus serving as a measure of the non-equilibrium. The value of N can vary from 0, no phase change, to 1, equilibrium phase change. We take N to be given by

$$N = X_{\text{act}} / X_{\text{equilibrium}} \quad (\text{G.1})$$

A value of $N = 20X_{\text{equilibrium}}$ was used by Henry for his water data; here $N = 4X_{\text{equilibrium}}$ yielded good predictions for Freon-11 (see Fig. 22). The following calculations give some credibility for this choice. By finding an upper bound to the actual quality, the magnitude of N can be limited in an increasing direction. Clearly, a lower bound is no-phase change or $N = 0X_{\text{equilibrium}}$.

Consider the flow to be homogeneous and the test-section pressure drop to consist of the following components:

$$P_o - P_e = \Delta P_{\text{total}} = \Delta P_{\text{entrance}} + \Delta P_{\text{friction}} + \Delta P_{\text{momentum}}, \quad (\text{G.2})$$

where

$$\Delta P_{\text{entrance}} = \frac{G^2}{2\rho} \left[1 + K_c \right] \quad (K_c \text{ is the entrance loss coefficient}) \quad (\text{G.3})$$

$$\Delta P_{\text{friction}} = \frac{G^2}{2\rho} \left[\frac{4fL}{D} \right];$$

$$\frac{1}{\sqrt{f}} = 1.74 - 2 \log \left[\frac{18.6}{\text{Re}\sqrt{f}} \right] \quad (\text{G.4})$$

$$\Delta P_{\text{momentum}} = G^2 \Delta v \quad (\text{this is the pressure change due to specific volume change}); \quad (\text{G.5})$$

$$\Delta v = [v_g - v_l] X_{\text{act, upper bound}}.$$

By solving for X_{act} , upper bound and using Eq. G.2 we find

$$X_{act, u.b.} = \frac{\Delta P_{momentum}}{G^2[v_g - v_l]}$$

$$= \frac{P_o - P_e - \Delta P_{ent} - \Delta P_{fric}}{G^2[v_g - v_l]}$$

In order for X_{act} , upper bound to be truly an upper bound, $\Delta P_{momentum}$, given by Eq. G.5 must be found in such a way as to maximize it. This can be done by calculating minimum values of ΔP_{ent} and ΔP_{fric} from Eq. G.2. These values are therefore calculated as all-liquid flow even though two-phase flow was observed. The results of these calculations are shown in Fig. 25, where X_{act} , upper bound is compared to $X = Nx_{eq}$ with $N = 4x_{eq}$. The data used in these calculations are from the 0.250-in.-ID, sharp-entrance test section. This is the same data that is compared with the Homogeneous Nonequilibrium Model in Fig. 22.

Fig. 25. Calculation of the Upper Bound of the Actual Exit Quality.

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