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BY

J. F. SCHIVELL AND D. J. GROVE

**PLASMA PHYSICS
LABORATORY** **PRINCETON UNIVERSITY
PRINCETON, NEW JERSEY**

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Thermal Stress Under High-Power Surface Heating

J. F. Schivell and D. J. Grove*

Plasma Physics Laboratory, Princeton University
Princeton, New Jersey 08540

ABSTRACT

Estimates are made of thermal stress in a wall subjected to transient heating on one side. A detailed calculation is carried out for a flat plate heated on one face and cooled on the other. These examples furnish a guide to power absorption limits for surfaces in high-power plasma devices. Figures of merit are tabulated for various possible materials.

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Introduction

The purpose of this note is to provide some simple guidelines for determining the importance of thermal stress in situations where large heat fluxes impinge upon metal surfaces.

This topic is particularly appropriate to the design of limiters for experimental tokamaks. In high-powered devices, the same considerations apply to the surfaces in the collection region of the divertors.

Recently, studies^{1,2} have appeared in which the rate of wall evaporation is considered as a limit on the amount of power that can be absorbed by a tokamak wall. As a result of our calculations it is seen that, depending on the parameters of the wall or other plasma impingement surfaces, the thermal stress can be a much more restrictive limitation.

In considering the maximum thermal stress that a body can withstand, we compare the stress with the tensile strength. If one takes into account plastic deformation under the stress, then a body could withstand a higher heat flux than we calculate. On the other hand, if plastic deformation is undesirable, then the flux must be lower than that determined from the tensile strength.

Approximate Treatment

Except in the very simple case of a free body with temperature a linear function of the rectangular coordinates, there will occur in a heated body a thermal stress having a maximum value

depending on the temperature difference between two parts of the body.³ That is

$$\sigma = k E \alpha \Delta T \quad (1)$$

where E is Young's modulus, α is the thermal expansion coefficient, and k is a constant, depending on the shape, of value ranging from 0.5 to 2.5. ΔT is the temperature difference measured in different ways depending on the particular problem, but representing generally the maximum difference in temperature between one part of the body and another.

$E \alpha \Delta T$ thus represents the order of magnitude of the thermal stress. For example, for stainless steel with $E = 28 \times 10^6$ psi and $\alpha = 17 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}$, the stress is 470 psi per degree C. The yield point of 36,000 psi is reached when temperature differences of only 75° exist within the material and with differences of 175°C the stresses exceed the tensile strength of 80,000 psi.

From the point of view of minimizing damage, tungsten is of course much more suitable as a limiter material, not only because of its superior strength at elevated temperatures but also because the stress is lower for the same heat flux. For example, at 1200°C, $E = 47 \times 10^6$ psi, $\alpha = 4.5 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}$, and the thermal stress is 210 psi per degree C. The stress thus equals the 1200°C strength of 60,000 psi at internal temperature differences of 280°C. Tungsten has the additional favorable property that its thermal conductivity is 6.5 times that of stainless steel so that for a given heat flux, the internal temperature differences are reduced substantially.

We now turn to estimates of temperature differences.⁴ One commonly treated case is that of a semi-infinite solid exposed to a fixed heat flux at the surface. The temperature rise at the surface is given by

$$T_s = \frac{2F}{K} \left(\frac{Dt}{\pi} \right)^{1/2} \quad (2)$$

where F is the (constant) heat flux, K the thermal conductivity, D the thermal diffusivity [= $(K/\rho c)$], and t the time in seconds after the heat flux is switched on.

If we use the value T_s for the ΔT in the stress formula, we obtain a maximum possible value for the thermal stress, which, as we shall see, is not too far from the actual value in cases of current interest.

Combining Eqs. (1) and (2) and assuming $k = 1$, we have an expression for the maximum flux which can be tolerated if the stress limit is to be just reached after a time t , viz.,

$$F_{\max} = \frac{K \sigma_{\max}}{2E\alpha} \left(\frac{\pi}{Dt} \right)^{1/2} \quad (3)$$

For tungsten (at 1200°C)

$$K = 0.25 \text{ cal cm}^{-1} \text{ sec}^{-1} \text{ } ^\circ\text{C}^{-1} ,$$

$$E = 47 \times 10^6 \text{ psi} ,$$

$$\alpha = 4.5 \times 10^{-6} \text{ } ^\circ\text{C}^{-1} ,$$

$$\sigma_{\max} = 6 \times 10^4 \text{ psi} ,$$

$$\rho = 19.3 \text{ gm cm}^{-3} ,$$

$$c = 3.6 \times 10^{-2} \text{ cal gm}^{-1} \text{ }^{\circ}\text{C}^{-1},$$

$$D = (K/\rho c) = 0.36 \text{ cm}^2 \text{ sec}^{-1},$$

$$\begin{aligned} F_{\max} &= 1.05 \times 10^2 t_{\text{sec}}^{-1/2} \text{ cal cm}^{-2} \text{ sec}^{-1} \\ &= 0.44 t^{-1/2} \text{ kilowatt cm}^{-2}. \end{aligned}$$

A typical tokamak value for t is ~ 0.1 sec and for this case $F_{\max} = 1.4 \text{ kW/cm}^2$. We shall see in what follows that, unless a surface is rather thin, a few kilowatts per cm^2 is the maximum that can be poured into even a tungsten surface without destroying it.

In the above we have assumed a semi-infinite solid and in setting the temperature difference equal to the surface temperature to get the maximum stress, we simply observe that for any time there is a distance into the solid sufficiently removed from the surface that the temperature there is negligible compared to the surface temperature.

More generally we will deal with situations where the heat flux is directed into the face of a plate of thickness ℓ . However, we can approximate this plate by the semi-infinite one as long as the temperature rise in the semi-infinite plate at depth ℓ is much smaller than the rise at the heated face. In the semi-infinite plate we have that⁴

$$\frac{T(\ell)}{T(0)} = \exp(-u^2) - 2u \int_u^{\infty} \exp(-\xi^2) d\xi \quad (4)$$

where $u = [\ell/(4Dt)]^{1/2}$. If we demand $T(\ell) < 0.1T(0)$ for validity of the approximation, we find that we must take $u > 1.0$. That is $(\ell^2/4Dt) > 1$, or $t < (\ell^2/4D)$. For $\ell = 1 \text{ cm}$ of

tungsten, $t < 0.7$ sec. As long as the power-on time is in this range, the approximation, and hence the power limit of 1.4 kW/cm^2 , is valid.

Detailed Calculation

Finally, we give one example of a detailed computation to indicate the accuracy of the estimates given above.

A constant and uniform heat flux F (per unit area) is switched on at $t = 0$, impinging on the $z = \ell/2$ face of a flat plate in the $x-y$ plane, the face at $z = -\ell/2$ being kept at zero temperature. (All edge effects are neglected so the extent of the plate in the x - and y -directions is immaterial.) We assume no radiation from the hot face.

The temperature distribution as a function of time is then given by

$$\frac{TK}{\ell F} = \left(\frac{z}{\ell} + \frac{1}{2} \right) - \frac{8}{\pi^2} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^2} \times \exp [(2n+1)^2 t / \tau] \sin \left[\frac{(2n+1)\pi}{2} \cdot \left(\frac{z}{\ell} + \frac{1}{2} \right) \right] \quad (5)$$

with the various terms defined as above, and $\tau = 4\ell^2 / \pi^2 D$.

The z -component of the thermal stress is zero but³

$$\sigma_x = \sigma_y = - \frac{E\alpha}{(1-\nu)} \left(T - \frac{1}{2c} \int_{-\ell/2}^{\ell/2} T dz - \frac{3z}{2c^3} \int_{-\ell/2}^{\ell/2} Tz dz \right) \quad (6)$$

where, again, the terms have been defined previously except for ν which is Poisson's ratio ($= 0.38$ for tungsten).

Combining Eqs. (5) and (6) and integrating we obtain

$$\begin{aligned} \frac{\pi^2 K(1-\nu)\sigma}{8\ell F E \alpha} = & \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^2} \exp [(2n+1)^2 t/\tau] \\ & \times \left\{ \sin \left[\frac{(2n+1)\pi}{2} \left(\frac{z}{\ell} + \frac{1}{2} \right) \right] - \frac{2}{(2n+1)\pi} \right. \\ & \left. - \frac{12}{(2n+1)\pi} \left(\frac{z}{\ell} \right) \left(-1 + \frac{4(-1)^n}{(2n+1)\pi} \right) \right\} \quad (7) \end{aligned}$$

where all constant factors have been put on the left-hand side.

Looking first at the temperature distribution from Eq. (5), we have plotted in Fig. 1(a) the dimensionless temperature $(KT/\ell F)$, as a function of z for a range of values of t/τ . In Fig. 1(b) we have the dimensionless stress $[\pi^2 K(1-\nu)\sigma/8\ell F E \alpha]$. An examination of the computed numbers shows that the maximum stress is at the hot face of the plate, and at a time $t/\tau = 0.1$. The maximum value is $[\pi^2 K(1-\nu)\sigma/8\ell F E \alpha] = 0.126$. Putting in the tungsten parameters gives

$$F_{\max} = 1.8 \text{ kW/cm}^2$$

for a 1 cm thick plate. For this plate $\tau = 1.0$ second.

Similar calculations for a hollow cylinder heated from the inside give comparable results. For example, a tungsten cylinder of radius 45 cm, wall thickness 1 cm, will reach the stress limit of 60,000 psi after 0.1 sec at an incident flux of 1.1 kW/cm^2 . (At this time in this hollow cylinder, the stress is still going up.)

For a solid cylinder heated from the outside, we have the following case for comparison: cylinder radius 0.8 cm, material tungsten, stress limit reached after 0.1 sec under 0.9 kW/cm^2 .

In one wall evaporation study² the conclusion was that a tungsten wall could take up to 2800 J/cm^2 without exceeding a reasonable rate of evaporation. But the energy dumping times considered were from 0.1 sec to 1.0 sec, and as we have seen, this implies a flux which would exceed, by possibly a large factor, that which would rupture the surface.

A possible way around the stress problem would be to make the surface thinner (with the back still cooled, as above), since the maximum stress is proportional to the thickness. However, in practice, one probably does not want to use a very thin wall if it contains coolant behind it, simply because there would be too much danger of a catastrophic failure in the event of an accidental local concentration of power.

Choice of Materials

In Table I we have gathered, for some representative materials, values of properties which are relevant to temperature rise and thermal stress. For the hot-face temperature rise we can see from Eq. (2) that T_s is proportional to $(K\rho c)^{-1/2}$. For ability to withstand heat flux we can define a figure of merit by inspection of Eq. (7). The right-hand side depends only on t/τ and z/l ; therefore, independently of the material the maximum stress is at $z = +l/2$ at $t = 0.1\tau$ and is one definite (dimensionless) value. If we set $\sigma = \sigma_{\max}$, then the

flux that a material can withstand is proportional to M , where M is given by

$$M = \frac{K(1 - \nu)\sigma_{\max}}{\alpha E}$$

if we compare material samples of the same thickness.

Examination of the Table shows that tungsten is undoubtedly one of the best materials with respect to temperature rise and thermal stress. Molybdenum can take less heat flux, but has the advantage of lower atomic number Z . Graphite emerges as a very good candidate, based on thermal stress, and in addition has very low Z . It is worth noting that titanium, which has possible application as a getter in plasma devices, is a relatively "weak" material from the point of view of thermal stress.

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References

- ¹R. Behrisch, Nuclear Fusion 12, 695 (1972).
- ²G. McCracken, private communication.
- ³S. Timoshenko and J. N. Goodier, Theory of Elasticity (McGraw-Hill, 1951).
- ⁴H. S. Carslaw and J. C. Jaeger, Conduction of Heat in Solids (Oxford University Press, 1959).
- ⁵"1973 Materials Selector," Materials Engineering 76, No. 4, (1972).
- ⁶Plenum Press Handbooks of High-Temperature Materials, No. 1, Materials Index, by Peter T. B. Shaffer (Plenum Press, 1964).
- ⁷R. Hultgren, et. al., Selected Values of Thermodynamic Properties of Metals and Alloys (Wiley, 1963).
- ⁸Proceedings of the Conference on Continuum Aspects of Graphite Design (Gatlinburg, Tennessee, 1970) USAEC Technical Information Center.

Table I.

Relevant Properties of Some Typical Materials at 1000 °C *

	Melting Point	K	ρ	c	$(K\rho c)^{-1/2}$	ν	$\sigma_{\max}$ Tensile	α	E	M
Units	°C	cal/°C cm sec	gm/cm ³	cal/°C gm			10 ³ lb/in ²	10 ⁻⁶	10 ⁶ lb/in ²	
W	3340	0.26	19.3	0.036	2.4	0.38	60	4.5	52	41
Mo	2610	0.24	10.2	0.076	2.3	0.32(r) ⁺	20	5.7	39	15
Ti	1660	0.037(r)	4.5	0.15	6.3	0.3 ?	20(300)	9.5	12(300)	5
Nb	2410	0.16	8.4	0.074	3.2	0.38(r)	13	8	6.5(500)	25
Ta	2850	0.15	16.6	0.036(r)	3.3	0.35 ?	15	11.7	22	6
Graphite [†]	3600	0.15	1.8	0.5	2.7	0.4 ?	3	5	1	54
WC	2870	~0.12	15.6			0.3 ?	50	5	90	9
Al ₂ O ₃	2015	0.015	4.0			0.25	35	9.5	50	0.8
MgO	2800	0.017	3.6	0.29	7.5	0.16(r)	11.5	14	21	0.6

* Taken from Refs. (1,5,6,7,8).

⁺ (r) = room temperature (300) = 300°C, etc.[†] Representative values — one grade only, not necessarily optimum.

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Fig. 1. Temperature (a) and stress (b) profiles in the flat plate as a function of depth z/l at several values of the time t/τ . Maximum stress occurs at $t/\tau = 0.1$.

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