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ARGONNE NATIONAL LABORATORY
P. O. Box 299
Lemont, Illinois

THERMODYNAMICS OF UNSTEADY FLOW PROCESSES
INVOLVING TRANSFER OF HEAT FOR
VENTED CONTAINMENT SYSTEMS

by

J. C. Heap

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ABSTRACT

The first law of thermodynamics for changes of state and quantity of the working substance in the case of unsteady flow is presented. Included are mass balance, forms of energy utilized, and definitions of stagnation enthalpy and stored energy. From the equations for unsteady flow the familiar equations for steady flow are deduced.

A special case of the first law of thermodynamics for changes of state and quantity is applied in the theoretical derivation on emptying of a vented containment structure having a constant heat source. The appropriate thermodynamic relations necessary for evolvment are stated and a resumé on nozzle flow is made. Graphs have been plotted for the air temperature, pressure, and mass with respect to time for the constant-pressure and constant-volume processes; for a large vent and a small vent where the same initial pressure is prevalent to the containment structure and atmosphere; and for heating the internal gas to a predetermined pressure and then releasing through a large vent or a small vent.

The resultant thermodynamic relations are applied to a hypothetical containment system to illustrate their generality. This hypothetical containment system consists of an extinguishing system, containment structure system and a venting system.

Tables of functions for various nozzle flow relations are given in an appendix. The basic first order equation is transformed in an appendix for use in conjunction with the tables and a desk computer. The essential equations for numerical integration and a numerical example are related in the appendices.

NOMENCLATURE

<u>English Letters</u>	<u>Units</u>
A = Area	ft ²
A _t = Net area of smallest cross section in vent (throat)	ft ²
A _{tT} = Net area of smallest cross section in sprinkler unit	ft ²
a = Acceleration	ft/sec ²
a,b,c, = Constants in general	
C = Constant for irreversible adiabatic process, $p\gamma^n = C$	
C ₁ = Mass flow rate constant, $\frac{A_t}{V} \sqrt{2g_c J c_p T_s}$	1/sec
C ₂ = Heat - time constant, $\frac{q}{c_v m_s T_s}$	1/sec
C* = Dimensionless constant, $\frac{q}{c_v \frac{A_t}{V} m_s T_s^{3/2} \sqrt{2 g_c J c_p}}$	
C ₀ * = Constant for special condition, $C^* \sqrt{\frac{n}{n-1}}$	
c _p = Mean specific heat at constant pressure	Btu/lbm-°R
c _v = Mean specific heat at constant volume	Btu/lbm-°R
E = Stored energy	Btu
e = Function, $r/m^* T^*$	
f(e) = Function, $e^{\frac{1}{n}} \sqrt{1 - e^{\frac{n-1}{n}}}$	
F = Force	lbf
g = Acceleration of gravity, in general and for the containment structure boundary	ft/sec ²

<u>English Letters</u>		<u>Units</u>
g_c	= Constant of proportionality	32.1740 ft-lbm/lbf-sec ²
h	= Specific enthalpy of working substance, in general and within the containment structure boundary	Btu/lbm
J	= Joule's constant	778 ft-lbf/Btu
j	= Function, $\sqrt{m^*}$	
k	= Ratio of specific heats, c_p/c_v	
m	= Mass of working substance, in general and within the containment structure boundary	lbm
m_1, m_2, m_3	= Masses of working substance within the containment structure at particular times t_1, t_2, t_3	lbm
m^*	= Dimensionless mass ratio	
n	= Constant in irreversible adiabatic process, $p\gamma^n = C$	
p	= Absolute unit pressure of working substance, in general and within the containment structure boundary	lbf/ft ²
p_a	= Absolute unit atmospheric pressure	lbf/ft ²
p_c	= Absolute unit critical pressure	lbf/ft ²
Q	= Total transferred heat	Btu
q	= Constant heat rate	Btu/sec
q^*	= Heat - time constant	1/sec
q_r	= Initial transferred heat rate	Btu/sec
R	= Gas constant	ft-lbf/lbm-°R
r	= Ratio of atmospheric pressure to initial pressure of working substance within containment structure	

<u>English Letters</u>		<u>Units</u>
T	= Absolute temperature of working substance, in general and within the containment structure boundary	$^{\circ}\text{R}$
T_a	= Absolute atmospheric temperature	$^{\circ}\text{R}$
T_1, T_2, T_3	= Absolute temperature of working substance within the containment structure at particular times t_1, t_2, t_3	$^{\circ}\text{R}$
T^*	= Dimensionless temperature ratio	
t	= Time	sec
u	= Specific internal energy of working substance, in general and within the containment structure boundary	Btu/lbm
V	= Volume, in general and of containment structure boundary	ft^3
v	= Velocity of working substance, in general and within the containment structure boundary	ft/sec
W	= Net work (shaft), in general and leaving the containment structure boundary	Btu
z	= Elevation above an arbitrary datum, in general and for the containment structure boundary	ft
<u>Greek Letters</u>		
α (alpha)	= Time constant for transferred heat	1/sec
Δ (delta)	= Step width	
Δt	= Time interval	sec
γ (gamma)	= Specific volume of working substance, in general and within the containment structure boundary	ft^3/lbm

Greek Letters

Units

η (eta) = Constant of irreversible adiabatic process
for extinguishing system

ρ (rho) = Density of working substance lbm/ft³

Subscripts

i = Inlet station of boundary

o = Outlet station of boundary

s = Initial working substance within the contain-
ment structure

T = Extinguishing system

v = Venting system

x = Any section in a vent

oT = An outlet station of boundary for extinguishing
system

iv = An inlet station of boundary for venting system

ov = An outlet station of boundary for venting system

st = Stagnation enthalpy

sa = Original working substance remaining in the
containment system

sT = Extinguishing working substance in the
containment system

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THERMODYNAMICS OF UNSTEADY FLOW PROCESSES INVOLVING TRANSFER OF HEAT FOR VENTED CONTAINMENT SYSTEMS

J. C. Heap

I. BACKGROUND

With the development of nuclear reactors, it is essential that the constituents resulting from an incident caused by a mistake in loading of fuel elements, major failure of a primary reactor vessel or in the reactor primary coolant system, etc., be confined to a limited sphere of catastrophe. Localizing the results of an incident is a basic function of a containment structure which serves as a restraint over and above the regulating and protecting mechanisms. In general, a containment structure is a configuration of any size restricting the working substance, or working substances, to a definite region. The containment structure in this report is considered the last outer envelopment restraining an incident and preventing the release of undesirable materials to the surroundings.

This inquiry has been developed in connection with an unforeseen incident inside a proposed critical experimental combination shielding-containment structure. One of the experiments planned for this containment structure could have a high-order explosive expansion, producing metal vapors and/or particles which would ignite upon exposure with the air inside the containment structure. The blast wave from the explosion, being of short duration, would not rupture the containment structure; however, an internal increase in temperature and pressure due to the burning of the metal could destroy the integrity of the rectangular parallelepiped concrete structure. From the safety aspect, the time-dependence of temperature and pressure of the gas in the containment structure is essential in analyzing the structural components. To relieve this pressure buildup, a venting system with a filtering unit to prevent release of the radioactive material is a possibility. The proper selection of filters requires knowledge of the temperatures, pressures, and flow rates.

To completely analyze such a design proposal, it is necessary to know the thermodynamic relations of the system. With the incorporation of the venting system, the mass of the gas remaining in the containment structure also varies; thus, mass, temperature, and pressure vary with time. Because of the generalized nature of the treatment, the resultant thermodynamic relations are applicable to a wide variety of problems and permit inclusion of a combination of system components. For example, the thermodynamic relations can be resolved for a containment system consisting of a containment structure system having a heat source, a venting system having a heat sink and filter, and an extinguishing system.

II. SYSTEM OF UNITS

In this report, the system of units will be based upon a unit of force - the pound force - since this basis is normally preferred by American and British engineers. The pound force is defined as the gravitational force on a mass of 1 lb at a selected standard location; thus, the weight of a body will have the same numerical value as its mass at the selected standard location. The selected standard location for establishing this standard will be on the earth's surface where the acceleration of gravity equals 32.1740 ft/sec^2 .

Expressing Newton's second law of motion as

$$F = ma/g_c, \quad (\text{II.1})$$

where

F = force in pounds force (lbf)

m = mass in pounds mass (lbm)

a = acceleration (ft/sec^2)

g_c = constant of proportionality,

there is obtained

$$[1 \text{ lbf}] = [1 \text{ lbm}] [32.1740 \text{ ft/sec}^2] / g_c \quad (\text{II.2})$$

or

$$g_c = 32.1740 (\text{ft-lbm} / \text{lbf-sec}^2). \quad (\text{II.3})$$

To maintain dimensional consistency in all equations, the unit of time will be the second and the unit of length will be the foot.

III. UNSTEADY AND STEADY-FLOW RELATIONS

III.1 Introduction

Since many thermodynamic processes involving the movement of the thermal working substance cannot be treated as a steady flow process, the first law of thermodynamics regarding changes of state and quantity is necessary. For example, applying heat to gas contained inside a containment structure having a vent, the gas flows through the vent during the process, but the pressure and temperature in the containment structure vary as the mass within the containment structure decreases. From an external viewpoint reciprocating engines may be considered as a steady flow machine; however, a change of state and quantity occur during the admission and release of the thermal working substance.

The first law of thermodynamics cannot be applied until the system and its boundary are defined. A system having a thermal working substance flowing at unsteady rates is considered and a delineation of this variable flow system is illustrated by Fig. 1. In order to state mathematically the first law of thermodynamics for changes on state and quantity, a circumventing boundary method is used to determine the energy changes between the system and its surrounding. A boundary is defined as a real or fictitious fixed, isolated area in space, at any instant confining the working substance, or working substances, of the system.

III.2 Unsteady and Steady-Flow Suppositions

- (a) The working substance while crossing a boundary experiences no change of state.
- (b) The forms of energy to be utilized are flow work, $mp\gamma/J$; internal energy, mu ; kinetic energy, $mv^2/2g_c J$; potential energy, $mzg/g_c J$; transferred heat, Q ; and work (shaft), W .

FIG. 1
VARIABLE FLOW SYSTEM

III.3 Unsteady Flow

For the illustrated variable flow system of Fig. 1, a material balance or mass balance at any instance requires that the increase, or decrease, of the mass flow rate within the boundary be equal to the net mass flow rates entering and leaving the boundary. Thus,

$$\frac{dm}{dt} = \frac{dm_i}{dt} - \frac{dm_o}{dt} \quad , \quad (\text{III.1})$$

which is the continuity equation for one inlet stream and one outlet stream.

The Law of Conservation of Energy states that energy cannot be created or destroyed; therefore, an energy balance of this variable flow system at any instant requires that the increase, or decrease, of the energy rate within the boundary be equal to the net energy rates entering and leaving the boundary. Then:

$$\begin{aligned} \frac{d}{dt} \left[m \left(u + \frac{v^2}{2g_c J} + \frac{zg}{Jg_c} \right) \right] &= \left(u_i + \frac{p_i \gamma_i}{J} + \frac{v_i^2}{2g_c J} + \frac{z_i g_i}{J g_c} \right) \frac{dm_i}{dt} \\ &- \left(u_o + \frac{p_o \gamma_o}{J} + \frac{v_o^2}{2g_c J} + \frac{z_o g_o}{J g_c} \right) \frac{dm_o}{dt} + \frac{dQ}{dt} - \frac{dW}{dt} \quad , \end{aligned} \quad (\text{III.2})$$

where the work rate (shaft) produced externally and the net transferred heat rates at the inlet and outlet stations are zero from the proper location of the boundary. It is to be noted that within the boundary a change of state and quantity occurs ($p\gamma/J$ varies); however, the flow work depends only upon the entering and exhaust states of the working substances flowing into and out of the boundary.

Defining the stored energy rate within the boundary as the sum of the internal energy, kinetic energy and potential energy, the following expression may be written:

$$\frac{dE}{dt} = \frac{d}{dt} \left[m \left(u + \frac{v^2}{2g_c J} + \frac{zg}{Jg_c} \right) \right] \quad . \quad (\text{III.3})$$

Substituting Eq. III.3 into Eq. III.2 and rearranging terms gives

$$\begin{aligned} \left(u_i + \frac{p_i \gamma_i}{J} + \frac{v_i^2}{2g_c J} + \frac{z_i g_i}{J g_c} \right) \frac{dm_i}{dt} + \frac{dQ}{dt} \\ = \left(u_0 + \frac{p_0 \gamma_0}{J} + \frac{v_0^2}{2g_c J} + \frac{z_0 g_0}{J g_c} \right) \frac{dm_0}{dt} + \frac{dW}{dt} + \frac{dE}{dt} \end{aligned} \quad (\text{III.4})$$

or

$$\begin{aligned} \left(h_i + \frac{v_i^2}{2g_c J} + \frac{z_i g_i}{J g_c} \right) \frac{dm_i}{dt} + \frac{dQ}{dt} = \left(h_0 + \frac{v_0^2}{2g_c J} + \frac{z_0 g_0}{J g_c} \right) \frac{dm_0}{dt} \\ + \frac{dW}{dt} + \frac{dE}{dt} , \end{aligned} \quad (\text{III.5})$$

where the specific enthalpy, $h = u + p\gamma/J$, has been used.

Defining stagnation enthalpy, h_{st} , as the summation of specific internal energy, flow work and kinetic energy, viz., $h_{st} = u + p\gamma/J + v^2/2g_c J$, eliminates the need for separate consideration of the kinetic energy of the discharge stream. Using the definition of stagnation enthalpy, Eq. III.5 may be written as

$$\left(h_{sti} + \frac{z_i g_i}{J g_c} \right) \frac{dm_i}{dt} + \frac{dQ}{dt} = \left(h_{st0} + \frac{z_0 g_0}{J g_c} \right) \frac{dm_0}{dt} + \frac{dW}{dt} + \frac{dE}{dt} . \quad (\text{III.6})$$

The extension of Eq. III.1, III.2, III.4, III.5, and III.6 to processes involving additional inlets and outlets of the working substances can be established readily by extending the one inlet-outlet case.

For many variable flow processes, the potential and kinetic energies are negligible; hence, Eq. III.2 becomes

$$\left(u_i + \frac{p_i \gamma_i}{J} \right) \frac{dm_i}{dt} + \frac{dQ}{dt} = \left(u_0 + \frac{p_0 \gamma_0}{J} \right) \frac{dm_0}{dt} + \frac{dW}{dt} + \frac{d}{dt}(\mu u) \quad (\text{III.7})$$

or

$$h_i \frac{dm_i}{dt} + \frac{dQ}{dt} = h_0 \frac{dm_0}{dt} + \frac{dW}{dt} + \frac{d}{dt}(\mu u) . \quad (\text{III.8})$$

If the inlet and outlet kinetic energies are significant, the stagnation enthalpy can be used; then,

$$h_{st_i} \frac{dm_i}{dt} + \frac{dQ}{dt} = h_{st_0} \frac{dm_0}{dt} + \frac{dW}{dt} + \frac{d}{dt} (mu) \quad . \quad (III.9)$$

III.4 Steady Flow

For steady flow the mass flow rates entering and leaving the boundary are equal; therefore, Eq. III.1 becomes

$$\frac{dm_i}{dt} = \frac{dm_0}{dt} \quad , \quad (III.10)$$

the steady-flow continuity equation for one inlet stream and one outlet stream.

The stored energy is zero for steady flow since the energies are in a state of equilibrium, i.e., the derivatives of constants equal zero; hence, the variable-flow equations reduce to steady-flow equations. From a less stringent point of view, if the states of the working substances at all stations within the system periodically and simultaneously become identical with previous existing states, it is apparent that steady flow prevails, but an analysis made for an interval within the cycle would require a variable treatment.

From Eq. III.7 the general steady-flow energy equation for one inlet stream and one outlet stream becomes

$$u_i + \frac{p_i \gamma_i}{J} + \frac{v_i^2}{2g_c J} + \frac{z_i g_i}{J g_c} + \frac{dQ}{dm_i} = u_0 + \frac{p_0 \gamma_0}{J} + \frac{v_0^2}{2g_c J} + \frac{z_0 g_0}{J g_c} + \frac{dW}{dm_i} \quad , \quad (III.11)$$

or from Eq. III.8 becomes

$$h_i + \frac{v_i^2}{2g_c J} + \frac{z_i g_i}{J g_c} + \frac{dQ}{dm_i} = h_0 + \frac{v_0^2}{2g_c J} + \frac{z_0 g_0}{J g_c} + \frac{dW}{dm_i} \quad . \quad (III.12)$$

IV. EMPTYING OF A VENTED CONTAINMENT STRUCTURE HAVING A CONSTANT HEAT SOURCE

IV.1 Introduction

The theoretical development of this problem is based upon the first law of thermodynamics for changes of state and quantity, and the flow of gases through nozzles. A special case of the first law of thermodynamics for changes of state and quantity as related in Section III.3 will be applicable for the theoretical analysis. Certain ideal gas relations are stated, but not evolved. If further information is desired, the references can be consulted. A schematic of the assemblage is illustrated by Fig. 2.

IV.2 Suppositions

- (a) The working substance within the containment structure is homogeneous in composition, state and velocity at any instant.
- (b) The working substance at any section of the vent is homogeneous in composition, state and velocity at any instant.
- (c) The working substance while crossing a boundary experiences no change of state.
- (d) The gas flow through the vent is a steady flow process at any instant.
- (e) The velocity of approach to the vent is nil. Note: The use of the stagnation enthalpy will give the same basic equation.
- (f) The perfect gas laws are applicable.
- (g) The transfer of heat through the walls of the containment structure is zero.
- (h) The forms of energy considered are flow work, $mp\gamma/J$; internal energy, mu ; kinetic energy, $mv^2/2g_c J$; transferred heat, Q ; and work (shaft), W . All other forms of energies are considered negligible.

FIG. 2
EMPTYING OF A VENTED CONTAINMENT STRUCTURE
HAVING A CONSTANT HEAT SOURCE

IV.3 Development

IV.3.1 Basic General Differential Equation

With the suppositions imposed in Section IV.2, potential and kinetic energies within the boundary considered negligible, the first law of thermodynamics for changes of state and quantity in differential form is given by Eq. III.8. Equation III.8 must be modified for the conditions particular to the problem and the assumptions made.

There being no inlet flow into the containment structure, viz.,

$$\frac{dm_i}{dt} = 0 \quad , \quad (IV.1)$$

the mass balance, using Eq. III.1, becomes

$$\frac{dm}{dt} = - \frac{dm_0}{dt} \quad , \quad (IV.2)$$

As the transferred heat rate is constant,

$$\frac{dQ}{dt} = q \quad . \quad (IV.3)$$

The enthalpies of the working substance in the containment structure and at the entrance section to the vent are the same, since homogeneity is assumed; hence,

$$h = h_0 \quad . \quad (IV.4)$$

Expressing the work produced externally in differential form,

$$\frac{dW}{dt} = \frac{p}{J} \frac{dV}{dt} = 0 \quad , \quad (IV.5)$$

since the volume of the containment structure remains constant, namely, $dV = 0$.

Substituting Eqs. IV.1, IV.2, IV.3, IV.4 and IV.5 into Eq. III.8 gives

$$q + h \frac{dm}{dt} = \frac{d(mu)}{dt} \quad . \quad (IV.6)$$

From the assumption of a perfect gas, appropriate thermodynamic relations for specific enthalpy and specific internal energy are as follows:

$$h = c_p T \quad , \quad (IV.7)$$

$$u = c_v T \quad , \quad (IV.8)$$

and

$$du = c_v dT \quad , \quad (IV.9)$$

where c_v and c_p are constant as associated with a perfect gas.

Substituting Eqs. IV.7, IV.8 and IV.9 into Eq. IV.6, rearranging terms, and recalling the definition for the ratio of specific heats ($k = c_p/c_v$), the following equation is obtained:

$$q = c_v m \frac{dT}{dt} + (1 - k) c_v T \frac{dm}{dt} \quad . \quad (IV.10)$$

This equation is important for the limiting cases which are the constant-volume process (volume and mass constant, nonflow) and the constant-pressure process (volume and pressure constant, flow). These processes are dealt with separately at a later time.

Modification of Eq. IV.10 is undertaken to acquire a differential equation for determining the general solution, viz.,

$$q = c_v m \frac{dT}{dm} \frac{dm}{dt} + (1 - k) c_v T \frac{dm}{dt} \quad . \quad (IV.11)$$

As dm/dt is the mass flow rate of gas passing a section, an expression must be attained in terms of T and m to solve the differential equation.

IV.3.2 Resumé on Nozzle Flow

In a nozzle, or vent, the transferred heat is practically zero ($dQ = 0$) and there is no work ($dW = 0$). Using the assumptions that the velocity of approach to the vent, the change of potential energies and the second-order effects of the working substance flowing through the vent (steady flow assumed) are insignificant, the velocity at any vent section becomes

$$v_x = \sqrt{2g_c J(h - h_x)} \quad , \quad (IV.12)$$

where the substitution of h for h_1 , h_x for h_0 , and v_x for v_0 were made in Eq. III.12.

As steady flow has been assumed throughout the vent at any instant, the mass flow rate is equal to the product of the cross-sectional area and velocity of the working substance divided by its specific volume for the vent section in question; thus, the mass balance, by extending Eq. IV.2, is

$$\frac{dm}{dt} = - \frac{dm_0}{dt} = - \frac{A_x v_x}{\gamma_x} \quad (IV.13)$$

The actual velocity and mass flow rate of a gas at any vent section will be less than the theoretical values computed for an isentropic expansion process. To compensate for the decrease in velocity and mass flow rate, two available methods are (1) the use of coefficients, viz., velocity coefficient, nozzle efficiency and discharge coefficient, and (2) replacing the isentropic process, $p\gamma^k = \text{Constant}$, by an irreversible adiabatic process, $p\gamma^n = C$, where n is an assigned value for the characteristics of the vent.

The second method with a proper value of n , is utilized. It can be proven that for an irreversible adiabatic expansion, such as occurring in a nozzle where the inlet temperature is greater than the outlet temperature, n must be less than k and greater than one to give a positive change in entropy, indicating an increase as required for an irreversible adiabatic process.

From the thermodynamic relation of Eq. IV.7, the change in enthalpy of a perfect gas is

$$h - h_x = c_p (T - T_x) \quad (IV.14)$$

and, the expansion of the gas being an irreversible adiabatic process,

$$T_x = T(p_x/p)^{\frac{n-1}{n}} \quad ; \quad (IV.15)$$

the velocity of the gas, on substitution of Eqs. IV.14 and IV.15 into IV.12, becomes

$$v_x = \sqrt{2g_c J_c p T \left[1 - (p_x/p)^{\frac{n-1}{n}} \right]} \quad (IV.16)$$

If the expansion in the vent is assumed to be an irreversible adiabatic process, then

$$\gamma_x = \gamma (p/p_x)^{1/n} \quad (IV.17)$$

and the mass flow rate of the gas, from Eqs. IV.13, IV.16 and IV.17, becomes

$$\frac{dm}{dt} = -\frac{A_x}{\gamma} \left[\frac{p_x}{p} \right]^{1/n} \sqrt{2g_c J_c p T \left[1 - (p_x/p)^{\frac{n-1}{n}} \right]} \quad (IV.18)$$

or

$$\frac{dm}{dt} = -A_x/\gamma \sqrt{2g_c J_c p T \left[(p_x/p)^{2/n} - (p_x/p)^{\frac{n+1}{n}} \right]} \quad (IV.19)$$

The mass flow rate for given inlet conditions depends in some measure on the pressure at section x since various ratios of p_x/p in Eq. IV.19 yield different values. The greatest mass flow rate per cross-sectional area must occur when

$$(p_x/p)^{2/n} - (p_x/p)^{\frac{n+1}{n}}$$

of Eq. IV.19 is a maximum. The maximum value is obtained by taking the derivative of this expression with respect to the pressure at section x and equating to zero i.e.,

$$\frac{d \left[(p_x/p)^{2/n} - (p_x/p)^{\frac{n+1}{n}} \right]}{dp_x} = 0 \quad (IV.20)$$

The differential and algebraic steps yield

$$p_x = p_c = p \left(\frac{2}{n+1} \right)^{\frac{n}{n-1}} \quad (IV.21)$$

where p_c , the critical pressure, is defined as the pressure where the flow of the working substance is a maximum for an adiabatic process.

Two important conclusions attached to the critical pressure are: (1) for all final pressures less than the critical pressures, the flow is a maximum and constant, and (2) for a final pressure greater than the critical pressure, the flow is below the maximum obtainable.

Figure 2 shows the vent to be a convergent-divergent nozzle. When the final pressure is larger than the critical pressure the nozzle need only be convergent for a proper expansion. If the final pressure is less than the critical pressure, a convergent-divergent nozzle is necessary to achieve proper expansion. In an actual containment structure the vent will probably resemble an orifice in a thick wall. The expansion will be decidedly irreversible for this arrangement. The length, cross sections, flow path, etc., of the vent will also affect the irreversibility of the expansion. As mentioned previously, an appropriate value of n or a coefficient of discharge with $n=k$ will account for this irreversibility.

The atmospheric pressure and smallest cross-sectional area of the vent are the logical choice for determining the mass flow rate where the vent is convergent, or an orifice, and $p_a > p_c$; hence,

$$\frac{dm}{dt} = -\frac{mA_t}{V} [p_a/p]^{1/n} \sqrt{2g_c J_{c_p} T \left[1 - (p_a/p)^{\frac{n-1}{n}} \right]} \quad (\text{IV.22})$$

where $\gamma = V/m$.

Since it is exceedingly difficult to evaluate Eq. IV.22 accurately when p is nearly equal to p_a , two equations derived by means of the binomial theorem will be mentioned. The first one,

$$\frac{dm}{dt} \approx -\frac{mA_t}{V} [p_a/p]^{1/n} \sqrt{2g_c J_{c_p} T \left(\frac{n-1}{n} \right) \left[\frac{p - p_a}{p} + \frac{1}{2n} \left(\frac{p - p_a}{p} \right)^2 \right]}, \quad (\text{IV.23})$$

is accurate when $p_a > 0.85p$. If the pressure drop is of the magnitude of 1% of the initial pressure, the second equation,

$$\frac{dm}{dt} \approx - A_t \sqrt{\frac{2g_c J_c p_m}{RV}} (p - p_a) \left(\frac{n-1}{n} \right) \quad , \quad (\text{IV.24})$$

can be used.

For the case where the final pressure is less than the critical pressure a convergent-divergent nozzle is necessary to accommodate the expanding gas properly and the maximum flow is given by

$$\frac{dm}{dt} = - \frac{mA_t}{V} \sqrt{2g_c J_c p T \left[\left(\frac{2}{n+1} \right)^{\frac{2}{n-1}} \right] \left[\frac{n-1}{n+1} \right]} \quad . \quad (\text{IV.25})$$

The following analysis considers only the condition where the final pressure is larger than the critical pressure. If the final pressure is less than the critical pressure, the required equations can be developed in a manner analogous to that for finding the equations with final pressure larger than the critical pressure.

IV.3.3 Characteristic Equation Relationship

To eliminate p from the flow equations in the case that the final pressure is larger than the critical pressure, the equation of state for a perfect gas is used. The gas in the containment structure at the beginning is described by

$$p_s V = m_s R T_s \quad (\text{IV.26})$$

and at a later time by

$$pV = mRT \quad . \quad (\text{IV.27})$$

From the last two equations, the relation

$$p = p_s (mT/m_s T_s) \quad (\text{IV.28})$$

is derived and Eq. IV.22 may be written as

$$\frac{dm}{dt} = - \frac{A_t m}{V} \left[\frac{p_a m_s T_s}{p_s m T} \right]^{1/n} \sqrt{2g_c J_c p T \left[1 - \left(\frac{p_a m_s T_s}{p_s m T} \right)^{\frac{n-1}{n}} \right]} \quad . \quad (\text{IV.29})$$

IV.3.4 General Differential Equations

Substitution of Eq. IV.29 into Eq. IV.11 leads to the following general differential equation:

$$\frac{dT}{dm} = \frac{-q}{c_v \frac{A_t m^2}{V} \left[\frac{p_a m_s T_s}{p_s m T} \right]^{1/n} \sqrt{2g_c J_c p T \left[1 - \left(\frac{p_a m_s T_s}{p_s m T} \right)^{\frac{n-1}{n}} \right]}} + \frac{(k-1)T}{m} \quad (\text{IV.30})$$

If the solution of a pair of simultaneous equations is desired, the following general differential equations may be derived by manipulation of Eqs. IV.11 and IV.29:

$$\left. \begin{aligned} \frac{dm}{dt} &= -\frac{A_t m}{V} \left[\frac{p_a m_s T_s}{p_s m T} \right]^{1/n} \sqrt{2g_c J_c p T \left[1 - \left(\frac{p_a m_s T_s}{p_s m T} \right)^{\frac{n-1}{n}} \right]} \\ \frac{dT}{dt} &= \frac{q}{c_v m} + (1-k) \frac{A_t T}{V} \left[\frac{p_a m_s T_s}{p_s m T} \right]^{1/n} \sqrt{2g_c J_c p T \left[1 - \left(\frac{p_a m_s T_s}{p_s m T} \right)^{\frac{n-1}{n}} \right]} \end{aligned} \right\} \quad (\text{IV.31})$$

Unfortunately, existing knowledge does not disclose a method for integrating these governing differential equations in closed form; hence, numerical integration must be performed for each set of parameters. Nondimensional variables will be introduced into the equations to procure a greater degree of adaptability.

IV.3.5 Nondimensional General Differential Equations

Using the following nondimensional variables and constant:

$$\left. \begin{aligned} T^* &\equiv \frac{T}{T_s} ; & dT^* &= \frac{dT}{T_s} \\ m^* &\equiv \frac{m}{m_s} ; & dm^* &= \frac{dm}{m_s} \\ r &\equiv \frac{p_a}{p_s} \end{aligned} \right\} , \quad (\text{IV.32})$$

Eq. IV.30 becomes

$$\frac{dT^*}{dm^*} = \frac{-C^* + (k - 1)T^* \left\{ m^* \sqrt{T^*} \left[\frac{r}{m^*T^*} \right]^{1/n} \sqrt{1 - \left(\frac{r}{m^*T^*} \right)^{\frac{n-1}{n}}} \right\}}{m^* \left\{ m^* \sqrt{T^*} \left[\frac{r}{m^*T^*} \right]^{1/n} \sqrt{1 - \left(\frac{r}{m^*T^*} \right)^{\frac{n-1}{n}}} \right\}} \quad (\text{IV.33})$$

where

$$C^* \equiv \frac{q}{c_v \frac{A_t}{V} m_s T_s^{3/2} \sqrt{2g_c J_{c_p}}}$$

Also Eqs. IV.31 become

$$\left. \begin{aligned} \frac{dm^*}{dt} &= -C_1 m^* \left[\frac{r}{m^*T^*} \right]^{1/n} \sqrt{T^* \left[1 - \left(\frac{r}{m^*T^*} \right)^{\frac{n-1}{n}} \right]} \\ \frac{dT^*}{dt} &= \frac{C_2}{m^*} + (1 - k) C_1 T^* \left[\frac{r}{m^*T^*} \right]^{1/n} \sqrt{T^* \left[1 - \left(\frac{r}{m^*T^*} \right)^{\frac{n-1}{n}} \right]} \end{aligned} \right\} \quad (\text{IV.34})$$

where

$$C_1 \equiv \frac{A_t}{V} \sqrt{2g_c J_{c_p} T_s}$$

and

$$C_2 \equiv \frac{q}{c_v m_s T_s}$$

It is noted that (1) at the beginning, time is zero and the dimensionless temperature and mass ratios are unity and that (2) the mass ratio is continually decreasing as time increases. The factor

$$\left[\frac{r}{m^*T^*} \right]^{1/n} \sqrt{1 - \left(\frac{r}{m^*T^*} \right)^{\frac{n-1}{n}}} = e^{1/n} \sqrt{1 - e^{\frac{n-1}{n}}} = f(e) \quad ,$$

where

$$e = r/m^*T^* \quad ,$$

appearing in Eqs. IV.33 and IV.34 has been computed and is given in air tables for numerous values (see Appendix A). Figure 3 is a plot of T^* as the ordinate and m^* as the abscissa for various constants C^* and r when $n = k = 1.4$.

IV.3.6 Special Nondimensional Differential Equation

Where the initial pressure of the cell is equal to the atmospheric pressure ($p_a = p_s$) and the pressure buildup in the containment structure is of an order not greater than 1% of the initial pressure ($p \leq 1.01 p_s = 1.01 p_a$), the following differential equation

$$\frac{dT^*}{dm^*} = \frac{-C_0^* + (k-1) T^* \sqrt{m^*(m^*T^* - 1)}}{m^* \sqrt{m^*(m^*T^* - 1)}} \quad (\text{IV.35})$$

may be derived, where

$$C_0^* \equiv \frac{q}{c_v A_t T_s \sqrt{\frac{2g_c m_s p_s J c_p}{VR} \left(\frac{n-1}{n}\right)}} = C^* \sqrt{\frac{n}{n-1}}$$

IV.3.7 Time Correlated to Gas Properties for the First-order Equation

To correlate time with respect to the gas properties (pressure, mass, and temperature), another formula is needed. Letting

$$q^* \equiv q / c_v m_s T_s \quad (\text{IV.36})$$

and utilizing Eq. IV.11, the following equation is obtained upon rearranging and integrating:

$$q^*(t_2 - t_1) = q^* \Delta t = (m_2^* T_2^* - m_1^* T_1^*) - k \int_{m_1^*}^{m_2^*} T^* dm^* \quad (\text{IV.37})$$

To evaluate the last expression in Eq. IV.37 assume

$$T^* = a + b m^* \quad (\text{IV.38})$$

or

$$T^* = a + b m^* + c(m^*)^2, \quad (\text{IV.39})$$

where the intervals have been taken small enough to maintain sufficient accuracy and the constants a , b , and c are attained by the solution of simultaneous equations from calculated data of T^* and m^* .

FIG. 3
 NONDIMENSIONAL VARIABLES OF MASS VERSUS
 TEMPERATURE FOR VARIOUS CONSTANTS, $n=k=1.4$

Placing expressions IV.38 and IV.39 in IV.37, integrating and transposing, the time interval equations are

$$\Delta t = \frac{1}{q^*} \left\{ (m_2^* T_2^* - m_1^* T_1^*) - k \left[\frac{1}{2} (m_2^* - m_1^*) (T_1^* + T_2^*) \right] \right\}, \quad (\text{IV.40})$$

where the constants a and b have been eliminated; or

$$\Delta t = \frac{1}{q^*} \left\{ (m_3^* T_3^* - m_1^* T_1^*) - k \left[a(m_3^* - m_1^*) + \frac{b}{2} (m_3^{*2} - m_1^{*2}) + \frac{c}{3} (m_3^{*3} - m_1^{*3}) \right] \right\} \quad (\text{IV.41})$$

where the following formulas can be used to determine the constants a, b and c;

$$\left. \begin{aligned} c &= \frac{(T_1^* - T_2^*) (m_1^* - m_3^*) - (T_1^* - T_3^*) (m_1^* - m_2^*)}{(m_1^* - m_2^*) (m_1^* - m_3^*) (m_2^* - m_3^*)} \\ b &= \frac{(T_1^* - T_3^*) (m_1^* - m_2^*) (m_1^* + m_2^*) - (T_1^* - T_2^*) (m_1^* - m_3^*) (m_1^* + m_3^*)}{(m_1^* - m_2^*) (m_1^* - m_3^*) (m_2^* - m_3^*)} \\ a &= T_1^* - b m_1^* - c (m_1^*)^2 \end{aligned} \right\} \quad (\text{IV.42})$$

The arrangement of Eq. IV.42 is especially useful at the start for the case where $r = 1$.

If m_1^* , m_2^* , T_1^* , T_2^* , $\left(\frac{dT^*}{dm^*}\right)_1$, and $\left(\frac{dT^*}{dm^*}\right)_2$ are known, the term $k \int_{m_1^*}^{m_2^*} T^* dm^*$ of Eq. IV.37 can be integrated by the Hermite quadrature formula for two points. This formula eliminates computation of the constants a, b, and c when determining the time interval. The time-interval equation, using the Hermite quadrature formula, for two points becomes

$$\Delta t = \frac{1}{q^*} \left\{ (m_2^* T_2^* - m_1^* T_1^*) - k \left\{ \frac{1}{2} (m_2^* - m_1^*) (T_1^* + T_2^*) + \frac{1}{12} (m_2^* - m_1^*)^2 \left[\left(\frac{dT^*}{dm^*}\right)_1 - \left(\frac{dT^*}{dm^*}\right)_2 \right] \right\} \right\} \quad (\text{IV.43})$$

IV.3.8 Special Cases of Constant Volume and Constant Pressure

Consideration is given to the special cases of constant volume (volume and mass constant, nonflow) and constant pressure (volume and pressure constant, flow). Detailed description of

attainment is omitted; however, principal qualifications and formulae are listed. As these processes have been integrated in closed form, limits are readily established for equivalent initial pressure in the containment structure and the atmosphere.

Constant Volume Process:

$$\begin{aligned}
 m_s &= m = \text{constant} \\
 m/m_s &= m^* = 1 \\
 \frac{dm}{dt} &= 0 \\
 A_t &= 0 \\
 q &= c_v m \frac{dT}{dt} \\
 T &= T_s + (q/c_v m_s) t \\
 p/p_s &= T/T_s \\
 p &= p_s \left(1 + \frac{q}{c_v m_s T_s} t \right)
 \end{aligned}
 \quad \left. \vphantom{\begin{aligned} m_s &= m = \text{constant} \\ m/m_s &= m^* = 1 \\ \frac{dm}{dt} &= 0 \\ A_t &= 0 \\ q &= c_v m \frac{dT}{dt} \\ T &= T_s + (q/c_v m_s) t \\ p/p_s &= T/T_s \\ p &= p_s \left(1 + \frac{q}{c_v m_s T_s} t \right) \right\} \quad (\text{IV.44})$$

Constant Pressure Process:

$$\begin{aligned}
 p_s &= p_a = p = \text{constant} \\
 mT &= m_s T_s \\
 m^* T^* &= 1 \\
 m \frac{dT}{dt} + T \frac{dm}{dt} &= 0 \\
 q &= c_v m \frac{dT}{dt} + c_v (1 - k) T \frac{dm}{dt} = c_p \frac{m_s T_s}{T} \frac{dT}{dt} \\
 T &= T_s \exp (q/c_p m_s T_s) t \\
 m &= m_s \exp (-q/c_p m_s T_s) t
 \end{aligned}
 \quad \left. \vphantom{\begin{aligned} p_s &= p_a = p = \text{constant} \\ mT &= m_s T_s \\ m^* T^* &= 1 \\ m \frac{dT}{dt} + T \frac{dm}{dt} &= 0 \\ q &= c_v m \frac{dT}{dt} + c_v (1 - k) T \frac{dm}{dt} = c_p \frac{m_s T_s}{T} \frac{dT}{dt} \\ T &= T_s \exp (q/c_p m_s T_s) t \\ m &= m_s \exp (-q/c_p m_s T_s) t \end{aligned}} \right\} \quad (\text{IV.45})$$

The constant-pressure process is physically impossible as far as present knowledge is concerned. The movement of a gas requires a pressure differential whereas for this case the pressure drop is zero. Equation IV.29 cannot be utilized, as the mass flow rate was developed for a pressure change and by using this equation the mass flow rate would be zero. The constant-pressure process serves as a limiting case: all other cases encountered tend to approach it after an

elapse of time. Differentiating the last expression of Eq. IV.45 gives the mass flow rate as,

$$-\frac{dm}{dt} = (q/c_p T_s) \exp(-q/c_p m_s T_s) t \quad (IV.46)$$

IV.4 Parameters Investigated

Three sets of parameters were investigated numerically:

Case 1. limiting cases for constant pressure and constant volume where the initial atmospheric pressure and containment structure pressure are equal;

Case 2. for conditions of a large vent opening and a small vent opening with the same initial pressure prevailing in the containment structure and atmosphere; and

Case 3. allowing the constant heat source to increase the internal energy of the gas at constant volume in the containment structure until a prescribed pressure is reached and then releasing the gas through a vent system by means of an instantaneous valve, rupture of a diaphragm, etc., for a large vent and a small vent.

IV.5 Conclusions

Upon plotting the pressure, temperature, and mass (gas properties) as the ordinates versus time as the abscissa (Figs. 4, 5, and 6), several pertinent conclusions can be related:

1. The constant-volume process gives maximum pressure.
2. The constant-pressure process yields maximum gas temperature after the system has been operating. The temperature for the constant-pressure process is the lowest during the initial stage, but, as the discharge continues, the temperature increases at a rapid rate and crosses over the constant-volume temperature rise.
3. The mass flow rate for the constant-volume process is zero, i.e., the mass remains constant inside the containment structure. For the same initial atmospheric and containment structure air pressures the constant-pressure process yielded the highest mass flow rate. The maximum occurred in the beginning. All other processes fall between these limiting processes.

FIG. 4
PROPERTIES OF AIR INSIDE A VENTED CONTAINMENT
STRUCTURE HAVING A CONSTANT HEAT SOURCE
-EMPHASIS ON LARGE VENT-

FIG. 5
PROPERTIES OF AIR INSIDE A VENTED CONTAINMENT STRUCTURE HAVING A CONSTANT HEAT SOURCE
-EMPHASIS ON SMALL VENT-

FIG. 6
PROPERTIES OF AIR INSIDE A VENTED CONTAINMENT STRUCTURE HAVING A CONSTANT HEAT SOURCE
-SMALL VENT-

4. With the exception of the constant-volume process, the gas properties (pressure, temperature, and mass) approach the constant-pressure process properties over a prolonged period of time.

5. For the second case, the pressure increases and then decreases. At the start, the constant volume is tangent to the pressure-time curves of this case, but as the time is extended this curve converges on that for constant pressure asymptotically. Initially the mass curve has the constant volume tangent to it; however, the constant-pressure limit is approached with the passing of time. The larger the opening, the less pressure buildup and the greater the amount of gas mass removed from the containment structure. The quantity of gas liberated has a pronounced effect on the temperature of the remaining gas since at the beginning the higher the discharge rate the lower will be the temperature. In time this tendency is reversed, the temperature rising at an increasing rate which crosses over the constant-volume temperature line, while at the onset it was lower.

6. The third case for both openings had decreases in mass and pressure at the start of release. For the large aperture, the gas discharge had an initial cooling effect, but the gas in the containment structure soon was reheated. The small aperture did not remove gas in an adequate amount; hence, the gas temperature increased. There is a possibility of an increase in pressure in this situation with a diminutive hole.

V. GENERALIZED CONTAINMENT SYSTEM

V.1 Introduction

A hypothetical containment system consisting of a number of components relates the generalization of the thermodynamic relations. Figure 7 is a schematic representation of this containment system.

In the event of a serious incident, it is postulated that a high-order explosive expansion will rupture the reactor vessel but will not impair the integrity of the containment system. The components of the venting system are protected during the explosive expansion by the closed gravitational door. Following the expansion, the burning of metal vapors will increase the temperature and pressure of the air inside the containment structure. At a predetermined temperature the sprinkling unit seals and the gravitational door supports will melt, releasing the extinguishing fluid and opening the venting system. The descending gravitational door trips the coolant release mechanism, permitting circulation through the heat sink to prevent overheating the filters.

This containment system is actually comprised of three separate systems which must be handled simultaneously. The thermodynamic relations for each system are given, but no attempt is made to analyze the complete hypothetical containment system.

V.2 Suppositions

(a) The working substance entering a boundary is homogeneous in composition, state, and velocity at any instant.

(b) The working substance within a boundary is homogeneous in composition, state, and velocity at any instant.

(c) The working substance leaving a boundary is homogeneous in composition, state, and velocity at any instant.

(d) The working substance, while crossing a boundary, experiences no change of state.

(e) The extinguishing fluid in the storage tank is a compressed gas at a pressure of several atmospheres.

FIG. 7
GENERALIZED CONTAINMENT SYSTEM

(f) The velocity of approach to the sprinkler unit, the venting system and the filters is nil.

(g) The flow of the working substance through the sprinkler unit and the filters is a steady flow process at any instant.

(h) The perfect gas laws are applicable.

(i) The transfer of heat through the walls of the containment structure is zero.

(j) The forms of energy considered are flow work, $mp\gamma/J$; internal energy, mu ; kinetic energy, $mv^2/2g_cJ$; transferred heat, Q ; and work (shaft), W . All other forms of energies, including the change of kinetic energies within the boundaries, are insignificant.

V.3 Development

V.3.1 Basic Considerations

The rate at which heat is transferred from the source inside the containment structure due to an incident is initially represented by

$$\frac{dQ}{dt} (\text{heat source}) = q_r [1 - \exp(-\alpha t)] \quad (V.1)$$

and the constant rate at which heat is extracted by the sink is

$$\frac{dQ}{dt} (\text{heat sink}) = -q \quad (V.2)$$

The work produced externally for the extinguishing system, containment structure system and venting system is zero, since the volumes remain constant, i.e.,

$$\int dW = \int p \, dV = 0, \quad (V.3)$$

because $dV = 0$.

Since homogeneity has been assumed, the enthalpy of the working substance inside the boundary of the system under consideration and the enthalpy of the working substance at the exit station of the boundary are normally considered the same. For the admission process, the enthalpy of the entering working substance is rarely equal to the enthalpy of the working substance inside the boundary of the

system. The extinguishing fluid in the storage tank for this hypothetical containment system is considered to have properties different from those of the air in the containment structure; thus, the enthalpies are not identical for the same temperature because of different specific heats at constant pressure.

The specific heats of the working substance for the extinguishing system and the containment structure system generally are unequal. This inequality requires consideration inside the containment structure system with progressing time. At any instant the specific heats at constant pressure and at constant volume for the containment structure system can be determined by

$$\left. \begin{aligned} c_p &= \frac{m_s T c_{pT} + m_{sa} c_{psa}}{m_s T + m_{sa}} = \frac{m_s T c_{pT} + m_{sa} c_{psa}}{m} \\ c_v &= \frac{m_s T c_{vT} + m_{sa} c_{vT}}{m_s T + m_{sa}} = \frac{m_s T c_{vT} + m_{sa} c_{vT}}{m} \end{aligned} \right\} \quad (V.4)$$

The initial mass flow rate and velocity for the working substance through the sprinkler are related by expressions similar to equations IV.25 and IV.16. Initially, the compressed gas is assumed to be at several atmospheres; hence, final pressure is less than the critical pressure and under a steady-flow condition the mass flow rate would theoretically be constant for any pressure above the critical pressure. The drop in storage tank pressure will eventually fall below the critical pressure and a mass flow rate expression analogous to equation IV.22 is required.

If the venting system were to be considered as analogous to a heat extraction in a steady flow in a constant cross-sectional duct, the flow cannot be completely described without Newton's second law of motion for fluid flow, since the flow is not adiabatic. Relations which may be used in the solution of one-dimensional, steady-flow problems are (1) mass balance, (2) first law of thermodynamics, (3) equation of state, or properties in tabular form,

(4) second law of thermodynamics, and (5) Newton's second law of motion. Four of these relations are required to determine the flow. In this report Newton's second law of motion for fluid flow has not been presented since the changes in velocity have been considered negligible except through the filter unit and the sprinkler unit.

Using procedures analogous to those developed in Section IV, the basic equations in order of mass balance, energy balance, mass flow rate and velocity for each system at commencement are set-up in preliminary form in the following sections.

V.4 Extinguishing System

V.4.1 Unsteady-flow Equations

$$\frac{dm_T}{dt} = - \frac{dm_{oT}}{dt} = - \frac{dm_i}{dt} \quad (V.5)$$

$$h_{oT} = h_T = c_{pT} T_{oT} = c_{pT} T_T; h_i = c_{pT} T_T (p/p_T)^{\frac{\eta-1}{\eta}} \quad (V.6)$$

$$0 = h_{oT} \frac{dm_i}{dt} + \frac{d}{dt} (m_T u_T) \quad (V.7)$$

V.4.2 Steady-flow Equation (Sprinkling Unit)

$$\frac{dm_i}{dt} = \frac{m_T A_{tT}}{V_T} \sqrt{2g_c J c_{pT} T_T \left[\left(\frac{2}{\eta+1} \right)^{\frac{2}{\eta-1}} \right] \left[\frac{\eta-1}{\eta+1} \right]} \quad (V.8)$$

$$v_i = \sqrt{2g_c J c_{pT} T_T \left[1 - (p/p_T)^{\frac{\eta-1}{\eta}} \right]} \quad (V.9)$$

V.5 Containment Structure System

V.5.1 Unsteady-flow Equations

$$\frac{dm}{dt} = \frac{dm_i}{dt} - \frac{dm_o}{dt} \quad (V.10)$$

$$\left(h_i + \frac{v_i^2}{2g_c J} \right) \frac{dm_i}{dt} + q_r [1 - \exp(-\alpha t)] \quad (V.11)$$

$$= h \frac{dm_o}{dt} + \frac{d}{dt} (mu)$$

V.6 Venting System

V.6.1 Unsteady-flow Equations

$$\frac{dm_v}{dt} = \frac{dm_{iv}}{dt} - \frac{dm_{ov}}{dt} = \frac{dm_0}{dt} - \frac{dm_{ov}}{dt} \quad (V.12)$$

$$h_0 = h_{iv}; c_p = c_{pv} \quad (V.13)$$

$$h \frac{dm_0}{dt} - q = h_{ov} \frac{dm_{ov}}{dt} + \frac{d}{dt} (m_v u_v) \quad (V.14)$$

V.6.2 Steady-flow Equation (Filters Unit)

$$\frac{dm_{ov}}{dt} = \frac{m_v A_t}{V_v} \left[\frac{p_a}{p} \right]^{1/n} \sqrt{2g_c J c_p T_v \left[1 - \left(\frac{p_a}{p} \right)^{\frac{n-1}{n}} \right]} \quad (V.15)$$

$$v_{ov} = \sqrt{2g_c J c_p T_v \left[1 - \left(\frac{p_a}{p} \right)^{\frac{n-1}{n}} \right]} \quad (V.16)$$

APPENDIX A. TABLES OF FUNCTIONS

Table I. Nine-place Table for $n = 1.4$ and $e = 1.0000$ to 0.9996

$e = \frac{r}{m^*T^*}$	$\frac{1}{e^n}$	$\frac{n-1}{e^n}$	$\sqrt{1 - \frac{n-1}{e^n}}$	$f(e)$
1.0000	1.000 000 000	1.000 000 000	0.000 000 000	0.000 000 000
0.9999	0.999 928 570	0.999 971 428	0.005 345 278	0.005 344 895
0.9998	0.999 857 139	0.999 942 853	0.007 559 563	0.007 558 484
0.9997	0.999 785 705	0.999 914 277	0.009 258 672	0.009 256 688
0.9996	0.999 714 270	0.999 885 698	0.010 691 211	0.010 688 157
0.9995	0.999 642 831	0.999 857 117	0.011 953 368	0.011 949 098
0.9994	0.999 571 392	0.999 828 534	0.013 094 503	0.013 088 890
0.9993	0.999 499 950	0.999 799 950	0.014 143 903	0.014 136 831
0.9992	0.999 428 506	0.999 771 364	0.015 120 714	0.015 112 073
0.9991	0.999 357 060	0.999 742 774	0.016 038 267	0.016 027 955
0.9990	0.999 285 613	0.999 714 184	0.016 906 094	0.016 894 016
0.9989	0.999 214 163	0.999 685 591	0.017 731 582	0.017 717 648
0.9988	0.999 142 710	0.999 656 996	0.018 520 367	0.018 504 490
0.9987	0.999 071 257	0.999 628 399	0.019 276 955	0.019 259 052
0.9986	0.998 999 800	0.999 599 800	0.020 004 999	0.019 984 990
0.9985	0.998 928 341	0.999 571 199	0.020 707 511	0.020 685 319
0.9984	0.998 856 882	0.999 542 596	0.021 387 005	0.021 362 558
0.9983	0.998 785 419	0.999 513 991	0.022 045 612	0.022 018 836
0.9982	0.998 713 955	0.999 485 383	0.022 685 171	0.022 655 997
0.9981	0.998 642 489	0.999 456 775	0.023 307 188	0.023 275 548
0.9980	0.998 571 021	0.999 428 163	0.023 913 114	0.023 878 942
0.9979	0.998 499 550	0.999 399 549	0.024 504 102	0.024 467 335
0.9978	0.998 428 077	0.999 370 934	0.025 081 188	0.025 041 762
0.9977	0.998 356 602	0.999 342 316	0.025 645 350	0.025 603 205
0.9976	0.998 285 125	0.999 313 697	0.026 197 385	0.026 152 460
0.9975	0.998 213 647	0.999 285 075	0.026 738 081	0.026 690 318
0.9974	0.998 142 166	0.999 256 452	0.027 268 077	0.027 217 417
0.9973	0.998 070 684	0.999 227 826	0.027 788 019	0.027 734 407
0.9972	0.997 999 199	0.999 199 199	0.028 298 428	0.028 241 808
0.9971	0.997 927 712	0.999 170 570	0.028 799 826	0.028 740 145
0.9970	0.997 856 224	0.999 141 937	0.029 292 712	0.029 229 915
0.9969	0.997 784 732	0.999 113 303	0.029 777 458	0.029 711 493
0.9968	0.997 713 240	0.999 084 667	0.030 254 471	0.030 185 286
0.9967	0.997 641 744	0.999 056 030	0.030 724 095	0.030 651 639
0.9966	0.997 570 247	0.999 027 389	0.031 186 712	0.031 110 936
0.9965	0.997 498 748	0.998 998 748	0.031 642 566	0.031 563 420
0.9964	0.997 427 247	0.998 970 104	0.032 091 993	0.032 009 428
0.9963	0.997 355 744	0.998 941 457	0.032 535 258	0.032 449 226
0.9962	0.997 284 239	0.998 912 810	0.032 972 564	0.032 883 019
0.9961	0.997 212 731	0.998 884 159	0.033 404 206	0.033 311 100
0.9960	0.997 141 221	0.998 855 506	0.033 830 371	0.033 733 657

Table II. Nine-place Table for $n = 1.4$ and $e = 1.0000$ to 0.9996

$e = \frac{r}{m^* T^*}$	$\frac{f(e)}{2e}$	$kf(e)$	$e^{1/2}$	$e^{3/2}$
1.0000	0.000 000 000	0.000 000 000	1.000 000 000	1.000 000 000
0.9999	0.002 672 715	0.007 482 853	0.999 949 999	0.999 850 004
0.9998	0.003 779 998	0.010 581 878	0.999 899 995	0.999 700 015
0.9997	0.004 629 733	0.012 959 363	0.999 849 989	0.999 550 034
0.9996	0.005 346 217	0.014 963 420	0.999 799 980	0.999 400 060
0.9995	0.005 977 538	0.016 728 737	0.999 749 969	0.999 250 094
0.9994	0.006 548 374	0.018 324 446	0.999 699 955	0.999 100 135
0.9993	0.007 073 367	0.019 791 563	0.999 649 939	0.998 950 184
0.9992	0.007 562 086	0.021 156 902	0.999 599 920	0.998 800 240
0.9991	0.008 021 197	0.022 439 137	0.999 549 899	0.998 650 304
0.9990	0.008 455 463	0.023 651 622	0.999 499 875	0.998 500 375
0.9989	0.008 868 579	0.024 804 707	0.999 449 849	0.998 350 454
0.9988	0.009 263 361	0.025 906 286	0.999 399 820	0.998 200 540
0.9987	0.009 642 061	0.026 962 673	0.999 349 789	0.998 050 634
0.9986	0.010 006 504	0.027 978 986	0.999 299 755	0.997 900 735
0.9985	0.010 358 197	0.028 959 447	0.999 249 719	0.997 750 844
0.9984	0.010 698 396	0.029 907 581	0.999 199 680	0.997 600 960
0.9983	0.011 028 166	0.030 826 370	0.999 149 638	0.997 451 084
0.9982	0.011 348 426	0.031 718 396	0.999 099 595	0.997 301 215
0.9981	0.011 659 928	0.032 585 767	0.999 049 548	0.997 151 354
0.9980	0.011 963 398	0.033 430 519	0.998 999 499	0.997 001 500
0.9979	0.012 259 412	0.034 254 269	0.998 949 448	0.996 851 654
0.9978	0.012 548 488	0.035 058 467	0.998 899 394	0.996 701 816
0.9977	0.012 831 114	0.035 844 487	0.998 849 338	0.996 551 985
0.9976	0.013 107 688	0.036 613 444	0.998 799 279	0.996 402 161
0.9975	0.013 378 606	0.037 366 445	0.998 749 218	0.996 252 345
0.9974	0.013 644 183	0.038 104 384	0.998 699 154	0.996 102 536
0.9973	0.013 904 746	0.038 828 170	0.998 649 088	0.995 952 735
0.9972	0.014 160 554	0.039 538 531	0.998 599 019	0.995 802 941
0.9971	0.014 411 867	0.040 236 203	0.998 548 947	0.995 653 155
0.9970	0.014 658 934	0.040 921 881	0.998 498 873	0.995 503 377
0.9969	0.014 901 943	0.041 596 090	0.998 448 797	0.995 353 606
0.9968	0.015 141 095	0.042 259 400	0.998 398 718	0.995 203 842
0.9967	0.015 376 562	0.042 912 295	0.998 348 636	0.995 054 086
0.9966	0.015 608 537	0.043 555 310	0.998 298 553	0.994 904 338
0.9965	0.015 837 140	0.044 188 788	0.998 248 466	0.994 754 596
0.9964	0.016 062 539	0.044 813 199	0.998 198 377	0.994 604 863
0.9963	0.016 284 867	0.045 428 916	0.998 148 286	0.994 455 137
0.9962	0.016 504 226	0.046 036 227	0.998 098 192	0.994 305 418
0.9961	0.016 720 761	0.046 635 540	0.998 048 095	0.994 155 707
0.9960	0.016 934 567	0.047 227 120	0.997 997 996	0.994 006 004

Table III. Five-place Table for $n = 1.4$ and $e = 1.000$ to 0.525

$e = \frac{r}{m^*T^*}$	$f(e)^a$	$\frac{f(e)}{2e}$	$e^{3/2}$	$kf(e)$
1.000	0	0	1	0
0.999	0.01689	0.008453	0.998504	0.02365
0.998	0.02388	0.011964	0.997001	0.03343
0.997	0.02923	0.014659	0.995503	0.04092
0.996	0.03373	0.016933	0.994006	0.04722
0.995	0.03770	0.018945	0.992509	0.05278
0.994	0.04127	0.020760	0.991014	0.05778
0.993	0.04455	0.022432	0.989521	0.06237
0.992	0.04760	0.023992	0.988024	0.06664
0.991	0.05046	0.025459	0.986530	0.07064
0.990	0.05316	0.026848	0.985039	0.07442
0.989	0.05573	0.028175	0.983545	0.07802
0.988	0.05818	0.029443	0.982054	0.08145
0.987	0.06052	0.030659	0.980563	0.08473
0.986	0.06277	0.031831	0.979074	0.08788
0.985	0.06494	0.032964	0.977585	0.09092
0.984	0.06703	0.034060	0.976095	0.09384
0.983	0.06906	0.035127	0.974609	0.09668
0.982	0.07102	0.036161	0.973122	0.09943
0.981	0.07293	0.037171	0.971636	0.10210
0.980	0.07478	0.038153	0.970150	0.10469
0.979	0.07658	0.039111	0.968666	0.10721
0.978	0.07834	0.040051	0.967182	0.10968
0.977	0.08006	0.040972	0.965699	0.11208
0.976	0.08174	0.041875	0.964217	0.11444
0.975	0.08338	0.042759	0.962735	0.11673
0.974	0.08498	0.043624	0.961255	0.11897
0.973	0.08655	0.044476	0.959775	0.12117
0.972	0.08809	0.045314	0.958296	0.12333
0.971	0.08960	0.046138	0.956817	0.12544
0.970	0.09108	0.046948	0.955339	0.12751
0.969	0.09253	0.047745	0.953862	0.12954
0.968	0.09396	0.048533	0.952386	0.13154
0.967	0.09537	0.049312	0.950911	0.13352
0.966	0.09675	0.050078	0.949436	0.13545

Table III (Cont'd.)

$e = \frac{r}{m^*T^*}$	$f(e)^a$	$\frac{f(e)}{2e}$	$e^{3/2}$	$kf(e)$
0.965	0.09811	0.050834	0.947962	0.13735
0.964	0.09944	0.051577	0.946489	0.13922
0.963	0.10076	0.052316	0.945017	0.14106
0.962	0.10205	0.052041	0.943545	0.14287
0.961	0.10333	0.053762	0.942074	0.14466
0.960	0.10459	0.054474	0.940604	0.14643
0.958	0.10705	0.055872	0.937666	0.14987
0.956	0.10945	0.057244	0.934731	0.15323
0.954	0.11178	0.058585	0.931800	0.15649
0.952	0.11406	0.059905	0.928871	0.15968
0.950	0.11628	0.061200	0.925945	0.16279
0.948	0.11844	0.062468	0.923023	0.16582
0.946	0.12056	0.063721	0.920104	0.16878
0.944	0.12264	0.064958	0.917187	0.17170
0.942	0.12467	0.066173	0.914274	0.17454
0.940	0.12665	0.067367	0.911364	0.17731
0.938	0.12860	0.068550	0.908457	0.18004
0.936	0.13051	0.069717	0.905553	0.18271
0.934	0.13238	0.070867	0.902650	0.18533
0.932	0.13421	0.072001	0.899754	0.18789
0.930	0.13601	0.073124	0.896860	0.19041
0.928	0.13778	0.074235	0.893968	0.19289
0.926	0.13952	0.075335	0.891080	0.19533
0.924	0.14123	0.076423	0.888194	0.19772
0.922	0.14291	0.077500	0.885312	0.20007
0.920	0.14456	0.078565	0.882433	0.20238
0.918	0.14618	0.079619	0.879557	0.20465
0.916	0.14778	0.080666	0.876684	0.20689
0.914	0.14936	0.081707	0.873815	0.20910
0.912	0.15091	0.082736	0.870948	0.21127
0.910	0.15243	0.083753	0.868085	0.21340
0.908	0.15393	0.084763	0.865224	0.21550
0.906	0.15541	0.085767	0.862367	0.21757
0.904	0.15687	0.086764	0.859513	0.21962
0.902	0.15830	0.087749	0.856663	0.22162

Table III (Cont'd.)

$e = \frac{r}{m^*T^*}$	$f(e)^a$	$\frac{f(e)}{2e}$	$e^{3/2}$	$kf(e)$
0.900	0.15972	0.088733	0.853815	0.22361
0.895	0.16317	0.091156	0.846710	0.22844
0.890	0.16651	0.093545	0.839624	0.23311
0.885	0.16973	0.095893	0.832559	0.23762
0.880	0.17285	0.098210	0.825513	0.24199
0.875	0.17588	0.100503	0.818488	0.24623
0.870	0.17881	0.102764	0.811482	0.25033
0.865	0.18165	0.105000	0.804497	0.25431
0.860	0.18440	0.107209	0.797531	0.25816
0.855	0.18707	0.109398	0.790586	0.26190
0.850	0.18966	0.111565	0.783661	0.26552
0.845	0.19218	0.113716	0.776757	0.26905
0.840	0.19463	0.115851	0.769870	0.27248
0.835	0.19701	0.117970	0.763009	0.27581
0.830	0.19932	0.120072	0.756166	0.27905
0.825	0.20157	0.122164	0.749343	0.28220
0.820	0.20375	0.124238	0.742542	0.28525
0.815	0.20588	0.126307	0.735760	0.28823
0.810	0.20795	0.128364	0.729000	0.29113
0.805	0.20996	0.130410	0.722260	0.29394
0.800	0.21191	0.132444	0.715542	0.29667
0.795	0.21381	0.134472	0.708844	0.29933
0.790	0.21566	0.136494	0.702167	0.30192
0.785	0.21746	0.138510	0.695512	0.30444
0.780	0.21921	0.140519	0.688877	0.30689
0.775	0.22091	0.142523	0.682264	0.30927
0.770	0.22256	0.144519	0.675672	0.31158
0.765	0.22417	0.146516	0.669102	0.31384
0.760	0.22573	0.148507	0.662553	0.31602
0.755	0.22725	0.150497	0.656025	0.31815
0.750	0.22873	0.152487	0.649519	0.32022
0.745	0.23016	0.154470	0.643035	0.32222
0.740	0.23155	0.156453	0.636572	0.32417
0.735	0.23290	0.158435	0.630131	0.32606
0.730	0.23421	0.160418	0.623712	0.32789

Table III (Cont'd.)

$e = \frac{r}{m^*T^*}$	$f(e)^a$	$\frac{f(e)}{2e}$	$e^{3/2}$	$kf(e)$
0.725	0.23548	0.162400	0.617315	0.32967
0.720	0.23671	0.164382	0.610940	0.33139
0.715	0.23791	0.166371	0.604587	0.33307
0.710	0.23906	0.168352	0.598257	0.33468
0.705	0.24018	0.170340	0.591948	0.33625
0.700	0.24126	0.172329	0.585662	0.33776
0.695	0.24231	0.174324	0.579398	0.33923
0.690	0.24332	0.176319	0.573157	0.34065
0.685	0.24429	0.178314	0.566938	0.34201
0.680	0.24523	0.180316	0.560742	0.34332
0.675	0.24614	0.182326	0.554569	0.34460
0.670	0.24701	0.184336	0.548419	0.34581
0.665	0.24785	0.186353	0.542291	0.34699
0.660	0.24866	0.188379	0.536187	0.34812
0.655	0.24943	0.190405	0.530105	0.34920
0.650	0.25017	0.192438	0.524047	0.35024
0.645	0.25088	0.194481	0.518012	0.35123
0.640	0.25155	0.196523	0.512000	0.35217
0.635	0.25220	0.198583	0.506011	0.35308
0.630	0.25281	0.200643	0.500047	0.35393
0.625	0.25340	0.202720	0.494106	0.35476
0.620	0.25395	0.204798	0.488188	0.35553
0.615	0.25447	0.206886	0.482295	0.35626
0.610	0.25496	0.208984	0.476425	0.35694
0.605	0.25542	0.211091	0.470579	0.35759
0.600	0.25585	0.213208	0.464758	0.35819
0.595	0.25625	0.215336	0.458960	0.35875
0.590	0.25662	0.217475	0.453188	0.35927
0.585	0.25696	0.219624	0.447499	0.35974
0.580	0.25727	0.221784	0.441715	0.36018
0.575	0.25756	0.223965	0.436015	0.36058
0.570	0.25781	0.226149	0.430341	0.36093
0.565	0.25803	0.228345	0.424691	0.36124
0.560	0.25823	0.230563	0.419065	0.36152
0.555	0.25840	0.232793	0.413466	0.36176

Table III.(Cont'd.)

$e = \frac{r}{m^*T^*}$	$f(e)^a$	$\frac{f(e)}{2e}$	$e^{3/2}$	$kf(e)$
0.550	0.25854	0.235036	0.407891	0.36196
0.545	0.25865	0.237294	0.402341	0.36211
0.540	0.25873	0.239565	0.396817	0.36222
0.535	0.25878	0.241850	0.391319	0.36229
0.530	0.25880	0.244151	0.385846	0.36232
0.525	0.25880	0.246476	0.380399	0.36232

^aData obtained from Keenan, J. and Kaye, J., "Thermodynamic Properties of Air" or "Gas Tables," John Wiley & Sons, Inc., New York, N.Y.

APPENDIX B An Alternate Method by Transformation of
Nondimensional Governing Differential Equation

For numerical analysis with a desk calculating machine, a transformation of the nondimensional governing differential equation is advantageous when the values of

$$\frac{r}{m^*T^*} \text{ and } \left[\frac{r}{m^*T^*} \right]^{\frac{1}{n}} \sqrt{1 - \left(\frac{r}{m^*T^*} \right)^{\frac{n-1}{n}}}$$

can be taken directly from compiled tables for each successive step. The transformation is accomplished by the functions

$$e = r/m^*T^* \quad (B.1)$$

and

$$f(e) = \left[\frac{r}{m^*T^*} \right]^{\frac{1}{n}} \sqrt{1 - \left(\frac{r}{m^*T^*} \right)^{\frac{n-1}{n}}} = e^{\frac{1}{n}} \sqrt{1 - e^{\frac{n-1}{n}}} \quad (B.2)$$

Taking the derivative of the Eq. B.1 with respect to m^* and rearranging terms, there is obtained

$$\frac{dT^*}{dm^*} = -\frac{T^*}{m^*} - \frac{T^*}{e} \frac{de}{dm^*} \quad (B.3)$$

Equating Eq. B.3 to Eq. IV.33, using the defined functions, eliminating T^* , and solving for the first derivative of e with respect to m^* gives

$$\frac{de}{dm^*} = \frac{e \left[C^* e^{3/2} \sqrt{m^*} - k r^{3/2} f(e) \right]}{m^* r^{3/2} f(e)} \quad (B.4)$$

Introducing the function

$$m^* \equiv j^2 \quad (B.5)$$

and taking the derivative yields

$$dm^* = 2j \, dj. \quad (B.6)$$

Substitution of Eqs. B.5 and B.6 into Eq. B.4 gives the transformed equation:

$$\frac{de}{dj} = \frac{2e \left[C^* e^{3/2} j - k r^{3/2} f(e) \right]}{r^{3/2} j f(e)} \quad (B.7)$$

For the case where r equals one ($r = 1$), Eq. B.7 must be inverted, as the numerical solution cannot be started with an infinite slope [when $r = 1$, $m^* = 1 = j$ and $T^* = 1 = e$; then $f(e) = 0$]; hence,

$$\frac{dj}{de} = \frac{r^{3/2}j[f(e)/2e]}{\left[C^*e^{3/2}j - r^{3/2}kf(e)\right]} \quad (B.8)$$

$$\equiv F(e,j) \quad .$$

To expedite the numerical solutions by formula B.8, tables of the functions $f(e)/2e$, $e^{3/2}$ and $kf(e)$ can be established. Such tables are given in Appendix A for $n = 1.4$.

The time dependence of gas properties will usually be computed from the temperature-mass-time equations developed under the original theoretical section, since the pressure, temperature, and mass are normally required. To use the Hermite quadrature, the following equation can be derived from the Eqs. B.1, B.3, B.5 and B.6:

$$\frac{dT^*}{dm^*} = -\frac{r}{j^3e} \left(\frac{1}{j} + \frac{1}{2e} \frac{de}{dj} \right) \quad (B.9)$$

APPENDIX C Equations for Numerical Analysis

The starting of the numerical solution for the case where r equals one ($r = 1$) is particularly troublesome. An exceedingly rapid change in the slope is encountered at the beginning, as the initial condition is a regular singular point. The Taylor's series is often used in establishing initial values; however, taking the derivative of Eq. B.8 yields

$$\frac{d^2 j}{de^2} = \frac{-kr^{9/2}j[f(e)]^3}{C*e^{3/2}j - r^{3/2}kf(e)} + \frac{2C*r^{3/2}je^{3/2}f(e)}{n} \\ - \frac{\left(\frac{n-1}{n}\right) \frac{C*r^{3/2}je^{5/2}e^{1/n}}{f(e)} + r^{3/2}j f(e) [5C*je^{3/2} - 2r^{3/2}kf(e)]}{\left\{2e[C*e^{3/2}j - r^{3/2}kf(e)]\right\}^2} \quad (C.1)$$

Values near the initial conditions give large negative values for the second derivative, as the third term in the numerator of expression C.1 has an infinite negative value at the beginning. This singularity of the second derivative is highly undesirable, as the function fails to be expandable in a Taylor's series.

A starting method for this particular case by the Taylor expansion is

$$j(1 - \Delta e) = j(1) - \frac{\Delta e}{1!} j'(1) + \frac{(\Delta e)^2}{2!} j''(1 - \theta \Delta e) + \dots, \quad (C.2)$$

where $\Delta e = 1 - e$ and $0 < \theta < 1$. For this problem Eq. C.2 reduces to

$$j(1 - \Delta e) = 1 + \frac{(\Delta e)^2}{2!} j''(1 - \theta \Delta e) + \dots, \quad (C.3)$$

since $j'(1) = 0$. The limit of the remainder for Δe approaching zero or e approaching one is given by the third term of Eq. C.1, namely,

$$\begin{aligned}
& \lim_{\Delta e \rightarrow 0} \frac{(\Delta e)^2}{2!} \left[\frac{-\left(\frac{n-1}{n}\right) \frac{C^* j^2 \bar{e}^{5/2} \bar{e}^{1/n}}{\bar{e}^{1/n} \sqrt{1 - \bar{e}^{\frac{n-1}{n}}}}}{\left\{2\bar{e} \left[C^* \bar{e}^{3/2} j - k f(\bar{e})\right]\right\}^2} \right] \\
& = \lim_{\Delta e \rightarrow 0} \frac{-\left(\frac{n-1}{n}\right) \frac{(\Delta e)^2}{\sqrt{1 - \bar{e}^{\frac{n-1}{n}}}}}{8C^*} \quad (C.4)
\end{aligned}$$

where $\bar{e} = 1 - \theta \Delta e$. Here r equals one, and for Δe equal to zero, j and $\bar{e}$, representing the initial condition, are each one. Expanding the terms under the radical in the denominator by the binomial series, there is obtained

$$\begin{aligned}
& \lim_{\Delta e \rightarrow 0} \frac{-\left(\frac{n-1}{n}\right) (\Delta e)^2}{8C^* \sqrt{1 - (1 - \theta \Delta e)^{\frac{n-1}{n}}}} \\
& = \lim_{\Delta e \rightarrow 0} \frac{-\left(\frac{n-1}{n}\right) (\Delta e)^2}{8C^* \sqrt{\left(\frac{n-1}{n}\right) \theta \Delta e + \frac{(n-1)}{n^2} \frac{(\theta \Delta e)^2}{2!}}}, \quad (C.5)
\end{aligned}$$

which reduces to

$$\lim_{\Delta e \rightarrow 0} -\frac{1}{8C^*} \sqrt{\frac{n-1}{n} \theta} (\Delta e)^{3/2} + \text{higher terms in } (\Delta e). \quad (C.6)$$

Because θ cannot be zero, a finite value is obtained. Therefore, in the neighborhood of e equals one; $j(1 - \Delta e)$ is finite and approximately

$$j(1 - \Delta e) \approx 1 + \text{constant } (\Delta e)^{3/2}. \quad (C.7)$$

An analogous analysis yields

$$j(1 - \Delta e) \approx \text{constant } (\Delta e)^{1/2}. \quad (C.8)$$

Hence, Δe versus $j'(e)$ is approximated by a parabola near e equals one, and the increment $\Delta j = 1 - j(1 - \Delta e)$ is given by

$$\Delta j = \int_{1-\Delta e}^1 j'(e) de = \int_0^{\Delta e} j'(1 - \Delta e) d\Delta e = \frac{2}{3}(\Delta e) j'(1 - \Delta e), \quad (C.9)$$

that is, the area under the parabolic curve (see Fig.8). As first trials for starting the numerical solution,

$$j = 1 \quad (C.10)$$

or

$$j = 1 - (\Delta e)^{3/2} \quad (C.11)$$

can be assumed. These values of j are improved by utilizing Eq. C.9 in conjunction with Eq. B.8, i.e.,

$$j \approx 1 - 2/3 (\Delta e) F(e = 1 - \Delta e, j = 1) \quad (C.12)$$

or

$$j \approx 1 - 2/3 (\Delta e) F(e = 1 - \Delta e, j = 1 - (\Delta e)^{3/2}) \quad (C.13)$$

If necessary these values can be further improved by iteration. A minute value of Δe is essential. In this report $\Delta e = 0.0002$ was used (see Numerical Analysis Example under Appendix F).

After the determination of a value for j near the initial conditions, conventional equations for the numerical analysis are employed. The equations used in this report have been adapted for backward integration and are as follows:

1. Determination of points 1 through 5

Kutta Formula

$$j_{n+1} = j_n - (1/6) (k_0 + 2k_1 + 2k_2 + k_3) + O(h^5) \quad (C.14)$$

where

$$k_0 = (\Delta e) F(e_n, j_n)$$

$$k_1 = (\Delta e) F(e_n - \frac{1}{2} \Delta e, j_n - \frac{1}{2} k_0)$$

FIG. 8
PLOTS OF STARTING VALUES

$$k_2 = (\Delta e) F(e_n - \frac{1}{2}\Delta e, j_n - \frac{1}{2}k_1)$$

$$k_3 = (\Delta e) F(e_n - \Delta e, j - k_2)$$

2. Checking of values determined by Kutta Formula

Newton Forward Difference Polynomial-Interpolation Formula

$$\left. \begin{aligned} j_1 &= j_0 - (\Delta e) \left[1 + \frac{1}{2}\Delta - \frac{1}{12}\Delta^2 + \frac{1}{24}\Delta^3 - \frac{19}{720}\Delta^4 + \frac{3}{160}\Delta^5 \dots \right] j_0' \\ j_2 &= j_0 - (\Delta e) \left[2 + 2\Delta + \frac{1}{3}\Delta^2 + 0\Delta^3 - \frac{1}{90}\Delta^4 + \frac{1}{90}\Delta^5 \dots \right] j_0' \\ j_3 &= j_0 - (\Delta e) \left[3 + \frac{9}{2}\Delta + \frac{9}{4}\Delta^2 + \frac{3}{8}\Delta^3 - \frac{3}{80}\Delta^4 + \frac{3}{160}\Delta^5 \dots \right] j_0' \\ j_4 &= j_0 - (\Delta e) \left[4 + 8\Delta + \frac{20}{3}\Delta^2 + \frac{8}{3}\Delta^3 + \frac{14}{45}\Delta^4 + 0\Delta^5 \dots \right] j_0' \end{aligned} \right\} \quad (C.15)$$

3. Determination of points from 6 on

Adams Interpolation Method

$$\left. \begin{aligned} \text{Prediction: } j_{n+1}^0 &= j_n - (\Delta e) \left[1 + \frac{1}{2}\nabla + \frac{5}{12}\nabla^2 + \frac{3}{8}\nabla^3 + \frac{251}{720}\nabla^4 + \frac{95}{288}\nabla^5 + \dots \right] j_n' \\ \text{Iteration: } j_{n+1} &= j_n - (\Delta e) \left[1 - \frac{1}{2}\nabla - \frac{1}{12}\nabla^2 - \frac{1}{24}\nabla^3 - \frac{19}{720}\nabla^4 - \frac{3}{160}\nabla^5 - \dots \right] j_{n+1}' \end{aligned} \right\} \quad (C.16)$$

An important factor in reducing the amount of the numerical work is the proper selection of the step width Δe . An estimate can be made by the formula

$$\Delta e \approx \frac{\rho_f}{a_{-1} \frac{\partial}{\partial j} \left(\frac{dj}{de} \right)} \quad (C.17)$$

where

ρ_f = convergence factor, normally taken about 0.1.

a_{-1} = factor depending upon method used (1 for Adams Interpolation Method).

The partial derivative of the differential Eq. B.8 with respect to j becomes

$$\frac{\partial}{\partial j} \left(\frac{dj}{de} \right) = \frac{-r^3 [kf(e)] [f(e)/2e]}{[C * e^{3/2} j - r^{3/2} kf(e)]^2} \quad (C.18)$$

A second criterion for the estimation of Δe is the requirement that the truncation error must be smaller than a predetermined quantity E . In most cases this means

$$\Delta e \frac{19}{720} |\nabla^4 j'_{n+1}| < E.$$

In this problem great care must be taken to insure that a stable method is selected. If usage of certain methods is desired, an inquiry should be made concerning the stability.

An investigation of Milne's methods was undertaken, as they have the advantage of smaller truncation errors when compared with Adams Interpolation Method. The smaller truncation errors permit greater step width; however, in this instance Milne's methods are unfavorable in regards to stability. Using the tactics as indicated in the section "The Special Case" of Ref. 4 in the Bibliography, pages 202-208, the parasitic solution will increase as the calculation proceeds and not damp out as required. Numerical calculations revealed that results obtained with Milne's methods oscillated and were unstable as predicted by the examination.

As can be anticipated, all numerical solutions tend to approach the limiting special case of constant pressure with the passing of time. The constant-pressure special case asymptote is when the containment structure is void of air.

APPENDIX D Runge-Kutta Simultaneous Equations for
Numerical Analysis

The following formulas can be utilized for numerical analysis of simultaneous Eqs. (IV.34 or IV.31):

$$\begin{aligned}
 \frac{dT^*}{dt} &= F(t, T^*, m^*) \\
 \frac{dm^*}{dt} &= G(t, T^*, m^*) \\
 T_{n+1}^* &= T_n^* + (1/6) (k_0 + 2k_1 + 2k_2 + k_3) \\
 k_1 &= F(t_n, T_n^*, m_n^*) \\
 k_2 &= F(t_n + \frac{1}{2}\Delta t, T_n^* + \frac{1}{2}k_1, m_n^* + \frac{1}{2}\ell_1) \Delta t \\
 k_3 &= F(t_n + \frac{1}{2}\Delta t, T_n^* + \frac{1}{2}k_2, m_n^* + \frac{1}{2}\ell_2) \Delta t \\
 k_4 &= F(t_n + \Delta t, T_n^* + k_3, m_n^* + \ell_3) \Delta t \\
 m_{n+1}^* &= m_n^* + (1/6) (\ell_1 + 2\ell_2 + 2\ell_3 + \ell_4) \\
 \ell_1 &= G(t_n, T_n^*, m_n^*) \\
 \ell_2 &= G(t_n + \frac{1}{2}\Delta t, T_n^* + \frac{1}{2}k_1, m_n^* + \frac{1}{2}\ell_1) \Delta t \\
 \ell_3 &= G(t_n + \frac{1}{2}\Delta t, T_n^* + \frac{1}{2}k_2, m_n^* + \frac{1}{2}\ell_2) \Delta t \\
 \ell_4 &= G(t_n + \Delta t, T_n^* + k_3, m_n^* + \ell_3) \Delta t
 \end{aligned} \tag{D.1}$$

APPENDIX E Data for Numerical Analysis

General Data for Parameters Investigated

$$c_p = 0.24 \text{ Btu/lbm} \cdot ^\circ\text{R}$$

$$c_v = 0.1715 \text{ Btu/lbm} \cdot ^\circ\text{R}$$

$$k = c_p/c_v = n = 1.4$$

$$p_a = 14.696 \text{ lbf/in.}^2 = 2116.2 \text{ lbf/ft}^2$$

$$q = 3.97 \times 10^4 \text{ Btu/sec}$$

$$R = 53.3 \text{ ft-lbf/}^\circ\text{R-lbm}$$

$$T_a = 520^\circ\text{R}$$

$$V = 60,000 \text{ ft}^3$$

$$\rho = 0.0765 \text{ lbm/ft}^3$$

$$m_s = 0.0765 \times 60,000 = 4590 \text{ lbm}$$

$$\sqrt{2g_c J c_p} = 109.61 \text{ ft/sec}(^\circ\text{R})^{1/2}$$

$$T_a^{3/2} = 11,857.824 \text{ }^\circ\text{R}^{3/2}$$

Vent Size at Throat

Large Vent

$$A_t = 50 \text{ ft}^2$$

Small Vent

$$A_t = 5 \text{ ft}^2$$

Initial Conditions of Gas for Commencing of Discharge

States 1 and 2

$$p_s = p_a = 14.696 \text{ lbf/in.}^2 = 2116.2 \text{ lbf/ft}^2$$

$$T_s = T_a = 520^\circ\text{R}$$

$$r = p_a/p_s = 1$$

State 3

$$r = p_a / p_s = 0.6$$

$$p_s = 14.696 \div 0.6 = 24.493 \text{ lbf/in.}^2$$

$$T_s = 520 \div 0.6 = 866.667^\circ\text{R}$$

APPENDIX F Numerical Analysis Example

Given: $A_t = 5 \text{ ft}^2$ and $r = 1$:

$$C^* = \frac{39,700}{0.1715(5/60,000)(4590)(11857.824)(109.61)} = 0.4656287$$

For starting, use step width $\Delta e = 0.0002$.

$$e = 0.9998$$

First iteration starting with Formula C.10:

$$\begin{aligned} j_{0.9998}^0 &= 1.000\,000\,000 \\ \left(\frac{dj}{de}\right)_{0.9998} &= \frac{1^{3/2}(1.000\,000\,000)(0.003\,779\,998)}{0.465\,628\,7(0.999\,700\,015)(1.000\,000\,000) - 1(0.010\,581\,878)} \\ &= 0.008\,309\,383\,75 \text{ (by formula B.8)} \\ j_{0.9998} &= 1 - (2/3)(0.0002)(0.008\,309\,383\,75) = 0.999\,998\,892 \text{ (by} \\ &\quad \text{formula C.12)} \end{aligned}$$

First iteration starting with Formula C.11:

$$\begin{aligned} j_{0.9998}^0 &= 1 - (0.0002)^{3/2} = 0.999\,997\,172 \\ \left(\frac{dj}{de}\right)_{0.9998} &= \frac{1^{3/2}(0.999\,997\,172)(0.003\,779\,998)}{0.465\,628\,7(0.999\,700\,015)(0.999\,997\,172) - 1(0.010\,581\,878)} \\ &= 0.008\,309\,384\,29 \text{ (by formula B.8)} \\ j_{0.9998} &= 1 - (2/3)(0.0002)(0.008\,309\,384\,29) = 0.999\,998\,892 \text{ (by} \\ &\quad \text{formula C.13)} \end{aligned}$$

The two methods produce the same value of $j_{0.9998}$ on the first iteration; hence,

$$\begin{aligned} \left(\frac{dj}{de}\right)_{0.9998} &= \frac{1^{3/2}(0.999\,998\,892)(0.003\,779\,998)}{0.465\,628\,7(0.999\,700\,015)(0.999\,998\,892) - 1(0.010\,581\,878)} \\ &= 0.008\,309\,383\,95. \end{aligned}$$

For $e = 0.9996, 0.9994, 0.9992$ and 0.9990 use formula C.14.

$$e = 0.9996$$

$$\begin{aligned} k_0 &= (0.0002)(0.008\,309\,384) = 0.000\,001\,662 \\ e_{0.9996} - 1/2\Delta e &= 0.9997; j_{0.9996} - 1/2k_0 = 0.999\,998\,061 \\ k_1 &= \frac{(0.0002)(0.999\,998\,061)(0.004\,629\,733)}{0.465\,628\,7(0.999\,550\,034)(0.999\,998\,061) - 0.012\,959\,363} \\ &= 0.000\,002\,046 \end{aligned}$$

$$e_{0.9998} - 1/2 \Delta e = 0.9997; j_{0.9998} - 1/2 k_1 = 0.999997869$$

$$k_2 = \frac{(0.002)(0.999997869)(0.004629733)}{0.4656287(0.999550034)(0.999997869) - 0.012959363}$$

$$= 0.000002046$$

$$e_{0.9996} = 0.9996; j_{0.9998} - k_2 = 0.999996846$$

$$k_3 = \frac{(0.0002)(0.999996846)(0.005346217)}{0.4656287(0.999400060)(0.999996846) - 0.014963420}$$

$$= 0.000002374$$

$$j_{0.9996} = 0.999998892 - 1/6 [0.000001662 + 2(0.000002046) + 2(0.000002374) + 0.000002377] = 0.999996855$$

$$\left(\frac{dj}{de}\right)_{0.9996} = \frac{1(0.999996855)(0.005346217)}{0.4656287(0.999400060)(0.999996855) - 0.014963420}$$

$$= 0.011870303$$

$$\underline{e = 0.9994}$$

$$k_0 = (0.0002)(0.011870303) = 0.000002374$$

$$e_{0.9996} - 1/2 \Delta e = 0.9995; j_{0.9996} - 1/2 k_0 = 0.999995668$$

$$k_1 = \frac{(0.0002)(0.999995668)(0.005977538)}{0.4656287(0.999250094)(0.999995668) - 0.016728737}$$

$$= 0.000002665$$

$$e_{0.9996} - 1/2 \Delta e = 0.9995; j_{0.9996} - 1/2 k_1 = 0.999995522$$

$$k_2 = \frac{(0.0002)(0.999995522)(0.005977538)}{0.4656287(0.999250094)(0.999995522) - 0.016728737}$$

$$= 0.000002665$$

$$e_{0.9994} = 0.9994; j_{0.9996} - k_2 = 0.999994190$$

$$k_3 = \frac{(0.0002)(0.999994190)(0.006548374)}{0.4656287(0.999100135)(0.999994190) - 0.018324446}$$

$$= 0.000002931$$

$$j_{0.9994} = 0.999996855 - (1/6) [0.000002374 + 2(0.000002665) + 2(0.000002665) + 0.000002931] = 0.999994194$$

$$\left(\frac{dj}{de}\right)_{0.9994} = \frac{1(0.999994194)(0.006548374)}{0.4656287(0.999100135)(0.999994194) - 0.018324446}$$

$$= 0.014653371$$

$$\underline{e = 0.9992}$$

$$k_0 = (0.002)(0.014653371) = 0.000002931$$

$$e_{0.9994} - 1/2 \Delta e = 0.9993; j_{0.9994} - 1/2 k_0 = 0.999992729$$

$$k_1 = \frac{(0.0002)(0.999992729)(0.007073367)}{0.4656287(0.998950184)(0.999992729) - 0.019791563}$$

$$= 0.000003177$$

$$e_{0.9994} - 1/2 \Delta e = 0.9993; j_{0.9994} - 1/2 k_1 = 0.999992605$$

$$k_2 = \frac{(0.0002)(0.999992605)(0.007073367)}{0.4656287(0.998950184)(0.999992605) - 0.019791563}$$

$$= 0.000003177$$

$$e_{0.9992} = 0.9992; j_{0.9994} - k_2 = 0.999991017$$

$$k_3 = \frac{(0.0002)(0.999991017)(0.007562086)}{0.4656287(0.998800240)(0.999991017) - 0.021156902}$$

$$= 0.000003407$$

$$j_{0.9992} = 0.999994194 - (1/6)[0.000002931 + 2(0.000003177) + 2(0.000003177) + 0.000003407] = 0.999991020$$

$$\left(\frac{dj}{de}\right)_{0.9992} = \frac{1(0.999991020)(0.007562086)}{0.4656287(0.998800240)(0.999991020) - 0.021156902}$$

$$= 0.017035064$$

$$\underline{e = 0.9990}$$

$$k_0 = (0.0002)(0.017035064) = 0.000003407$$

$$e_{0.9992} - 1/2 \Delta e = 0.9991; j_{0.9992} - 1/2 k_0 = 0.999989316$$

$$k_1 = \frac{(0.0002)(0.999989316)(0.008021197)}{0.4656287(0.998650304)(0.999989316) - 0.022439137}$$

$$= 0.000003625$$

$$e_{0.9992} - 1/2 \Delta e = 0.9991; j_{0.9992} - 1/2 k_1 = 0.999989207$$

$$k_2 = \frac{(0.0002)(0.999989207)(0.008021197)}{0.4656287(0.998650304)(0.999989207) - 0.022439137}$$

$$= 0.000003625$$

$$e_{0.9990} = 0.9990; j_{0.9992} - k_2 = 0.999987395$$

$$k_3 = \frac{(0.0002)(0.999987395)(0.008455463)}{0.4656287(0.998500375)(0.999987395) - 0.023651622}$$

$$= 0.000003832$$

$$j_{0.9990} = 0.999991020 - (1/6)[0.000003407 + 2(0.000003625) + 2(0.000003625) + 0.000003832] = 0.999987397$$

$$\left(\frac{dj}{de}\right)_{0.9990} = \frac{1(0.999987397)(0.008455463)}{0.4656287(0.998500375)(0.999987397) - 0.023651622}$$

$$= 0.019161283$$

A check for $e = 0.9996, 0.9994, 0.9992$ and 0.9990 will be made using equation C.15.

e	j	$\frac{dj}{de} = j'$	$\Delta^I_{j'}$	$\Delta^{II}_{j'}$	$\Delta^{III}_{j'}$	$\Delta^{IV}_{j'}$
1.0000	1.000000000	0.000000000				
			0.008309384			
0.9998	0.999998892	0.008309384		-0.004748465		
			0.003560919		0.003970614	
0.9996	0.999996855	0.011870303		-0.000777851		-0.003594138
			0.002783068		0.000376476	
0.9994	0.999994194	0.014653371		-0.000401375		-0.000230575
			0.002381693		0.000145901	
0.9992	0.999991020	0.017035064		-0.000255474		
			0.002126219			
0.9990	0.999987397	0.019161283				

$$j_{0.9996} = 0.999998892 - (0.0002) [0.008309384 + 1/2(0.003560919) \\ - 1/12(-0.000777851) + 1/24(0.000376476) \\ - 19/720(-0.000230575)]$$

$$= 0.999996857$$

$$j_{0.9994} = 0.999998892 - (0.0002) [2(0.008309384) + 2(0.003560919) \\ + 1/3(-0.000777851) + 0(0.000376476) \\ - 1/90(-0.000230575)]$$

$$= 0.999994195$$

$$j_{0.9992} = 0.999998892 - (0.0002) [3(0.008309384) \\ + 9/2(0.003560919) + 9/4(-0.000777851) \\ + 3/8(0.000376476) - 3/80(-0.000230575)]$$

$$= 0.999991022$$

$$j_{0.9990} = 0.999998892 - (0.0002) [4(0.008309384) + 8(0.003560919) \\ + 20/3(-0.000777851) + 8/3(0.000376476) \\ + 14/15(-0.000230575)]$$

$$= 0.999987398$$

For values of $j = 0.9988$ and thereon, formula C.16 will be utilized.

$$e = 0.9988$$

$$\begin{aligned} j_{0.9988}^0 &= 0.999\,987\,397 - (0.0002) [0.019\,161\,283 + 1/2 (0.002\,126\,219) \\ &\quad + 5/12 (-0.000\,255\,474) + 3/8 (0.000\,145\,901) \\ &\quad + 251/720 (-0.000\,230\,575)] \\ &= 0.999\,983\,379 \end{aligned}$$

$$\begin{aligned} \left(\frac{dj}{de}\right)_{0.9988} &= \frac{(0.999\,983\,379)(0.009\,263\,361)}{0.465\,628\,7(0.998\,200\,540)(0.999\,983\,379) - 0.025\,906\,286} \\ &= 0.021\,106\,622 \end{aligned}$$

e	j	$\frac{dj}{de} = j'$	$\Delta_{j'}^I$	$\Delta_{j'}^{II}$	$\Delta_{j'}^{III}$	$\Delta_{j'}^{IV}$
0.9994						-0.000 230 575
0.9992					0.000 145 901	
0.9990	0.999 987 397	0.019 161 283	0.002 126 219	-0.000 255 474	0.000 074 594	-0.000 071 307
0.9988		0.021 106 622	0.001 945 339	-0.000 180 880		

$$\begin{aligned} j_{0.9988} &= 0.999\,987\,397 - (0.0002) [0.021\,106\,622 - 1/2 (0.001\,945\,339) \\ &\quad - 1/12 (-0.000\,180\,880) - 1/24 (0.000\,074\,594) \\ &\quad - 19/720 (-0.000\,071\,307)] \\ &= 0.999\,983\,367 \end{aligned}$$

$$\begin{aligned} \left(\frac{dj}{de}\right)_{0.9988} &= \frac{(0.999\,983\,367)(0.009\,263\,361)}{0.465\,628\,7(0.998\,200\,540)(0.999\,983\,367) - 0.025\,906\,286} \\ &= 0.021\,106\,622 \text{ (This is in agreement with the previously} \\ &\quad \text{computed } \frac{dj}{de}; \text{ therefore, iteration is completed.)} \end{aligned}$$

Values for j at $e = 0.9986, 0.9984$, etc., are computed in a manner analogous to $e = 0.9988$. As the computations progress, the step width can be increased. Use is made of the transformed equation B.7 as the calculations proceed. Convergency becomes difficult with the inverted equation B.8, as the slope increases and approaches infinitive as a limit.

The values m^* and T^* are calculated directly from the values of e and j (see table at end of this section).

Calculation of Time Interval

The time interval is evolved by equations IV.41-42 and IV.40 or IV.43.

$$q^* = \frac{39700}{0.1715 (4590) (520)} = 0.096\,986\,291 \frac{1}{\text{sec.}}$$

Time from $m^* = 1.000\,000\,000$ to $0.999\,993\,710$

$$c = \frac{(1.0 - 1.000\,202\,21)(1.0 - 0.999\,993\,710) - (1.0 - 1.000\,406\,45)(1.0 - 0.999\,997\,784)}{(1.0 - 0.999\,997\,784)(1.0 - 0.999\,993\,710)(0.999\,997\,784 - 0.999\,993\,710)}$$

$$= -6\,536\,955.82$$

$$b = \frac{(1.0 - 1.000\,406\,45) [(1.0)^2 - (0.999\,997\,784)^2] - (1.0 - 1.000\,202\,21) [(1.0)^2 - (0.999\,993\,710)^2]}{(1.0 - 0.999\,997\,784)(1.0 - 0.999\,993\,710)(0.999\,997\,784 - 0.999\,993\,710)}$$

$$= 13\,073\,805.91$$

$$a = 1.0 - 13\,073\,911.64 (1.0) - 6\,536\,955.82 (1.0)^2 = -6\,536\,849.09$$

$$\Delta t = \frac{1}{0.096\,986\,291} \left\{ \left[(0.999\,993\,710)(1.000\,406\,45) - (1.0)(1.0) \right] \right.$$

$$- 1.4 \left[(-6\,536\,849.09)(0.999\,993\,710 - 1.0) \right.$$

$$+ \frac{13\,073\,805.91}{2} (\overline{0.999\,993\,710^2} - \overline{1.0^2})$$

$$\left. - \frac{6\,536\,955.82}{3} (\overline{0.999\,993\,710^2} - \overline{1.0^2}) \right\}$$

$$= 0.004\,217 \text{ sec.}$$

TABLE IV Numerical Example

Values of Mass, Temperature, Pressure and Time from $e = 1.0000$ to $e = 0.9960$

e	j	$m^* = j^2$	$T^* = r/em^*$	$m = m_S m^*$	$T = T_S T^*$	$p = p_S m^* T^*$	Δt	t
1.0000	1.000 000 000	1.000 000 000	1.000 000 00	4590.000	520.000	14.696	0.000 000	0.000 000
0.9998	0.999 998 892	0.999 997 784	1.000 202 21	4589.995	520.105	14.699		
0.9996	0.999 996 855	0.999 993 710	1.000 406 45	4589.971	520.211	14.702	0.004 217	0.004 217
0.9994	0.999 994 194	0.999 988 388	1.000 611 97	4589.947	520.318	14.705	0.002 141	0.006 358
0.9992	0.999 991 020	0.999 982 040	1.000 818 61	4589.918	520.426	14.708	0.002 157	0.008 515
0.9990	0.999 987 397	0.999 974 794	1.001 026 23	4589.884	520.534	14.711	0.002 171	0.010 686
0.9988	0.999 983 367	0.999 966 734	1.001 234 74	4589.847	520.642	14.714	0.002 183	0.012 869
0.9986	0.999 978 963	0.999 957 926	1.001 444 09	4589.807	520.751	14.717	0.002 195	0.015 064
0.9984	0.999 974 208	0.999 948 417	1.001 654 22	4589.763	520.860	14.720	0.002 206	0.017 270
0.9982	0.999 969 122	0.999 938 245	1.001 865 11	4589.716	520.970	14.723	0.002 216	0.019 486
0.9980	0.999 963 721	0.999 927 443	1.002 076 70	4589.667	521.080	14.725	0.002 226	0.021 712
0.9978	0.999 958 018	0.999 916 038	1.002 289 00	4589.615	521.190	14.728	0.002 236	0.023 948
0.9976	0.999 952 025	0.999 904 052	1.002 501 95	4589.560	521.301	14.731	0.002 245	0.026 193
0.9974	0.999 945 752	0.999 891 507	1.002 715 55	4589.502	521.412	14.734	0.002 254	0.028 447
0.9972	0.999 939 208	0.999 878 420	1.002 929 79	4589.442	521.523	14.737	0.002 263	0.030 710
0.9970	0.999 932 400	0.999 864 805	1.003 144 64	4589.379	521.635	14.740	0.002 271	0.032 981
0.9968	0.999 925 335	0.999 850 676	1.003 360 09	4589.315	521.747	14.743	0.002 280	0.035 261
0.9966	0.999 918 020	0.999 836 047	1.003 576 12	4589.247	521.860	14.746	0.002 288	0.037 549
0.9964	0.999 910 461	0.999 820 930	1.003 792 74	4589.178	521.972	14.749	0.002 296	0.039 845
0.9962	0.999 902 662	0.999 805 333	1.004 009 93	4589.106	522.085	14.752	0.002 304	0.042 149
0.9960	0.999 894 629	0.999 789 269	1.004 227 68	4589.033	522.198	14.755	0.002 311	0.044 460

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