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Bootstrap Scheme for Particle Interactions Based on an Eigenvalue Condition for the Fine Structure Constant*

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We propose a bootstrap scheme for particle interactions based on an eigenvalue condition for the fine structure constant α in non-abelian gauge theories. It is assumed that the new renormalization-group equations derived by Weinberg, if we impose the proposed bootstrap conditions, allow a non-trivial bootstrap solution besides the trivial free field solution. All the dimensionless quantities in the bootstrap scheme then become calculable in principle. In particular, there is a direct way of calculating α assuming perturbation methods are applicable. The validity of the bootstrap assumptions and the reliability of the perturbation methods can be tested in deep-inelastic lepton-hadron scattering experiments. We also discuss the possible origin of the various interactions, the spontaneous breaking of the gauge symmetries as well as the unification of the leptons with the hadrons.

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DISCLAIMER

I. Introduction

The idea that the fundamental dimensionless coupling constants of a theory can be determined within the framework of the theory is a very attractive one. It is certainly a requirement of any bootstrap theory.[1] Historically there have been many interesting attempts[2] to determine the fine structure constant α. In their classic paper on renormalization-group methods,[3] Gell-Mann and Low discovered a remarkable eigenvalue condition in quantum electrodynamics (QED), although it is an eigenvalue condition on the unrenormalized coupling constant α_o rather than α. Their analysis has been further developed by Johnson, Baker and Willey in the program of finite quantum electrodynamics.[4] Recently Adler extended these earlier works and established the possibility of an electrodynamic determination of α through an eigenvalue condition.[2]

The expected difficulty involved in actually carrying out such a fundamental calculation[5] has prevented a direct test of the validity of the proposed eigenvalue condition so far. In the meantime the renormalization-group techniques were applied to the non-abelian gauge field theories and culminated in the establishment of their asymptotic freedom by Gross and Wilczek[6,7] and Politzer.[8] Georgi and Glashow[9] further pointed out the attractive possibility, in a SU(5) invariant gauge theory, that α is the only coupling constant for all the elementary particle forces, the strong interactions being associated with the infrared behavior of the non-abelian gauge theories.[7.10] It is then a natural question to ask whether α can be determined through an eigenvalue condition of a non-abelian gauge theory rather than an eigenvalue condition of QED alone.

In the following we shall show that the non-abelian gauge theories offer a unique possibility that both α and α_o are determined through eigenvalue conditions. These eigenvalue conditions can serve as the basis for a bootstrap scheme. Specifically we assume that the new renormalization-group equations derived by Weinberg, if we impose these eigenvalue conditions, allow a non-trivial bootstrap solution besides the trivial free field solution. All the dimensionless quantities in the bootstrap scheme then become calculable in principle. In particular, there is a direct way of calculating α assuming perturbation methods are applicable. The validity of the bootstrap assumptions and the reliability of the perturbation methods can be tested in deep-inelastic lepton-hadron scattering experiments. We also discuss the possible origin of the various interactions, the spontaneous breaking of the gauge symmetries as well as the unification of the leptons with the hadrons.

II. The Renormalization-Group Equations

Consider a renormalizable field theory characterized by a single unrenormalized coupling constant g and a single unrenormalized mass m. (We shall see that these limitations can be eliminated in a bootstrap situation.) We also assume that there are no elementary spin-zero mesons in the theory for the technical reason that Weinberg's approach be applicable. We shall follow Weinberg's formulation of the new renormalization-group equations[11] and use the same notations and refer the reader to the original article for systematic developments. The renormalization procedure he used is based on prescriptions for the value of certain selected Green's functions for $m = 0$ (even though we are considering the case when m may not be zero) at certain selected non-zero four momenta characterized by a single arbitrary scale parameter μ with the dimension of mass. Weinberg[11] showed that the "mass-zero" renormalization prescription has the crucial advantage that the renormalization-group equations derived are exact equations not just asymptotic equations even when $m \neq 0$, if there are no spin-zero mesons in the theory. The conclusions we draw from these equations are therefore independent of which way of summing the perturbation series is correct. This is particularly important for QED, where as shown by Adler,[2] summing the perturbation series "vacuum-polarization-insertion-wise" or "loopwise" leads to different results.

Using Weinberg's renormalization prescription at "zero mass", the renormalized coupling constant g_R and mass m_R are given by[11]

$$g_R = g_R(g, \mu/\Lambda) \tag{1}$$

$$m_R = m\, Z_\theta^{-1}\ (g, \mu/\Lambda) \tag{2}$$

where θ is the mass operator in the Lagragian and Λ is the cut-off. In order to facilitate the comparison between non-abelian gauge theories and QED, we introduce the following notation:

$$\alpha_o \equiv \frac{e_o^2}{4\pi} \equiv \frac{g^2}{4\pi} \tag{3}$$

$$\alpha \equiv \frac{e^2}{4\pi} \tag{4}$$

$$g_\infty \equiv g_R(e_o,\ x = \infty) \tag{5}$$

$$e \equiv g_R(e_o,\ x = 0) \tag{6}$$

where $x = \mu/\Lambda$. In QED, because of the Källén-Lehmann[12] spectral representation for the photon propagator and the positivity condition, we have

$$e_o = g_\infty \tag{7}$$

and

$$\alpha_o \geq \alpha\ . \tag{8}$$

The equality in (8) holds only if $\alpha_o = \alpha = 0$. We also list explicitly the conventional functional form for g_R:

$$\text{for QED:} \quad g_R(e_o,x) = Z_3^{\frac{1}{2}}(e_o,x)\, e_o \tag{9}$$

$$\text{for non-abelian gauge theories:} \quad g_R(e_o,x) = Z_3^{3/2}(e_o,x)\, Z_1^{-1}(e_o,x)\, e_o \tag{10}$$

For definitions of the Z's see Ref. 11 and Ref. 7.

Given any unrenormalized Green's function $\Gamma(p,g,m,\Lambda)$, we can form a Λ-independent renormalized Green's function:[11]

$$\Gamma_R(p,g_R,m_R,\mu) = Z_\Gamma(g,\mu/\Lambda)\, \Gamma(p,g,m,\Lambda) \; . \tag{11}$$

The renormalization-group equation for Γ_R, valid for all K (a momentum scale variable), in the Landau gauge is[11]

$$\{K \frac{\partial}{\partial K} - \beta(g_R) \frac{\partial}{\partial g_R} + [1 + \gamma_\theta(g_R)]\, m_R \frac{\partial}{\partial m_R} - D_\Gamma + \gamma_\Gamma(g_R)\}\, \Gamma_R(Kp_o,g_R,m_R,\mu) = 0 \tag{12}$$

where

$$\beta(g_R) = x \frac{\partial}{\partial x} g_R(g,x) \tag{13}$$

$$\gamma_\theta(g_R) = x \frac{\partial}{\partial x} \ln Z_\theta(g,x) \tag{14}$$

$$\gamma_\Gamma(g_R) = x \frac{\partial}{\partial x} \ln Z_\Gamma(g,x) \tag{15}$$

and D_Γ is the dimension of Γ in powers of mass, p_o is a set of fixed momenta. As stressed by Weinberg[11] the coefficients (13)-(15) must be independent of $x = \mu/\Lambda$ and Eqs. (12)-(15) are exact for arbitrary momenta even if $m \neq 0$.

III. The Eigenvalue Conditions and the Bootstrap Assumptions

The eigenvalue conditions are obtained from the Gell-Mann-Low equation,[3] Eq. (13), which can be put in the Gell-Mann-Low form:

$$\ln \frac{x}{x_o} = \int_{g_R(g,x_o)}^{g_R(g,x)} \frac{dz}{\beta(z)} \quad . \tag{16}$$

Integrating the right-hand side of (16), we have

$$\ln \frac{x}{x_o} = F[g_R(g,x)] - F[g_R(g,x_o)] \tag{17}$$

or

$$F[g_R(g,x)] - \ln x = F[g_R(g,x_o)] - \ln x_o \quad . \tag{18}$$

The left-hand side of (18) is independent of x, therefore it must be a function of g only:

$$F[g_R(g,x)] - \ln x = \frac{1}{f(g)} \quad . \tag{19}$$

The exact functional form of f(g) is not known, but one may be able to study it in perturbation theory (see discussions below). Inverting to solve for g_R, we have

$$g_R(g,x) = F^{-1}(y), \quad y \equiv \frac{1}{f(g)} + \ln x \ . \tag{20}$$

Follow the general discussions of Wilson,[13] finite limits of F^{-1} at $y = +\infty$ $(x = \infty)$ and $y = -\infty$ $(x = 0)$ exists if there are ultraviolet (UV) - and infrared (IR) - stable fixed points of the renormalization group. They are the zeroes of β. If we denote the UV - and IR - stable fixed points by α_+ and α_- respectively, we have (if $\alpha_\pm$ eixst)

$$g_\infty = F^{-1}(+\infty) = \alpha_+ \tag{21}$$

$$e = F^{-1}(-\infty) = \alpha_- \tag{22}$$

and

$$\beta(\alpha_\pm) = 0 \quad . \tag{23}$$

We would like to point out that there is an additional possibility where the unrenormalized coupling constant $e_0 \equiv g$ satisfies an eigenvalue condition

$$f(e_0) = 0 \quad . \tag{24}$$

Then from Eq. (20) we have the situation where $g_R(g,x)$ becomes independent of x and identically equal to one of the zeroes of β:

$$g_R(g,x) = g_\infty = e \tag{25}$$

$$\beta(e) = 0 \tag{26}$$

The possibility was characterized by Adler[2] as type—1 asymptotic behavior in the context of QED. Unfortunately because of the restrictions of (7) and (8), which holds for QED, we see that type —1 asymptotic behavior is possible for QED only if[14]

$$e = e_o = 0 \tag{27}$$

which means, among other things, that for QED $f(e_o = 0) = 0$. This latter fact is, of course a general property of $f(g)$. When $g = 0$, we have a free field Lagrangian, which clearly has type —1 asymptotic behavior, because we can solve the theory exactly without even introducing Λ. The obvious property of $f(g)$ that

$$f(g = 0) = 0 \quad , \tag{28}$$

however, will turn out to be useful in the following.

Whether type —1 asymptotic behavior is possible in non-abelian gauge theories for a non-trivial value of the physical coupling constant e is an open question, because the requirements (7) and (8) may not necessarily be true for non-abelian gauge theories in general. If (7) is not ture, then e_o does not have to be the same as e even if type —1 asymptotic behavior is assumed. If (8) is not ture, then e_o can be smaller than e. It then allows the possibility of a real bootstrap situation with $e > 0$ but $e_o = 0$! From (28), we must have type —1 asymptotic behavior in this case.

We conjecture that the bootstrap possibility is the one chosen by nature. In other words, we assume that both e and e_o satisfy eigenvalue conditions:

$$\beta(e) = 0 \tag{29}$$

and

$$f(e_o) = 0 \quad . \tag{30}$$

The free-field solution $e_o = e = 0$ always satisfies these bootstrap conditions. The existence and uniqueness of non-trivial solutions depends on the number of non-trivial zeroes of the functions f and β. The most attractive possibility is (i) for β to have one (and hopefully only one) non-trivial zero given by the fine structure constant:

$$\beta(e) = 0 \qquad \frac{e^2}{4\pi} = \alpha \approx 1/137 \tag{31}$$

for a particular non-abelian gauge theory (and hopefully in no other theory will β have a non-trivial zero), and (ii) for $f(e_o)$ to have no zero other than $e_o = 0$. The non-trivial bootstrap solution is then unique. In this bootstrap theory, we have the type –1 asymptotic behavior:

$$g_R(e_o,x) = e \neq 0 \quad . \tag{25}$$

We would like to illustrate the difference between the bootstrap theory and the asymptotically free theory (which shall be denoted as type –2 asymptotic behavior, following Adler[2]) by an elementary example. Consider the simple form

$$g_R(e_o,x) = \frac{e}{1 + e_o\, x^e} \tag{32}$$

which correspond to $\beta(g_R) = g_R(g_R - e)$ and $f(e_o) = \frac{e}{\ln e_o}$! If $e_o > 0$, we have the physical coupling constant $g_R(e_o, x = 0) = e^o$ and $g_\infty = 0$, namely an asymptotically free theory (the power rather than logarithmic behavior of g_R as $x \to \infty$ comes from the simple zero of β we used in order to have a closed form expression for g_R) with type —2 asymptotic behavior: $g_R(e_o, x)$ depends on x. In this case e_o is an arbitrary parameter. But if $e_o = 0$, then $g_R(e_o, x) = e$ for all x, and we have type —1 asymptotic behavior. In both cases α is given by the zero of β.

The unrenormalized gauge parameter ξ is introduced solely to fix the gauge[7] when the gauge invariant interaction is non-zero. If the bootstrap conditions are satisfied with $e_o = 0$, clearly there is no need for introducing a gauge parameter at all. Hence, the Landau gauge where the renormalized gauge parameter ξ_R and ξ both vanish is the correct gauge to use for bootstrap theories. For type —2 asymptotic beahvior where $e_o \neq 0$ is an arbitrary parameter, in order to have a unique condition for calculating α, we must demand $Z_3^{-1}(e_o, \mu/\Lambda) \to 0$ at infinite cut-off so that

$$\xi_R = Z_3^{-1}(e_o, \mu/\Lambda)\, \xi \to 0 \text{ as } \Lambda \to \infty$$

independent of ξ![7] Again the Landau gauge gives the correct result.

The bootstrap conditions have now fixed all dimensionless constants in the theory. The only remaining parameter is the unrenormalized mass m. To study m, we examine the behavior of Z_θ when type —1 asymptotic behavior is assumed. From (14)

$$Z_\theta(e_o,\mu/\Lambda) = C(e_o)(\mu^2/\Lambda^2)^{\gamma_\theta(e)/2} \quad . \tag{33}$$

Therefore if $\gamma_\theta > 0$, $Z_\theta \to 0$ in the limit of infinite cut-off. The un-renormalized mass will vanish for finite m_R:

$$m = m_R\, Z_\theta = 0 \quad . \tag{34}$$

This is the possibility pointed out by Johnson, Baker and Willey in finite QED.[4]

Together with the conditions on the coupling constants and gauge parameters we have the remarkable bootstrap situation where e_o, m, ξ all vanish. In other words we could shart from a free-field Lagrangian for massless particles! One can perhaps understand the situation in the following way. There is always the free-field solution to the renormalization group equations (12)-(15), namely $e = e_o = 0$ and $m_R = m = 0$. But when there is a non-trivial zero of β, we have the alternative non-trivial bootstrap solution where $e \neq 0$ and $m_R \neq 0$ although $e_o = 0$ and $m = 0$. We shall come back to this point later to discuss the possible origin of symmetry breaking.

When we have the type —1 asymptotic behavior, the renormalization group equation (12) simplifies:

$$\{K \frac{\partial}{\partial K} + [1 + \gamma_\theta(e)]\, m_R \frac{\partial}{\partial m_R} - D_\Gamma + \gamma_\Gamma(e)\}\, \Gamma_R\,(Kp_o,e,m_R,\mu) = 0 \ . \tag{35}$$

Similar situation holds for the experimentally more interesting quantities, the coefiicient functions in a Wilson operator-product expansion.[15] Following Weinberg[16] we divide the external lines of a Green's function Γ into two sets, A and B, with corresponding sets of four momenta k and q. The Wilson operator-product expansion, when the various components of k tend to infinity with fixed ratios and with q fixed, is

$$\Gamma_{AB,R}(k,q,g_R,m_R,\mu) \sim \sum_{O} U_{AO}(k,g_R,m_R,\mu)\ \Gamma_{BO}(q,g_R,m_R,\mu) \tag{36}$$

where O runs over all local renormalized operators. The renormalization group equation satisfied by the coefficient function U_{AO} has been derived by Weinberg:[11]

$$\{K\frac{\partial}{\partial K} - \beta(g_R)\frac{\partial}{\partial g_R} + [1+\gamma_\theta(g_R)]\,m_R\frac{\partial}{\partial m_R} - D_{AO} + \gamma_A(g_R) - \gamma_O(g_R)\}\,U_{AO} = 0 \ . \tag{37}$$

When type—1 asymptotic behavior is assumed, we have

$$\{K\frac{\partial}{\partial K} + [1+\gamma_\theta(e)]\,m_R\frac{\partial}{\partial m_R} - D_{AO} + \gamma_A(e) - \gamma_O(e)\}\,U_{AO} = 0 \tag{38}$$

The solution to Eqs. (35) and (38) for all K is

$$\Gamma_R(Kp_o,e,m_R,\mu) = K^{D_\Gamma - \gamma_\Gamma(e)}\ \Gamma_R(p_o,e,m(K),\mu) \tag{39}$$

and

$$U_{AO}(Kk_o,e,m_R,\mu) = K^{D_{AO} - \gamma_A(e) + \gamma_O(e)}\ U_{AO}(k_o,e,m(K),\mu) \tag{40}$$

where

$$m(K) = K^{-[1+\gamma_\theta(e)]}\ m_R \ . \tag{41}$$

IV. The Validity of Perturbation Expansion, the Origin of Symmetry Breakings and Interactions

The derivation of all the results up to this point depends only on the renormalizability of the theory but not on the convergence of the perturbation series. In order to test the bootstrap conjecture theoretically (in the sense of formulating a definite strategy for calculating α) and experimentally, we shall now assume the perturbation expansion for β and the γ's about the origin converges rapidly and with a radius of convergence $r > e$. Although we cannot prove this drastic assumption (it is not ture for QED[2]), we want to argue that it can turn out to be true in non-abelian gauge theories.

In QED there are two reasons for the perturbation method of estimating β (hence probably the γ's also) to break down when type -1 asymptotic behavior is assumed. First of all type -1 asymptotic behavior implies that β vanishes with a zero of infinite order, namely an essential singularity, if it vanishes at all at a non-zero value of its argument. This is proven by Adler[2], the proof however depends on Z_3 being a constant independent of x when $e = e_o \neq 0$. The full photon propagator then becomes exactly equal to the free photon propagator (with coupling constant e_o). For non-abelian gauge theory Z_3 does not have to be a constant even if type -1 asymptotic behavior is assumed, because in the definition of g_R, we have the product $Z_3^{3/2}\ Z_1^{-1}$ not $Z_3^{\frac{1}{2}}$ alone as in QED. The second reason for the break down of the perturbation estimate in QED, is the explicit form of β[13]

$$\beta(x) = \frac{1}{6\pi^2} \left[x^2 + \frac{3}{16\pi} x^3 + \ldots\right] .$$

The first two terms are of the same sign and the magnitude of the coefficient of the second term is much smaller than that of the first term. Therefore even if the series were convergent, many higher order term must contribute to give β a zero at α since α is so small. The lowest order term of β (which is independent of the renormalization prescriptions) for general non-abelian gauge theories has been calculated by Gross and Wilczek[6,7] and Politzer[8]

$$\beta(g_R) = -\frac{g_R^3}{16\pi^2} \left[\frac{11}{3} C_2(G) - \frac{4}{3} T(R)\right] + O(g_R^5) \tag{42}$$

where the notation is that of Gross and Wilczek.[7] The first term in the square bracket is the contribution from the gauge vector bosons and the second term is the contribution from the fermions in the theory. As pointed out by Gross and Wilczek, we clearly can have the situation, for some special gauge group and fermion representations, that the two contributions almost cancel each other completely. If there is no similar cancelation at higher orders the coefficient of the g_R^3 term may become as small as α times the coefficient (not yet calculated) of the g_R^5 term and with opposite sign. In this case it is possible for β to have a zero at α and a rapidly convergent perturbation series.

From this point of view one can understand why the symmetry must be broken, if one assumes α is the <u>only</u> possible non-trivial zero of the function β. Because if the symmetry is not broken, then all the fermion fields in the Lagrangian are exactly the same. If there are N fermion fields in the theory, then the gauge group is SU(N) (and U(1) which correspond to the fermion number gauge group) and the fermion fields form a single irreducible fundamental representation of SU(N). From (42), with $C_2(G) = N$ and $T(R) = \frac{1}{2}$ for this case,[7] the contribution of the gauge bosons is far from being canceled by the contribution of the fermions whatever the value of N is. The only way to achieve the near complete cancelation needed for a small α, is to have a much smaller gauge group $SU(N')$, $N' < N$ with different types of fermion fields grouping into several separate rerpresetations of $SU(N')$. Notice that we cannot directly break SU(N) to the actual experimental situation where the only exact gauge group is the abelian U(1) group corresponding to charge conservation. We already showed that QED does not allow a non-trivial type—1 solution to the renormalization group equations. Hence $N' > 1$. The symmetry breaking must come in two stages:

At the first stage the gauge symmetry $SU(N')$ is exact: Fermion fields belong to the same representation have the same m_R and the same physical mass m_p, which is determined by the pole position of the corresponding full fermion propagator. Different fermion representations are differentiated by their differnce in m_R and m_p. We can describe the relation between m_R and m_p qualitatively. The full fermion propagator $S_F(p)$ for a given representation is given by (with the "mass-zero" renormalization prescription):

$$S_F^{-1}(p) = \not{p} - \Sigma(p) \tag{43}$$

where $\Sigma(p)$ is the corresponding fermion self-energy part. For "mass-zero" renormalization prescription Weinberg showed[11] that the cut-off dependent part of

$\Sigma(p)$ is independent of m and get multiplied by m, hence can be absorbed into m_R:

$$S_F^{-1}(p) = \not{p} - m_R\, \Sigma_1(p^2) - \not{p}\, \Sigma_2(p^2)$$

where Σ_1 and Σ_2 are cut-off independent. Hence

$$m_p = m_R \left. \frac{\Sigma_1(p^2)}{1 - \Sigma_2(p^2)} \right|_{p^2 = m_p^2} .$$

In particular $m_R = 0$ implies $m_p = 0$ (but not to converse, since Σ_1 may vanish). The gauge vector bosons are massless. The physical coupling constant is now fixed at α.

At the next stage, the gauge symmetry $SU(N')$ is broken spontaneously but dynamically.[17] (We do not have spin-zero Higgs bosons). The physical mass m_p within a given representation will now be different, because Σ_1 and Σ_2 develop parts that violate $SU(N')$ symmetry dynamically. The gauge vector bosons, except the photon, also acquire heavy masses which are related to the fermion mass differences.[17] The crucial fact is that m_R remain the same for a given representation, as it must, because m_R must be able to absorb the cut-off dependence independent of whether the symmetry is broken or not. This is the key to the renormalizability for the spontaneously broken gauge theories.[18]

What we have in mind is for the gauge vector bosons to mediate the weak, the electromagnetic and other weaker interactions but <u>not</u> the strong interactions. We associate, instead, the "strong" interactions of a given fermion field with the magnitude of its m_R: The larger the magnitude of m_R the stronger the interaction strength. The quotation marks on the word strong are to allow for the case when m_R is very small but non-zero for some fermions. The $SU(N')$ invariance of the "strong" interactions (m_R being the same for a given fermion representation) from this point of view is a natural consequence of the requirement that α satisfies the eigenvalue condition.

To study the feasibility of the origin of "strong" interactions suggested above, we turn to the infrared limit when $K \to 0$. Attributing the strong interactions to the infrared behavior of a non-abelian gauge theory where the effective coupling constant can become very large (infrared slavery) has been advocated by Weinberg[10] and Gross and Wilczek[7] and is crucial to the SU(5) gauge theory of Georgi and Glashow.[9] For the type—1 asymptotic behavior considered here, however, the coupling constant is fixed at α. But as emphasized by Weinberg[11], m_R plays only the role of a coupling constant in the "mass-zero" renormalization prescription. (We have already seen that m_R does not determine the pole position m_p, except when $m_R = 0$.) From (41) if $[1+\gamma_\theta(e)] > 0$ and $m_R \neq 0$, then

$$m(K) = m_R\, K^{-[1+\gamma_\theta(e)]} \sim \infty \quad \text{as } K \to 0.$$

Hence there is the possibility for $m(K)$ to serve as an effective coupling constant of "strong" interactions. The strength of the "strong" interactions of a given fermion will then depend on the magnitude of its m_R. Since m_R is proportional to m_p, it agrees with the general trend that hadrons are heavier than the leptons. However, because $m_R = 0$ implies $m_p = 0$, leptons, if by which we mean particles with <u>absolutely</u> no "strong" interactions, must be massless. Hence we cannot simply define "heavy leptons" as heavy particles with no strong interactions. The leptons and the hadrons are unified in the bootstrap scheme, the difference between the leptons and the hadrons is the difference in the magnitude of their m_R's. The muon and the electron being massive will have some "strong" interactions. The neutrinos will have no "strong" interactions only if they are strictly massless and do not belong to the same $SU(N')$ representation containing the electron or the muon. The best place to look for these exotic effects (presumably suppressed by ratios of m_R of leptons to m_R of quarks at least at low momentum) is obviously the muon, especially if it does not belong to the same $SU(N')$ representation containing the electron. The recent e^+e^- annihilation experimental results[19] and the muon-proton scattering experiment[20] are particularly intriguing in this respect.

V. Experimental Tests of the Bootstrap Assumptions and the Validity of the Perturbation Methods

We would like now to turn to the crucial question of testing the more conventional consequences of the bootstrap conditions experimentally. Assuming that the perturbation series converges rapidly, then the anomalous dimensions will take the form:[7]

$$\gamma(e) = \alpha \times (\text{power series in } \alpha)$$

with the coefficients of the power series exactly calculable and depending on the value of N' of the gauge group $SU(N')$ and the nature of the fermion representations. In any case the γ's are of order α:

$$|\gamma(e)| \sim \alpha \ll 1 \quad .$$

Hence $m(K) \sim 0$ as $K \to \infty$ and $m(K) \sim \infty$ as $K \to 0$.

The asymptotic behavior of the coefficient function as $K \to \infty$ becomes

$$U_{AO}(Kk_o, e, m_R, \mu) \sim K^{D_{AO} - \gamma_A(e) + \gamma_O(e)} \{U_{AO}(k_o, e, 0, \mu) + O[m(K)]\} \ .$$

(Weinberg[11] showed that the first three terms of an asymptotic expansion in $m(K)$ around $m(K) = 0$ are finite.) This is the behavior of broken scale invariance, although the deviation from canonical scaling is small because α is small.

In deep-inelastic lepton hadron scattering, $\gamma_A = 0$ since we are dealing with conserved or partially conserved currents.[7] The dominant operators in the scaling region are those of twist (equal to dimension minus spin) 2.

The leading term is the energy-momentum tensor $T_{\mu\nu}$, which has canonical dimensions, hence $\gamma_T = 0$. The leading term in the structure function therefore shows Bjorken scaling. Correction terms at high energy will exist (the correction terms proportional to m(K), being suppressed by a power of K: $K^{-[1 + \gamma_\theta(e)]}$, should be important only at low energy), if there are other operators $\tilde{O} \neq T_{\mu\nu}$ of twist 2 in the bootstrap scheme. In this case, the correction term will be of the form

$$U_{A\tilde{O}} \sim K^{D_{A\tilde{O}} + \gamma_{\tilde{O}}(e)}.$$

The anomalous dimension $\gamma_{\tilde{O}}(e)$ is calculable in a given model, and in general, if the perturbation estimates are correct, should be of order α.

Since the type —2 asymptotic behavior of the asymptotically free theories (UV - stable fixed point at $\alpha_+ = 0$, IR - stable fixed point at $\alpha_- = \alpha$) have correction terms involving powers of ℓn K rather than powers of K, we can distinguish the two cases experimentally. There is another possibility (although not a very attractive one in our opinion) that we should distinguish from the type—1 behavior. As shown above the correction terms in type —1 asymptotic behavior involve powers of K controlled by $\gamma(e) \approx \alpha$ independent of how β vanishes at e. The power behavior of the correction terms can be simulated if the UV - stable fixed point α_+ is a simple zero of β. However, in this case the physical coupling constant α_- must be different from α_+ in contrast to the type —1 case. There is then no reason for α_+ to be the same as α. The possibility that $\alpha_+ \neq \alpha$ is a simple zero of β will be denoted by type —3. Experimentally, we can distinguish type —1 from type —3 by measuring the magnitude of the γ's.

We can have the following interpretation of the possible experimental outcome. The SLAC - MIT results[21] suggest that at the highest SLAC energy the correction terms proportional to m(K) are no longer important. Thus

(i) Between say 2 GeV/c to 20 GeV/c: If the correction terms to Bjorken sealing seem to go away by a power of K rather than a power of $\ln K$, then depending on the measured value of γ_θ either type—1 or type—3 are favored. Precocious scaling would seem to suggest this possibility. If the correction terms behave more like a power of $\ln K$, then it suggests the type—2 behavior of asymptotically free theory.

(ii) From 20 GeV/c to highest available energy: If there are not new deviation from Bjorken scaling, it would correspond to the unlikely situation that there are no twist 2 operators with non-trivial anomalous dimensions, then we cannot differentiate type—1 from type—3 here. If there are deviations like a small power (or order α) of K, then type—1 asymptotic bahavior is the natural explaination. It would also suggest that perturbation expansions for β and the γ's are probably convergent. Needless to say, the deviation will be quite small if the power is as small as α. For example, between $k_1 = 20$ GeV/c and $k_2 = 300$ GeV/c, $(k_2/k_1)^\alpha \approx 1.02$, while $(\ln k_2/\ln k_1) \approx 1.90$. Hence, accurate experiments here can distinguish between all three types of asymptotic behavior.

VI. Summary

We propose a bootstrap scheme for particle interactions based on an eigenvalue condition for α in non-abelian gauge theories. Specifically we assume that in a bootstrap situation we can actually start from a free field Lagrangian of N massless fermion fields (in the sense that all the unrenormalized quantities e_o, m and ξ can be set equal to zero after renormalization). The N fermion fields must group into separate representations of a gauge group SU(N′) with $1 < N' < N$ in order for the function β to have a non-trivial zero. The physical coupling constant α is now fixed at the non-trivial zero of β. This eigenvalue condition serves to determine uniquely not only α but N, N′ and the nature of the fermion representations. The gauge symmetry SU(N′) is then spontaneously broken[17] to leave only a U(1) gauge group of electric charge unbroken. Fermion masses within the same representation are now split. The gauge vector bosons, except the photon, acquire heavy masses related to the fermion mass differences.

The bootstrap theory has type —1 asymptotic behavior. The validity of the bootstrap assumptions can therefore be tested experimentally in deep-inelastic lepton-hadron scattering experiments. One should also look for any evidence that the muon and the electron may have interactions that are qualitatively similar to the strong interaction.

Theoretically the important tasks are: (i) The difficult calculation of the coefficient of the g_R^5 term in the perturbation expansion of β using the "mass-zero" renormalization prescription. It will give us a good idea whether the function β can vanish when its argument is of the order α and what the values of N and N' might be. (ii) Study the mechanism with which SU(N) and SU(N') are broken spontaneously to obtain quantitative relations between the ratios of m_R between the different representations (when m = 0, one of the non-vanishing m_R's is arbitrary) and the relations between m_R and m_p within a given fermion representation.

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