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POI - A COMPUTER PROGRAM IN FORTRAN IV

(Title Unclassified)

NON-NERVA

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POI - A Computer Program in FORTRAN IV

ABSTRACT

It is assumed that events occur randomly in time, e.g. emission by a radioactive source. During any time interval Δt the average number of events is $\langle B \rangle$. The probability, $P_{\langle B \rangle}(T)$, that T events will occur in a time interval Δt is:

$$P_{\langle B \rangle}(T) = \frac{e^{-\langle B \rangle}}{T!} \langle B \rangle^T,$$

this is the Poisson Formula. In this report a program is described that allows one to calculate: a) the probability, $P_{\langle B \rangle}(\geq T)$, that T or more than T events will occur during a time interval Δt , b) the probability, $P_{\langle B \rangle}(\leq T)$, that T or less than T events occur during a time interval Δt , and c) the probability, $P_{\langle B \rangle}(T)$, that T events will occur during a time interval Δt .

I. STATISTICAL ANALYSIS

Suppose the average number of events during a time interval Δt is $\langle B \rangle$. The probability, $P_{\langle B \rangle}(\geq T)$, that T or more than T events will occur during a time interval Δt is

$$P_{\langle B \rangle}(\geq T) = \sum_{m=T}^{\infty} P_{\langle B \rangle}(m) = \sum_{m=T}^{\infty} \frac{e^{-\langle B \rangle}}{m!} \langle B \rangle^m \quad (1)$$

or

$$P_{\langle B \rangle}(\geq T) = e^{-\langle B \rangle} \left\{ \frac{\langle B \rangle^T}{T!} + \frac{\langle B \rangle^{T+1}}{(T+1)!} + \frac{\langle B \rangle^{T+2}}{(T+2)!} + \frac{\langle B \rangle^{T+3}}{(T+3)!} + \dots \right\}, \quad (2)$$

or

$$P_{\langle B \rangle}(\geq T) = \frac{e^{-\langle B \rangle}}{T!} \langle B \rangle^T \left\{ 1 + \frac{\langle B \rangle}{T+1} + \frac{\langle B \rangle^2}{(T+1)(T+2)} + \frac{\langle B \rangle^3}{(T+1)(T+2)(T+3)} + \dots \right\} \quad (3)$$

It is assumed that $B < T$ and Equation 3 has been solved numerically* by truncating the series, i.e. we have solved

$$P_{\langle B \rangle}(\geq T) = \frac{e^{-\langle B \rangle}}{T!} \langle B \rangle^T \left\{ 1 + \frac{\langle B \rangle}{T+1} + \dots + \frac{\langle B \rangle^n}{(T+1)(T+2)\dots(T+n)} \right\} \quad (4)$$

where the n th term in the brackets is the first term that is equal to or less than ϵ which is an input quantity in the computer program. By choosing ϵ small enough we can calculate the probability as accurately as desired. For the results presented in this report $\epsilon = 10^{-12}$. Since all of the terms in the series are positive, Equation 4 will yield a probability slightly less than the true probability. Comparing probabilities that are calculated with $\epsilon = 10^{-3}$

*The computer program is described in Appendix I.

and 10^{-5} the values agree to 3 significant figures; with $\epsilon = 10^{-5}$ and 10^{-7} the values agree to 5 places; with $\epsilon = 10^{-7}$ and 10^{-9} the values agree to 7 places; and with $\epsilon = 10^{-9}$ and 10^{-12} the values agree to 9 places which is the number of figures the main program prints out.

A probability, $P_{\langle B \rangle}(\geq T)$, which is greater than the true probability is given by

$$P_{\langle B \rangle}(\geq T) \equiv \frac{e^{-\langle B \rangle}}{T!} \langle B \rangle^T \left\{ \frac{1}{1 - \frac{\langle B \rangle}{T}} \right\} \quad (5)$$

Equation 5 is obtained from Equation 3 by letting

$$\left\{ 1 + \frac{\langle B \rangle}{T+1} + \frac{\langle B \rangle^2}{(T+1)(T+2)} + \dots \right\} \rightarrow \left\{ 1 + \frac{\langle B \rangle}{T} + \left(\frac{\langle B \rangle}{T} \right)^2 + \dots \right\}$$

Hence Equations 4 and 5 bound the true probability (remember $\langle B \rangle < T$).

Both Equations 4 and 5 are calculated with the computer program. In addition to ϵ one inputs a set of $\langle B \rangle$ values and a set of T 's. If one uses the main program that is given with the POI subroutine, it will solve Equations 4 and 5 for all combinations of $\langle B \rangle$ and T 's in these sets. Some results are presented in Table I.

It can be seen from Table I that Equation 5 is quite a good approximation for many of the values of $\langle B \rangle$ and T that have been considered. For example when $\langle B \rangle = 100$ and $T = 130$, Equation 5 gives a probability that is 9% greater than that calculated by Equation 4 with $\epsilon = 10^{-12}$. As $\frac{T}{\langle B \rangle}$ increases the agreement improves; when $\frac{T}{\langle B \rangle} = 1.4$ and $\langle B \rangle = 100$, $P_{\langle B \rangle}(\geq T)$ is 5% greater than $P_{\langle B \rangle}(\geq T)$, and when $\frac{T}{\langle B \rangle} = 2$, and $\langle B \rangle = 100$, $P_{\langle B \rangle}(\geq T)$ is only

1% greater than $P_{\langle B \rangle}(\geq T)$. Also as $\langle B \rangle$ increases the agreement improves; when $\frac{T}{\langle B \rangle} = 1.4$ and $\langle B \rangle = 1000$: $P_{\langle B \rangle}(\geq T)$ is 0.6% greater than $P_{\langle B \rangle}(\geq T)$.

Equation 5 is much easier to solve of course than Equation 4 and can often provide a quick estimate of the true probability. It is also more convenient for mathematical manipulation.

The probability $P_{\langle B \rangle}(\leq T)$ that T or less than T events will occur is⁺

$$P_{\langle B \rangle}(\leq T) = \sum_{m=0}^T P_{\langle B \rangle}(m) = \sum_{m=0}^T \frac{e^{-\langle B \rangle}}{m!} \langle B \rangle^m \quad (6)$$

or
$$P_{\langle B \rangle}(\leq T) = e^{-\langle B \rangle} \left\{ 1 + \langle B \rangle + \frac{\langle B \rangle^2}{2!} + \frac{\langle B \rangle^3}{3!} + \frac{\langle B \rangle^4}{4!} + \dots + \frac{\langle B \rangle^T}{T!} \right\} \quad (7)$$

or
$$P_{\langle B \rangle}(\leq T) = \frac{e^{-\langle B \rangle}}{T!} \langle B \rangle^T \left\{ 1 + \frac{T}{\langle B \rangle} + \frac{T(T-1)}{\langle B \rangle^2} + \frac{T(T-1)(T-2)}{\langle B \rangle^3} + \dots + \frac{T!}{\langle B \rangle^T} \right\} \quad (8)$$

One could, of course, truncate this series (for $T < \langle B \rangle$) by including only those terms whose values are greater than some ϵ , however, the values of T in which we are interested are sufficiently small that all terms in the series are included in the calculation. This would be true for most applications, e.g. even if T were as great as 10^6 it would not be too great a task for a high-speed computer (e.g. the CDC 6600) to make the enumeration. A probability, $P_{\langle B \rangle}(\leq T)$, greater than the true probability is given by

⁺For the input and also in the printouts $\langle D \rangle$ is used rather than $\langle B \rangle$ to represent the average number of events whenever finding the probability that T or less than T events occur.

$$P_{\langle B \rangle}(\leq T) \equiv \frac{e^{-\langle B \rangle}}{T!} \langle B \rangle^T \left\{ \frac{1}{1 - \frac{T}{\langle B \rangle}} \right\} \text{ for } T < \langle B \rangle. \quad (9)$$

Table II is a printout of some results obtained by solving Equations 8 and 9. Sets of values of $\langle B \rangle$ and T are input, and Equations 8 and 9 are solved for all combinations. This is only true if one uses the main program provided with the subroutine POI; for each entry, to the subroutine POI, of any $\langle B \rangle$ and T it will return with $P_{\langle B \rangle}(\leq T)$ and $P_{\langle B \rangle}(\geq T)$. In Table II one can see to what extent Equation 9 is a good approximation to Equation 8 for the values of $\langle B \rangle$ and T that have been examined.

As already pointed out, Equation 4 is used only for $T > \langle B \rangle$. However one can obtain $P_{\langle B \rangle}(\geq T)$ where $\langle B \rangle \geq T$ by obtaining

$$P_{\langle B \rangle}(\geq T) = 1 - P_{\langle B \rangle}(\leq T - 1). \quad \langle B \rangle > T \quad (10)$$

Also

$$P_{\langle B \rangle}(\leq T) = 1 - P_{\langle B \rangle}(\geq T + 1) \quad \langle B \rangle < T \quad (11)$$

One can obviously also calculate the probability of obtaining from x to y counts, etc.

The program will also make a direct calculation using the Poisson Formula. It uses Equation 12 below for this type of calculation.

$$P_{\langle B \rangle}(T) = \frac{e^{-\langle B \rangle}}{T!} \langle B \rangle^T. \quad (12)$$

$\langle B \rangle$ and T may have any reasonable values, i.e. $\langle B \rangle$ does not have to be less than T . Examples of this calculation are presented in Table III.

The CDC 6600 computer time required to obtain the results given in Table I, II, and II was 14 seconds.

The input instructions for the computer program are given in Appendix II, and the output is described in Appendix III. The program listing is presented in Appendix IV*.

*The subroutine FALOG was programmed by L. L. Moran.

EPS= 1.0000000E-12

NP=0

Table I

R	T	T/R	NO. TERMS	N TH TERM	P-SERIES	P-CALCULATE	P(S)/P(C)	P(S)**2
10	11	1.1000	32	2.84104101E-13	4.16960250E-01	1.25110036E+00	.33327482	1.73855850E-01
10	12	1.2000	31	3.40924921E-13	3.03223854E-01	5.68681981E-01	.53320461	9.19447055E-02
10	13	1.3000	30	4.43202398E-13	2.08443524E-01	3.15934434E-01	.65976830	4.34487025E-02
10	14	1.4000	29	6.20483357E-13	1.35535577E-01	1.82269866E-01	.74359838	1.83698927E-02
10	15	1.5000	28	9.30725035E-13	8.34584729E-02	1.04154209E-01	.80129717	6.96531670E-03
10	16	1.6000	28	3.46316292E-13	4.87404033E-02	5.78634494E-02	.84233491	2.37562691E-03
10	17	1.7000	27	5.88737697E-13	2.70416098E-02	3.09982765E-02	.87235849	7.31248660E-04
10	18	1.8000	27	2.40847239E-13	1.42776136E-02	1.59549952E-02	.89486793	2.03850250E-04
10	19	1.9000	26	4.57609755E-13	7.18650460E-03	7.87900999E-03	.91210756	5.16458484E-05
10	20	2.0000	25	9.15219510E-13	3.45434198E-03	3.73216263E-03	.92556041	1.19324785E-05
10	25	2.5000	23	5.99760870E-13	4.69493814E-05	4.87818483E-05	.96243548	2.20424442E-09
10	30	3.0000	21	8.72137986E-13	2.50995120E-07	2.56735760E-07	.97763989	6.29985503E-14
10	40	4.0000	19	3.47115079E-13	7.34163631E-13	7.41905942E-13	.98956430	5.38996238E-25
10	50	5.0000	17	5.58728318E-13	1.85472688E-19	1.86590841E-19	.99400746	3.44001181E-38
10	100	10.0000	13	4.72655818E-13	5.39858973E-63	5.40516576E-63	.99878338	2.91447711E-125
100	110	1.1000	75	7.12733783E-13	1.70559898E-01	2.57648044E-01	.66198794	2.90906788E-02
100	120	1.2000	67	8.72406463E-13	2.82303940E-02	3.33663893E-02	.84607279	7.96955143E-04
100	130	1.3000	61	6.68041315E-13	2.28209305E-03	2.49276163E-03	.91548788	5.20794871E-06
100	140	1.4000	56	5.19387997E-13	9.16465143E-05	9.67185697E-05	.94755862	8.39908358E-09
100	150	1.5000	51	7.24443918E-13	1.88421047E-06	1.95334814E-06	.96460555	3.55024908E-12
100	160	1.6000	47	8.41923995E-13	2.05069754E-08	2.10408995E-08	.97462447	4.20536041E-16
100	170	1.7000	44	7.19782793E-13	1.22116958E-10	1.24486053E-10	.98096900	1.49125515E-20
100	180	1.8000	41	8.79633747E-13	4.10485158E-13	4.16641862E-13	.98522303	1.68498065E-25
100	190	1.9000	39	6.57159062E-13	8.01716461E-16	8.11284292E-16	.98820656	6.42749283E-31
100	200	2.0000	37	6.27269802E-13	9.34315007E-19	9.43394121E-19	.99037612	8.72944533E-37
100	250	2.5000	30	5.39699952E-13	1.90948894E-36	1.91784791E-36	.99564173	3.64614983E-72
100	300	3.0000	26	4.11409553E-13	1.81872182E-58	1.82322398E-58	.99753066	3.30774905E-116
100	400	4.0000	21	5.42742538E-13	7.73743012E-113	7.74597995E-113	.99889622	5.98678248E-225
100	500	5.0000	18	9.68589354E-13	3.80875423E-178	3.81112583E-178	.99937772	0.
100	1000	10.0000	13	9.25263226E-13	0.	0.	0.00000000	0.
1000	1100	1.1000	167	8.18936223E-13	9.62630406E-04	1.04488387E-03	.92127981	9.26657298E-07
1000	1400	1.4000	78	6.81253267E-13	5.10227996E-33	5.13370670E-33	.99387835	2.60332608E-65
1000	1800	1.8000	47	9.97388070E-13	1.86139028E-114	1.86428610E-114	.99844669	3.46477379E-228
1000	2000	2.0000	41	6.05242371E-13	3.05819208E-170	3.06124116E-170	.99900397	0.

EPS= 0.

NP=1

Table II

D	T	D/T	NO. TERMS	N TH TERM	P-SERIFS	P-CALCULATE	P(S)/P(C)	2*P(S)
10	15	.6667	16	1.30767437E-03	9.51259597E-01	0.	0.00000000	1.90251919E+00
10	14	.7143	15	8.71782912E-04	9.16541527E-01	0.	0.00000000	1.83308305E+00
10	13	.7692	14	6.22702080E-04	8.64464423E-01	0.	0.00000000	1.72892885E+00
10	12	.8333	13	4.79001600E-04	7.91556476E-01	0.	0.00000000	1.58311295E+00
10	11	.9091	12	3.99168000E-04	6.96776146E-01	0.	0.00000000	1.39355229E+00
10	10	1.0000	11	3.62880000E-04	5.83039750E-01	0.	0.00000000	1.16607950E+00
10	9	1.1111	10	3.62880000E-04	4.57929714E-01	1.25110036E+00	.36602157	9.15859429E-01
10	8	1.2500	9	4.03200000E-04	3.32819679E-01	5.62995161E-01	.59115904	6.65639358E-01
10	7	1.4286	8	5.04000000E-04	2.20220647E-01	3.00264086E-01	.73342320	4.40441293E-01
10	6	1.6667	7	7.20000000E-04	1.30141421E-01	1.57638645E-01	.82556800	2.60282842E-01
10	5	2.0000	6	1.20000000E-03	6.70859629E-02	7.56665496E-02	.88660000	1.34171926E-01
10	4	2.5000	5	2.40000000E-03	2.92526881E-02	3.15277290E-02	.92784000	5.85053762E-02
10	3	3.3333	4	6.00000000E-03	1.03360507E-02	1.08095071E-02	.95620000	2.06721014E-02
10	2	5.0000	3	2.00000000E-02	2.76939572E-03	2.83749561E-03	.97600000	5.53879143E-03
10	1	10.0000	3	0.	4.99399227E-04	5.04443664E-04	.99000000	9.98798455E-04
100	150	.6667	151	5.71338396E-38	9.99998767E-01	0.	0.00000000	1.99999753E+00
100	140	.7143	141	1.34620125E-39	9.99935987E-01	0.	0.00000000	1.99987197E+00
100	130	.7692	131	6.46685549E-41	9.98293160E-01	0.	0.00000000	1.99658632E+00
100	120	.8333	121	6.68950291E-42	9.77330671E-01	0.	0.00000000	1.95466134E+00
100	110	.9091	111	1.58824554E-42	8.52862652E-01	0.	0.00000000	1.70572530E+00
100	100	1.0000	101	9.33262154E-43	5.26562199E-01	0.	0.00000000	1.05312440E+00
100	90	1.1111	91	1.48571596E-42	1.71385119E-01	2.50389446E-01	.68447421	3.42770239E-01
100	80	1.2500	81	7.15694570E-42	2.26491766E-02	2.59892706E-02	.87148181	4.52983533E-02
100	70	1.4286	71	1.19785717E-40	9.71444028E-04	1.03520299E-03	.93840921	1.94288806E-03
100	60	1.6667	61	8.32098711E-39	1.08122182E-05	1.11767869E-05	.96738162	2.16244363E-05
100	50	2.0000	51	3.04140932E-36	2.40159224E-08	2.44628433E-08	.98173062	4.80318447E-08
100	40	2.5000	41	8.15915283E-33	7.51964669E-12	7.59898332E-12	.98955957	1.50392934E-11
100	30	3.3333	31	2.65252860E-28	1.99179001E-16	2.00352006E-16	.99414528	3.98358002E-16
100	20	5.0000	21	2.43290201E-22	1.90555874E-22	1.91133673E-22	.99697699	3.81111748E-22
100	10	10.0000	11	3.62880000E-14	1.13768795E-30	1.13905912E-30	.99879622	2.27537590E-30
1000	1500	.6667	1501	0.	1.00000000E+00	0.	0.00000000	2.00000000E+00
1000	1000	1.0000	1001	0.	5.08409367E-01	0.	0.00000000	1.01681873E+00
1000	500	2.0000	501	0.	8.30383407E-69	8.32031095E-69	.99801968	1.66076681E-68
1000	100	10.0000	101	9.33262154E-143	6.04252493E-293	6.04326909E-293	.99987686	1.20850499E-292
500	1500	.3333	1501	1.68779197E+66	1.00000000E+00	0.	0.00000000	2.00000000E+00
500	1000	.5000	1001	4.31161413E+132	1.00000000E+00	0.	0.00000000	2.00000000E+00
500	500	1.0000	501	3.99398443E-216	5.11891122E-01	0.	0.00000000	1.02378224E+00
500	100	5.0000	101	1.18305033E-112	7.52308846E-106	7.52776131E-106	.99937925	1.50461769E-105

EPS= 0.

NP=2

Table III

R	T	T/R	NO. TERMS	N TH TERM	P-SERIES	P-CALCULATE	P(S)/P(C)	P(S)**2
100	2000	20.0000	1	1.00000000E+00	0.	0.	0.00000000	0.
100	1500	15.0000	1	1.00000000E+00	0.	0.	0.00000000	0.
100	1000	10.0000	1	1.00000000E+00	0.	0.	0.00000000	0.
100	800	8.0000	1	1.00000000E+00	0.	0.	0.00000000	0.
100	400	4.0000	1	1.00000000E+00	5.80948496-113	5.80948496-113	1.00000000	3.37501155-225
100	200	2.0000	1	1.00000000E+00	4.71697060E-19	4.71697060E-19	1.00000000	2.22498117E-37
100	110	1.1000	1	1.00000000E+00	2.34225495E-02	2.34225495E-02	1.00000000	5.48615823E-04
100	100	1.0000	1	1.00000000E+00	3.98609968E-02	3.98609968E-02	1.00000000	1.58889907E-03
100	90	.9000	1	1.00000000E+00	2.50389446E-02	2.50389446E-02	1.00000000	6.26948748E-04
100	60	.6000	1	1.00000000E+00	4.47071474E-06	4.47071474E-06	1.00000000	1.99872903E-11
100	10	.1000	1	1.00000000E+00	1.02515321E-30	1.02515321E-30	1.00000000	1.05093911E-60
100	1	.0100	1	1.00000000E+00	3.72007598E-42	3.72007598E-42	1.00000000	1.38389653E-83
500	2000	4.0000	1	1.00000000E+00	0.	0.	0.00000000	0.
500	1500	3.0000	1	1.00000000E+00	4.22124085-284	4.22124085-284	1.00000000	0.
500	1000	2.0000	1	1.00000000E+00	1.65241513E-86	1.65241513E-86	1.00000000	2.73047575-172
500	800	1.6000	1	1.00000000E+00	1.38572892E-35	1.38572892E-35	1.00000000	1.92024463E-70
500	400	.8000	1	1.00000000E+00	4.30870346E-07	4.30870346E-07	1.00000000	1.85649255E-13
500	200	.4000	1	1.00000000E+00	5.62174672E-53	5.62174672E-53	1.00000000	3.16040362-105
500	110	.2200	1	1.00000000E+00	3.45574653E-99	3.45574653E-99	1.00000000	1.19421841-197
500	100	.2000	1	1.00000000E+00	6.02220905-106	6.02220905-106	1.00000000	3.62670019-211
500	90	.1800	1	1.00000000E+00	3.87367917-113	3.87367917-113	1.00000000	1.50053903-225
500	60	.1200	1	1.00000000E+00	7.42650468-138	7.42650468-138	1.00000000	5.51529718-275
500	10	.0200	1	1.00000000E+00	1.91732643-197	1.91732643-197	1.00000000	0.
500	1	.0020	1	1.00000000E+00	3.56228820-215	3.56228820-215	1.00000000	0.
1000	2000	2.0000	1	1.00000000E+00	1.53062058-170	1.53062058-170	1.00000000	0.
1000	1500	1.5000	1	1.00000000E+00	1.05485478E-49	1.05485478E-49	1.00000000	1.11271862E-98
1000	1000	1.0000	1	1.00000000E+00	1.26146113E-02	1.26146113E-02	1.00000000	1.59128419E-04
1000	800	.8000	1	1.00000000E+00	6.58315164E-12	6.58315164E-12	1.00000000	4.33378855E-23
1000	400	.4000	1	1.00000000E+00	7.92690985-104	7.92690985-104	1.00000000	6.28358997-207
1000	200	.2000	1	1.00000000E+00	6.43619890-210	6.43619890-210	1.00000000	0.
1000	110	.1100	1	1.00000000E+00	3.19595350-283	3.19595350-283	1.00000000	0.
1000	100	.1000	1	1.00000000E+00	5.43894218-293	5.43894218-293	1.00000000	0.
1000	90	.0900	1	1.00000000E+00	0.	0.	0.00000000	0.
1000	60	.0600	1	1.00000000E+00	0.	0.	0.00000000	0.
1000	10	.0100	1	1.00000000E+00	0.	0.	0.00000000	0.
1000	1	.0010	1	1.00000000E+00	0.	0.	0.00000000	0.

APPENDIX I - POISSON DISTRIBUTION CALCULATIONS

Some care must be taken in calculating $P_{\langle B \rangle}(\geq T)$ with a computer (using Equation 4) because the terms $\langle B \rangle^T$ and $T!$ are usually too large to store in a computer word, when $\langle B \rangle$ and T are reasonable numbers, e.g. $\langle B \rangle = 100$ and $T = 200$. Since Equation 4 contains the ratio of $\langle B \rangle^T$ to $T!$ and this ratio is a small number, relative to $\langle B \rangle^T$ to $T!$, it is obvious that $P_{\langle B \rangle}(\geq T)$ should be calculated using logarithms. If the logarithm of both members of Equation 4 is taken, the result is

$$\log_{10} P_{\langle B \rangle}(\geq T) \cong -\langle B \rangle \log_{10} e + T \log_{10} \langle B \rangle - \log_{10} T! + \log_{10} X \quad (1.1)$$

$$\text{where } X = \left\{ 1 + \frac{\langle B \rangle}{(T+1)} + \frac{\langle B \rangle^2}{(T+1)(T+2)} + \dots + \frac{\langle B \rangle^n}{(T+1)(T+2)\dots(T+n)} + \dots \right\}$$

In this form these large terms can be handled properly. Because the terms $T \log_{10} \langle B \rangle$ and $-\log_{10} T!$ have opposite signs it is convenient to take the antilogarithm of $\log_{10} P_{\langle B \rangle}(\geq T)$ directly rather than handling the characteristic and the mantissa separately. To calculate $P_{\langle B \rangle}(\geq T)$ the computer program uses Equations 1.1 and 1.2.

$$P_{\langle B \rangle}(\geq T) \cong 10^{\log_{10} P_{\langle B \rangle}(\geq T)} \quad (1.2)$$

X is calculated using a truncated form of the above infinite series.

It is calculated by

$$X = 1 + \sum_{i=1}^n \frac{\langle B \rangle^i}{(T+1)(T+2)\dots(T+i)} \quad (1.3)$$

where the nth term is the first term that is less than ϵ and ϵ is input to the program, e.g. $\epsilon = 1.0 \times 10^{-12}$.

$P_{\langle B \rangle}(\leq T)$ is calculated in a similar fashion except the series corresponding to X above is a finite series, and all of the terms are included in its calculation.

APPENDIX II - INPUT PREPARATION FOR THE MAIN PROGRAM

Card No. 1

Punch in cols.	1-5	NB	The number of $\langle B \rangle$'s for a $P_{\langle B \rangle}(\geq T)$ calculation, the number of $\langle D \rangle$'s for a $P_{\langle D \rangle}(\leq T)$ calculation or the number of $\langle B \rangle$'s for a $P_{\langle B \rangle}(T)$ calculation.
	6-10	NT	The number of T's.
	11-15	NP	= 0 $P_{\langle B \rangle}(\geq T)$ calculation. = 1 $P_{\langle D \rangle}(\leq T)$ calculation. = 2 $P_{\langle B \rangle}(T)$ calculation.

The program will do all combinations of the input quantities NB, and NT, e.g. if NB = 2, and NT = 3 it will do 6 calculations of the type requested. If one does not want this type of output he may use the POI subroutine with his own main program. The arguments to the POI subroutine are defined at the end of these input instructions.

Card Group No. 2

FORMAT (7E10.4)	$\langle B \rangle$'s or $\langle D \rangle$'s depending upon the value of NP above. There will be NB numbers entered.
-----------------	--

Card Group No. 3

FORMAT (7E10.4)	T's	There will be NT numbers entered. If option NP = 0 is used above, every T here must be greater than every $\langle B \rangle$ in Card Group 2.
-----------------	-----	--

Card No. 4

FORMAT (E10.4)	EPS is the ϵ and it is used in the calculation of the X series described in Appendix I. Although EPS is only used when doing the $P_{\langle B \rangle}(\geq T)$ calculation and not for the $P_{\langle D \rangle}(\leq T)$ or $P_{\langle B \rangle}(T)$ calculations, it must be input to the program. The program will set EPS equal to zero when doing the $P_{\langle D \rangle}(\leq T)$ and $P_{\langle B \rangle}(T)$ calculations regardless of the input quantity.
----------------	--

Repeat card groups 1 to 4 to run multiple cases.

Subroutine POI

If the POI subroutine is used by itself, the following CALL statement should be made:

```
CALL POI(BB,TT,EPS,PS,PC,XC,QT,NP)
```

where BB is $\langle B \rangle$ or $\langle D \rangle$ depending upon which calculation is being done,

TT is T,

EPS is ϵ usually $1.0E-12$,

PS is $P_{\langle B \rangle}(\geq T)$, $P_{\langle D \rangle}(\leq T)$ or $P_{\langle B \rangle}(T)$,

PC is $\phi_{\langle B \rangle}(\geq T)$, $\phi_{\langle D \rangle}(\leq T)$ or $P_{\langle B \rangle}(T)$,

XC is the number of terms used in the X series calculation. For $P_{\langle D \rangle}(\leq T)$ calculation XC will equal $T + 1$.

QT is the value of the XCth term in the X series calculation. It should always be less than ϵ .

NP = 0 for $P_{\langle B \rangle}(\geq T)$ calculation,

= 1 for $P_{\langle D \rangle}(\leq T)$ calculation,

= 2 for $P_{\langle B \rangle}(T)$ calculation.

If one uses option 2 (NP = 2) the program sets XC and QT equal to 1.0 and PS and PC both contain the answer $[P_{\langle B \rangle}(T)]$.

APPENDIX III - DESCRIPTION OF OUTPUT

Tables I, II, and III are examples of the programs output.

B or D is the average count. In other words the average number of events during some time interval.

T is the random variable in the Poisson Distribution.

T/B or D/T The ratio of T to $\langle B \rangle$ or $\langle D \rangle$ to T depending upon NP.

NO. TERMS If NP = 0 this is the number of terms in the truncated series X. If NP = 1 this is equal to T + 1. If NP = 2 this value is 1.

N th TERM If NP = 0 this is the value of the last term in the truncated series X. If NP = 1 this is the value of the (T+1) term. If this is equal to zero it means that the last term in the series became so small that the computer could not store it properly ($< 10^{-294}$). If NP = 2 this value is 1.

P-SERIES is $P_{\langle B \rangle}(\geq T)$, $P_{\langle D \rangle}(\leq T)$, or $P_{\langle B \rangle}(T)$.

P-CALCULATE is $\mathcal{P}_{\langle B \rangle}(\geq T)$, $\mathcal{P}_{\langle D \rangle}(\leq T)$, or $P_{\langle B \rangle}(T)$.

P(S)/P(C) is P-SERIES/P-CALCULATE

$P(S)**2$ is $[P-SERIES]^2$ only if NP = 0 or 2.

$2*P(S)$ is $2[P-SERIES]$ only if NP = 1.

These last two quantities are printed out because they are useful for a particular type of calculation carried out at WANL.

If the P-SERIES calculation is printed out as a zero, it means that the results are less than 10^{-294} and the computer could not store a small number.

If the P-CALCULATE calculation is printed out as a zero, it could be because of one of two reasons. One is similar to the above when the P-SERIES calculation goes to zero, and the other is that the calculation is not possible, e.g. let $\langle D \rangle = 10$ and $T = 15$ and calculate $\mathcal{P}_{\langle D \rangle}(\leq T)$ using Equation 9.

The results will be some negative number because $\frac{T}{\langle D \rangle}$ will be greater than 1.

P(S)/P(C) is printed out as 0.0 if either P-SERIES or P-CALCULATE is printed as a 0.0.

APPENDIX IV

```

PROGRAM POISSON (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)

C
000003      DIMENSION R(100),T(100),D(100)
C
000003      EQUIVALENCE (R,D)
C
000003      DATA MR,MD,MTH,MDT,MRF,MDH/1HR,1HD,3HT/B,3HD/T,7HP(S)**2,6H2*P(S)/
C
000003      10 READ (5,10001) NR,NT,NP
000015      10001 FORMAT(14I5)
000015      READ (5,10002) (R(I),I=1,NR)
000030      10002 FORMAT(7E10.4)
000030      READ (5,10002) (T(I),I=1,NT)
000043      READ (5,10002) EPS
000051      IF(NP.NE.0) EPS=0.0
000053      MM=MR
000055      MMM=MTH
000056      MMMM=MRF
000060      IF(NP.NE.1) GO TO 20
000062      MM=MD
000063      MMM=MDT
000065      MMMM=MDH
000066      20 WRITE (6,20001) EPS,NP
000076      20001 FORMAT(5H1EPS=,E14.7,10X,3HNP=,I1)
000076      WRITE (6,20002) MM,MMM,MMMM
000110      20002 FORMAT(3H1 ,A1,5X,1H1,4X,A3,3X,9HNO. TERMS,3X,9HNT TERM,6X,
18HP-SERIES,8X,11HP-CALCULATE,4X,9HP(S)/P(C),6X,A7//)
000110      LINE=1
000111      IF(NP.EQ.1) GO TO 40
000113      DO 30 IH=1,NP
000115      BB=R(IH)
000117      DO 30 IT=1,NT
000120      IF(LINE.LT.41) GO TO 25
000122      WRITE (6,20002) MM,MMM,MMMM
000134      LINE=1
000135      25 KK=T(IT)
000137      TT=KK
000141      TR=TT/BB
000143      CALL POI (BB,TT,EPS,P,PP,XJ,QT,NP)
000152      POVPP=0.0
000153      IF(PP.NE.0.0) POVPP=P/PP
000156      PSQ=P**2
000160      NN=XJ
000161      WRITE (6,20003) BB,TT,TH,NN,QT,P,PP,POVPP,PSQ
000207      20003 FORMAT(1X,F4.0,2X,F4.0,  F8.4,2X,I4,2X,3E16.8,F12.8,2X,E15.8)
000207      LINE=LINE+1

```

```

000211      30 CONTINUE
000216      GO TO 10
000216      40 DO 70 ID=1,NB
000220          DD=D(ID)
000222          DO 70 IT=1,NT
000223              IF(LINE.LT.41) GO TO 50
000225              WRITE (6,20002) MM,MMN,MMMM
000237              LINE=1
000240      50 KK=T(IT)
000242          TT=KK
000244          DT=DD/TT
000245          CALL POI (DD,TT,EPS,P,PP,XJ,QT,NP)
000255          POVPP=0.0
000256          IF(PP.NE.0.0) POVPP=P/PP
000261          PSQ=2.0*P
000263          NN=XJ+1.0
000266          WRITE (6,20003) DD,TT,DT,NN,QT,P,PP,POVPP,PSQ
000313          LINE=LINE+1
000315      70 CONTINUE
000322      GO TO 10
000322      END
  
```

PROGRAM LENGTH INCLUDING I/O BUFFERS
004771

FUNCTION ASSIGNMENTS

STATEMENT ASSIGNMENTS

10	-	000003	20	-	000066	25	-	000135	40	-	000216
50	-	000240	10001	-	000335	10002	-	000337	20001	-	000342
20002	-	000346	20003	-	000363						

BLOCK NAMES AND LENGTHS

VARIABLE ASSIGNMENTS

B	-	000400	BB	-	000730	D	-	000400	DD	-	000745
DT	-	000746	EPS	-	000722	I	-	000721	IB	-	000727
ID	-	000744	IT	-	000731	KK	-	000732	LINE	-	000726
MB	-	000710	MBH	-	000714	MD	-	000711	MDH	-	000715
MDT	-	000713	MM	-	000723	MMM	-	000724	MMMM	-	000725
MTR	-	000712	NR	-	000716	NN	-	000743	NP	-	000720
NT	-	000717	P	-	000735	POVPP	-	000741	PP	-	000736
PSQ	-	000742	QT	-	000740	T	-	000544	TB	-	000734
TT	-	000733	XJ	-	000737						

START OF CONSTANTS
000324

START OF TEMPORARIES
000374

START OF INDIRECTS
000377

UNUSED COMPILER SPACE
015500

```

SUBROUTINE POI (BB,TT,EPS,P,PP,XJ,QT,NP)
000011      KK=TT
000012      XJ=1.0
000013      QT=1.0
000014      CC=0.0
000015      X=1.0
000016      MP=NP+1
000020      GO TO (10,40,70),MP
000026      10 CC=BB/TT
000030      RNUM=BB
000031      TDEN=TT+1.0
000033      QT=RNUM/TDEN
000035      X=1.0+QT
000036      XJ=2.0
000037      20 TDEN=TT+XJ
000041      XJ=XJ+1.0
000043      QT=QT*RNUM/TDEN
000045      X=X+QT
000047      IF(QT.GE.EPS) GO TO 20
000051      GO TO 70
000051      40 CC=TT/BB
000053      TNUM=TT
000054      DDEN=BB
000055      QT=TNUM/DDEN
000057      X=1.0+QT
000060      XJ=1.0
000061      DO 60 I=2, KK
000062      QT=QT*(TNUM-XJ)/DDEN
000066      XJ=XJ+1.0
000067      60 X=X+QT
000074      70 Q=-BB*0.4342944819032518+TT*ALOG10(BB)-FALOG(KK,10)
000113      P=10.0** (Q+ALOG10(X))
000125      PP=0.0
000126      IF(CC.LT.1.0) PP=10.0** (Q-ALOG10(1.0-CC))
000144      RETURN
000145      END

```


SUBPROGRAM LENGTH
000210

FUNCTION ASSIGNMENTS

STATEMENT ASSIGNMENTS

10	-	000026	20	-	000037	40	-	000051	70	-	000074
----	---	--------	----	---	--------	----	---	--------	----	---	--------

BLOCK NAMES AND LENGTHS

VARIABLE ASSIGNMENTS

RNUM	-	000202	CC	-	000177	DDEN	-	000205	I	-	000206
KK	-	000176	MP	-	000201	NP	-	000001	Q	-	000207
QT	-	000000	TDEN	-	000203	TNUM	-	000204	X	-	000200

START OF CONSTANTS
000147

START OF TEMPORARIES
000155

START OF INDIRECTS
000176

UNUSED COMPILER SPACE
016200

```

000004      FUNCTION FALOG(FACTOR,OPTION)
000004      INTEGER FACTOR,OPTION
000004      DIMENSION VAL(100)
      DOUBLE PRECISION ACC,A,FUDGE,PAR

C
C      OPTION = 1 FOR NATURAL LOGARITHMS (BASE E).
C      OPTION = 10 FOR LOGARITHMS TO THE BASE 10.
C      THE LARGEST ARGUMENT FALOG WILL EVALUATE IS LOG OF 2010 FACTORIAL.
C

000004      DATA      (VAL(I),I=1,50) /18.386124616877623,47.911645068155849,
181.920174849390150,118.854727722499319,157.970003654715583,
2198.825393847218947,241.129109988696655,284.673456240681844,
3329.302971424793213,374.896888640039833,421.358669542130883,
4468.609368705647285,516.583220982611238,565.224592047015904,
5614.485803043771739,664.325536874152021,714.707643784655374,
6765.600228506718850,816.974940563632117,868.806414177724946,
7921.071820316650701,973.750504141640704,1026.823688424570719,
11080.274228577924077,1134.086408535127703,1188.245769304863643,
91242.738963911477185,1297.553633832474588,1352.678303092201531,
11408.102286966277461,1463.815612860737019,1519.808951401530067,
11576.073556138588174,1632.601210558917955,1689.384181333603919,
11746.415176908121794,1803.687310693589097,1861.194068237251486,
11918.929277848539641,1976.887084237634554,2035.061924789966724,
12093.448508155226591,2152.041794875258347,2210.836979813902872,
12269.829476183804218,2329.014900993031915,2388.389061756912270,
12447.947944340732647,2507.687701815273613,2567.604644222126808/
000004      DATA      (VAL(I),I=51,100)/2627.695229157470749,2687.956053094399977,
12748.383843372663250,2808.975450792684569,2869.727842758322367,
22930.638096918046358,2991.703395260381512,3052.921018623557757,
33114.288341583727743,3175.802827689782134,3237.462025015658583,
43299.263562004343839,3361.205143579936703,3423.284547506482340,
53485.499620974369464,3547.848277396784397,3610.328493400200387,
63672.938305994612165,3735.675809910098906,3798.539155087855761,
73861.526544314445346,3924.636230989475735,3987.866517017086153,
84051.215750813105842,4114.682325419795234,4178.264676721242722,
94241.961281752504874,4305.770657096727518,4369.691357364325086,
14433.721973749285098,4497.861132657475537,4562.107494402880548,
14626.459751967195189,4690.916629819403170,4755.476882791350363,
14820.139295006403700,4884.902678857906722,4949.765874034434091,
15014.727746589487651,5079.787188053043792,5144.943114582361886,
15210.194466150365770,5275.540205769182649,5340.979318747034995,
15406.510811976826517,5472.133713254675968,5537.847070626710774,
15603.649951763072750,5669.541443357069511,5735.520650549064158/
      PAR=0.
      IF (FACTOR.GE.0.AND.FACTOR.LE.2010) GO TO 10
      WRITE(6,20000) FACTOR

```

```

000025      20000 FORMAT( 95H1***ERROR STOP*** THE ARGUMENT TO THE FALOG FUNCTION IS
              1 NOT AN ACCEPTABLE VALUE.      ARGUMENT = ,I7)
000025      STOP
000026      10 ACC=0.0
000030      FUDGE=1.0
000032      IF(OPTION.LT.5) FUDGE=2.30258509299404568402
000036      IF(FACTOR.EQ.0) GO TO 50
000037      I=FACTOR/20
000042      II=FACTOR-(20*I)
000044      IF(II.GT.10) GO TO 20
000047      IF(I.NE.0) PAR=VAL(I)
000052      IF(II.EQ.0) GO TO 50
000053      ISTOP=FACTOR
000054      ISTART=20*I+1
000060      GO TO 30
000060      20 PAR=VAL(I+1)
000063      ISTOP=20*(I+1)
000066      ISTART=FACTOR+1
000067      30 DO 40 I=ISTART,ISTOP
000071      A=I
000073      40 ACC=DLOG10(A)+ACC
000110      IF(II.GT.10) ACC=-ACC
000115      50 FALOG=(PAR+ACC)*FUDGE
000131      RETURN
000132      END

```

SUBPROGRAM LENGTH
000343

FUNCTION ASSIGNMENTS

STATEMENT ASSIGNMENTS

10	-	000026	20	-	000060	30	-	000067	50	-	000115
20000	-	000136									

BLOCK NAMES AND LENGTHS

VARIABLE ASSIGNMENTS

A	-	000331	ACC	-	000327	FALOG	-	000162	FUDGE	-	000333
I	-	000337	II	-	000340	ISTART	-	000342	ISTOP	-	000341
PAR	-	000335	VAL	-	000163						

START OF CONSTANTS
000134

START OF TEMPORARIES
000155

START OF INDIRECTS
000161

UNUSED COMPILER SPACE
015500