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THE EFFECT OF FUEL PARTICLE SIZE
ON THE TRANSIENT BEHAVIOR
OF A HOMOGENEOUS GRAPHITE REACTOR

WRITTEN BY

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~~CONFIDENTIAL~~ ABSTRACT

Heat transfer from the fuel to the moderator during a sudden power increase in a uranium-impregnated graphite-moderated reactor is investigated in detail to determine whether the use of relatively large fuel particles inhibits effective operation of the negative temperature coefficient of reactivity. For the Low Power Research Reactor designed by North American Aviation, Inc., the time lags in heat transfer to the moderator after the sudden introduction of 2 per cent excess reactivity (reactor period, 80 milliseconds) are found to be 3 milliseconds for uranium oxide particles 100 microns in diameter and 11 milliseconds for particles 200 microns in diameter.

The fraction of fission fragments escaping into the moderator is 20 per cent in the former case (100 microns) and 10 per cent in the latter (200 microns). For comparison, all the fission fragments escape if the particles are less than a few microns in diameter; this obviates the heat transfer problem but intensifies radiation damage in the graphite.

Since, for relatively large fuel particles, the moderator temperature follows the power rise without appreciable time lag, it may be concluded that much radiation damage could be prevented without inhibiting the action of the temperature coefficient.

This report is based upon studies conducted for the Atomic Energy Commission under Contract AT-11-1-GEN-8.

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I. INTRODUCTION

If the core of an enriched, homogeneous, graphite-moderated reactor is fabricated by means of chemical impregnation methods, the uranium oxide particles are sufficiently small (less than 1 micron in diameter) that most of the fission fragments escape from the fuel and dissipate a large fraction of their energy in the graphite. The moderator temperature would therefore respond quickly to an abrupt increase in power level, thereby providing the intrinsic safety feature of a large, negative, transient temperature coefficient of reactivity which is not available in a heterogeneous reactor because of the time lag in heat transfer from the fuel to the moderator.

However, concurrent with this, the rate of radiation damage in the graphite would be large because of its direct exposure to fission fragments as well as fast neutrons. If the fuel particles could be made large enough to contain most of the fission fragments, the radiation damage could be reduced considerably, although the temperature coefficient of reactivity would then depend on the details of heat transfer to the moderator.

It is the purpose of this report to investigate the volume average graphite temperature during an exponential power rise for fuel particles whose diameter is much larger than the range of fission fragments in uranium oxide. This is represented by an idealization in which all sources of heat are confined to the fuel particles. The result is then compared with a calculation in which the heat sources are distributed uniformly throughout the graphite. The former will be referred to as the "cell calculation" and the latter as the "homogeneous calculation," though the neutron physics treats both as homogeneous.

II. ANALYSIS

The procedure to be followed is to consider each uranium oxide particle as being embedded in a sphere of graphite. Each fuel particle is thus associated with a "cell" whose dimensions are prescribed by the overall fuel-to-moderator ratio and the average size of the fuel particles.

Before considering this cell calculation in detail, it is helpful to investigate the simpler problem of a point source of heat in an infinite medium.

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~~CONFIDENTIAL~~A. Point Source in an Infinite Medium

The temperature at any point in an infinite medium having thermal diffusivity D with a point source $\phi(t)$ at the origin is a function of position and time given by

$$T(r, t) = \frac{1}{(4\pi D)^{3/2}} \int_0^t \phi(t') \exp \left[-\frac{r^2}{4D(t-t')} \right] \frac{dt'}{(t-t')^{3/2}} \quad \dots (1)$$

where the source is zero for $t < 0$ and where the initial temperature is zero everywhere. The source is an idealization of a fuel particle of volume V and power per unit volume P , and may be written as

$$\phi(t) = P(t) V / \rho c$$

where the density ρ and specific heat c are characteristic of the infinite medium.

If for simplicity the power density is assumed to be $P_0 e^{\lambda t}$, the substitutions $x = t - t'$ and $A = P_0 V / [\rho c (4\pi D)^{3/2}]$ lead to

$$T(r, t) = A e^{\lambda t} \int_0^t x^{-3/2} \exp(-\lambda x - r^2/4Dx) dx \quad \dots (2)$$

Now the integrand as a function of x vanishes at $x = 0$ and $x = \infty$. It has no roots for x positive and finite, and the maximum is at

$$x = \frac{3}{4\lambda} \left[\left(1 + \frac{4\lambda r^2}{9D} \right)^{1/2} - 1 \right]$$

If the point of observation is sufficiently close to the source or if the power rise is sufficiently rapid, the radical may be approximated by the first two terms of its binomial expansion to yield

$$x \approx r^2/6D.$$

The integral in Eq. (2) is therefore a monotonic function of time which for points near the source rises very rapidly toward its asymptotic value. For any time greater than intervals of the order of magnitude of r^2/D , the upper limit of the integral may be taken as infinite. The integral then becomes a function of r only, and the temperature becomes a product of two functions

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a function of time only and the other a function of position only.

As an example, the volume ratio of fuel to moderator in the Low Power Research Reactor (LPRR) designed by North American Aviation, Inc.,^{1,2} is 4.5×10^{-4} . For relatively large fuel particles of a diameter of 200 microns (radius 0.010 centimeter), the miniature cell associated with each particle has a radius of 0.1305 centimeter. The thermal diffusivity of graphite may be taken as $0.50 \text{ cm}^2/\text{sec}$. The interval r^2/D at the furthest point from the source is then 34 milliseconds, so that the variables of time and position are essentially separable within 34 milliseconds after the initiation of a runaway. For points closer to the source the time interval is even less.

For comparison, the period $1/\lambda$, resulting from the sudden introduction of 2 per cent excess reactivity in the LPRR, would be 80 milliseconds. The separation of variables is then a good approximation except for a fraction of the pile period at the beginning of the runaway.

Of course, the point source approximation fails in that it neglects that portion of the heat which raises the temperature of the fuel particle. The point source problem has been discussed here only because it indicates a plausible approach to the more complicated cell calculation using a finite source.

B. Cell Calculation

The partial differential equation to be solved, subject to appropriate boundary conditions, is

$$\frac{\partial T}{\partial t} = D \nabla^2 T + \frac{P}{\rho c} \quad \dots (3)$$

The cell is composed of two regions: (1) a sphere of uranium oxide of radius a , and (2) a spherical shell of graphite of inner radius a and outer radius b . The ratio b/a is fixed by the fuel-to-moderator ratio in the core. In region 1, the power density is $P_0 e^{\lambda t}$; in region 2, it is zero.

Consider a solution of the form

$$T(r, t) = e^{\lambda t} [\phi(r)].$$

Substitution into Eq. (3) yields

$$D \nabla^2 \phi - \lambda \phi + Q = 0$$

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where $Q = P_o / \rho c$. If we now write

$$\phi(r) = u(r) + Q/\lambda$$

we obtain

$$D \nabla^2 u = \lambda u \quad \dots (4)$$

The differential Eq. (3) is thus satisfied by

$$T(r, t) = e^{\lambda t} [u(r) + Q/\lambda] \quad \dots (5)$$

where u is a solution of Eq. (4). For region 1 ($r \leq a$) we have

$$T_1 = e^{\lambda t} [u_1 + Q/\lambda]$$

where u_1 is a solution of $\nabla^2 u_1 = \beta_1^2 u_1$, with $\lambda \equiv \beta_1^2 D_1$ and $D_1 = K_1 / \rho_1 c_1$ and where K is the thermal conductivity. For region 2 ($a \leq r \leq b$) we have

$$T_2 = e^{\lambda t} u_2$$

where u_2 is a solution of $\nabla^2 u_2 = \beta_2^2 u_2$ with $\lambda \equiv \beta_2^2 D_2$ and $D_2 = K_2 / \rho_2 c_2$.

The nature of the appropriate boundary condition at the surface $r = b$ is not obvious. In the usual lattice calculation of steady-state temperatures, one assumes a cylindrical cell which is independent of its neighbors since the heat generated in that cell is carried away by coolant flowing along the axis. This is not the type of cell we are using.

Nevertheless, in the time-dependent problem we are considering, the power rises very rapidly to values much larger than those in the steady state. Indeed, the steady-state condition holding for $t < 0$ is to be represented by the somewhat artificial situation of uniform temperature and zero power density. While the power is rising rapidly, the action of the coolant will not be very significant, so that the assumption of no heat flow between the cells (which ignores the steady-state flow) is applicable during the runaway.

The conditions on the functions $T_1(r, t)$ and $T_2(r, t)$ which must be met for all finite positive values of t are:

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$$\left. \begin{aligned}
 \text{I. } T_1(O, t) \text{ is finite} \\
 \text{II. } T_1(a, t) &= T_2(a, t) \\
 \text{III. } K_1 \frac{\partial T_1}{\partial r} \Big|_{r=a} &= K_2 \frac{\partial T_2}{\partial r} \Big|_{r=a} \\
 \text{IV. } \frac{\partial T_2}{\partial r} \Big|_{r=b} &= 0
 \end{aligned} \right\} \dots (6)$$

The conditions (6) are satisfied by:

$$\left. \begin{aligned}
 \text{I. } u_1(O) \text{ is finite} \\
 \text{II. } u_1(a) + Q/\lambda &= u_2(a) \\
 \text{III. } K_1 u_1'(a) &= K_2 u_2'(a) \\
 \text{IV. } u_2'(b) &= 0
 \end{aligned} \right\} \dots (7)$$

where the prime signifies the first derivative with respect to r .

Condition I may be applied immediately, and the solutions are of the form

$$\left. \begin{aligned}
 u_1 &= \frac{A_1}{r} \sinh \beta_1 r \\
 u_2 &= \frac{A_2}{r} \sinh \beta_2 r + \frac{B_2}{r} \cosh \beta_2 r
 \end{aligned} \right\} \dots (8)$$

The other three conditions serve to determine the three constants A_1 , A_2 and B_2 . The inhomogeneous set of three linear equations is:

$$\left. \begin{aligned}
 A_1 \sinh \beta_1 a - A_2 \sinh \beta_2 a - B_2 \cosh \beta_2 a &= -Qa/\lambda \\
 K_1 A_1 (\beta_1 a \cosh \beta_1 a - \sinh \beta_1 a) - K_2 A_2 (\beta_2 a \cosh \beta_2 a - \sinh \beta_2 a) \\
 &\quad - K_2 B_2 (\beta_2 a \sinh \beta_2 a - \cosh \beta_2 a) = 0 \\
 A_2 (\beta_2 b \cosh \beta_2 b - \sinh \beta_2 b) + B_2 (\beta_2 b \sinh \beta_2 b - \cosh \beta_2 b) &= 0
 \end{aligned} \right\} \dots (9)$$

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The determinant of the coefficients is:

$$\Delta = \begin{vmatrix} \sinh \beta_1 a & -\sinh \beta_2 a & -\cosh \beta_2 a \\ K_1(\beta_1 a \cosh \beta_1 a - \sinh \beta_1 a) & -K_2(\beta_2 a \cosh \beta_2 a - \sinh \beta_2 a) & -K_2(\beta_2 a \sinh \beta_2 a - \cosh \beta_2 a) \\ 0 & \beta_2 b \cosh \beta_2 b - \sinh \beta_2 b & \beta_2 b \sinh \beta_2 b - \cosh \beta_2 b \end{vmatrix} \dots (10)$$

Solution of the set of linear equations, Eq. (9), yields for the constants in Eq. (8) the following:

$$\left. \begin{aligned} A_1 &= \frac{QaK_2}{\lambda \Delta} \left[(\beta_2^2 ab - 1) \sinh \beta_2(b-a) + \beta_2(b-a) \cosh \beta_2(b-a) \right] \\ A_2 &= \frac{QaK_1}{\lambda \Delta} \left[(\beta_1 a \cosh \beta_1 a - \sinh \beta_1 a) (\beta_2 b \sinh \beta_2 b - \cosh \beta_2 b) \right] \\ B_2 &= -\frac{QaK_1}{\lambda \Delta} \left[(\beta_1 a \cosh \beta_1 a - \sinh \beta_1 a) (\beta_2 b \cosh \beta_2 b - \sinh \beta_2 b) \right] \end{aligned} \right\} \dots (11)$$

Define two functions, $\psi_1(r)$ and $\psi_2(r)$, as follows:

$$\psi_1(r) \equiv \left[(\beta_2^2 ab - 1) \sinh \beta_2(b-a) + \beta_2(b-a) \cosh \beta_2(b-a) \right] \sinh \beta_1 r$$

$$\psi_2(r) \equiv (\sinh \beta_1 a - \beta_1 a \cosh \beta_1 a) \left[\beta_2 b \cosh \beta_2(b-r) - \sinh \beta_2(b-r) \right]$$

Upon expanding the determinant of the coefficients, Eq. (10) becomes

$\Delta = K_1 \psi_2(a) - K_2 \psi_1(a)$. Substituting the constants from Eq. (11) into Eq. (8) yields

$$\left. \begin{aligned} u_1 &= \frac{QaK_2}{\lambda [K_1 \psi_2(a) - K_2 \psi_1(a)]} \cdot \frac{\psi_1(r)}{r} \\ u_2 &= \frac{QaK_1}{\lambda [K_1 \psi_2(a) - K_2 \psi_1(a)]} \cdot \frac{\psi_2(r)}{r} \end{aligned} \right\} \dots (12)$$

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The temperatures are

$$\left. \begin{aligned} T_1(r, t) &= e^{\lambda t} \left[u_1(r) + \frac{Q}{\lambda} \right] \\ T_2(r, t) &= e^{\lambda t} u_2(r) \end{aligned} \right\} \dots (13)$$

with $\lambda = \beta_1^2 D_1 = \beta_2^2 D_2$

We have now obtained solutions which satisfy the boundary conditions exactly, but which fail to satisfy an initial condition of uniform temperature. This may be compared with the point source solution Eq. (2) with the upper limit of the integral replaced by infinity. An interpretation is that the exact solutions for the cell would contain transients in the functions $u_1(r)$ and $u_2(r)$, much as the integral in Eq. (2) which quickly loses its time dependence as the asymptotic value of the integral is approached. By analogy with the point source problem, Eq. (13) may be accepted as representing the temperatures in the cell except for short-lived transients in the radial functions.

C. Volume Average Graphite Temperature

Since the neutron mean free path is much larger than the miniature cells, the temperature T_2 averaged over the volume contained in $a \ll r \ll b$ may be taken as the effective moderator temperature. We must evaluate

$$\bar{u}_2 = \frac{QaK_1}{\lambda [K_1 \psi_2(a) - K_2 \psi_1(a)]} \left[\frac{\psi_2(r)}{r} \right]$$

using

$$\left[\frac{\psi_2(r)}{r} \right] = \frac{\int_a^b r \psi_2 dr}{\int_a^b r^2 dr}$$

After performing the indicated operations and inserting the explicit expressions for ψ_1 and ψ_2 , we obtain

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$$\bar{u}_2 = \frac{\frac{3Qa}{\lambda \beta_2^2 (b^3 - a^3)}}{\frac{K_2/K_1}{\beta_1^a \coth \beta_1^a - 1} + \frac{\beta_2^b \coth \beta_2(b-a) - 1}{\beta_2^{(b-a)} \coth \beta_2(b-a) - 1 + \beta_2^{2ab}}} \dots (14)$$

If the conductivity of the moderator is sufficiently larger than that of the fuel, we have

$$\bar{u}_2 \approx \frac{3Qa [\beta_1^a \coth \beta_1^a - 1]}{\lambda \beta_2^2 (b^3 - a^3) K_2/K_1}$$

Since $\beta_2^2 = \lambda/D_2 = \rho_2 c_2 \lambda/K_2$ and since $a^3 \ll b^3$, this becomes

$$\bar{u}_2 \approx \frac{3Qa [\beta_1^a \coth \beta_1^a - 1]}{\rho_2 c_2 \lambda^2 b^3/K_1} \dots (15)$$

Note that this expression is independent of the moderator thermal conductivity. The approximation used here introduces less than 0.2 per cent error into the calculation of $\bar{u}_2$ for fuel particles less than 300 microns in diameter. For larger particles the approximation is not as good.

D. Comparison with Homogeneous Case

Fig. 1 shows the probability of escape from uranium oxide particles for an average light fission fragment and for an average heavy fragment for various particle sizes.³ The preceding analysis, applicable to the larger particles, is to be compared with a corresponding analysis wherein the power density is uniformly distributed throughout the graphite. In other words, the particle concept is now to be suppressed except insofar as the particles must be sufficiently small that most of the fission fragment energy is converted to heat in the graphite.

We write

$$\frac{dT}{dt} = \frac{P'}{\rho_2 c_2}$$

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where $P' = (a^3/b^3)P$ since the source is distributed throughout the cell.
If $P' = P_o e^{\lambda t}$, the temperature is

$$T = \frac{P_o'}{\rho_2 c_2 \lambda} (e^{\lambda t} - 1) \quad \dots (16)$$

If the term not involving t is dropped, the effect is similar to the previous calculation in that the initial condition is not satisfied exactly, but the result becomes a good approximation in a very short time. We may compare this with Eq. (13) and note that $P_o'/\rho_2 c_2 \lambda$ plays the role of $u_2(r)$ in the cell calculation.

In fact the method of performing the cell calculation gains considerable justification upon taking the limit of $\bar{u}_2$ in Eq. (15) as $a \rightarrow 0$ with b/a fixed.

Using

$$\lim_{a \rightarrow 0} \left[\beta_1 a \coth \beta_1 a - 1 \right] = \lim_{a \rightarrow 0} \beta_1^2 a^2 / 3$$

together with $\beta_1^2 = \lambda/D_1 = \rho_1 c_1 \lambda/K_1$ and $Q = P_o'/\rho_1 c_1$,

we obtain

$$\bar{u}_2 \approx \frac{P_o' a^3}{\rho_2 c_2 \lambda b^3} \approx \frac{P_o'}{\rho_2 c_2 \lambda}$$

We are now in a position to compare the average temperature in a cell $\bar{u}_2 e^{\lambda t}$, for various size fuel particles keeping b/a fixed, with the temperature in the completely homogeneous case $u_o e^{\lambda t}$ where $u_o = P_o'/\rho_2 c_2 \lambda$. The average temperature in a cell will lag behind the temperature for the homogeneous case, thus delaying the effect of the temperature coefficient of reactivity. This effectively gives the runaway a "head start" by the amount of the time lag. Since both temperatures are exponential with identical periods, the time lag is independent of t .

The two functions will be equal at different times:

$$\bar{u}_2 e^{\lambda t} = u_o e^{\lambda t'}$$

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The time lag for the cell temperature is therefore

$$\Delta t = t - t' = \frac{1}{\lambda} \ln u_o / \bar{u}_2 \quad \dots (17)$$

where $u_o = P'_o / \rho_2 c_2 \lambda$ and $\bar{u}_2$ is given by Eqs. (14) or (15). Note that the time lag is independent of the steady-state power level.

E. Thermal Resistance Between Fuel and Moderator

The results may easily be modified to include the effect of a thermal resistance at the interface between the uranium oxide and the graphite. For simplicity, assume a temperature discontinuity ΔT at $r = a$ satisfying the relation

$$K \frac{\Delta T}{\delta} = -K_1 \left. \frac{\partial T_1}{\partial r} \right|_{r=a} \quad \dots (18)$$

where K is the conductivity of a thin gas-filled gap of thickness δ . Although Eq. (18) is written for a thin film ($\delta \ll a$), it will yield a fairly good estimate of the effect of a larger gap whose effective thermal resistance is δ/K (temperature difference per unit heat flux).

Assuming $\Delta T = \Delta u e^{\lambda t}$, we have

$$K \Delta u / \delta = K_1 u'_1(a) \quad \dots (19)$$

The boundary condition II of Eq. (6) must be replaced by

$$T_1(a, t) = T_2(a, t) + \Delta T.$$

Condition II of Eq. (7) becomes

$$u_1(a) + Q/\lambda = u_2(a) + \Delta u \quad \dots (20)$$

where Δu is obtained by solving Eq. (19).

The solutions have the same form as in Eq. (19), but the constants A_1 , A_2 and B_2 will be different. Making use of Eqs. (8) and (19), it is seen that an additional term will appear in the first of the set of boundary equations, Eq. (9); the term

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$$\frac{K_1 \delta}{K a} \left[\beta_1 a \cosh \beta_1 a - \sinh \beta_1 a \right] A_1$$

must be added to the left-hand side. The 1, 1 term $\sinh \beta_1 a$ in the determinant of the coefficients (Eq. (10)) is replaced by

$$\left(1 - \frac{K_1 \delta}{K a} \right) \sinh \beta_1 a + \frac{K_1 \beta_1 \delta}{K} \cosh \beta_1 a$$

The constants A_1 , A_2 and B_2 are still given by Eq. (11) except for the modification in Δ . This is a consequence of the fact that the only alteration in Eq. (9) occurs in the one equation containing an inhomogeneous term.

The volume average temperature in the graphite has the same form as before except for the additional terms in the determinant Δ . The net effect is an additional term in the denominator of the expression for $\bar{u}_2$. Eq. (14) becomes

$$\bar{u}_2 = \frac{\frac{3Qa}{\lambda \beta_2^2 (b^3 - a^3)}}{\frac{K_2/K_1}{\beta_1 a \coth \beta_1 a - 1} + \frac{K_1 \delta}{K a} + \frac{\beta_2 b \coth \beta_2 (b-a) - 1}{\beta_2 (b-a) \coth \beta_2 (b-a) - 1 + \beta_2^2 ab}} \dots (21)$$

III. NUMERICAL RESULTS

Combining Eqs. (14) and (17), with $\rho c \lambda = K \beta^2$ for each region, the time lag may be expressed as:

$$\Delta t = \frac{1}{\lambda} \ln \left\{ \frac{K_1 \beta_1^2 a^2 (1 - a^3/b^3)}{3K_2} \left[\frac{K_2/K_1}{\beta_1 a \coth \beta_1 a - 1} + \frac{\beta_2 b \coth \beta_2 (b-a) - 1}{\beta_2 (b-a) \coth \beta_2 (b-a) - 1 + \beta_2^2 ab} \right] \right\} \dots (22)$$

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The ratio b/a is kept constant and equal to 13.05. This is determined from $a^3/(b^3 - a^3) = 4.5 \times 10^{-4}$, the ratio of graphite volume to uranium oxide volume in the core. Densities, specific heats, and thermal conductivities are as follows: for uranium oxide $\rho_1 = 10.8 \text{ gm/cm}^3$, $c_1 = 0.0635 \text{ cal/gm}^\circ\text{C}$, $K_1 = 3.5 \times 10^{-4} \text{ cal/cm-sec-}^\circ\text{C}$; for graphite $\rho_2 = 1.67 \text{ gm/cm}^3$, $c_2 = 0.30 \text{ cal/gm}^\circ\text{C}$, $K_2 = 0.25 \text{ cal/cm-sec-}^\circ\text{C}$. The reactor period is assumed to be 80 milliseconds.

The time lag is plotted as a function of the fuel particle diameter $2a$ in Fig. 2. Note that this time lag is overestimated, especially for the smaller fuel particles, since the calculation was made under the assumption that all sources of heat were confined to the fuel. Actually those fission fragments which escape from the fuel will dissipate some fraction of their energy in the moderator.

Comparing Eqs. (14) and (21), the effect of a thermal resistance (gas film of conductivity K and thickness δ) is to add a term $K_1 \delta / Ka$ inside the square brackets of Eq. (22). The core of the Low Power Research Reactor is sealed with a helium atmosphere, so that a space between the fuel and the relatively porous moderator would be filled with helium ($K = 3.4 \times 10^{-4} \text{ cal/cm-sec-}^\circ\text{C}$). But even if the void had a thickness $\delta = a$, the additional term in Eq. (22) would have negligible effect. For example, for fuel particles 500 microns in diameter, * the term $K_2/(K_1 \beta_1 a \coth \beta_1 a - 1) = 245$, while $K_1 \delta / Ka \approx 1$. For smaller fuel particles, the disparity is even greater.

IV. CONCLUSIONS

The time lags as given by Fig. 2 are not effective immediately because of the transient effect mentioned in connection with Eq. (13). This is an effect which is more serious for the larger cells, but a near-optimum situation is represented by fuel particles 100 microns in diameter. Here, the fraction of fission fragments escaping is about 20 per cent, and the

* Fuel elements containing such large uranium oxide particles would probably be fabricated directly from a mixture of powdered graphite and uranium oxide pellets, in contrast with chemical impregnation methods whereby tiny fuel particles are deposited in the pores between crystallites. Insofar as heat transfer from the fuel to the moderator is concerned, the former situation is probably much better because of better thermal contact.

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potentiality for radiation damage is even less than 20 per cent because of energy lost along the path before emerging from the fuel.

From Fig. 2, the time lag is about 3 milliseconds. This is a small fraction of the 80-millisecond period. From Eq. (14) $\bar{u}_2 = 4.95 \times 10^{-3}$, and if the initial temperature is 100°C , the temperature at any time $t > 0$ will be*

$$T = 100 + 4.95 \times 10^{-3} e^{12.5t} \quad \dots (23)$$

where time is measured in seconds. It is seen that the temperature remains less than 105°C for 0.55 seconds. By this time the transient effect is wiped out and the temperature functions are correctly given by Eq. (13). The power has already multiplied about 1000 times, and the effect of the 3-millisecond time lag due to heat transfer is negligible. The temperature now begins to rise rapidly as it would in the homogeneous case (very small fuel particles), and the temperature coefficient of reactivity comes into play. The net effect of the time lag is to allow the peak power to assume a value slightly higher than that attained with very small fuel particles. (The additional multiplication is $e^{\lambda \Delta t}$ or about 1.04).

The temperature as a function of time as computed from Eq. (23) is plotted in Fig. 3 together with the function

$$T = 100 + 5.15 \times 10^{-3} (e^{12.5t} - 1) \quad \dots (24)$$

which is the homogeneous case as given by Eq. (16). The curves continue to rise steeply because the temperature coefficient of reactivity has not been taken into account.

The effect of a helium-filled gap between the fuel and the moderator has been shown to be negligible. Radiative heat transfer across such a gap has not been considered; its effect would be to shorten any time lag.

It may be concluded that a large fraction of the radiation damage associated with homogeneous graphite reactors may be prevented by using large fuel particles (of the order of 100 microns in diameter) without sacrificing any of the intrinsic safety characteristics.

* This is for a steady-state power of 135 kilowatts; core volume, 10^6 cm^3 . Present specifications for the Low Power Research Reactor are only slightly different.

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Figure 1. Fraction of Fission Fragments Escaping from a Uranium Oxide Particle vs Particle Diameter

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Figure 2. Time Lag in Heat Transfer to Moderator For Various Size Fuel Particles During Initial Stage of Runaway (Pile Period = 80 Milliseconds)

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Figure 3. Graphite Temperature as a Function of Time During Initial Stage of Runaway (Temperature Coefficient of Reactivity Neglected)

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1. LPRR Engineering Design Report, to be published.
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