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THEORY OF CORRELATION MEASUREMENT
IN TIME AND FREQUENCY DOMAINS WITH ^{252}Cf *

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ABSTRACT

MASTER

The theory of cross-correlation measurements between the pulses from an ionization counter containing a ^{252}Cf neutron source, which furnished the initiators of fission chains in a neutron-multiplying assembly, and the pulses from a detector observing the particles from the fission chains is reviewed. The expression for the detector response in this type of measurement is first presented in the time domain. Then the cross-power spectral densities in the frequency domain measurements are developed. The theory of the usual two-detector noise measurement is reformulated to incorporate a modified Diven factor that includes the effects of neutrons from external sources such as spontaneous fission of the californium and the reactor fuel. Finally, the equivalence between both the time and frequency domain expressions is demonstrated.

A new method of reactivity determination is proposed which avoids some of the difficulties of the present methods, such as dependence on detection efficiency and inherent source strength, and the requirement for a calibration near delayed criticality.

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INTRODUCTION

The chain-reacting nature of nuclear processes in a reactor gives rise to a nonrandom distribution of detected counts because the individual counts are dependent on other neutrons in the fission chain. Thus, the statistical properties of the count sequence depend on the dynamic properties of the nuclear system. This dependence is the basis of a variety of measurement methods in both the time⁽¹⁻⁸⁾ and frequency⁽⁷⁻¹²⁾ domains from which the reactivity has been obtained. The domain in which the measurements are performed is selected for convenience and efficiency of data collection. Basically these methods determine the reactivity in two ways: from the prompt neutron decay constant or a related quantity, or from an amplitude associated with the amplitude of the correlated component of the count sequence. Pulsed neutron methods have also been used to obtain the prompt neutron decay constant.^(13,14) All of these existing methods of reactivity determination depend on a calibration of the method at some known reactivity usually near delayed criticality and also require corrections for the changes in prompt neutron lifetime or detection efficiency from the calibration state to the state of interest.

Consider a reactor with three detectors: a fission chamber which contains a spontaneous fission isotope, ^{252}Cf (detector 1); and a pair (2 and 3) that detect neutrons emitted by fission chains propagated by the reactor fuel. The neutron induced fission rate in the chamber with ^{252}Cf is negligible compared with the spontaneous fission rate of ^{252}Cf for fluxes attainable in reactors. This paper describes the theory of cross-correlation measurements between the pulses from the ionization counter containing a ^{252}Cf neutron source, which furnished the initiators of fission chains in a neutron-multiplying assembly, and the pulses from a detector detecting the

particles from the fission chains. The expression for the detector response in this type of measurement is first developed in the time domain⁽¹⁵⁾ and the results are compared with those from a Rossi- α measurement with one or two detectors. Then the cross-power spectral densities (CPSD) in the frequency domain measurements are derived and compared with those from a conventional two-detector CPSD measurement. Finally, the equivalence between both expressions is demonstrated.

From this theoretical development a new method of reactivity determination is proposed which avoids some of the difficulties of the present methods; such as dependence on detection efficiency, inherent source strength, and the requirement for a calibration near delayed criticality. Possible uses of this method are also discussed.

THEORY OF TIME DOMAIN MEASUREMENTS

In the following discussion of the theory for a randomly pulsed neutron measurement⁽¹⁵⁾ with a ^{252}Cf source it is assumed that particles from the source do not interact directly with the detector. In practice, this assumption is valid if the detector is placed sufficiently far from the californium source. An ideal Type-I time analyzer⁽¹⁶⁾ (i.e., one which responds to all pulses in the trigger channel as triggers and to all pulses in the detection channel as counts) is triggered each time a ^{252}Cf nucleus fissions, and thus, it measures the time distribution of neutrons from the fission chains initiated by ^{252}Cf fission neutrons at time $t = 0$.

An expression for the correlated counts in a randomly pulsed neutron measurement is obtained as follows: If τ is the mean life of a neutron to fission and if λ is the prompt-neutron lifetime, the rate of change of the prompt-neutron population, N , is

$$\frac{dN}{dt} = -\frac{N}{\ell} + \bar{\nu} \frac{N}{\tau} = \frac{k_p - 1}{\ell} N = -\alpha N, \quad (1)$$

where

α = prompt-neutron decay constant

k_p = prompt-neutron multiplication factor

$\bar{\nu}$ = average number of neutrons per fission in the reactor.

If there are N_0 neutrons in the reactor at time $t = 0$, the average number of neutrons at a later time, t , is $N_0 \exp(-\alpha t)$. The average number of fissions produced in dt at time t is $N dt / \tau$ or $N_0 [\exp(-\alpha t)] dt / \tau$, and the resulting number of neutrons is $\bar{\nu} N_0 [\exp(-\alpha t)] dt / \tau$. Since the N_0 neutrons are produced by ^{252}Cf fission, $N_0 = \nu_c I_c / I$, where ν_c is the number of neutrons per ^{252}Cf fission, I_c is the importance of a neutron from ^{252}Cf fission and I is the average importance of a neutron from reactor fission.

If S is the number of reactor fissions per ^{252}Cf fission and W' is the detection efficiency for the detector detecting particles from reactor fission in counts per ^{252}Cf fission, then $W'/S = W$ where W is the detection efficiency in counts per reactor fission. The number of correlated counts, dC , in dt at time t after a ^{252}Cf fission is

$$dC = \frac{\bar{\nu} I_c W k_p}{\bar{\nu} \ell I} \exp(-\alpha t) dt. \quad (2)$$

The total correlated count rate, C_{RPN} , collected by an ideal Type-I analyzer, is the product of the integral of dC over the analysis time ($\gg \alpha^{-1}$) and the spontaneous fission rate of the ^{252}Cf , F_c , or

$$C_{RPN} = \frac{\bar{v} \frac{1}{c} F W_R}{\bar{v} \Delta t} P. \quad (3)$$

The uncorrelated count rate in a randomly pulsed neutron measurement is the product of the counting rate, $W'F_c$, and the time per second that a given channel is open, $F_c \Delta t$, where Δt is the channel width (sec). Thus, the ratio of the time-correlated to the uncorrelated count rate is inversely proportional to the ^{252}Cf spontaneous fission rate.

The total correlated counts, $C_{R-\alpha}$, collected per unit time by a Type-I analyzer in a Rossi- α measurement ⁽¹⁾ is

$$C_{R-\alpha} = \frac{(W)^2 F R N K_P^2}{2\alpha^2 \lambda^2} \quad (4)$$

where F is the reactor fission rate, and X is the neutron dispersion number, $\overline{v(v-1)}/\bar{v}^2$. (A modified form of the dispersion number which accounts for source fission is derived in the theory of the frequency domain measurement.) The factor R , which corrects point-reactor kinetics for spatial effects, is equal to

$$\int P(r) dr \int P(r) I^2(r) dr : [\int P(r) I(r) dr]^2, \quad (5)$$

where

$$P(r) = \int \Sigma_f(rE) \phi(rE) dE,$$

$$I(r) = \int X(rE) \phi^+(rE) dE,$$

$$X(rE) = \text{energy, } E, \text{ distribution of neutrons from fission at point } r,$$

$$\Sigma_f(rE) = \text{macroscopic fission cross section,}$$

$$\phi(rE) = \text{fundamental spatial mode flux,}$$

$$\phi^+(rE) = \text{adjoint flux. (17,18,19)}$$

The ratio, A, of correlated counts in a randomly pulsed neutron measurement to those in a Rossi- α measurement is

$$A = \frac{2\bar{v}_c I_c F_c \alpha \ell}{\sqrt{VX} k_p W' I R F} = \frac{2\bar{v}_c}{\sqrt{VXW'}} \frac{1}{I} \frac{1 - k_p}{k_p} \frac{F_c}{F} \quad (6)$$

This ratio of correlated count rates, A, can be used to assess which method of measurement accumulates correlated information at a faster rate. As the multiplication factor or the detector efficiency approaches zero, the ratio becomes infinite; as the prompt-neutron multiplication factor approaches one, this ratio approaches zero. For a wide range of detection efficiency the Rossi- α method accumulates correlated counts faster at high reactivities, while at low reactivities the reverse holds. The ratio is independent of the prompt-neutron lifetime, and for a given reactor it depends only on the prompt-neutron multiplication factor, the location of the ^{252}Cf source, the efficiency of the detector, and the ratio of the fission rate in the randomly pulsed neutron measurements to that in the Rossi- α measurements. Thus, if

$$W' < \frac{2\bar{v}_c I_c}{\sqrt{VX} I} \frac{1 - k_p}{k_p} \frac{F_c}{F}, \quad (7)$$

the randomly pulsed neutron measurement with ^{252}Cf will accumulate more correlated counts per unit time than the Rossi- α technique. If the Rossi- α measurements can be performed at high fission rates, i.e., $F \gg F_c$, or with high detection efficiency, then they may be more practicable. Large values of F may arise from the initiation of fission chains by a large number of delayed neutrons in assemblies at or above delayed criticality or by large inherent neutron sources such as those present in systems containing a large amount of ^{240}Pu .

The ratio of uncorrelated count rates in the randomly pulsed neutron measurement to that in the Rossi- α measurement is $W'F_c^2/W^2F^2$. If the Rossi- α measurement is made simultaneously with the randomly pulsed neutron measurement, then, $W'F_c = WF$, and this ratio of uncorrelated count rates is the californium fission rate, F_c , divided by the detector count rate, WF . When the Rossi- α measurement is made with two detectors, the efficiency in Eqs. (6) and (7) is that of the detector not common to both measurements since the product of detection efficiencies would appear in Eq. (4).

In the randomly pulsed neutron measurement, the californium fission rate is limited presently by the capability of pulse-counting techniques.

At present, for ionization chambers where discrimination against pulses from alpha particles allows the detection of >99.5% of the californium fissions, the minimum pulse width achievable is about 10 nsec (full width at 1/10 maximum). The amount of californium employed on a chamber operating in the pulse mode is limited by coating loss considerations. Use of 15 μg of ^{252}Cf will result in losses of 5-10%.

In obtaining all of the preceding relations, fundamental-mode decay and no loss of triggers or counts by the detection systems and the time analysis equipment were assumed.

REACTIVITY DETERMINATION FROM TIME DOMAIN MEASUREMENTS

The subcriticality has been determined previously from the prompt-neutron time behavior in a variety of ways. In applying the Sjostrand,²⁰ Gozani,²¹ or the Garablis-Russell²² methods for determining the subcriticality, the contribution of the background due to the random nature of the source should be subtracted before analysis. In obtaining the

subcriticality, the usual problems of interpreting measurements with assemblies not decaying on the fundamental mode still exist. In addition to these previously described method of obtaining the reactivity a new method based on the performance of both the Rossi- α and the randomly pulsed neutron measurement is possible.

Consider the ratio AW' is the ratio of the correlated counts per californium fission from the randomly pulsed neutron measurement to the correlated counts per detector count in the Rossi- α measurement. The following expression, ⁽²³⁾

$$AW' = \frac{2\bar{v}_c I_c}{\bar{v}RXI} \left(\frac{1 - k_p}{k_p} \right), \quad (8)$$

can be used to determine the prompt reactivity in k units rather than in dollars and to compare calculated multiplication factor directly with a measured value. The values of $\bar{v}$, R , and X are easily calculated or obtained from independent measurements and do not change much with reactivity. For example these values change only a few percent for a change in a typical liquid metal fast breeder reactor from the delayed critical to the fully shutdown configuration. The neutron importance values can be obtained from independent measurements. ⁽²²⁾ The use of Eq. (8) to obtain a reactivity does not depend on determination of properties of the reactor at delayed criticality nor does it require a calibration by other reactivity determination methods. Thus, it could be particularly useful in the initial startup of a reactor when the properties of the reactor at delayed critical are not known or a calibration near delayed critical cannot be made.

For a delayed critical system $k_p = 1 - \beta$, where β is the effective delayed neutron fraction, i.e., the increment of the multiplication factor between delayed and prompt criticality. Thus, for a delayed critical system, the ratio $(AW')_{dc}$ is

$$(AW')_{dc} = \frac{2\bar{v}_c I_c}{\bar{v} R X I} \frac{\beta}{1 - \beta} \quad (9)$$

Equation (9) is a special case of Eq. (8) which can be used to determine the effective delayed neutron fraction.

THEORY OF FREQUENCY DOMAIN MEASUREMENTS

The limitation of the californium source size imposed by pulse mode operation makes it difficult to apply time domain measurements to reactors with large inherent sources such as an LMFBR. For practical applications in an LMFBR the size of the californium source precludes time domain measurements and requires current mode operation of the californium chamber and frequency domain techniques. The theory for this type of measurement is now presented.

The standard expression for noise source spectral density, $G_s^{(25)}$, was modified to include terms that account for the inherent or external fission source in the reactor, i.e., the spontaneous fission of ^{240}Pu , and for the spontaneous fission of ^{252}Cf . This source spectral density is

$$G_s = \frac{n}{\Lambda} \left[(1 - \beta) \left(\frac{\bar{v}^2 - \bar{v}}{\bar{v}} \right) + 2\beta - \rho \right] + \frac{\bar{v}_i^2 F_i I_i}{I} + \frac{\bar{v}_c^2 F_c I_c}{I} \quad (10)$$

where

F_i = fission rate of the inherent or the external source,

- v_i = number of prompt neutrons per inherent or external source fission,
 I = average neutron importance of neutrons from reactor fission,
 I_i = average importance of neutrons from spontaneous fission of the inherent or external source,
 ρ = reactivity,
 Λ = prompt-neutron generation time,
 n = neutron density.

For an external source producing one neutron per disintegration, $\overline{v_i^2 I_i}$ is replaced by the source disintegration rate. It has been assumed that fluctuations in the importance of the neutrons from fission are small and can be neglected. The magnitude of this effect is related to the dependence of the energy spectrum of neutrons from fission on the number emitted in each fission.

To explain the theory for this proposed measurement technique and compare the results with those from a two-detector CPSD measurement, consider a reactor with the three detectors described previously. A schematic diagram of the frequency domain measurement with ^{252}Cf is shown in Fig. 1. The pair of detectors (2 and 3) detect neutrons emitted by fission chains initiated by neutrons from the external source or from the spontaneous fission of the inherent source by neutrons from the spontaneous fission of ^{252}Cf and by delayed neutrons. Since extra constants and corrections for the frequency response of the instrumentation are necessary in equations that describe the operation of instrumentation in the current mode, it is assumed in the following derivations that the instrumentation is being operated in the current mode. This leads to a more general form of the theory and specialization to pulse mode detection systems is made by dropping these constants.

The input noise source spectral density, Eq. (10), can be expressed as the sum of three terms: from reactor fission G_f ; from inherent or external source fission G_i ; and from ^{252}Cf fission G_c . The substitution of the fission rate in reactor fuel, F , for $\frac{n}{\beta} \frac{(1-\beta)}{\bar{v}}$ has been made in the first term. The equations for these three terms are

$$G_f = F \left[\bar{v}^2 - \bar{v} + \bar{v} \frac{(2\beta - 1)}{1 - \beta} \right], \quad (11)$$

$$G_i = \bar{v}_i^2 F_i I_i / I, \quad (12)$$

and

$$G_c = \bar{v}_c^2 F_c I_c / I. \quad (13)$$

The transfer function of the detection system is factored into the product of response function for the detector $U_2(\omega)$ and the response function for the detection system electronics $h_2(\omega)$. Now

$$U_2 = \frac{W_2 \bar{q}_2}{\bar{v}}. \quad (14)$$

The power spectral density of the detector noise is expressed as

$$G_2 = F W_2^2 \bar{q}_2^2, \quad (15)$$

where q_2 is the charge released per neutron interaction in detector 2, and W_2 is the detection efficiency in counts per reactor fission for detector 2.⁽²⁵⁾ It is assumed that U_2 and G_2 are independent of frequency ω and all frequency dependence is assigned to $h_2(\omega)$. Similar expressions are valid for detector 3.

Fig. 1. Schematic diagram of correlation experiment with ^{252}Cf .

For the ^{252}Cf chamber the current resulting from the Cf fission rate is $F_c \overline{q_c}$, and the ratio of current to fission rate is $U_1 = \overline{q_c}$. Since Cf also undergoes alpha decay, there is an alpha emission source with a power spectral density $G_1 = 31 F_c \overline{q_\alpha^2}$ (there are ~ 31 alpha disintegrations for each spontaneous fission, ⁽²⁶⁾ and q_α is the charge collected from an alpha event in the chamber).

The auto-power spectral densities (APSDs) are

$$C_{11} = |h_1(\omega)|^2 [G_1 + |U_1|^2 B_{cF_c}] \quad (16)$$

and

$$G_{22} = |h_2(\omega)|^2 [G_2 + |U_2|^2 |H_s(\omega)|^2 G_s], \quad (17)$$

where $B_c = \overline{q_c^2} / \overline{q_c^2}$. The cross-power spectral densities (CPSDs) are

$$G_{23} = h_2^*(\omega) h_3(\omega) U_2^* U_3 |H_s(\omega)|^2 G_s \quad (18)$$

and

$$G_{12} = h_1^*(\omega) h_2(\omega) U_1^* U_2 H_s(\omega) G_c' \quad (19)$$

where $G_c' = \overline{V_c F_c I_c} / I$, and $H_s(\omega)$ is the source transfer function of the reactor (neglecting delayed neutrons) which equals $[(\alpha + i\omega)\Lambda]^{-1}$.

The spectral densities G_{33} and G_{13} are defined by equations analogous to Eqs. (17) and (19). Substitution into Eqs. (16) to (19) from previous equations yields

$$G_{11} = |h_1(\omega)|^2 F_c \left(31 \overline{q_\alpha^2} + \overline{q_c^2} \right), \quad (20)$$

$$G_{22} = |h_2(\omega)|^2 \left[F W_2 \overline{q_2^2} + \frac{W_2^2 \overline{q_2^2}}{V^2} |H_s(\omega)|^2 G_s \right], \quad (21)$$

$$G_{23} = h_2^*(\omega) h_3(\omega) \frac{\overline{W_2 W_3 q_2 q_3}}{\overline{V}^2} |H_s(\omega)|^2 G_s, \quad (22)$$

and

$$G_{12} = \overline{q_c} h_1^*(\omega) h_2(\omega) \frac{\overline{W_2 q_2}}{\overline{V}} H_s(\omega) G_c', \quad (23)$$

with expressions similar to Eqs. (21) and (23) for G_{33} and G_{13} . As expected, CPSD G_{12} is proportional to the source transfer function of the reactor.

The assumption has been made that the neutrons from fission of the inherent and ^{252}Cf sources are not detected by the detectors 2 and 3. This assumption is usually valid for the ^{252}Cf source since it can usually be located some distance from detectors 2 and 3. If the reactor core is sufficiently large, only a small fraction of the material containing the inherent source is near detectors 2 and 3. If these assumptions are not valid, frequency-independent background terms should be added to the CPSDs given in Eqs. (22) and (23). The absence of these terms can be determined in practice by verification of the $1/\omega^2$ rolloff of the CPSDs at high frequency.

The source spectral density can be factored into the following form:

$$G_s = \overline{V}^2 XFR \left[1 + \frac{V}{XR} + \frac{F_i I_i}{FI} \frac{\overline{V_i^2}}{XRV^2} + \frac{F_c I_c}{F I} \frac{\overline{V_c^2}}{XRV^2} \right], \quad (24)$$

or

$$G_s = \overline{V}^2 X'FR, \quad (25)$$

where X is the usual Diven factor, X' is a modified neutron dispersion number, and $V = (2\beta - \rho)[(1 - \beta)\bar{V}]^{-1}$. The expression in brackets in Eq. (24) expresses the deviation from the standard dispersion factor and approaches unity for a reactor with high neutron multiplication, since $F \gg F_c$ or F_i , and V approaches $2\beta/\bar{V}$.

The square of the coherence between detectors 1 and 2, i.e., $|G_{12}|^2/G_{11}G_{22}$, can be written as

$$\gamma_{12}^2 = B_c \frac{1}{(1 + C_\alpha)} \left(\frac{\bar{V}_c I_c}{\bar{V} I} \right)^2 \frac{F_c}{F} \frac{1}{RX'} [1 + 1/Q_2(\omega)]^{-1}, \quad (26)$$

where

$$Q_2 = \frac{W_2 X' R}{B_2 \Lambda^2} \frac{1}{(\alpha^2 + \omega^2)}, \quad (27)$$

and

$$C_\alpha = 31 q_\alpha^2 / q_c^2 \quad B_2 = q_2^2 / q_2^2. \quad (28)$$

The square of the coherence between detectors 2 and 3, γ_{23}^2 , is $\{[1 + 1/Q_2(\omega)][1 + 1/Q_3(\omega)]\}^{-1}$, and an expression similar to Eq. (26) gives the coherence between detectors 1 and 3. By use of the ^{252}Cf chamber the two factors in the expression for γ_{23}^2 can be determined separately:

$$\frac{G_{12}G_{23}}{G_{13}G_{22}} = [1 + 1/Q_2(\omega)]^{-1}, \quad (29)$$

and

$$\frac{G_{13}G_{23}^*}{G_{12}G_{33}} = [1 + 1/Q_3(\omega)]^{-1}. \quad (30)$$

Values of the low-frequency ($\omega \ll \alpha$) coherence for the Cf measurement γ_{12} and the two-detector measurement γ_{23} were calculated for reactivities from 1 to 100 dollars subcritical and for detection efficiencies from 10^{-6} to 10^{-2} counts per reactor fission. The values of the parameters in these calculations were typical of the Fast Flux Test Facility reactor and were $X' = 0.8$, $R = 1.1$, $\beta = 0.003$, $\bar{V} = 2.91$, $\bar{V}_c = 3.73$, $C_\alpha = 0.10$, $B_c = B_2 = 1.2$, $I_c/I = 1.15$, and $F_c \gg F_1$. The calculated results are plotted in Fig. 2. With these coherence values the measurement time can be estimated from the following relation for the relative standard deviation of the power spectral density: (27)

$$\epsilon_{ij} \approx \left[\frac{0.5 + \frac{1}{\gamma_{ij}^2}}{2\Delta f T} \right]^{1/2}, \quad (31)$$

where Δf is the bandwidth of the analyzer and T is the total sampling time. In a digital Fourier analyzer, $\Delta f T$ is the total number of samples. For a near-critical or a slightly subcritical reactor, the coherence for the Cf measurement γ_{12} is less than the coherence γ_{23} for the two-detector noise measurement; thus the measurement with Cf will require more time to achieve the same precision in the CPSD. With a reactor at far subcritical, a measurement with Cf will take less time. Near criticality, where $[1 + 1/Q_2]^{-1}$ is at its maximum, the coherence γ_{12}^2 is low because of the relatively small value of F_c/F , which results from the relatively high multiplication of the reactor. As the reactivity is reduced, the decrease in $[1 + 1/Q_2]^{-1}$ tends to be compensated by the increase in F_c/F . Thus the coherence for the Cf measurement does not vary (factor of <4) with reactivity as much as the coherence for the two-detector noise measurement.

(factor from 10 to 10^3). Although it is advantageous to have the Cf source stronger than the inherent source, further increases in the size of the Cf source are not beneficial because there is no further improvement in the ratio F_c/F . The estimated measurement times to achieve a given precision, obtained from these values of the coherence (γ_{12}), are somewhat longer than those given by Seifritz.

REACTIVITY DETERMINATION FROM FREQUENCY DOMAIN MEASUREMENTS

The methods for determination of the reactivity from the power spectral densities given by Eqs. (20) to (23) can be classified into three categories: (1) the existing methods related to the conventional two-detector auto- and cross-power spectral densities, G_{22} , G_{33} , and G_{23} ; (2) those that determine parameters which have been used previously for reactivity determinations from the cross power spectral densities with Cf, G_{12} ; and (3) new methods that determine the reactivity without a calibration near delayed criticality.

The use of previously described methods, ^(9,10,11,12,28) such as the determination of the reactivity from the breakfrequency of APSDs and CPSDs or from the coherence amplitude, γ_{23} , requires a knowledge of the breakfrequency and coherence amplitude at some reference state, usually near delayed criticality, for which the reactivity is known, as well as the change in the prompt-neutron generation time or detection efficiency between the reference state and the reactivity state of interest. The breakfrequency noise analysis methods require corrections for the frequency response functions of the detection system, $h_2(\omega)$ and $h_3(\omega)$, which are usually obtained from measurements in which the detection system is exposed to the random emission of a neutron source [this input

Fig. 2. Coherence as a function of reactivity for various detection efficiencies.

is a white noise source, and thus, the measured frequency response will be $h_1(\omega)$. Both methods of reactivity determination are difficult to apply in the initial startup of a reactor because they require a calibration near delayed criticality.

After a correction is made for the frequency response of the detection systems, the CPSD, G_{12} , is proportional to the source transfer function of the reactor, $[(\alpha + i\omega)\Lambda]^{-1}$, and $|G_{12}|^2$ can be fitted by a least-squares method to determine the value of α . Since the source transfer function can be written as $(\alpha - i\omega)/(\alpha^2 + \omega^2)\Lambda$, G_{12} has a real and an imaginary part which can be obtained by present digital Fourier analyzers. Since the values of ω are known, the product of the ratio of the real to the imaginary part of G_{12} and the frequency ω is constant and equal to the prompt-neutron decay constant. In principle, a high precision measurement at one frequency should yield the prompt-neutron decay constant. The subcritical reactivity can be obtained from the ratio of this decay constant to that for the calibration state after it has been corrected for the change in the neutron lifetime from the calibration state to the subcritical state of interest. In practice, this product would be obtained over a wide range of frequencies. The constancy can be used to determine the decay constant, without least-squares analysis, and to validate the point kinetics model since the presence of higher modes would cause the value of α determined in this way to be a function of frequency. The constancy of the ratio would also verify whether the correction for the frequency response of the instrumentation is correct.

Various combinations of spectral densities were examined in order to determine if any additional information could be obtained from the

simultaneous measurement of all four spectral densities represented by Eqs. (20), (22), and (23). To develop a method for measuring the reactivity, consider the following combination of spectral densities:

$$\frac{G_{12}^* G_{13}}{G_{11} G_{23}} = \frac{1}{B_c (1 + C_\alpha)} \left(\frac{\bar{v}_c I_c}{\bar{v} I} \right)^2 \frac{1}{RX'} \frac{F_c}{F} . \quad (32)$$

Note that γ_{12}^2 , as given in Eq. (26), is the product of this expression [Eq. (32)] and $[1 + 1/Q_2(\omega)]^{-1}$.

The ratio of spectral densities given in Eq. (32) is independent of frequency. Thus, the constancy of Eq. (32) as a function of frequency could be used to validate the model as well as to determine whether the experimental data from the noise analysis measurement have adequate precision. Since $F\bar{I}\bar{v} = (F_c I_c \bar{v}_c + F_i I_i \bar{v}_i) \frac{k}{1-k}$, Eq. (32) can be rearranged to yield the subcritical reactivity ρ in Δk units, where k is the neutron multiplication factor including the effect of delayed neutrons. Thus,

$$\frac{1-k}{k} = \frac{G_{12}^* G_{13}}{G_{11} G_{23}} \frac{(F_c I_c \bar{v}_c + F_i I_i \bar{v}_i)}{(F_c I_c \bar{v}_c)} \frac{\bar{I}\bar{v}}{I_c \bar{v}_c} B_c (1 + C_\alpha) X' R . \quad (33)$$

No correction for the frequency response of the instrumentation is required, because the transfer functions for the detection system electronics are canceled in forming the ratio given by Eq. (32).

Eq. (33) does not depend on detection efficiency, but it does depend on the properties of the detection system for ^{252}Cf fission which can be determined outside the reactor. The ratio of the effective neutron production rates can be determined from the change in count rate when the ^{252}Cf source is inserted into the subcritical reactor. The values of R

and X' can be determined from calculation and other measurements; both are insensitive to large changes in reactivity. The values of the numbers of neutrons per fission are known and the relative importance I/I_c can be calculated or obtained from other measurements. Thus, solution of Eq. (33) gives the neutron reactivity or multiplication factor in k units. This method requires neither a calibration near delayed criticality nor corrections for neutron lifetime and detection efficiency changes, which other noise analysis methods do require.

For a pulse mode electronics system and a ^{252}Cf chamber in which all the alpha pulses can be discriminated, $C_\alpha = 0$ and $B_c = 1$ and Eq. (32) reduces to $\left(\frac{\bar{v} I_c}{\bar{v} I}\right)^2 \frac{1}{RX'} \frac{F_c}{F}$. Thus, for pulse mode operation of the californium ionization chamber the reactivity does not depend on the properties of the californium fission detection system.

The reactivity can be determined in another way by considering the ratio of the real part of G_{12} to the magnitude of G_{23} . The CPSD's G_{12} require correction for the frequency response of the instrumentation, which is measured with the californium source and detection system 2 (isolated from the reactor or significant amounts of moderator that could introduce a frequency dependence other than that of the detection system electronics). The corrected spectral density is

$$G_{c12} = \frac{\bar{v} F_c I_c J_2 H_s(\omega)}{\bar{v} F I J_{A_2}} \quad (34)$$

where J_2 and J_{A_2} are the current from detection system 2 in the reactor measurement and the calibration measurement, respectively.

The squared magnitude of the CPSD G_{23} corrected for the frequency response of the instrumentation is

$$|G_{c23}|^2 = \left(\frac{J_2 J_3}{F} RX' \right)^2 \frac{\bar{q}_{A_2}}{J_{A_2} J_{A_3} \bar{q}_{A_2}^2} \frac{\bar{q}_{A_3}}{\bar{q}_{A_3}^2} |H_S(\omega)|^4. \quad (35)$$

The ratio of the real part of G_{c12} to the magnitude of G_{c23} is independent of frequency and is given by

$$\frac{\text{Re}(G_{c12})}{|G_{c23}|} = \left(\frac{1 - k_p}{k_p} \right) \frac{I_c \bar{v}_c}{I \bar{v}} \left[\frac{J_{A_3} \bar{q}_{A_2}^2 \bar{q}_{A_3}^2}{J_{A_2} \bar{q}_{A_2} \bar{q}_{A_3}} \right]^{1/2} \frac{1}{RX'} \frac{F_c}{\bar{q}_3^{FW_3}}, \quad (36)$$

where k_p is the prompt-neutron multiplication factor, and $\alpha\Lambda = (1 - k_p)/k_p$.

Equation (36) can be reduced to an expression obtained previously in the theory of the time domain measurements with pulse mode electronics except for the modified dispersion number X' , which was not incorporated in the theory of the time domain measurement. For pulse mode electronics all values of q equal unity and no correction for instrumentation response is required in the time domain measurement since fission chamber response function is not a function of frequency up to frequencies beyond the range of interest. Thus, the term in brackets is unity. Now since $W_3 F = W_3^1 F_c$, this ratio, Eq. (36), is

$$\frac{\bar{v}_c}{\bar{v}} \frac{I_c}{I} \frac{1}{RX' W_3^1} \left(\frac{1 - k_p}{k_p} \right), \quad (37)$$

which is within a factor of two of the ratio of correlated counts per Cf fission to the correlated counts per count in a two-detector Rossi- α

measurement obtained previously [Eq. (8)]. The factor of two arises since the transformation of the cross-correlation function to the frequency domain removes the factor of two present in the time correlated term of the cross-correlation factor.

To obtain the ratio, Eq. (36), with better precision, the integrals of G_{12} and G_{23} over frequency can be used. This ratio can be large for a low detection efficiency. The unknown quantities in Eq. (36) are the current $(\bar{q}_3 F' W'_3)$ and the currents and charges in the calibration measurement, which can be measured. The other quantities on the right side of Eq. (36) are known constants, except for the spatial effects factor R and the ratio of the importance of the neutrons from Cf fission to those from reactor fission.

CONCLUSIONS

The new methods of reactivity determination proposed in this paper enable the determination of subcriticality without a knowledge of the properties of the reactor or critical assembly at delayed criticality.

Measurement in the time domain requires a determination of the ratio of correlated counts per californium fission to correlated counts per count in a Rossi- α measurement. This measurement also requires knowledge of the spontaneous fission rate of the californium source, and, thus the intensity of the californium source is limited by counting losses and the ability of the instrumentation system to distinguish between alpha decay and spontaneous fission of californium. In addition values of $\bar{v}_c$, $\bar{v}$, I_c , I , R , and X' , which can be obtained from other measurements, are needed to determine the prompt reactivity. The limit of source intensity ($\sim 10 \mu\text{g}$, or 6×10^6 fissions/sec) restricts this method

to applications in critical experiments or reactors with small inherent sources. For application to LMFBRs, the source intensity would have to be $>200 \mu\text{g}$, which would necessitate operation in the current mode, thus eliminating the possibility of time domain measurements.

The frequency domain measurements that are analogous to the time domain results yield the ratio of the real part of the CPSD with californium to the absolute value of the CPSD between the pair of neutron detectors. In addition to the parameter required for the time domain measurement, this method requires a calibration measurement with a white noise source to determine the frequency response functions for the instrumentation system. These response functions could be measured outside the reactor before the instrumentation is installed in the reactor, but once installed, the response functions must not change. With current mode electronics, higher intensity, larger sources could be used and the method would be useful for reactors with inherent sources of higher intensity.

The difficulty of the preceding method caused by the requirement of measurement of the frequency response functions for the instrumentation systems can be avoided by the other frequency domain method that measures the ratio $G_{12}^* G_{13} / G_{11} G_{23}$. This ratio is frequency independent, and the required properties of the californium chamber can be determined outside the reactor. This method also requires the ratio of the neutron production rates with and without the californium source in the reactor. This ratio can be determined from the ratio of count rates from the source range monitoring system for the reactor with and without the californium source in the reactor.

These methods assume that the application of the point kinetics equations to the subcritical state of interest is valid. However, they do provide verification of this assumption in each measurement.

Since these methods do not require calibration at a known reactivity state, they can be used in the initial loading of a reactor where subcriticality determinations cannot depend on some calibration near delayed criticality. They may also be useful in determining the reactivity of assemblies where sufficient fissionable material to achieve criticality is not available. These methods do require a detector with high efficiency.

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