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A COMPUTER TO SOLVE ONE FORM OF VOLTERRA'S INTEGRAL EQUATION

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ABSTRACT

This computer was first proposed by Hessemer and Williams in SC Tech Memo 207-54-54. Its purpose is to find one function of the integrand of the convolution integral when the other function of the integrand and the integral itself are known. Such an equation is a special case of Volterra's equation. The present memo gives particular attention to the operation speed and to time constants, and suggests some modifications of the original computer. Some examples are considered to illustrate the operation of the computer.

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Dr. R. A. Hessemer of 5412 prepared the example of Paragraph 4.1. Discussions of numerous points with him were of great assistance to the author.

Messrs. F. O. J. Lane and R. P. Crabb of 5242 simulated the operation of the computer described in this memo. This made it possible to solve several problems of the type described in Paragraph 4 much more rapidly than was possible by the hand method described in that paragraph. Of more importance was the experience obtained in operating the simulator. This led to the considerations expressed in Paragraph 5.

945 002

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REFERENCES

Hessemer & Williams, SC Tech Memo 207-54-54

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945 004

A COMPUTER TO SOLVE ONE FORM OF VOLTERRA'S INTEGRAL EQUATION

1.0. Introduction

The integral

$$f(t) = \int_0^t g(t - T) h(T) dT \quad (1.0-1)$$

can be approximated¹ by

$$f_k = \Delta t \sum_{i=1}^k g_{k-i+1} h_i, \quad (1.0-2)$$

where

$$f_k \triangleq f(k\Delta t),$$

$$g_k \triangleq g\left(\left\{k - \frac{1}{2}\right\} \Delta t\right),$$

$$h_k \triangleq h\left(\left\{k - \frac{1}{2}\right\} \Delta t\right),$$

provided the interval Δt is sufficiently small. For the sake of simplicity, we shall consider the special case in which six ordinates are used to define $f(t)$, three to define $g(t)$, and hence four to define $h(t)$. More general cases readily follow. It is implicit in (1.0-2) and the definitions of f_k , g_k , and h_k that the f_k are values of $f(t)$ at intervals Δt while g_k and h_k are values of $g(t)$ and $h(t)$ at the midpoints of such intervals. Figure 1 illustrates these functions.

Equation (1.0-2) can be written

¹Hessemer & Williams, SC Tech Memo 207-54-54. The conditions on the functions f , g , h are described in this memo; they are repeated in the present memo in equations (2.1-3). See also footnote p. 9.

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Fig. 1

$$\begin{aligned}
 f_1 &= \Delta t (g_1 h_1) \\
 f_2 &= \Delta t (g_2 h_1 + g_1 h_2) \\
 f_3 &= \Delta t (g_3 h_1 + g_2 h_2 + g_1 h_3) \\
 f_4 &= \Delta t (g_3 h_2 + g_2 h_3 + g_1 h_4) \\
 f_5 &= \Delta t (g_3 h_3 + g_2 h_4) \\
 f_6 &= \Delta t (g_3 h_4)
 \end{aligned}
 \tag{1.0-3}$$

In SC Tech Memo 207-54-54, it is stated that the first four of equations (1.0-3) are sufficient to determine h_1 , h_2 , h_3 , and h_4 , if $f(t)$ and $g(t)$ are known, and that the h_k so determined will satisfy the remaining equations. If this is true, the 6 equations of (1.0-3) are not independent; in fact, only four of them are independent. In the application under consideration, $f(t)$ represents an amplitude function of time which is obtained at the output of an electrical system whose response to unit impulse is $h(t)$ and whose input is $g(t)$. These functions are related by the convolution integral (1.0-1) and hence by equations (1.0-3). Thus, any four of the equations should suffice to find h_1 , h_2 , h_3 , and h_4 , if the f_k and g_k are known, since the f_k were generated by the g_k and h_k .

In the applications for which the computer is intended, the f_k and the g_k are determined by measurement. Thus, it is impossible to know the f_k and g_k exactly, and hence the equations (1.0-3) are independent. That is, there do not exist values of h_1 , h_2 , h_3 , and h_4 which will simultaneously satisfy all six equations. (In addition to the uncertainty in knowing f_k and g_k , there is also the possibility that the mathematical model does not exactly fit the physical system, i.e., the convolution integral (1.0-1) may itself be only an approximation.)

2.0 Solution of the Problem

Our problem is to find values of h_k which satisfy (1.0-3) in some sense. If we had reason to suppose that a certain four of the six equations were more creditable than the others, we could solve these four for the h_k . However, our assumption is that all of the equations are equally creditable and hence should all be used in finding the h_k . Therefore, we choose to find the h_k by the following method.

We define a function $F(h_1, h_2, h_3, h_4)$ as follows:

$$\begin{aligned}
 F(h_1, h_2, h_3, h_4) = & \left[f_1 - \Delta t(g_1 h_1) \right]^2 \\
 & + \left[f_2 - \Delta t(g_2 h_1 + g_1 h_2) \right]^2 \\
 & + \left[f_3 - \Delta t(g_3 h_1 + g_2 h_2 + g_1 h_3) \right]^2 \\
 & + \left[f_4 - \Delta t(g_3 h_2 + g_2 h_3 + g_1 h_4) \right]^2 \\
 & + \left[f_5 - \Delta t(g_3 h_3 + g_2 h_4) \right]^2 \\
 & + \left[f_6 - \Delta t(g_3 h_4) \right]^2
 \end{aligned} \quad (2.0-1)$$

We note that, if $F = 0$, we have in fact satisfied all six of equations (1.0-3), since every quantity in brackets [] must be zero. Since in general the equations (1.0-3) are independent, we will not be able to make $F = 0$. However, we will minimize F by varying the h_k . The set of h_k which produce the minimum in F represents the solution. This means that we have fitted the quantities on the right-hand side of (1.0-3) to those on the left, i.e., to the f_k , in a least-squares sense.

2.1 Uniqueness of Solution

The h_k which produce the minimum must simultaneously satisfy the equations

$$\frac{\partial F}{\partial h_1} = 0, \quad \frac{\partial F}{\partial h_2} = 0, \quad \frac{\partial F}{\partial h_3} = 0, \quad \frac{\partial F}{\partial h_4} = 0.$$

Now F is of second degree in the h_k ; hence, the equations $\partial F / \partial h_k = 0$ represent linear equations in the h_k , and there are as many equations as there are h_k . Such a system of linear equations has only one solution; that is, there is only one set of h_k that produces a minimum of $F(h_1, h_2, h_3, h_4)$.

In general, we have

$$F = \sum_{k=1}^M \left[f_k - \Delta t \sum_{i=1}^k g_{k-i+1} h_i \right]^2 \quad (2.1-1)$$

where M is the number of ordinates used to define $f(t)$. Then

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$$\frac{\partial F}{\partial h_r} = \sum_{k=1}^M \left\{ 2 \left[f_k - \Delta t \sum_{i=1}^k g_{k-i+1} h_i \right] \left[- \Delta t g_{k-r+1} \right] \right\}, \quad (2.1-2)$$

where

$$r = 1, 2, \dots, M - N + 1.$$

Since¹

$$\left. \begin{aligned} f_k &= g_k = h_k = 0, \text{ for } k < 1, \\ f_k &= 0, \text{ for } k > M \geq N, \\ g_k &= 0, \text{ for } k > N, \\ h_k &= 0, \text{ for } k > M - N + 1, \end{aligned} \right\} \quad (2.1-3)$$

we can extend the sum on i in (2.1-2) up to M since the term $g_{k-i+1} = 0$ for $i > k$. We then have

$$\frac{\partial F}{\partial h_r} = - 2 \Delta t \left[\sum_{k=1}^M f_k g_{k-r+1} - \Delta t \sum_{i=1}^M \left\{ \sum_{k=1}^M g_{k-i+1} g_{k-r+1} \right\} h_i \right].$$

Setting $\frac{\partial F}{\partial h_r} = 0$ leads to

$$\sum_{i=1}^M \left\{ \sum_{k=1}^M g_{k-i+1} g_{k-r+1} \right\} h_i = \frac{1}{\Delta t} \sum_{k=1}^M f_k g_{k-r+1}. \quad (2.1-4)$$

Due to the conditions (2.1-3), the limits of summation of (2.1-4) can be changed to obtain

¹ Hessemer & Williams, SC Tech Memo 207-54-54. Although the functions f , g , and h are taken as positive here and in the memo of reference, this restriction is not necessary. Examination of the computer (Fig. 2) makes it clear that f and g can be made negative by changing the polarity of the batteries; $h(t)$ can be made negative by amplifiers of gain -1 in series with the variable conductances.

$$\sum_{i=1}^{M-N+1} \left\{ \sum_{k=r}^{N+r-1} g_{k-i+1} g_{k-r+1} \right\} h_i = \frac{1}{\Delta t} \sum_{k=r}^{N+r-1} f_k g_{k-r+1} \quad (2.1-5)$$

where

$$r = 1, 2, \dots, M - N + 1.$$

Due to the range of r , (2.1-5) represents $M - N + 1$ equations. These equations are linear in the h_i and there are $M - N + 1$ of the h_i . Hence, there is only one solution.

3.0 The Computer

The computer described in SC Tech Memo 207-54-54 minimizes the function $F(h_1, h_2, h_3, h_4)$. A diagram of the computer for the special case considered in the present memo is shown in Figure 2. The quantities $(\Delta t G g_1)$, $(\Delta t G g_2)$, and $(\Delta t G g_3)$ are inserted as voltages as indicated; h_1, h_2, h_3 , and h_4 are conductances. The rotating decks 10 and 14 of switches 11 and 13, respectively, are coupled mechanically as indicated in Figure 2 so that they advance in position together. They are motor-driven and rotate continuously.*

Let the position of the switches shown in Figure 1 be called Position 1. Let the position occupied by the rotating decks after the contacts have advanced one position clockwise from that shown be called Position 2; etc. The following table gives the potentials at various points in the system for each switch position.

* The variable t in the functions $f(t)$, $g(t)$, $h(t)$ is not directly related to real time as it concerns speed of operation of the computer.

Fig. 2

Switch Position	Potential at 12 with respect to ground	Potential at 15 with respect to ground	Voltage output of summing amplifier
1	0	0	0
2	$-\Delta t(g_1 h_1)$	f_1	$a[f_1 - \Delta t(g_1 h_1)]$
3	$-\Delta t(g_2 h_1 + g_1 h_2)$	f_2	$a[f_2 - \Delta t(g_2 h_1 + g_1 h_2)]$
4	$-\Delta t(g_3 h_1 + g_2 h_2 + g_1 h_3)$	f_3	$a[f_3 - \Delta t(g_3 h_1 + g_2 h_2 + g_1 h_3)]$
5	$-\Delta t(g_3 h_2 + g_2 h_3 + g_1 h_4)$	f_4	$a[f_4 - \Delta t(g_3 h_2 + g_2 h_3 + g_1 h_4)]$
6	$-\Delta t(g_3 h_3 + g_2 h_4)$	f_5	$a[f_5 - \Delta t(g_3 h_3 + g_2 h_4)]$
7	$-\Delta t(g_3 h_4)$	f_6	$a[f_6 - \Delta t(g_3 h_4)]$

Note: a is voltage gain of summing amplifier.

The thermocouple and galvanometer M square and integrate the output of the summing amplifier, provided the time constant is several times the period of revolution of the switch.* Hence, the galvanometer reading is proportional to $F(h_1, h_2, h_3, h_4)$ which is defined by equation (2.0-1).

The operator will adjust the h_k in sequence to find a minimum of F . The set of h_k which yields the minimum represents the desired solution for $h(t)$.

4.0 Some Illustrations

The following examples were solved in order to have some better idea of the behavior of the computer.

4.1 Solution Using a Generated $f(t)$

Let the functions $g(t)$ and $h(t)$ be represented by the ordinates:

$$\begin{array}{ll}
 g_1 = 3 & h_1 = 5 \\
 g_2 = 6 & h_2 = 3 \\
 g_3 = 2 & h_3 = 2 \\
 & h_4 = 1
 \end{array}$$

* See paragraph 5.1

Convolution of these two by means of (1.0-3) generates* $f(t)$ as follows:

$$\begin{array}{ll} f_1 = 15 & f_4 = 21 \\ f_2 = 39 & f_5 = 10 \\ f_3 = 34 & f_6 = 2 \end{array}$$

If the mathematics that are used to describe the physical system are correct, the output pulse $f(t)$ is generated in just this fashion. Then knowing $f(t)$ and the input $g(t)$, we seek $h(t)$.

This problem may be solved in the same manner that the computer will solve it. The function $F(h_1, h_2, h_3, h_4)$ as defined by (2.0-1) is formed. Initially h_2, h_3 , and h_4 are taken as zero. Then from $\partial F / \partial h_1 = 0$ we find a value of h_1 . This value of h_1 is used to determine F , and then from $\partial F / \partial h_2 = 0$, with h_3 and h_4 still zero, a value for h_2 is found. This process is continued until values for h_1, h_2, h_3 , and h_4 are obtained. This is the conclusion of the first run.

Next, using the values of h_2, h_3 , and h_4 as found in the first run we find a new value for h_1 from $\partial F / \partial h_1 = 0$. We use this new value of h_1 and the values from the first run for h_3 and h_4 and find a new value for h_2 from $\partial F / \partial h_2 = 0$. Etc.

The process is repeated until each new value for an h_k is the same as (or differs little from) the previous value. Then we have the minimum value for F . The result of six runs with the functions defined above is tabulated here:

Run	1	2	3	4	5	6	Known correct values
h_1	7.1	5.1	5.3	5.0	4.8	5.0	5
h_2	2.6	2.6	3.1	3.3	3.0	2.9	3
h_3	2.6	1.4	1.5	2.2	2.1	2.1	2
h_4	0.7	1.7	1.3	0.8	0.94	0.95	1

It is evident that the h_k are converging rapidly to the known correct values. (It should be noted that, in this case, since $f(t)$ was generated by $g(t)$ and $h(t)$, it would be possible to find h_1, h_2, h_3 , and h_4 to satisfy all six of equations (1.0-3).)

* Here and in paragraph 4.2, Δt is taken as unity. This only represents a scaling of the h_k .

4.2 Solution Using Arbitrary $f(t)$

In order to test the computer's method under more unfavorable conditions, an arbitrary function $f(t)$ was chosen. The ordinates of $f(t)$ and $g(t)$ are:

$$\begin{array}{lll} f_1 = 1 & f_5 = 3 & g_1 = 3 \\ f_2 = 3 & f_6 = 2 & g_2 = 6 \\ f_3 = 5 & f_7 = 1 & g_3 = 4 \\ f_4 = 4 & & g_4 = 2 \end{array}$$

This problem is somewhat more difficult than the previous since $f(t)$ and $g(t)$ are represented by more ordinates; further, $f(t)$ is arbitrary.

Twenty-four runs were made. Some of them are tabulated here:

Run	1	2	6	12	18	23	24
h_1	.754	.354	.350	.313	.294	.280	.278
h_2	.357	.371	.404	.476	.523	.549	.553
h_3	.294	.287	.197	.131	.082	.059	.055
h_4	.188	.225	.261	.310	.332	.342	.343
F	11.836	1.291	1.092	.913	.852	.831	.829

The function $F(h_1, h_2, h_3, h_4)$ which the process minimizes is tabulated along with the h_k . Note that at the 24th run, the changes in the h_k are:

$$\Delta h_1 = - .002, \Delta h_2 = + .004, \Delta h_3 = - .004, \Delta h_4 = + .001.$$

The greatest change (.004) is about 1 per cent of the maximum h_k (.553). The greatest relative change is in h_3 where it is $(.004)/(.059) \doteq 7$ per cent. Additional runs would lead to greater accuracy.

In this problem, we have no known h_k with which to compare the results as in paragraph 4.1. We compare by convolving the determined h_k with the g_k to find $f_{k?}$ (i.e., the right-hand side of equations (1.0-3)) which may be compared to the known f_k . These results of runs 6, 12, 18, and 24 are shown graphically in Figure 3. Both f_k and $f_{k?}$ are plotted as well as the difference $f_k - f_{k?}$. The function F which is minimized is proportional to the square of the area between $f_k - f_{k?}$ and the base line.

Fig. 3

5.0 Computer Design Considerations

Messrs. F. O. J. Lane and R. P. Crabb of 5242 simulated the operation of the computer described herein through use of analogue computer components. Operation of this simulator provided information concerning design of the computer. These points are discussed below.

5.1 Speed of Switch and Time Constant of Thermocouple and Galvanometer

In obtaining the minimum of the function $F(h_1, h_2, h_3, h_4)$ through adjustment of the variable conductances representing the h_k , it is essential that a change in an h_k be reflected as quickly as possible in F as read on the output galvanometer. Otherwise, the operator becomes disconcerted. This confusion arises because the expected output change does not immediately follow a change of h_k . Let us assume, then, that T_R is the time allowed for the change in the galvanometer reading to become 87 per cent complete, following a change in an h_k . Suitable values of T_R will have to be determined by experience; probably a value of 0.5 sec is sufficiently short. In any case, after T_R is selected, the switch speed and the time constant of the thermocouple and galvanometer are determined as will be shown below.

Now let T_M represent the time constant of the thermocouple-galvanometer combination. Then, if the system consisting of these elements is not oscillatory, the deflection of the galvanometer M will be given by

$$d_M(t) = kE_{in}^2 (1 - e^{-t/T_M})$$

when

$$e_{in}(t) = E_{in} u(t),$$

where $u(t)$ is a unit step at $t = 0$, $e_{in}(t)$ is the input voltage applied to the thermocouple, k is a constant of proportionality, and t is actual time.

We can relate T_R and T_M as follows. A change of input to the thermocouple will cause the deflection d_M to change; this change in d_M will be 87 per cent complete in time $2T_M$. Hence

$$T_R = 2T_M \text{ sec,}$$

or

$$T_M = T_R/2 \text{ sec.} \quad (5.1-1)$$

This equation may be used to design or select T_M when T_R has been chosen.

The thermocouple and galvanometer combination must square the input and integrate it over a period of revolution of the switch. In order to accomplish the integration, the time constant T_M must be appreciably longer than

the period of revolution of the switch. If S is the switch speed in revolutions per second, the period is $1/S$ and we may state

$$T_M \gg 1/S \text{ sec.}$$

Specifically, if we set

$$T_M = 10/S \text{ sec,} \quad (5.1-2)$$

we can assume that the error in integration is less than 10 per cent. In practice, this error will show up in a variation of the deflection d_M of the galvanometer during a switch revolution. If the condition (5.1-2) is met, this variation will be less than 10 per cent of the total deflection.

Combining (5.1-2) with (5.1-1) gives

$$S = 20/T_R \text{ rev/sec.} \quad (5.1-3)$$

Hence, if T_R is chosen to be 0.5 sec (this is the time required for a change in galvanometer deflection to be 87 per cent complete following a change in an h_k), $S = 40$ rps, or 2400 rpm. Since some difficulty may be experienced with a switch speed of this magnitude, a larger value of T_R may have to be accepted.

Equations (5.1-1) and (5.1-3),

$$T_M = T_R/2 \text{ sec,} \quad (5.1-1)$$

$$S = 20/T_R \text{ rev/sec,} \quad (5.1-3)$$

are the design equations for the time constant of the thermocouple-galvanometer combination, and the speed of the switch, when T_R has been selected.

5.2 Spurious Minima of F

For certain values of the h_k , changes in F due to changes in the h_k may be quite small. Such a set of values of the h_k might be taken for a minimum when in fact it is not.

Now F is the d-c component of the voltage output of the thermocouple (see Fig. 2), that is, it is the square of the area under $f_k - f_{k?}$ as illustrated in Figure 3. Since only the changes in F are of interest, it is possible to introduce a voltage between the thermocouple and galvanometer M which will balance out a large part (or all) of F . Then the difference may be greatly amplified so that the reading on M , a ΔF due to a change Δh_k in an h_k , will be large. In this manner changes in F due to changes in the h_k produce readily perceptible changes of the output, so that the operator is unlikely

to mistake a set of h_k for which changes in F are small when the h_k are varied, for the true minimum of F . (There is only one true minimum as proved in paragraph 2.1).

In the table of paragraph 4.2, the change in F from the end of run 23 to the end of run 24 is only 0.002. Since F at the end of run 24 is 0.829, this represents a relative change of 0.24 per cent. Even in this case where no spurious minimum is involved, it would be better to be able to amplify changes in F rather than F itself.

6.0 A Different Output Circuit

To avoid the use of the thermocouple, which is expensive and easily damaged, the bridge rectifier circuit shown in Figure 4 may be used instead. The volt-ampere characteristic of this circuit is shown in Figure 5. Figure 5(a) is the characteristic when R is negligible so that the characteristic is essentially that of two diodes in series. In Figure 5(b), the characteristic includes the effect of R .

Figure 5 illustrates the fact that i is unidirectional regardless of the polarity of e ; thus, the circuit rectifies. If R is large compared to the diode resistance, the characteristic is essentially linear so that i has the form $i = a|e|$. However, by proper choice of diodes and resistance R , it should be possible to make the characteristic approximate

$$i = ae^2,$$

at least for small values of e . (Toward the end of the solution of a problem, e will be small as Figure 3 shows; e is proportional to $f_k - f_{k?}$.) Then the galvanometer may be placed in series with R . Its deflection is proportional to i and hence to e^2 . The time constant T_M (see paragraph 5.1) is determined solely by the mechanical inertia of the galvanometer.

If the galvanometer has an appreciable effect on the current i , it may be necessary to isolate the galvanometer by an amplifier. The input to the amplifier will be the voltage iR across R .

7.0 Improved Speed of Response

Equation (5.1-3) indicated that a high-speed switch is needed if the response time T_R is to be kept short. To avoid these high speeds, the circuit of Figure 6 is suggested. In this circuit, the voltage on capacitor C_1 is proportional to the integral of e_g , since

$$e_{C_1} = \frac{1}{C_1} \int_0^t i_p dt,$$

and

$$i_p = g_m e_g,$$

Fig. 4

Fig. 5

- (a) Volt-ampere characteristic of circuit of Fig. 4, $R = 0$,
- (b) Volt-ampere characteristic of circuit of Fig. 4, $R \neq 0$.

where g_m is constant. (The voltage e_g is the output of the bridge rectifier or thermocouple, depending on which is used.) The period of integration extends over one revolution of the switch. At the end of this period, switch S_1 closes momentarily and the voltage on capacitor C_2 rises to that of C_1 , since $C_2 \ll C_1$. S_1 then opens and S_2 closes momentarily so that the voltage on C_1 may drop to zero. After S_2 opens, the circuit is ready for another period of integration. The switches S_1 and S_2 are synchronized with the main rotating switch so that the proper time relationships are maintained.

The voltage e_o on capacitor C_2 is read on a voltmeter of very high input impedance so that C_2 is not discharged. The voltage e_o is, of course, proportional to the function $F(h_1, h_2, h_3, h_4)$ defined by equation (2.0-1) due to the squaring by the bridge rectifier (or thermocouple) and the integration by the circuit of Figure 6.

With this circuit, if a change is made in an h_k , the voltage e_g (which is proportional to $(f_k - f_{k'})^2$) changes. Then the change in e_o will be complete within two revolutions of the switch at most. Thus, in order to have the response time $T_R = 0.5$ sec, the switch speed S is only $1/0.25 = 4$ rev/sec, or 240 rpm. This is only one-tenth of the speed required to make $T_R = 0.5$ sec with the system described in paragraph 5.1.

The circuit of Figure 6 has one disadvantage. In some cases a change in an h_k will not cause any change at all in e_o until the switch has made one full revolution. The earlier circuits have the merit that the output begins to change as soon as an h_k is changed.

8.0 Convolutions Performed by the Computer

In this memo and in SC Tech Memo 207-54-54, the ability of the computer to find $h(t)$ when $f(t)$ and $g(t)$ are known has been described. (See paragraph 1.0 for notation.) It has not been specifically stated, although it has certainly been implied, that the computer may be used also to perform convolutions. That is, if the functions $g(t)$ and $h(t)$ are known, $f(t)$ can be found. No minimizing is required to find $f(t)$. The functions $g(t)$ and $h(t)$ are inserted as voltages and conductances, respectively, as shown in Figure 2. Then $-f(t)$ can be read at point 12 of Figure 2.

The computer, through its ability to perform convolutions, should find wide use in determining the response $f(t)$ of linear systems to an input $g(t)$ when the system's response $h(t)$ to unit impulse is known.

9.0 Conclusions

The computer which is discussed in this memo was first suggested in SC Tech Memo 207-54-54 (Reference 1). A computer based on the one described in Reference 1 is under construction at the Physical Science Laboratory of the New Mexico College of Agriculture and Mechanic Arts and it will be used in the reduction of terrain return data. The actual building

of the computer has required more detailed thought about the design of the computer, and this memo presents the results of this thinking.

One evident problem remains to be considered. This is the effect on the solution if the output of the bridge rectifier is not the square of the input. That is, referring to the notation of paragraph 6.0, what is the meaning of the minimum of $F(h_1, h_2, h_3, h_4)$ if $i = ae + be^2 + ce^3 + de^4$? In this connection, note that F will have to be redefined since each of the squared terms of equation (2.0-1) will have to be represented by series of the type above for i .

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Fig. 6

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