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INTERRUPTION OF NORMAL OPERATION

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POWER AND TEMPERATURE OF TBR FOLLOWING
INTERRUPTION OF NORMAL OPERATION

SUMMARY

Calculations have been performed to determine the behavior of the TBR under certain conditions involving interruption of normal operation. The situations considered and the results are as follows:

Case I. Sudden Stoppage of Circulation in the Core Circuit.

For this case the core temperature rises over 40°C in the first minute and continues afterwards to rise at a lower rate.

Case II. Sudden Cessation of Heat Removal in the Heat Exchanger.

The core temperature rises over 30°C in the first minute and continues afterward to rise at a lower rate.

Case III. Circulating Pump Failure.

The core temperature rises about 6°C but returns to the normal operating temperature in less than one minute. After this short transient period the core remains critical at about 60 Mw with solution flow by natural circulation.

The rapid temperature rise for Cases I and II results in the solution temperature quickly exceeding the level at which phase separation occurs, with subsequent hazard from danger of accumulation of a critical mass. No danger appears to be involved in the condition of Case III.

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In addition to the preceding, curves of the energy release rate from fission product decay are presented, and a plot of the flow rate of solution in the core circuit versus time after pump stoppage is given. About 7 seconds is required for flow to decay to the natural circulation rate following a pump failure

INTRODUCTION

Two types of incidents which could occur during operation of a homogeneous reactor are considered here: interruption of heat removal in the heat exchanger; interruption of circulation of the core solution. In either event the rate of heat generation in the core will change and the temperature of the fuel solution will rise. To permit assessment of the hazards associated with these occurrences, calculations have been performed for three situations which involve sudden changes in reactor conditions. The design figures employed are those of the present concept of the TBR, with the exception that the core vessel volume was taken as 104 cu ft rather than the 65 cu ft now planned (see DISCUSSION).

Estimates of core power and temperature, following interruption of normal conditions, are obtained by application of the kinetic equations describing nuclear behavior of a homogeneous reactor. Knowledge of fission product energy release rates and the time-dependent behavior of flow in a reactor circuit are required; these are presented and discussed separately in case they may be of interest independently of the other calculations.

Fission Product Decay

In the normal, steady operation of a reactor with no removal of fission products approximately 6% of the power comes from beta and gamma decay energy. Following a reduction in reactor power the fission products accumulated at the earlier power level continue to release energy at a rate decreasing with time. Allowance for this delayed power must be included in calculations of the subsequent power levels.

The curves of Fig. 1 summarize the rates of energy release from fission-product decay that are used in the calculations which follow. Fig. 9 indicates the results of operation for various periods of time. In Appendix A the accuracy of the data and their limitations are discussed.

With continuous removal of fission products by processing of the fuel solution the decay energy stored in the core will be less than postulated in the computations. If all insoluble products (about 3/4 of the total) were removed instantaneously on formation, the decay power would be of the order of 1/4 of the values in Fig. 1. Removal of all fission products over twelve hours old would reduce the fission-product energy release rate during the first hundred seconds by about 1/4; elimination of all over one hour old would only reduce it by less than half. This can be seen by noting from Fig. 9 that after 100 seconds of decay the energy release by fission products within the given age ranges is as follows:

<u>Age of F.P., hrs.</u>	<u>Power from F.P. in Given Range, Mw.</u>
0 - 1	6.1
1 - 12	2.6
<u>12 - ∞</u>	<u>3.3</u>
0 - ∞	12.0

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Heat Release Rate

Time after Fissioning Ceases (sec)

Fig. 1

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For a separatory process which removed insoluble fission products on a one-day cycle, the total energy released in the core by fission products during the first 100 seconds would be over $2/3$ of that released if all fission products were retained.

The preceding indications of the effect of continuous fission product removal on decay energy apply only for the first few minutes after fissioning ceases. As can be seen from Fig. 9, the curves for long and short periods of operation diverge rapidly with increasing decay time. Continuous processing greatly affects both the rate of heat release after a decay period longer than several hours and the total energy released.

Flow Decay in Core Circuit

Following a sudden stoppage of the core circulating pump, flow in the core circuit would decrease as the momentum of the fluid was dissipated. However, the temperature difference between the two vertical segments of the loop would continue to provide a buoyancy driving force for circulation at a lower rate. The flow decrease would continue until equilibrium at this natural circulation value was reached.

In Appendix B the method of calculation of the flow rate as a function of time is derived. The results of calculations for two temperature-difference driving forces are plotted in Fig. 2. These calculations reveal that the flow in the TBR core circuit would decrease to the natural circulation rate in about 7 seconds. In the derivations and the calculations the temperature difference

Fig. 2

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between the hot and cold legs is assumed to remain constant throughout the period considered. The initial flow rate and pressure drop employed in the calculations were obtained from reference 1.

Equations for natural circulation under steady conditions are derived in Appendix C and the results of calculations for the TBR core system given in Fig. 10.

Time-Dependent Behavior of Reactor Power

For all cases considered in this memorandum the power of the reactor is always below prompt critical. This permits simplification of the problem by the valid approximation that the kinetic behavior of the reactor is controlled by the delayed neutrons. In addition, a single group of delayed neutrons having an average decay constant is employed to approximate the behavior of the several groups which are known to exist (see Appendix E).

A change in the solution flow rate in a homogeneous reactor results in a change of the delayed-neutron precursor concentration in the core. The manner of allowing for this in the calculations is approximate but in all cases is conservative, the predicted power being higher than would actually result.

The fission power in a sub-critical reactor at any instant is dependent on the history of the reactor behavior and the instantaneous temperature of the core. Thus simultaneous solution of the kinetic equations for reactor power and the energy balance involving core temperature is required. The energy release rate used in the energy balance must include power from fission-product decay as well as prompt fissions. As a result, closed solutions cannot be obtained

The kinetic equations for the three cases considered are presented in Appendix D and the method of solution indicated.

In order to evaluate the hazards associated with the interruption of normal operation in a large homogeneous reactor, three extreme cases of incidents involving change in heat removal rate or solution flow rate are analyzed. Curves are presented of temperature and power versus time following the changes in condition. The methods of calculation are discussed in the preceding sections and in Appendices B, D, and E; the reactor parameters and conditions used in the calculations are summarized in Appendix F.

This, the extreme case of change in the circulation rate, is similar to the condition of the reactor if circulation stopped during a dump. As shown in Fig. 3, following flow stoppage fissioning decreases rapidly and in 60 seconds becomes negligible compared to fission-product decay. In the first minute the core solution temperature rises about 40°C (Fig. 4) continuing to rise at a reduced rate after fissioning ceases.

The assumptions made in this situation are that heat exchange between the core solution and the exchanger

Fig. 3

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Fig. 4

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ceases suddenly and completely but circulation in the core system continues at the normal rate. Case II represents the extreme situation resulting from interruption of heat-exchanger performance; if the design of the heat exchanger permitted rapid drainage of the shell side, the condition which would result from quick discharge of boiler water is similar to this.

As shown in Figs. 5 and 6 the rate of temperature rise for Case II is less than for Case I. This occurs because with continued pump operation the heat capacity of the entire circulating system is available for storage of energy and, in addition, some delayed-neutron precursors are transported out of the core and decay in the external system. The lower rate of temperature rise results in the fission power holding up longer than in Case I in spite of the same delayed-neutron reactivity.

Case III. Circulating Pump Failure.

The condition postulated is that the core circulating pump stops suddenly but heat removal continues in the heat exchanger with the temperature on the shell side remaining unchanged. Flow in the core circuit decays as the momentum of the fluid is dissipated, decreasing rapidly until the natural circulation rate is reached.

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Fig. 5

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Temperature of TBR Core
Following Sudden Cessation
of Heat Removal in Heat
Exchanger — with Pump
Continuing to Operate

In calculating the flow rate, the temperature difference between hot and cold leg is assumed to be constant at 75°C . This results in the calculated values of core power being more than would actually occur. The core temperature might rise to a slightly higher maximum than predicted, but no more than a few degrees. In the power and temperature calculations, the solution entering the core is assumed to remain at 250°C for 7 seconds (the fluid in the line between heat exchanger and core when the pump stops); then to fall, reaching 225°C (the steam side temperature) at 10 seconds; and to remain at that temperature thereafter.

The behavior of core power and temperature for Case III is shown in Figs. 7 and 8. Following pump stoppage the temperature initially rises, since the decreased flow rate results in a lower rate of heat removal from the core. This temperature rise results in the power decreasing. The temperature therefore falls again and an equilibrium power of the order of 60 Mw is reached at which the reactor remains critical, circulating at a flow rate of about 400 lb/sec.

DISCUSSION

Because of the high power density of the TBR the rate of temperature rise of the core following interruption of normal operation may be very rapid. The results shown in Figs. 4 and 6 indicate

Fig. 7

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Temperature of TBR Core
Following Sudden Pump Stoppage
for Case of Continued Heat
Removal in Heat Exchanger

Mean Core Temperature, T ($^{\circ}\text{C}$)

300

290

280

270

260

0 2 4 6 8 10 12 14 16 18 20 22 24 26 28

Time after Pump Stoppage, t (sec)

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that within a few minutes after either stoppage of flow in the core circuit or complete cessation of heat removal in the heat exchanger the solution temperature can rise above the level at which phase separation occurs. Situations which could result in either of these conditions are thus hazardous if the fuel-rich phase can accumulate and attain criticality. Examples of possibly hazardous events are stoppage of circulation during a dump or sudden drainage of the shell side of the heat exchanger.

The behavior of the system following a pump failure alone does not appear hazardous as long as the stationary impeller does not offer a high resistance to flow. The momentary core temperature rise for this situation (illustrated in Fig. 8) is not significantly high, and it is likely the reactor would rapidly attain a stable critical condition, with flow of fuel solution by natural circulation.

In all of the temperature rise calculations in this memorandum the core heat capacity was based on a core tank volume of 10⁴ cu ft, rather than the 65 cu ft (5 ft diameter) presently planned for the TBR. The nuclear characteristics employed, however, are all applicable to the TBR with 5 ft diameter core. The difference in heat capacity has the following effect on the results: Case I. The rate of temperature rise after a sudden flow stoppage would be greater in the TBR than shown in Fig. 4; in the first minute the temperature would rise higher than given, but by less than 60% (the difference in heat capacities) and probably by about 30%. Case II. The behavior of the TBR after sudden interruption of heat removal would be as indicated in Figs. 5 and 6, since the heat capacity of the total

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circulating system is employed in this calculation and the value used is applicable. Case III. Following a pump failure the momentary temperature rise would be somewhat higher than shown in Fig. 8 (less than 60% higher) and the system would return to steady operation as indicated. In Cases I and III the more rapidly rising temperature causes the fission power to be below what is plotted in Figs. 3 and 7.

To be conservative, the gamma and beta decay heat release has been taken as the maximum possible by assuming all fission products to remain in the reactor. Continuous fission-product removal would not significantly affect the results obtained for the first few minutes following interruption of normal operation (see INTRODUCTION: Fission-Product Decay). However, for longer decay periods the accumulative energy release in the fuel solutions would be appreciably reduced by processing.

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APPENDIX AHEAT RELEASE RATE FROM FISSION-PRODUCT DECAY

The amount of energy available from beta and gamma emission in fission-product decay and the rate at which it is released depend on the operating history of the fuel. The curves of Fig. 1 apply to a reactor core system which has been operated for an infinite time at 357 Mw with no fission products removed. In Fig. 9 is shown the results of operation for shorter periods. At 10^4 seconds the values for infinite time are higher than those for one month of operation by 30% (within the estimated accuracy of the data); the difference is less for longer operating times or shorter decay times.

For periods of operation at powers other than 357 Mw, the rate of energy release can be obtained by multiplying the curves of Figs. 1 and 9 by $P_o/357$.

The data of Figs. 1 and 9, obtained from reference 4, are for fission products of U^{235} but are applicable to U^{233} . The values plotted are estimated by the authors to have the following accuracy:

Under 1 second	- large error
1 to 10^2 seconds	- $\pm 50\%$
10^2 to 10^4 seconds	- $\pm 30\%$
10^4 to 10^6 seconds	- $\pm 10\%$.

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Fig. 9

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APPENDIX B

TIME DEPENDENCE OF CIRCULATION IN A REACTOR SYSTEM
FOLLOWING PUMP STOPPAGE

For turbulent flow a "one-dimensional" flow model (i.e., all flow is axial and is uniform across the channel at any cross section) is a satisfactory approximation. Writing a force balance on a differential length of pipe yields the following equations.

$$\frac{1}{g_c} \frac{\partial(\rho AU)}{\partial t} = -A \frac{\partial p}{\partial x} - b \tau_w - \frac{1}{g_c} \frac{\partial(\rho AU^2)}{\partial x} - \frac{g}{g_c} \rho A \cos \xi \quad (1)$$

In equation (1) the first term represents the rate of change of momentum of the control volume and is equated to the algebraic sum of forces from the static pressure gradient, frictional resistance, momentum transport and gravitational attraction. Equation (2) is obtained by dividing equation (1) by A, replacing ρAU by W, and integrating around the circuit.

$$\begin{aligned} \frac{1}{g_c} \int \left(\frac{1}{A} \frac{\partial W}{\partial t} \right) dx = & - \int \left(\frac{\partial p}{\partial x} \right) dx - \int \left(\frac{b \tau_w}{A} \right) dx - \frac{1}{g_c} \int \left(\frac{1}{A} \frac{\partial(WU)}{\partial x} \right) dx \\ & - \frac{g}{g_c} \int \rho \cos \xi \, dx \end{aligned} \quad (2)$$

Use is made of the approximation* that the frictional resistance is proportional to the square of the flow rate, and the steady condition for forced flow is indicated by the subscript zero (noting A and b are not functions of W).

* For turbulent flow in straight pipes the wall shear force is actually proportional to W^n where n lies between 1.8 and 2.0. The pressure loss in bends, enlargements, and contractions is generally taken as proportional to W^2 . Thus the approximation used should not be greatly in error. (see reference 3, p. 145, 146)

$$\int_0^L \frac{bT_w}{A} dx = \left[\int_0^L \frac{bT_w}{A} dx \right]_0 \left(\frac{W}{W_0} \right)^2 \quad (3)$$

Application of the equation of continuity (i.e., $\frac{\partial W}{\partial x} = 0$) reveals the fourth term in equation (2) to be zero. At steady state with the pump operating the first term is zero and the buoyancy term can be neglected. Integration around the circuit can be from pump discharge to pump intake (i.e., neglecting the pump in the friction term).

$$- \int_0^L \frac{\partial p}{\partial x} dx = \Delta p_0 = \left[\int_0^L \frac{bT_w}{A} dx \right]_0 \quad (4)$$

Substitution of equation (4) into equation (3) relates the frictional resistance at any instant to the flow rate and initial conditions.

$$\int_0^L \frac{bT_w}{A} dx = \Delta p_0 \left(\frac{W}{W_0} \right)^2 \quad (5)$$

Returning to equation (2), if the pump is not operating, the second term is zero. The buoyancy term can be expressed as a function of the temperature difference between the hot and cold legs.

$$\int_0^L \rho \cos \xi dx = (\rho_2 - \rho_1) \Delta z = \left(\frac{\partial \rho}{\partial T} \right)_m (T_2 - T_1) \Delta z \quad (6)$$

In the first term of equation (2), $\partial W / \partial t$ is not a function of x .

$$\frac{1}{g_c} \int_0^L \frac{1}{A} \frac{\partial W}{\partial t} dx = \frac{1}{g_c} \frac{\partial W}{\partial t} \int_0^L \frac{dx}{A} = \frac{1}{g_c} \frac{\partial W}{\partial t} \sum x/A \quad (7)$$

Substituting equations (5), (6), and (7) into (2), and eliminating the second and fourth terms which are zero yields the following.

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$$\frac{1}{g_c} \frac{\partial W}{\partial t} \sum x/A = - \Delta p_o \left(\frac{W}{W_o} \right)^2 - \frac{g}{g_c} \left(\frac{\partial p}{\partial T} \right)_m (T_2 - T_1) \Delta Z \quad (8)$$

Replacing $\partial W/\partial t$ with dW/dt , dividing by W_o and rearranging gives a total differential equation.

$$\frac{d(W/W_o)}{dt} = - \frac{g_c \Delta p_o}{W_o \sum x/A} \left(\frac{W}{W_o} \right)^2 - \frac{g \Delta Z}{W_o \sum x/A} \left(\frac{\partial p}{\partial T} \right)_m (T_2 - T_1) \quad (9)$$

Making the approximation that $T_2 - T_1$ is constant, the third term is constant.

$$Y \equiv W/W_o$$

$$K_1 \equiv \frac{g_c \Delta p_o}{W_o \sum x/A}$$

$$K_2 \equiv \frac{g \Delta Z}{W_o \sum x/A} \left(\frac{\partial p}{\partial T} \right)_m (T_2 - T_1)$$

Equation (9) is simplified by substitution of the preceding definitions.

$$\frac{dY}{dt} = -K_1 Y^2 - K_2 \quad (10)$$

$$\int_0^t dt' = - \int_1^Y \frac{dy}{K_1 y^2 + K_2} \quad (11)$$

$$t = \frac{1}{\sqrt{-K_1 K_2}} \left[\tanh^{-1} \left(\frac{-K_2}{Y \sqrt{-K_1 K_2}} \right) - \tanh^{-1} \left(\frac{-K_2}{\sqrt{-K_1 K_2}} \right) \right] \quad (12)$$

The result of application of equation (12) to the TBR for different values of $T_2 - T_1$ is seen in Fig. 2.

APPENDIX C

NATURAL CIRCULATION IN A REACTOR SYSTEM

The natural circulation rate in a reactor system under steady conditions can be obtained by setting equation (9) equal to zero.

$$0 = - \frac{g_c \Delta p_o}{W_o \sum x/A} \left(\frac{W}{W_o} \right)^2 - \frac{g \Delta Z}{W_o \sum x/A} \left(\frac{\partial \rho}{\partial T} \right)_m (T_2 - T_1) \quad (13)$$

Like quantities are cancelled and equation (13) rearranged to give equation (14).

$$\frac{W}{W_o} = \left[- \frac{g}{g_c} \left(\frac{\partial \rho}{\partial T} \right)_m \frac{(T_2 - T_1)}{\Delta p_o} \right]^{1/2} \quad (14)$$

An equation reducible to (14) could have been obtained by setting equation (10) equal to zero.

$$Y = \left(- \frac{K_2}{K_1} \right)^{1/2} \quad (15)$$

The buoyancy force is equal to the pressure difference, Δp_b , which would exist between the hot and cold legs if there were no flow.

$$\Delta p_b \equiv - \frac{g}{g_c} \left(\frac{\partial \rho}{\partial T} \right)_m (T_2 - T_1) \quad (16)$$

Thus equation (14) can be written in the simple form.

$$\frac{W}{W_o} = \left(\frac{\Delta p_b}{\Delta p_o} \right)^{1/2} \quad (17)$$

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0.2

0.18

0.16

0.14

0.12

0.10

0.08

0.06

0.04

0.02

0

Ratio of Natural Circulation Flow
Rate to Normal Pumped Flow in
FBR vs. Temperature Difference
in Hot and Cold Legs

Based on -

$$\Delta p_o = 68.1 \text{ psi}$$

$$W_o = 3180 \text{ lb/sec}$$

$\tau_w \equiv$ Wall shear force

$$\frac{W}{W_o}$$

$$\tau_w \sim W^2$$

$$\tau_w \sim W^{1.8}$$

If the frictional resistance in equation (3) had been taken as proportional to the 1.8 power of the flow rate rather than the square, the final natural circulation equation would be similar to (17) but with a different exponent.

$$\frac{W}{W_o} = \left(\frac{\Delta P_b}{\Delta P_o} \right)^{1/1.8} \quad (18)$$

A comparison between the use of equations (17) and (18) for the TBR can be seen in Fig. 10.

APPENDIX DEQUATIONS FOR TIME DEPENDENCE
OF REACTOR POWER AND TEMPERATURE

The kinetic behavior of a homogeneous reactor is approximated by the following pair of equations. Equation (19) relates core power to the delayed-neutron precursor concentration and equation (20) is the kinetic equation for the precursors.

$$\lambda \dot{P} = -(\bar{\beta} + \theta) P + P_d \quad (19)$$

$$\dot{P}_d + \bar{\lambda} P_d = \bar{\lambda} \bar{\beta} P \quad (20)$$

The model of one group of delayed-neutron precursors is employed in obtaining the preceding equations and the use of $\bar{\beta}$ allows for transport of precursors from the core.

For a reactor which is below critical, the first term in equation (19) is small compared to the other terms and can be set equal to zero. This has the physical significance that the prompt neutron generation time is so small that the kinetic behavior of the core is controlled by the delayed neutrons.

$$0 = -(\bar{\beta} + \theta) P + P_d \quad (21)$$

Eliminating P from equations (20) and (21) in the manner of reference 2, equation (22) is obtained.

$$\frac{\dot{P}_d}{P_d} = -\bar{\lambda} \frac{\theta}{\bar{\beta} + \theta} \quad (22)$$

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Integrating from time zero to t and applying boundary conditions yields the equations for fission power.

$$P_d = P_{d0} e^{-\bar{\lambda} \int_0^t \frac{\theta}{\bar{\beta} + \theta} dt'} \quad (23)$$

$$P_d = (\bar{\beta} + \theta) P \quad (21)$$

$$\theta(0) = 0$$

$$P(0) = 0$$

$$P_d(0) = P_{d0} = \bar{\beta} P_0 \quad (24)$$

$$P_f = \frac{\bar{\beta}}{\bar{\beta} + \theta} P_0 e^{-\bar{\lambda} \int_0^t \frac{\theta}{\bar{\beta} + \theta} dt'} \quad (25)$$

The symbol P_f has been substituted for P in equation (25) to indicate that only the power corresponding to the rate of fissioning then occurring is included; the heat released by fission products from an earlier power level is not included in P_f . Note that θ is a function of the power-time history of the reactor incident.

Equation (25) is applicable to a change in reactor operating conditions for which $\bar{\beta}$ is not changed, i.e., the circulation rate in the core system remains constant. If the flow were suddenly stopped, $\bar{\beta}$ in equation (20) would be instantly replaced by β , but the value of $\bar{\beta}$ in equation (21) would change to β over a period of time. To be conservative, for this situation the initial value of $\bar{\beta}$ is retained in equation (21), but the replacement of $\bar{\beta}$ by β is made in equation (20). The equations are combined and integrated

as before.

$$P_f = \frac{\bar{\beta}}{\bar{\beta} + \theta} P_o e^{-\bar{\lambda} \int_0^t \left(1 - \frac{\beta}{\bar{\beta} + \theta}\right) dt'} \quad (26)$$

For a situation in which the circulation rate is reduced, equation (26) gives results which are conservative in the sense that the power is actually lower than predicted.

The form of the energy balance which describes the change of temperature with time depends on the specific system. Three different situations are considered below and the appropriate energy balance is given for each. Closed solutions to the simultaneous equations cannot be obtained; therefore the appropriate pairs of equations are put in finite difference form for solution by numerical methods.

Case I. Sudden Stoppage of Flow

The energy balance and fission power equation for a sudden, complete stoppage of flow are as follows.

$$\frac{dT}{dt} = S_c P_T(t) = S_c [P_f(t) + P_{fp}(t)] \quad (27)$$

$$T(t_{n+1}) - T t_n = S_c [P_T(t_n)] (t_{n+1} - t_n) \quad (28)$$

$$P_f(t_n) = P_o \frac{\bar{\beta}}{\bar{\beta} + \theta(t_n)} e^{-\bar{\lambda} \int_0^{t_n} \left(1 - \frac{\beta}{\bar{\beta} + \theta}\right) dt} \quad (29)$$

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Case II. Sudden Cessation of Heat Removal in Heat Exchanger with Pumping Continued

If the time required for passage of fluid through the external system (approximately 4 sec. for the TBR) is neglected, the rate of temperature rise can be expressed in terms of the heat capacity of the entire circulating system. The prime designation indicates power in the total circulating system.

$$\frac{dT}{dt} = S_s P'_T(t) = S_s [P_f(t) + P_{fp}(t)] \quad (30)$$

$$T(t_{n+1}) - T(t_n) = S_s [P'_T(t_n)] (t_{n+1} - t_n) \quad (31)$$

$$P_f(t_n) = \frac{\bar{\beta}}{\bar{\beta} + \theta(t_n)} P_o e^{-\bar{\lambda} \int_0^{t_n} \left(\frac{\theta}{\bar{\beta} + \theta} \right) dt} \quad (32)$$

Case III. Sudden Pump Failure with Heat Removal in Heat Exchanger Continued

In this situation the circulation rate following pump stoppage is a function of time and can be calculated, using the method of Appendix B. The energy balance involves transport of energy from the core.

$$\frac{dT}{dt} = S_c [P_t(t) - C_p W(t) (T_2 - T_1)] \quad (33)$$

The following approximations are made:

$T_1(t)$ is known

$T_2(t) = T$

$W(t)$ is known

It is possible to approximate $T_1(t)$ during the important initial period (about 7 sec. for the TBR) since the solution in the pipe between heat exchanger and core is at the temperature existing before pump stoppage. After the flow has slowed appreciably the solution leaves the heat exchanger at near the steam temperature.

The validity of the assumption that the solution leaves the reactor at the mean core temperature, T , is dependent on the degree of mixing in the core. During the initial slowing-down period the flow rate is little affected by the temperature difference (see Fig. 2). However the flow rate after the initial period is determined by the temperature difference and in the calculation a conservative value was therefore selected. The following equations then apply:

$$\frac{dT}{dt} = S_c \left[P_T(t) - C_p W(t) (T - T_1) \right] \quad (34)$$

$$T(t_{n+1}) - T(t_n) = S_c \left\{ P_T(t_n) - C_p W(t_n) [T(t_n) - T_1(t_n)] (t_{n+1} - t_n) \right\} \quad (35)$$

$$P_f(t_n) = \frac{\bar{\beta}}{\bar{\beta} + \theta(t_n)} P_o e^{-\bar{\lambda} \int_0^{t_n} \left(1 - \frac{\beta}{\bar{\beta} + \theta} \right) dt} \quad (36)$$

APPENDIX EEQUATIONS FOR EQUILIBRIUM DELAYED-NEUTRON PRECURSOR
CONCENTRATION AND AVERAGE DECAY CONSTANT

To determine the equilibrium precursor concentration in the core the rate of change of P_1 with time is equated to zero.

$$\dot{P}_1 = \beta_1 \lambda_1 P - \lambda_1 P_1 - \frac{P_1}{t_c} \left[1 - e^{-\lambda_1 t_p} \right] = 0 \quad (37)$$

$$t_c = V_c \rho / W$$

$$t_p = V_p \rho / W$$

Complete mixing of the core fluid is assumed in equation (37).

Allowance is made for transport of precursors from the core, decay in the external system, and return to the core. Equation (37) can be rearranged and solved for P_1 , yielding equation (38).

$$P_1 = \frac{\beta_1}{1 + \frac{1}{\lambda_1 t_c} \left(1 - e^{-\lambda_1 t_p} \right)} P_0 = \bar{\beta}_1 P_0 \quad (38)$$

$$\bar{\beta}_1 = \frac{\beta_1}{1 + \frac{1}{\lambda_1 t_c} \left(1 - e^{-\lambda_1 t_p} \right)} \quad (39)$$

$$P_d = \sum_i P_i \quad (40)$$

$$\bar{\beta} = \sum_i \bar{\beta}_i \quad (41)$$

$$\bar{\lambda} = \frac{\sum P_i}{\sum P_i / \lambda_i} \approx \frac{P_0 \sum \beta_i}{P_0 \sum \beta_i / \lambda_i} = \frac{\beta}{\sum \beta_i / \lambda_i} \quad (42)^*$$

* A definition of $\bar{\lambda}$ using $\bar{\beta}$ rather than β could be used but the method employed in the more conservative.

APPENDIX FVALUES OF QUANTITIES EMPLOYED IN CALCULATIONS FOR TBR*

Geometry of System:

$$V_c = 104 \text{ cu ft}$$

$$V_s = 344 \text{ cu ft}$$

$$\sum x/A = 30.2 \text{ ft}^{-1}$$

$$\Delta Z = 22 \text{ ft}$$

Normal Reactor Conditions:

$$P_o = 357 \text{ Mw}$$

$$t_c = 1.85 \text{ sec}$$

$$t_p = 4.3 \text{ sec}$$

$$T_1 = 250^\circ\text{C}$$

$$T_2 = 300^\circ\text{C}$$

$$T = 280^\circ\text{C}$$

$$T_b = 225^\circ\text{C}$$

$$W_o = 3180 \text{ lb/sec}$$

$$\Delta p_o = 68.1 \text{ psi}$$

Physical Properties of Fuel Solution:

$$c_p = 1.28 \text{ Btu/lb-}^\circ\text{F} = 2.43 \times 10^{-3} \text{ Mw-sec/lb-}^\circ\text{C}$$

$$\rho = 56.8 \text{ lb/cu ft}$$

$$(\partial\rho/\partial T)_m = -0.11 \text{ lb/cu ft-}^\circ\text{C}$$

* Most of the design values listed were obtained from reference 1.

Characteristics of System:

$$S_c = 0.070 \text{ } ^\circ\text{C/Mw-sec}$$

$$S_s = 0.0212 \text{ } ^\circ\text{C/Mw-sec}$$

$$\alpha = -\partial k_e / \partial T = 1.5 \times 10^{-3} \text{ } ^\circ\text{C}^{-1}$$

$$\beta = 2.42 \times 10^{-3}$$

$$\bar{\beta} = 1.03 \times 10^{-3}$$

$$\bar{\lambda} = 0.061 \text{ sec}^{-1}$$

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REFERENCES

1. "HRP Quar. Prog. Rep. for Period Ending April 30, 1955", ORNL-1895, 1955.
2. Hill, J. W., Jr., "HRT Power and Fuel Temperature After Fuel Circulation Stops", ORNL-CF-54-9-64, Sept. 9, 1954.
3. McAdams, W. H., Heat Transmission, Third Ed., McGraw-Hill Book Co., New York, 1954.
4. Untermeyer, S., and J. T. Weills, "Heat Generation in Irradiated Uranium", ANL-4790, 1952.

NOMENCLATURE

- a Fraction of neutrons causing fission times the energy released per fission.
- A Cross-section of channel normal to direction of flow (sum of cross-sections for multi-channel flow).
- b Perimeter of channel (sum of perimeters for multi-channel flow).
- C_p Specific heat of fuel solution.
- g Local gravitational acceleration.
- g_c Conversion factor, 32.2 lb mass-ft/lb force-sec² in engineering units.
- K_1 Dimensional constant $\frac{g_c \Delta p_o}{w_o \sum x/A}$ having units of reciprocal time.
- K_2 Dimensional constant $\frac{g_c \Delta Z}{w_o \sum x/A} \left(\frac{\partial \rho}{\partial T} \right)_m T_2 - T_1$.
- k Multiplication constant for an infinite reactor; number of neutrons produced per absorption.
- k_e Multiplication constant for a finite reactor; neutrons produced in one generation per neutron produced in preceding generation.
- ℓ Lifetime of prompt neutrons.
- N_i Number of delayed neutron precursors of species i in reactor core.
- P Power (heat release rate) in reactor core; P_o for total power at initial condition; P_f for fission power; P_{fp} for fission product heat release rate; $P_T \equiv P_f + P_{fp}$.
- P_i Power associated with delayed-neutron precursors $ak_e \lambda_i N_i / k$.
- P_d $\sum_i P_i$; P_{do} at initial conditions.
- p Static pressure.
- S_c Reciprocal heat capacity of reactor core.
- S_s Reciprocal heat capacity of entire core circulating system.

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T	Mean temperature of reactor core; T_0 for initial condition.
T_1	Temperature of solution entering reactor core.
T_2	Temperature of solution leaving reactor core.
T_b	Boiling point of water on shell-side of heat exchanger.
t	Time variable; time from beginning of incident; t' for dummy variable of integration replacing t .
t_c	Average time required for solution to pass through core under normal conditions. (see equation 37 ff.).
t_p	Average time required for solution to pass through the external system from core exit to entrance under normal conditions (see equation 37 ff.).
U	Mean velocity at a cross-section in circulating system.
V_c	Volume of core tank.
V_p	Volume of circulating system external to core.
W	Mass flow rate of core solution; W_0 for initial condition or design condition.
x	Axial distance in circulating system.
Y	W/W_0 ; y for dummy variable of integration replacing Y .
α	Negative of temperature coefficient of reactivity, $-\partial k_e / \partial T$.
β	Total fraction of delayed neutrons which are delayed, $\sum_i \beta_i$.
$\bar{\beta}$	Effective value of β allowing for transport of delayed-neutron precursors from core, $\sum_i \bar{\beta}_i$.
β_i	Fraction of neutrons which are delayed that have decay constant λ_i .
$\bar{\beta}_i$	Effective value of β_i allowing for transport of delayed-neutron precursors from core.
Δp_o	Pressure drop across core circulating system (between pump discharge and intake) at mass flow rate W_0 .
Δp_b	Buoyancy pressure driving force which would exist under condition of no flow.

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39.

- $(\partial \rho / \partial T)_m$ Mean temperature coefficient of density over range of interest.
- θ Negative of reactivity change resulting from temperature change of reactor core, $\alpha(T-T_0)$.
- λ_1 Decay constant of delayed-neutron precursor of species 1.
- $\bar{\lambda}$ Average delayed-neutron precursor decay constant (see equation 42).
- ξ Angle between direction of flow and vertical up.
- ρ Density of fuel solution.
- τ_w Wall shear force.

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