**FUEL PROGRAMMING
FOR SODIUM GRAPHITE REACTORS**

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ABSTRACT

This report concerns the effect of fuel programming, i.e., the scheme used for changing fuel in a core, on the reactivity and specific power of a sodium graphite reactor. Fuel programs considered include replacing fuel a core-load at a time or a radial zone at a time, replacing fuel to maintain the same average exposure of fuel elements throughout the core, and replacing and transferring fuel elements to maintain more highly exposed fuel in the center or at the periphery of the core. Flux and criticality calculations show the degree of power flattening and the concurrent decrease in effective multiplication which results from maintaining more exposed fuel toward the core center. Converse effects are shown for the case of maintaining more exposed fuel near the core periphery. The excess reactivity which must be controlled in the various programs is considered. Illustrative schedules for implementing each of these programs in an SGR are presented.

I. INTRODUCTION

Fuel programming refers to the sequence and schedule for removing spent fuel elements, inserting new fuel elements, and if desirable, rearranging partially spent fuel elements in the core of a reactor. There are many nuclear power cost factors which are related to fuel programming. These include the average exposure of discharged fuel elements, initial enrichment requirements, fuel inventory outside the core, number of fuel handling operations, average power density in the fuel, etc. A discussion of some alternative fuel programs for a sodium-graphite reactor (SGR) has been given in a previous report¹. The present work examines the effect of these programs on core reactivity and power distribution.

A fuel program is necessarily influenced by the characteristics of the reactor under consideration. An SGR, for example, is a solid-fuel, heterogeneous reactor. The fuel elements are widely spaced in comparison to those in hydrogen-moderated reactors. Present designs require uranium fuel enrichments of about 3 to 4 at.% U^{235} . Although SGR designs are not necessarily limited to these features, it has been assumed here that the reactor must be shut down in order to make any change in fuel element arrangement, and that each fuel element is as long as the core, i.e., fuel elements are not segmented. To the extent that these characteristics influence fuel programming, the present work is so limited.

Slightly enriched uranium fuel generally shows a continuous decline in reactivity as fission takes place. Due to the high microscopic fission cross section of Pu^{239} , the microscopic fission cross section of the fuel may increase initially, even though the conversion ratio is well below unity. However, the microscopic absorption cross section also increases and the net effect is a continuous decline in reactivity. In this characteristic it differs from a natural-uranium or very slightly enriched uranium reactor which can show an initial increase in reactivity due to plutonium formation. The values of reactivity and power distributions which are presented in this report are based on a fuel enrichment of 4 at.% U^{235} and an initial conversion ratio of 0.6. Since the interest here is primarily on relative changes in core characteristics for different fuel programs, the absolute values of these characteristics are secondary.

Except for the unique condition of all-new fuel, any core is fueled by an array of fuel elements of varying exposure. The particular array is determined by the quantity of energy that has been extracted from the core, the flux or power distribution in the core, the isotopic changes in the fuel and, of course, any changes made in the position of fuel elements. The qualitative effects of various distributions of exposed fuel may be easily summarized. If the fuel toward the center of core is more exposed than that toward the periphery, then the flux or power distribution in the core is flatter than for the condition of all new fuel elements. This situation permits higher power extraction, assuming that the core power is limited by some "hot spot." Furthermore, this type of array, with more exposed fuel at the center, is the one which comes about naturally as burning proceeds from the condition of all-new fuel. A second effect of having more highly exposed fuel toward the center of the core is that the reactivity is less than that for a core containing uniformly exposed fuel of the same average exposure. The effect of having the more highly exposed fuel toward the periphery of the core is, of course, just the opposite. The peak-to-average-power ratio is higher, and the reactivity of the reactor is higher than for the case of uniformly exposed fuel.

Before proceeding with the analysis of various fuel programs, it will facilitate the discussion to have a nomenclature by which to refer to these programs. The programs to be analyzed in the report are named and briefly described in Table I. Each program is then analyzed in some detail in the following pages.

TABLE I
NOMENCLATURE OF FUEL PROGRAMS

Program Number	Name	Description
I	Fuel reloading by core	Reload entire core with new fuel elements each time fuel is changed.
II	Fuel reloading by zone	Replace all elements within a radial zone as the most exposed elements within the zone reach allowable exposure. Zone includes all fuel elements which will reach allowable exposure before next shutdown.
III*	Fuel reloading for uniformly graded exposure	Fuel elements are replaced in radial zones at a rate proportional to the fuel element power density in that zone. The zones are defined by the number of burning periods a fuel element within that zone can remain in the core. In this program, at steady state, the average exposure of fuel elements within the core is nearly independent of radial position.
IV	Radially graded exposure - new fuel at center	Exposed fuel elements are moved from central positions before reaching the limiting exposure. New fuel elements are loaded into central positions; spent elements are discharged from outer positions.
V	Radially graded exposure - new fuel at periphery	Exposed fuel elements are moved from outer radial positions before reaching the limiting exposure. New fuel elements are loaded into peripheral positions; spent elements are discharged from central positions.

*In order to establish this program starting from the all-new-fuel condition, a major reshuffling of fuel elements is required.

II. DISCUSSION

A. PROGRAMS CONSIDERED

1. Program I - Fuel Reloading by Core

The simplest fuel program conceivable would be to load the core entirely with new fuel elements and operate until the most exposed fuel element has reached some exposure which is regarded as its radiation-damage limit from the viewpoint of fuel element structural integrity. Alternatively, the criterion for shutting down might simply be loss of sufficient core reactivity to operate with adequate control. In either of these cases, upon shutting down the reactor the entire core load of fuel elements is removed and replaced by new fuel elements, and the cycle repeated.

There are obvious reasons why one should be able to improve on this simple program. In the first place, the reactivity of the discharged fuel elements are, on the average, above the most exposed elements. Many, therefore, have further serviceability. To discard them is to use fuel elements inefficiently. Since the nuclear fuel cost items which are proportional to the rate of use of fuel elements (e.g., fuel fabrication, transportation, and reprocessing) form a significant portion of SGR power costs, there is a considerable incentive to use fuel elements efficiently. A typical power distribution for an SGR reactor with new fuel shows a radial peak-to-average power ratio of about 1.75. During exposure there is some flattening due to xenon and samarium poisoning and to faster burning of center fuel elements. Even so, fuel elements would be used in the above simple cycle about 60% faster than in the ideal case where each fuel element is discharged at the irradiation damage or reactivity limit.

2. Program II - Fuel Reloading by Zone

The first modification of Program I is also obvious. When the reactor is shut down to replace fuel the first time, only the centermost elements which are most exposed should be changed. The new elements replace the discharged fuel, and the partially burned elements are not moved. Naturally, all fuel elements which would become "over-exposed" if left in for the next burning period must be removed. For this reason the fuel elements removed are not all at the allowable

exposure limit. Nevertheless, by this type of fuel program, a high average fuel element exposure on discharge can be achieved, usually about 90% of the maximum allowable value. There are, however, serious objections to such a fuel program.

In order to make the discussion more specific, an illustration of Program II is given in Table II. It is based on a fuel element radial power distribution shown in Figure 1. According to the program, no fuel is removed from the reactor, initially loaded with all new fuel, until the end of the third period (each burning period is identical in energy extracted from the core). At this time the centermost fuel is exposed to the allowable limit. All fuel which will reach the

Figure 1. Reference SGR Power Distribution (all new fuel)

allowable limit before the end of the fourth period must also be removed. The boundary of the central zone is, therefore, set at $\phi/\phi_c = 0.75$, and, according to Figure 1, $R/R_c = 0.52$. At the end of the fourth period all fuel which will reach the limit before the end of the fifth period is removed. In this manner, the core is divided into seven regions. The fraction of the fuel elements within each zone is indicated in Table II. Table II also shows the exposure of each region at the beginning and end of each burning period and

the schedule for changing the various zones. It is important to note that this schedule is based on a constant radial power distribution (Figure 1). Since the variation in fuel exposure across the core is continually changing, the radial power distribution is also changing, and the constant distribution is only a first approximation. The magnitude of the change in power distribution change with burning time is discussed below. For the present arguments regarding Program II, Table II is sufficiently accurate.

The principal objection to Program II is that the particular array of fuel element exposures in the core changes every burning period. The result is that the reactor operator will encounter times when reactivity is relatively high and other times when it is low. During some periods the fuel toward the center is

TABLE II

ILLUSTRATION OF PROGRAM II - FUEL RELOADING BY ZONE

Core Zone Numbers		1	2	3	4	5	6	7	% of Core Replaced
% of Total Fuel		27.0	17.9	12.8	9.5	8.4	9.0	15.4	
Burning Period No.	Time	Fuel Element Exposure, Fraction of Allowable							% of Core Replaced
1	End	0.33	0.25	0.20	0.17	0.14	0.12	0.11	
2	End	0.67	0.50	0.40	0.33	0.29	0.25	0.22	0
3	End	1.00	0.75	0.60	0.50	0.43	0.38	0.33	27.0
4	Begin	0.00	0.75	0.60	0.50	0.43	0.38	0.33	17.9
	End	0.33	1.00	0.80	0.67	0.57	0.50	0.45	
5	Begin	0.33	0.00	0.80	0.67	0.57	0.50	0.45	12.8
	End	0.67	0.25	1.00	0.83	0.71	0.62	0.56	
6	Begin	0.67	0.25	0.00	0.83	0.71	0.62	0.56	36.5
	End	1.00	0.50	0.20	1.00	0.86	0.75	0.67	
7	Begin	0.00	0.50	0.20	0.00	0.86	0.75	0.67	8.4
	End	0.33	0.75	0.40	0.17	1.00	0.88	0.78	
8	Begin	0.33	0.75	0.40	0.17	0.00	0.88	0.78	26.9
	End	0.67	1.00	0.60	0.33	0.14	1.00	0.89	
9	Begin	0.67	0.00	0.60	0.33	0.14	0.00	0.89	42.4
	End	1.00	0.75	0.80	0.50	0.29	0.12	1.00	
10	Begin	0.00	0.25	0.80	0.50	0.29	0.12	0.00	12.8
	End	0.33	0.50	1.00	0.67	0.43	0.25	0.11	
11	Begin	0.33	0.50	0.00	0.67	0.43	0.25	0.11	0
	End	0.67	0.75	0.20	0.83	0.57	0.38	0.22	
12	Begin	0.67	0.75	0.20	0.83	0.57	0.38	0.22	54.4
	End	1.00	1.00	0.40	1.00	0.71	0.50	0.33	
13	Begin	0.00	0.00	0.40	0.00	0.71	0.50	0.33	0
	End	0.33	0.25	0.60	0.17	0.86	0.62	0.45

more exposed with a resultant power flattening; during others the converse is the case. For example, at the end of the twelfth period, 54% of the core fuel elements located in the central 67% of fuel positions, are at the exposure limit. This is a time of low reactivity but relatively flat power. On the other hand, at the beginning of the thirteenth period one would expect to find high core reactivity but also a high peak-to-average power. Unfortunately, the reactor designer, in specifying required fuel enrichment, must consider the period of minimum core reactivity. At the same time, in specifying the heat transfer requirements, he must consider the period of maximum peak-to-average power. Moreover, the shim-rod system must have sufficient worth to handle this resulting wide range of reactivity changes. One is led to look for a fuel program which will avoid these reactivity minima and peak-to-average maxima.

Figure 2. Changes in SGR Core Characteristics as Fuel Burnup Proceeds from All-New-Fuel Condition

Figures 2, 3, 4, and 5 show these effects in a more quantitative manner. The variation in reactivity and power distribution as burning proceeds from the all-new-fuel condition is shown in Figures 2 and 3. The discontinuity which would

Figure 3. Changes in SGR Core Radial Power Distribution as Fuel Burnup Proceeds from All-New-Fuel Condition

Figure 4. All-New-Fuel Condition Changes in Core Characteristics under Program II when Central 27% of Fuel is Replaced

occur under Program II, when the center zone of the core was reloaded with new fuel elements, is shown in Figures 4 and 5. As would be expected, the magnitude of the discontinuity is a function of the allowable maximum exposure of the fuel. It can be seen that power-flattening effect which develops as the center fuel is more highly exposed is lost as soon as the central fuel is replaced. In fact, Figure 4 shows that the power peaking becomes worse than in the all-new-fuel case.

Figure 5. Change in Radial Power Distribution when Central 27% of Fuel is Replaced

3. Program III - Fuel Reloading for Uniformly Graded Exposure

It has already been mentioned that any core contains an array of fuel elements of varying exposure. The only exceptions to this are the case of all new fuel elements and the unlikely case of a perfectly uniform power distribution. However, it is possible to program the loading and discharge of fuel elements so that the average exposure of a representative sample of fuel elements in every zone of the reactor is nearly equal to the average exposure in the entire core.¹ Spinrad² discusses such a condition as "uniformly graded exposure" for the situation in which fuel can be changed continually. Even for the present case where fuel must be changed periodically, it is possible to program a good approximation

to this case. Such a program is outlined in Table III. The plan is that fuel elements be replaced in any zone of the core at a rate proportional to the power and the number of fuel positions in that zone. The effect of this program is that, on the average, the fuel in each zone is at 50% of the allowable exposure. Actually, as may be seen from the exposure values in Table III, the fuel in each zone is less than 50% exposed at the beginning of a burning period and over 50% at the end.

The particular program shown in Table III is based on the power distribution given in Figure 1. The core is divided into zones just as for Program II, in Table II. Fuel elements stay in the central zone for three burning periods, in the next zone for four periods, and so on. The choice of a life of three periods for the fuel elements in the center region is arbitrary. In an actual case it is determined by the power density (Mw/T) in the fuel, the plant load factor, the desired interval between shutdowns and the allowable exposure of a fuel element. Since the fuel elements in Zone 2 must remain in the core for 4 periods, they must be burning at a rate no greater than 75% of Zone 1. The zone boundary is then set by reference to Figure 1. The entire core is subdivided in the same way. Within a zone there is a power variation so that the average fuel element is discharged at less than the allowable exposure. The calculation of average exposure in a zone is likewise very simple. In Zone 1, for example, one-third of the elements are discharged and replaced by new elements at the end of each burning period. At the beginning of a burning period, therefore, one-third of the fuel elements are new, one-third are one-third exposed, and one-third are two-thirds exposed. The average exposure for the zone is therefore 0.33. Due to the fact that the average-to-peak exposure within Zone 1 is 0.87, the actual average exposure for the zone is adjusted to 0.29. The average exposure at the end of the burning period is calculated in a similar manner. This procedure is repeated for all zones. These figures apply only after the fuel replacement routine has been established. The period following initial startup of the reactor is discussed later in this section.

The question that must be answered is, "What are the characteristics of the core with the array of fuel elements resulting from Program III compared with the characteristics which result from Program II?" On the average, the fuel element exposure at discharge would be about the same under either program.

TABLE III
ILLUSTRATION OF PROGRAM III
FUEL RELOADING FOR UNIFORMLY GRADED EXPOSURE
 (Radial Power Distribution from Figure 1)

Core Zone Number	1	2	3	4	5	6	7
Fuel element residence (no. of burning periods)	3	4	5	6	7	8	9
Zone peak power (relative to center)	1.00	0.75	0.60	0.50	0.43	0.38	0.33
Zone outer boundary R/R_c	0.52	0.67	0.76	0.82	0.87	0.92	1.0
Fuel elements in zone (% of core)	27.0	17.9	12.8	9.5	8.4	9.0	15.4
Fuel elements replaced during each shutdown (% of core)	9.0	4.5	2.6	1.6	1.2	1.1	1.7
Average exposure of discharged fuel, E/E_o * (also zone average- to-peak power)	0.87	0.89	0.92	0.92	0.93	0.93	0.93
Average exposure of fuel, E/E_o							
Beginning of burning period	0.29	0.33	0.38	0.38	0.40	0.41	0.41
End of burning period	0.58	0.56	0.56	0.59	0.53	0.52	0.52

* E_o refers to the maximum value that the exposure will be permitted to attain.

The fuel element handling is the same, i.e., each fuel element occupies a single lattice position during its entire life. The two characteristics of the reactor which differ under the two programs are the peak-to-average power distribution and the minimum reactivity of the reactor for a given discharge exposure of the fuel. The former determines the design power output of the core assuming the power to be limited by a "hot spot" at or near the point of maximum power generation. The condition of minimum core reactivity is the one that determines the initial enrichment of the fuel, i.e., the initial enrichment or initial reactivity of the fuel must be sufficient to permit the reactor to remain critical and controllable at the condition of minimum core reactivity encountered during the lifetime of the reactor.

On both of these points, peak-to-average power and core reactivity, the superiority of Program III over Program II may be seen almost by inspection. The increase in peak-to-average power that occurs at the time of first fuel change under Program II is shown in Figure 4, and it is seen to be considerably higher than at the initial all-new-fuel condition. Under Program III, however, the fuel exposure tends to be about the same in each region (see Table III) and, therefore, the power distribution will closely resemble that for the all-new-fuel condition. It is true that at the beginning of a burning period under Program III the fuel toward the center is slightly less exposed than that toward the periphery. One would expect an increase in peak-to-average power as a result. Although no calculations have been performed to determine the amount of this increase, it is expected that it would be slight. The effect of less exposed fuel in the central core zones on power distribution is discussed under Program IV.

With regard to core reactivity, a comparison of Tables II and III shows that, under Program II, the core will encounter periods of reactivity considerably less than would be encountered under Program III. In Table II it may be seen that at the end of the 12th burning period the average exposure of the fuel is about 75% of allowable. Table III, on the other hand, indicates that at the minimum reactivity condition, the fuel in Program III has an average exposure of about 55% of allowable. Clearly, a greater initial fuel enrichment would be required to meet the minimum reactivity condition encountered in Program II.

A more detailed comparison of reactivity under Programs II and III is given in Figure 6. The reactivity of the core (relative to its reactivity with all new fuel) is plotted as a function of discharge fuel exposure. For Program II the plot is the same as given in Figure 2 and represents the reactivity of the core at the time fuel is initially discharged (end of burning period 3 in Table II). For Program III, the reactivity is that of the core at the end of a burning period. The core reactivity is higher for Program III although the difference in the two cases is not great. In line with the remarks in the above paragraph, however, it should be noted that in Program III, the core-reactivity, allowable exposure relation is one that holds for as long as the program is maintained; under Program II, subsequent fuel loadings will produce periods of lower reactivity than shown in Figure 6.

Of definite concern in the implementation of Program III is the fact that

Figure 6. Effect of Fuel Program on Core Reactivity

new and old fuel elements occupy adjacent lattice positions. They are, therefore, in the same gross flux of the reactor and the question arises as to their relative heat generation rate. If new fuel elements, with a higher fission cross section, generate heat at a higher rate, adjacent "older" fuel elements would then be operating below maximum heat generation. If these elements are located at the center of the core, there would be a consequent increase in peak-to-average power generation. In the

present case, this effect appeared to be small because the maximum variation of fission cross section of the fuel was about 8% for an exposure range from 0 to 24,000 Mwd/T. (This value follows closely the lattice cell parameter, $\epsilon v \Sigma_f$, plotted in Figure 11.) The magnitude of this effect should be evaluated in each case, however.

A basic difference between the two programs is that Program II, Fuel Reloading by Zone, starts from an all-new-fuel condition, while Program III, Fuel Reloading for Uniformly Graded Exposure, assumes that a uniformly graded array is in existence and then maintains that condition. Since, presumably, a reactor must start on all new fuel elements, it will be necessary to rearrange the fuel

after some burning has occurred in order to get into the "beginning-of-burning-period" situation indicated in Table III. An example of how this might be done is illustrated in Table IV. Starting from the all-new-fuel condition, the reactor is operated for two burning periods as indicated in Table II, illustrating Program II. At that point the average exposure of the fuel in each zone is equalized by distributing the fuel elements from each zone among all zones in proportion to the number of lattice positions in that zone. The required transfers are shown in Table IV. The transfers involve 83% of the core elements. Naturally, compromises will be necessary in reducing the given core fraction values to integral numbers of fuel elements. In Table IV, it is demonstrated that these transfers do result in a uniformly graded exposure in the core. Program III is now initiated, and at the end of the next (third) burning period the fuel replacements indicated in Table III are made. This switching of fuel appears to be the price required to get into Program III; for the gains over Program II, the price is well justified.

4. Program IV - Radially Graded Exposure - (new fuel at center)

The previous work has shown that when more exposed fuel lies toward the center of the core, the power distribution tends to flatten while the core reactivity is somewhat less than if the fuel elements of the same average exposure were distributed uniformly. It is of interest to demonstrate whether a fuel program can be devised which would result in the central core regions always containing fuel of lower average exposure. Such an arrangement would be more reactive and, therefore, require a lower fuel enrichment. On the other hand, it will have a higher peak-to-average power. Some consideration will show that if all fuel elements are to be discharged at or near the allowable exposure, then the maintenance of a lower average fuel exposure in the center will require transfers of fuel from one lattice position to another. New fuel should be loaded near the center of the core and moved toward the periphery when only partially burnt. Spent fuel elements are discharged from the positions at or near the periphery of the core.

There are countless combinations of fuel moves which will maintain a radial exposure gradient. Table V shows one such program, used here for illustrative purposes. Just as in previous cases, the core is divided into zones according to the number of periods a fuel element could remain in that zone before reaching the allowable exposure. The program in Table V requires that, on the average,

TABLE IV

ESTABLISHMENT OF PROGRAM III

At end of second burning period of Program II (see Table II), make the following fuel element transfers:

From Zone No.	To Zone Number							Total Moves
	1	2	3	4	5	6	7	
1	-	4.8*	3.5	2.6	2.3	2.4	4.1	19.7
2	4.8	-	2.3	1.7	1.5	1.6	2.8	14.7
3	3.5	2.3	-	1.2	1.1	1.1	2.0	11.2
4	2.6	1.7	1.2	-	0.8	0.8	1.5	8.6
5	2.3	1.5	1.1	0.8	-	0.8	1.3	7.8
6	2.4	1.6	1.1	0.8	0.8	-	1.4	8.1
7	4.1	2.8	2.0	1.5	1.3	1.4	-	13.1

Results in following average exposure (in sample zones)

ZONE NO. 1				ZONE NO. 7			
Fuel Elements*		E/E_o^{**}		Fuel Elements*		E/E_o^{**}	
7.3	x	0.67	= 4.9	4.1	x	0.67	= 2.75
4.8		0.50	2.4	2.8		0.50	1.4
3.5		0.40	1.4	2.0		0.40	0.8
2.6		0.33	0.86	1.5		0.33	0.5
2.3		0.29	0.67	1.3		0.29	0.38
2.4		0.25	0.69	1.4		0.25	0.35
4.2		0.22	0.92	2.4		0.22	0.53
27.1			11.8	15.5			6.7
$E/E_o = 11.8/27.1 = 0.44^{**}$				$\bar{E}/E_o = 6.7/15.5 = 0.43^{**}$			

* Numbers represent percent of core fuel elements.

** Must be reduced by average-to-peak factor for each zone.

TABLE V
ILLUSTRATION OF PROGRAM IV
RADIALLY GRADED EXPOSURE - NEW FUEL AT CENTER

(Radial Power Distribution from Figure 1)

Zone No.	Zone Peak Power	Zone Boundary R/R_c	Zone Average-to-peak Power	Fuel Elements (% of Core)	Fuel Exposure E/E_o		Fuel Movement			
					Burning Period Beginning	End	New Add.	Transfers In	Out	Discharge
1.0	1.0	0.52	0.87	17	0.00	0.29	17		6	
				11	0.29	0.58			11	
				<u>28</u>	0.11	0.40				
2.0	0.75	0.67	0.89	6	0.00	0.22	6			
				6	0.22	0.44			3	
				3	0.44	0.66				
				3	0.66	0.88				3
				<u>18</u>	0.26	0.48				
3.0	0.60	0.76	0.92	6	0.29	0.47		6		
				6	0.47	0.65			6	
				<u>12</u>	0.38	0.56				
4.0	0.50	0.82	0.92	11	0.58	0.73		11	11	
5.0	0.43	0.87	0.93	3	0.44	0.57		3		
				3	0.57	0.70				
				3	0.70	0.83				3
				<u>9</u>	0.57	0.70				
6.0	0.38	0.92	0.93	6	0.65	0.77		6	6	
7.0	0.33	1.00	0.93	11	0.73	0.83		11		11
				6	0.77	0.87		6		6
				<u>17</u>	0.74	0.84				
		TOTALS		101			<u>23</u>	<u>43</u>	<u>43</u>	<u>23</u>

each fuel element be transferred twice. Since the zoning of the core depends on the power distribution and the power distribution depends on the distribution of fuel exposure in the core, the calculation of a fuel program is an iterative process. An initial exposure distribution was obtained from the power distribution in Figure 1 and the program in Table V. Flux and reactivity calculations were made for such a core distribution. The exposure values used and resulting power distribution are shown in Figure 7 for the case of an allowable fuel exposure of 24,000 Mwd/T. It can be seen that the power peaking is more accentuated than for the reference case (Figure 1). Some change in power distribution takes place during the burning period. In all cases the distribution at the start of a burning period is given since it has the

Figure 7. Effect of Fuel Program IV on Fuel Power Distribution

highest peak-to-average. The core reactivity is shown in Figure 6, and may be seen to be somewhat higher than in other fuel programs. The peak-to-average power is plotted as a function of allowable fuel exposure in Figure 8. As expected, it is seen to increase with increasing allowable exposure. The power distribution shown in Figure 5 is not a final solution since a second iteration using this distribution to estimate fuel exposures should now be performed. However, this effect is small and did not justify the computational effort.

Just as with Program III, some initial shuffling of fuel elements would be required to establish this program. Such a shuffling has been worked out in detail for Program III (Table IV). In the case of Program IV the same procedure could be used. At a shutdown after starting from the all-new-fuel condition, fuel elements are rearranged, with the more exposed fuel being placed toward the core periphery. At subsequent shutdowns the fuel transfer schedule indicated in Table V could be followed.

5. Program V - Radially Graded Exposure (new fuel at periphery)

The final fuel program to be evaluated is that in which a radial exposure gradient is established with the more exposed fuel at the center. From previous

calculations, it is to be expected that such a program will result in a flatter power distribution and a lower core reactivity for a given value of discharge fuel exposure. Two programs are described in Table VI. In Program V-A, the number of fuel element transfers is restricted to 25% of the core each shutdown. The core is divided arbitrarily into two regions, each containing 50% of the lattice positions. Each element stays in the core for four burning periods so that 25% of the core's fuel elements are discharged each shutdown. During a shutdown, those elements which have been in the inner zone for two burning periods are discharged. They are replaced by fuel elements which have been in the outer zone for two periods. These, in turn, are replaced by new fuel elements. It is presumed that in each region the fuel elements of different exposure are "mixed" so that the flux and reactivity are similar during each burning period.

Program V-B is based on a very simple but extensive fuel element position change. The core is divided into four radial regions of equal area. At shutdown all the fuel in the central region is discharged. All the fuel in the second zone is moved into the first, the third into the second, and the fourth into the third. New fuel is placed in the outer zone. Every fuel element is moved every shutdown.

The effect of Program V on power distribution is shown in Figures 9 and 10. It can be seen that a considerable flattening of the radial power distribution results. Again the calculation of the power distribution is an iterative process just as in Program IV. A power distribution is postulated, and the resulting exposure distribution is calculated. Then a new power distribution is calculated and the process repeated until the distribution converges. The power distributions shown in Figures 9 and 10 are not completely converged. Figure 9 represents a second calculation, while Figure 10 represents a first trial in which the exposure is based on a postulated flat power. This fact may be recognized in that the relative exposure at the beginning of the burning period is 0, 25, 50, and 75%, in the successive zones starting from the outside. This array would only result from a

Figure 8. Effect of Fuel Program on Power Distribution

TABLE VI

ILLUSTRATION OF PROGRAM V

RADIALLY GRADED EXPOSURE - NEW FUEL AT PERIPHERY

Zone Number	Program V-A		Program V-B			
	1	2	1	2	3	4
Fuel element residence (no. of burning periods)	2	2	1	1	1	1
Zone outer boundary, R/R_c	0.71	1.00	0.50	0.71	0.87	1.00
Fuel elements in zone (% of core)	50	50	25	25	25	25
Fuel elements discharged each shutdown (% of core)	25	0	25	0	0	0
Fuel element position changes (% of core)	0	25	0	25	25	25
New fuel elements added each shutdown (% of core)	0	25	0	0	0	25
Zone peak-to-average power						
$E_o = 12,000 \text{ Mwd/T}$	1.18	1.30	1.05	1.11	1.14	1.14
$E_o = 18,000 \text{ Mwd/T}$	1.11	1.24	1.01	1.03	1.11	1.12
Average exposure of discharged fuel, E/E_o						
$E_o = 12,000 \text{ Mwd/T}$	0.81	-	0.90	-	-	-
$E_o = 18,000 \text{ Mwd/T}$	0.86	-	0.94	-	-	-
<u>Average Exposure of Fuel, $\bar{E}/E_o$</u>						
Beginning of burning period	0.44	0.08	0.75	0.50	0.25	0.00
End of burning period	0.69	0.24	1.00	0.75	0.50	0.25

flat power distribution. The final power distribution should be somewhat less flat than the value shown. In the case of Program V-A the difference between the final distribution and the distribution given in Figure 9 is slight.

The power distribution, expressed as peak-to-average power, is given as a function of allowable fuel exposure for Program V in Figure 8. The values are based on Program V-A. Program V-B should result in somewhat lower peak-to-average power values, but considerably more fuel handling is involved. The core reactivity at the end of the burning period is given in Figure 6. Program V is seen to result in a lower reactivity relative to the all-new-fuel condition than either Program III or IV. The reason for this is the concentration of exposed fuel in the center.

B. NUCLEAR CALCULATIONS

The reactivity and power distribution are based on the two-group equations of the form

$$D_1 \nabla^2 \phi_1 - \frac{D_1}{\tau} \phi_1 = \epsilon v \Sigma_{f2} \phi_2$$

$$D_2 \nabla^2 \phi_2 - \Sigma_{a2} \phi_2 = p \frac{D_1}{\tau} \phi_1$$

where the subscripts 1 and 2 refer to the fast and thermal groups, respectively. The other terms have their usual meaning:

Figure 9. Effect of Fuel Program V-A on Fuel Power Distribution

Figure 10. Effect of Fuel Program V-B on Fuel Power Distribution

- D = Diffusion coefficient
- p = Resonance escape probability
- ϵ = Fast fission factor
- τ = Fermi age
- ν = Neutrons per fission
- Σ_{a2} = Absorption cross section for thermal group
- Σ_{f2} = Fission cross section for thermal group

Only the terms Σ_{a2} and $\epsilon\nu\Sigma_{f2}$ were considered to vary with fuel isotopic composition. The estimation of these lattice constants for exposed SGR fuel has been described¹. Briefly, the isotopic change differential equations are solved for a range of fuel exposure. The resulting effects on intracell flux and the lattice parameters Σ_{a2} and $\epsilon\nu\Sigma_{f2}$ are then calculated. The particular values used in this study are plotted in Figure 11 as a function of exposure. It is then postulated that an SGR core from which energy has been extracted may be represented as a several-radial-region core, each region having a set of lattice constants corresponding to the exposure of the fuel in that region. When a fuel program requires that a single radial region contains fuel elements of widely different exposures, it is assumed that a weighted-average set of lattice constants would give a reasonable approximation of the reactivity and gross flux distribution in the core. Multiregion, one-dimensional radial solutions of the above two-group equations were obtained by use of the WANDA code³.

Since the variation in exposure of fuel with position is a function of flux distribution, which is in turn a function of the region lattice constants, an iterative calculation is required by this technique. The calculation of power

Figure 11. Variation of Lattice Parameter Values with Fuel Exposure

distributions for Programs IV and V require this iterative calculation. A power distribution is assumed, the exposure of fuel throughout the core calculated, and then a new power distribution calculated for a particular array of exposed fuel as required by the program. In the case of a core starting from the all-new-fuel condition, in which there is a single value of fuel exposure at any radial position, the CANDLE code⁴ performs sequential flux-criticality and isotope change calculations. This code was used to obtain flux distributions and reactivity in Program II.

All results presented here have been based on one-dimensional radial calculations. The resulting values of reactivity and power distribution do not apply in an absolute sense to a core in which fuel exposure varies in the axial direction. Two-dimensional burnup and criticality calculations would be required to obtain such values. For the purpose of comparison of the different fuel programs, the much simpler one-dimensional calculations are regarded as adequate.

C. SUMMARY AND CONCLUSIONS

Various schemes or programs for fueling SGR reactors have been considered and the relative effects of each on fuel enrichment, control requirements, core power distribution, and fuel element exposure at discharge have been estimated. With one exception, the programs have been cyclic or repetitive in nature. That is, the reactor is operated on a repetitive burning period - shutdown cycle, and the fuel replacement and fuel transfers within the core are nearly identical from one cycle to the next. The single exception has been Program II, Fuel Reloading by Zones, in which the fuel is replaced in radial zones as the fuel in that zone approaches limiting exposure. The information in this report indicates that this latter type of program is undesirable and that the reactor operator should seek to place the reactor on a repetitive program. Briefly, the argument against the zone reloading is that during the history of the reactor operation, periods of poor power distribution, and low reactivity will be encountered. These conditions must be considered in establishing design criteria, and they place an unnecessarily severe requirement on fuel enrichment, control system reactivity capacity, and fuel power density. The repetitive fuel program, on the other hand, avoids the periods of poorest power distribution and lowest reactivity and, therefore, relaxes certain design requirements. The repetitive program will require some additional

fuel handling in order to establish the program and may require additional fuel handling on a continuing basis. The additional fuel handling is that associated with switching the lattice positions of partially spent fuel.

A qualitative comparison of the various fuel programs is summarized in Table VII. As would be expected, each program gains an advantage in some features at a sacrifice in others. The weighing of the advantages and sacrifices can only be done by a designer or operator for specific cases where fuel handling costs, fuel processing costs, etc., can be estimated. However, certain generalizations regarding the effects of different fuel programs appear justified. These are discussed below.

1. Shim-Rod Worth Required By Fuel Burnup

One of the chief advantages of fuel enrichment is that long fuel reactivity life can be achieved, with an attendant reduction of fuel fabrication and processing costs. Uranium dioxide or carbide fuel elements appear to have the necessary resistance to irradiation damage to permit the realization of long fuel life. However, the very enrichment which makes long reactivity-lifetimes possible creates a considerable problem of high reactivity in the case of a core loaded with all new fuel. In the case of SGR reactors with uranium dioxide fuel, it appears that this initial reactivity will be high. The initial reactivity is disadvantageous from several points of view: safety, the high number of shim-control rods with their drives and ancillary equipment, the number of lattice positions preempted by these rods, and the distortion of the power distribution by a "high worth" shim rod bank. One way of meeting this problem is to include a burnable poison in the fuel. This expedient has the disadvantage of reducing neutron economy and of introducing a new quality control requirement to fuel production. At least a partial answer to this problem appears to lie in the judicious selection of a fuel program. This point is illustrated by the schematic diagram of the reactivity history of the core in Figure 12. After a repetitive program is established, the reactivity of the core is bounded between the same limits each burning period. In a non-repetitive program such as Program II, the reactivity ranges between considerably wider limits. The result is that for the latter case, more shim-control rods are required.

TABLE VII
SUMMARY COMPARISON OF FUEL PROGRAMS

Program		I	II	III	IV	V
		Reloading by Core	Reloading by Zone	Uniformly Graded Exposure	Radially Graded Exposure (new fuel at center)	Radially Graded Exposure (new fuel at periphery)
Feature	Rating No. 1 Indicates					
Shim-rod worth required by fuel burnup	Lowest	4	5	2	3	1
Radial power distribution	Flattest	2	5	3	4	1
Average exposure of discharged fuel (relative to maximum allow- able)	Highest	3	1	2	2	2
Fuel handling	Lowest	2	1	1*	3	3
Fuel enrichment	Lowest	4	5	2	1	3

*Neglects nonrepetitive fuel handling in connection with establishment of program.

Figure 12. Effect of Fuel Program on Reactivity Changes

What is suggested here is that design specifications regarding number and worth of control rods, drives, etc., be established on the basis of an established repetitive fuel program. The all-new-fuel case, since it would occur only once, could be treated by some special expedient which would not detract from reactor performance during its entire life. No study of possible expedients has been made in this work. One possibility, however, would be to substitute dummy plugs for fuel elements in the center of the core. These plugs might have to have some parasitic absorption in order to prevent flux peaking. In fact, they might simply be the poison column of a control rod, but without any drive mechanism. They could only be removed, like the fuel elements, during a shutdown. The heat transfer deficiency resulting from omission of fuel elements from these lattice positions could be made up either by addition of elements at the core periphery where reactivity worth is low, or by making the absorption cross section and position of the dummy elements such as to achieve radial power flattening. These dummy elements could be removed at the time of establishment of the repetitive fuel program.

In Table VII, a distinction is made between the Programs III, IV, and V with respect to shim-rod worth requirement. This distinction relates to the fact that the reactivity change for a given energy extraction from a core is related to the power distribution. The higher the peak-to-average power, the more the negative effects of energy extraction (e.g., fission products, U^{235} and burnup) will be concentrated in the center and, therefore, the greater the reactivity drop during a given energy extraction. For this reason, Program V, with the flattest radial power distribution, has the lowest reactivity change during a burning period. The differences between these repetitive programs are small, however, compared to the difference between the repetitive programs and Program II.

2. Radial Power Distribution in the Core

The results of this work show that the radial power distribution in the core is significantly influenced by the fuel program. Briefly, it appears that Program III, Fuel Reloading for Uniformly Graded Exposure, will give a power distribution roughly equal to the all-new-fuel case throughout the reactor life. The requirement that fuel elements of different exposure occupy adjacent lattice positions in the center of the core will result in some increase in peak-to-average power due to different heat generation rates. As previously discussed, this effect should be small. Program II, Fuel Reloading by Zones, will result in periods both of a flatter distribution and more peaked distribution than the all-new-fuel case. Program IV, Radially Graded Exposure (new fuel at center), results in a higher peak-to-average power as indicated in Figure 7. Program V, Radially Graded Exposure (new fuel at periphery), results in power flattening shown in Figure 9. In cases where the power output of the core is limited by a hot spot at or near the peak power density, the influence of fuel program on power distribution is important to the designer. Generally, the basis for meeting the heat power requirements of the core is the all-new-fuel condition. It is important to know whether a more peaked distribution than that will exist during the reactor life. The ratings in Table VII are made on the basis of the "worst" condition to be encountered.

3. Average Exposure of Discharged Fuel

One of the primary objectives of fuel programming is to obtain exposures of discharged fuel as close to allowable as possible. Fuel elements are so expensive that it is costly to use them inefficiently. The sole criterion for discharge and replacement of fuel in Program II is to maximize discharged fuel exposure subject only to the restriction of an allowable exposure and required burning period time. It is to be expected that this program would result in highest discharge fuel exposure. It appears that the average exposure of discharged fuel will be at least 90% of allowable under Program II. In the repetitive fuel programs (Programs III, IV and V) the objective of controlling the fuel exposure distribution and power distribution in the core may require some compromise on discharged fuel exposure. These programs do not differ greatly from Program II in this regard. The illustration of Program III in Table III shows an average discharged fuel exposure of almost 90% of allowable. In the illustrations of Programs IV

and V that are shown in Tables V and VI, respectively, the discharged fuel exposures work out to about 85% of allowable. Insufficient study of this matter has been made to tell whether this difference is inherent in these programs. To some extent, the number of fuel transfers between lattice positions influences the discharged fuel exposure. It appears that the differences in discharged-fuel exposure, if real, are small enough that they would not play an important part in selection of a fuel program.

4. Fuel Handling

Fuel handling in this context refers to the operations of moving new fuel from storage to the core, moving spent fuel from the core to hot storage, and moving partially spent fuel from one lattice position to another. Each of the repetitive programs involves placing fuel elements of different exposure within the same radial zone in the reactor, even in adjacent lattice positions. This situation is not one that will come about as burning proceeds from the all-new-fuel condition without transfers of fuel elements between lattice positions. The fuel handling for the repetitive programs breaks down into the moves necessary to establish a program (probably a one-time job) and the moves necessary each shutdown to maintain the program, a repetitive job. The moves necessary to establish Program III are illustrated in Table IV. It can be seen that it would be necessary to change the positions of 83% of the fuel in the core during that shutdown. There are undoubtedly many other ways in which a program could be established.

As for the fuel handling necessary to maintain a program, Programs II and III are identical, except that in Program II the number of elements replaced changes each shutdown (Table II). In both of these programs a fuel element occupies the same lattice position for its entire life in the core. Since the average exposure of discharged fuel is about the same in both programs, the number of fuel element moves must, on the average, be the same. In Programs IV and V a number of fuel-element lattice position changes is required each shutdown in order to maintain the program. For the illustration of Program IV given in Table V, 43% of the core fuel elements must be transferred each shutdown, in addition to replacement of 23%. In the illustrations of Program V given in Table VI, fuel element transfers of 25% and 75% of the core are required for Programs V-A and V-B, respectively. The relation between the number of fuel element position changes and

the degree of attainment of the program objective (e.g., reactivity conservation and power flattening) is complex and has hardly been touched here. It is a matter for optimization in a specific case.

In any case, the maintenance of a repetitive program with a radial exposure gradient (i.e., Programs IV or V) will require some fuel element position changes, and the expense accruing to this incremental fuel handling must be offset by some advantage gained, either reactivity, power flattening or shorter residence time of the fuel elements in the core.

5. Fuel Enrichment Requirements

One possible advantage to be secured by adjusting the array of exposed fuel in a core is to increase the minimum reactivity that a core would encounter and, therefore, decrease the enrichment requirements of the fuel. One would expect that maximum reactivity from a given core load of fuel elements would be obtained by placing the most reactive (newest) fuel in the central positions and the least reactive at periphery. Program IV is designed to effect such a distribution. The resulting minimum reactivity of the core as a function of allowable fuel exposure is shown in Figure 6. The gain in reactivity by this program over Programs III and V may be seen. The gain computed here seems to be rather small for a designer to consider this program favorably in view of the increase in fuel handling, and increase in peak-to-average power. Likewise the cost in reactivity for maintenance of Program V, with the newer fuel at the core periphery, does not seem large in view of the power flattening that this program effects. These choices, however, must be evaluated on the basis of similar calculations for a specific case.

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