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A GLAUBER-DECK MODEL FOR 3π PRODUCTION ON NUCLEI

by

R. T. Cutler

Department of Physics
University of Illinois
Urbana, Illinois 61801

Abstract

A Glauber theory is developed for the reaction $\pi^- A \rightarrow \pi^+ \pi^- \pi^- A$, where A is a nucleus, using a Reggeized Deck amplitude for 3π production on a nucleon. A simple absorption amplitude for the 3π system is proposed, and the resulting amplitude calculated for Carbon at 15.1 GeV/c, using a Monte Carlo approach. Alternative absorption schemes are discussed, including a multi-impact parameter model. The reaction $\pi^- C \rightarrow \pi^+ \pi^- \pi^- C_{4.44}^*$ is also calculated using an effective one particle excitation parametrization.

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I. INTRODUCTION

Several experimental groups have begun to explore pion production on nuclei at high energy. From CERN comes data on $\pi^- A \rightarrow \pi^+ \pi^- \pi^- A$ and $\pi A \rightarrow 5\pi A$ at $15.1 \text{ GeV}/c$ ^{1/}, where A, the target nucleus, ranges from Be to Pb. When a simple multiple scattering parametrization is applied to the 3π data^{2/}, one can extract the effective cross section, $\sigma_{3\pi}$, for the interaction between the 3π system and a nucleon, with the rather surprising result that $\sigma_{3\pi} \approx \sigma_{\pi} \approx 25 \text{ mb}$. Several models have been proposed to explain this anomalously low absorption.^{3/} These models typically take the view that drastic departures from conventional theory are required, and often have enough free parameters to fit almost any experimental data.

An Illinois group is analyzing data on $\pi^- C \rightarrow \pi^+ \pi^- \pi^- C^*$ at $6 \text{ GeV}/c$ ^{4/}, where C^* is the $J^P = 2^+$ excited state of Carbon at 4.44 MeV. The final state of the nucleus is observed by its characteristic gamma emission in coincidence with the event. In a previous article^{5/} (hereafter referred to as I) a Reggeized Deck model was used to calculate $\pi C \rightarrow 3\pi C^*$ (and also $\pi C \rightarrow 3\pi C$), resulting in a pleasing agreement with experiment for the shapes of the $M_{3\pi}$ and t distributions. However, this approach has two unattractive features; it ignores the known structure of the nucleus, treating Carbon as an elementary particle, and it does not treat correctly the absorption of the outgoing 3π system by nuclear matter.

In this paper, the Deck model will be applied to pion production on Carbon using Glauber multiple diffraction theory, in the hope that the simplifications of I may be removed. In Section II of this paper, a Deck amplitude for 3π production on nucleons will be used to derive an amplitude for $\pi C \rightarrow 3\pi C$, and the absorption of the 3π system treated in a plausible way which introduces no free

parameters. This approach will be seen to remove part of the discrepancy between theory and experiment for the coherent part of $\pi C \rightarrow 3\pi C$ in Section III. Alternative treatments of the 3π absorption will be discussed in Section IV. A simple parametrization for the excitation of the nucleus will be used to derive amplitudes for $\pi C \rightarrow \pi C^*$ and $\pi C \rightarrow 3\pi C^*$ in Section V. The excitation form factor will be fit to experimental results of $\pi C \rightarrow \pi C^*$, and the reaction $\pi C \rightarrow 3\pi C^*$ calculated in Section VI.

II. PION PRODUCTION IN GLAUBER THEORY, A SIMPLE MODEL

The Glauber theory of multiple diffraction^{6/} has been quite successful in calculating reactions like $\pi A \rightarrow \pi A$. To review briefly, the Glauber amplitude for elastic scattering at high energy and small momentum transfer is

$$F_{\pi A}(\vec{q}) = \frac{ik}{2\pi} \int d^2b \left(\prod_{k=1}^A d^3r_k \right) e^{i\vec{q} \cdot \vec{b}} |u(\vec{r}_1, \dots, \vec{r}_A)|^2 \left\{ 1 - \prod_{\ell=1}^A [1 - \gamma(\vec{b} - \vec{s}_\ell)] \right\}, \quad (1)$$

$$\gamma(b) \equiv \frac{1}{2\pi i k} \int f_{\pi N}(\vec{q}) e^{-i\vec{q} \cdot \vec{b}} d^2q, \quad (2)$$

where b is an impact parameter, $\vec{r}_k = (\vec{s}_k, z_k)$ are the coordinates of the k^{th} nucleon, and q is the transverse component of the momentum transfer, $t \approx -q^2$. If one makes the following approximations, valid for large nuclei and used throughout this report,

$$|u(\vec{r}_1, \dots, \vec{r}_A)|^2 \approx \prod_{m=1}^A \rho(r_m), \quad (3)$$

$$\begin{aligned} \int \gamma(\vec{b} - \vec{s}) \rho(\vec{s}, z) d^2s dz &\approx \int \gamma(s) d^2s \int \rho(\vec{b}, z) dz \\ &= \frac{2\pi i}{k} f(0) \frac{T(b)}{A} \\ &= \frac{\sigma(1-i\alpha)}{2} \frac{T(b)}{A}, \end{aligned} \quad (4)$$

with

$$T(b) \equiv A \int_{-\infty}^{\infty} \rho(b, z) dz,$$

$$\alpha \equiv \frac{\text{Re } f(0)}{\text{Im } f(0)},$$

$$1 = \int d^3r \rho(r),$$

$$\sigma' \equiv \sigma(1-i\alpha) \quad (5)$$

and uses

$$\left(1 - \frac{x}{A}\right)^A \approx e^{-x}, \quad (6)$$

the scattering amplitude becomes

$$F_{\pi A}(q) = \frac{ik}{2\pi} \int d^2b e^{i\vec{q} \cdot \vec{b}} \left(1 - e^{-\frac{\sigma'}{2} T(b)}\right). \quad (7)$$

Note that σ , σ' , $f(q)$, and α all refer to $\pi N \rightarrow \pi N$, with N a nucleon. This amplitude results in t distributions determined by the size of the entire nucleus, since the scattering from the nucleons is summed coherently. The incoherent process, where the final state of the nucleus is summed over, will not be considered in this paper, since it can be distinguished in the experimental data from the coherent contribution by its t distribution; it is, of course, absent in the case where C^* is observed.

Kölbig and Margolis^{7/} have suggested an extension of Glauber theory to reactions of the type $aA \rightarrow bA$, where a and b are different particles (e.g. $a = \pi$, $b = \rho$). They assume that $\sigma_{ab} \ll (\sigma_a \text{ and } \sigma_b)$, where σ_{ab} is the cross section for $aN \rightarrow bN$, and σ_a (σ_b) is the total cross section for particle a (b) on a nucleon, and thus they treat the production of b in first order.

The theory reported here is similar to that of Kölbig and Margolis, except for two assumptions designed to treat the 3π production on nucleons by a Deck mechanism:

- 1) A Deck amplitude^{8/} is used for $\pi N \rightarrow 3\pi N$.
- 2) The absorption amplitude for the 3π system is assumed to be the product of three separate one pion amplitudes, all at the same impact parameter. Alternatives to this assumption will be discussed in Section IV.

The starting point for this calculation is, then,

$$F_{3\pi C} = \frac{ik}{2\pi} \sum_{j=1}^A \int d^2b \left(\prod_{k=1}^A d^3r_k \right) e^{i\vec{q} \cdot \vec{b}} |u(\vec{r}_1, \dots, \vec{r}_A)|^2 \Gamma_{\pi, 3\pi}(\vec{b} - \vec{s}_j) \prod_{z_i < z_j} [1 - \gamma(\vec{b} - \vec{s}_i)] \prod_{z_\ell > z_j} [1 - \Gamma_{3\pi, 3\pi}(\vec{b} - \vec{s}_\ell)] , \quad (8)$$

with

$$1 - \Gamma_{3\pi, 3\pi}(\vec{b}) = [1 - \gamma(\vec{b})]^3 , \quad (9)$$

$$\Gamma_{\pi, 3\pi}(b) = \frac{1}{2\pi i k} \int f_{3\pi N}(q) e^{-i\vec{q} \cdot \vec{b}} d^2q . \quad (10)$$

In order to use a Deck amplitude for $\Gamma_{\pi, 3\pi}$, $f_{3\pi N}$ must be converted to an invariant amplitude. The nucleon will be treated as spinless in the following, and all momenta and angles will be in the lab system unless otherwise specified. The cross section for $\pi N \rightarrow 3\pi N$ is

$$d\sigma_{3\pi N} = \frac{1}{(2\pi)^8 2^{10} (P_{\pi N} M_N)^2} |m_{3\pi N}|^2 d^8\tau , \quad (11)$$

with

$$d^8\tau = \frac{dM_{3\pi}}{M_{3\pi}} dt d\alpha d\cos\beta d\gamma ds_1 ds_2 d\varphi \equiv dt d\varphi d^6\tau , \quad (12)$$

where t is the momentum transfer to the 3π system, α , $\cos\beta$, and γ are Euler angles describing the orientation of the three pions in the 3π center of mass, s_1 and s_2 are the Dalitz plot variables, and φ is the azimuthal angle of the 3π system. Then, changing variables from t to the production angles of the 3π system, one finds

$$\begin{aligned}
d\sigma_{3\pi N} &= \left(\frac{1}{(2\pi)^4 2^5 p_{\pi N} M_N} \right)^2 2p_{\pi} p_{3\pi} |\mathcal{M}_{3\pi N}|^2 d\Omega_L d^6\tau \\
&= |f_{3\pi N}|^2 d\Omega_L d^6\tau,
\end{aligned} \tag{13}$$

and so one has

$$\Gamma_{\pi, 3\pi}(\vec{b}) = \frac{1}{2\pi i k} \frac{\sqrt{2p_{\pi} p_{3\pi}}}{(2\pi)^4 2^5 p_{\pi N} M_N} \int \mathcal{M}_{3\pi N}(q) e^{-i\vec{q} \cdot \vec{b}} d^2q. \tag{14}$$

A Deck amplitude^{8,5/} is now used for $\mathcal{M}_{3\pi N}$:

$$\mathcal{M}_{3\pi N} = \sum_{\text{diagrams}} \mathcal{M}_{\pi\pi\pi} \mathcal{R} \mathcal{M}_{\pi N}, \tag{15}$$

$$\mathcal{R} = \frac{\left(\frac{s_R - u_R}{2s_0} \right)^{\alpha_{\pi}(t_R)} e^{-i \frac{\pi}{2} \alpha_{\pi}(t_R)}}{t_R - \mu^2},$$

$$\alpha_{\pi}(t_R) = t_R - \mu^2,$$

$$s_0 = 1 \text{ GeV}. \tag{16}$$

The diagrams summed over and conventions of notation are illustrated in Fig. 1.

A fit to experimental data, discussed in Ref. 8, is used for $\mathcal{M}_{\pi\pi\pi}$. Noting that

$$\gamma(b) = \frac{-i}{(2\pi)^2 2^2 p_{\pi N}^{CM} M_{\pi N}} \int \mathcal{M}_{\pi N}(q) e^{-i\vec{q} \cdot \vec{b}} d^2q, \tag{17}$$

where $p_{\pi N}^{CM}$ refers to the CM of the lower (πN) vertex, one finds

$$\frac{ik}{2\pi} \Gamma_{\pi, 3\pi}(b) = \frac{i\sqrt{2}}{(2\pi)^4 2^3} \sqrt{\frac{p_{3\pi}}{p_{\pi}}} \frac{p_{\pi N}^{CM} M_{\pi N}}{M_N} \sum_{\text{diagrams}} \mathcal{M}_{\pi\pi\pi} \mathcal{R} \gamma(b). \tag{18}$$

It has been assumed that $\sigma(\pi N \rightarrow \pi N)$ is a constant in the range of $M_{\pi N}$ of interest, and this model is thus similar in this respect to FIT 3 in I.

If one approximates the 3π absorption amplitude (9) by $(1-\gamma)^3 \approx 1-3\gamma$, then $F_{3\pi C}$ has the same form as that of Ref. 7, except that an explicit production mechanism is included. In this case $\sigma_{3\pi}/\sigma_{\pi} = 3$, which disagrees with the experimental value^{2/} of 1.0 to 1.2. However, the higher order terms may be evaluated by employing an approximate form for $f_{\pi N}(q)$ and using (2) through (5). If one writes

$$1 - \frac{\sigma' T(b)}{2A} \Delta \equiv \int [1 - \gamma(\vec{b} - \vec{s})]^3 \rho(s, z) d^2 s dz, \quad (19)$$

$$\sigma' \equiv (1 - i\alpha)\sigma,$$

and uses

$$f_{\pi N} = \frac{i\sigma' k}{4\pi} e^{-\frac{a}{2} q^2}, \quad (20)$$

then one finds

$$\Delta = 3 - \frac{3\sigma'}{8\pi a} + \frac{(\sigma')^2}{48\pi^2 a^2}. \quad (21)$$

The real part of Δ may be identified with $\sigma_{3\pi}/\sigma_{\pi}$ of the first order theory, and interpreted as an effective number of pions in the outgoing system. For $\sigma = 25 \text{ mb}$, $\alpha = -.2$, and $a = 8 \text{ GeV}^{-2}$, one finds $\Delta = 2.17 - 0.14i$. The 3π production amplitude (8) now becomes, using (18), (19) and (6) and summing the series as in Ref. 7,

$$F_{3\pi C}(\tau) = \frac{i\sqrt{2}}{(2\pi)^4 2^3} \sqrt{\frac{p_{3\pi}}{p_{\pi}}} \sum_{\text{diagrams}} \left(\frac{p_{\pi N}^{\text{CM}} M_{\pi N}}{M_N} \right) \mathcal{M}_{\pi\pi\pi} \mathcal{R} Z(t), \quad (22)$$

$$Z(-q^2) = \int d^2b \left(\frac{e^{\frac{\sigma' T(b)}{2}} - e^{-\Delta \frac{\sigma' T(b)}{2}}}{\Delta - 1} \right) e^{i\vec{q} \cdot \vec{b}}.$$

The cross section is then given by

$$d\sigma = \frac{1}{(2\pi)^8 2^6} \left| \sum_{\text{diagrams}} \left(\frac{p_{\pi N}^{CM}}{p_{\pi N}^M} \right) \mathcal{M}_{\pi\pi} \mathcal{R} Z(t) \right|^2 d^8\tau. \quad (23)$$

III. CALCULATION OF $\pi^- C \rightarrow \pi^+ \pi^- \pi^- C$

The primary t dependence of $F_{3\pi C}$ may be displayed by calculating $Z(t)$. $T(b)$ is calculated from a modified Gaussian wave function for Carbon^{9/}

$$\rho(r) = \frac{1}{\rho_0} \left(1 + \alpha \frac{r^2}{R^2}\right) e^{-\frac{r^2}{R^2}},$$

$$\rho_0 = \pi^{3/2} R^3 \left(1 + \frac{3}{2} \alpha\right), \quad (24)$$

with $\alpha = 1.12$ and $R = 1.71$ fm. The integration is done numerically and the results shown in Fig. 2. If the $\Delta = 1.1$ curve is identified with experiment, the theory is seen to remove a part of the discrepancy between the simplest theory, $\Delta = 3$, and experiment.

As in I, a Monte Carlo approach is used to display the full content of (23). The Monte Carlo calculation generates a random sample of 3π events which are distributed according to $d\sigma/d^8\tau$. These events can be analyzed in exactly the same way one would treat experimental data. Events were generated in $M_{3\pi}$ bins of 100 MeV, from .8 to 2.0 GeV at incident pion momentum 15.1 GeV/c. Approximately 1500 events were generated in each bin at about 5% efficiency. The resulting $M_{3\pi}$ distribution is compared to experiment in Fig. 3. The theoretical $M_{3\pi}$ spectrum has been multiplied by 3.0, in order to make the peaks of the two curves coincide. The shape of the $M_{3\pi}$ spectrum is seen to be well predicted.

Figure 4 shows the theoretical and experimental distributions of coherent events in t' , with $.9 < M_{3\pi} < 1.9$ GeV. The incoherent contribution to the experiment has been approximated by $d\sigma_I/dt' = I_0 e^{bt'}$, with $I_0 = 8.45 \text{ mb} \cdot \text{GeV}^{-2}$ and $b = 8.53 \text{ GeV}^{-2}$, and has been subtracted from the total experimental data. This

incoherent cross section is a fit by eye to the data with $t' > 1.4 \text{ GeV}^2$, as shown in Fig. 4, and should be considered an estimate. The resulting coherent cross section is, for $t' \gtrsim .05 \text{ GeV}^2$, very sensitive to the incoherent fit. However, there does seem to be evidence for a dip in the experimental data at $t' \approx .075 \text{ GeV}^2$, which (c.f. Fig. 2) is consistent with $\Delta = 1.1$, as claimed in Ref. 2. The ratio between calculation and experiment is roughly the same as the ratio of curves b) and d) in Fig. 2, as expected.

The Monte Carlo calculation allows other distributions to be displayed, but there is little difference in most spectra between this calculation and those reported in I and Ref. 8. In particular, the ρ band is very prominent, 56% of the Monte Carlo events having at least one $\pi^+ \pi^-$ mass within 100 MeV of $M_\rho = 770 \text{ MeV}$.

IV. ALTERNATIVES FOR 3π ABSORPTION

It may be argued that ρ production is so dominant that 3π absorption should be calculated as $\pi\rho$ absorption.^{10/} If the ρ -nucleon interaction^{11/} is approximated by $\sigma_{\rho N} = \sigma_{\pi N}$, then $(1 - \Gamma_{3\pi, 3\pi}(b)) = (1 - \gamma(b))^2$, resulting in $\Delta = 1.68 - .06i$. Curve c) in Fig. 2 shows $|Z(t)|^2$ for this value of Δ . A Monte Carlo calculation can now be done where the lower value of Δ is used for those diagrams where the $\pi^+ \pi^-$ from the upper vertex are in a rather wide ρ band, $670 < M_{\pi^+ \pi^-} < 870$ MeV, and the higher value of Δ is used otherwise. Approximately 2500 events were generated in each $M_{3\pi}$ bin, and the resulting t' distribution for $.9 < M_{3\pi} < 1.9$ GeV is shown in Fig. 4. For $t' \gtrsim .05$, the statistics of the Monte Carlo calculation are not good enough to cleanly separate the two theoretical curves. The effect of including $\pi\rho$ absorption is seen to be rather small, but again in the correct direction to reconcile theory and experiment.

It is also possible to formulate a more complicated theory by removing the assumption that all three pions leave the nucleus with the same impact parameter. It seems plausible to assign different impact parameters ($\vec{b}_1, \vec{b}', \vec{b}''$) to the produced pions, and another ($\vec{b}_0$) to the incident pion, as shown in Fig. 5. The usual identification $\ell = kb + \frac{1}{2} \approx kb$, valid for $k \gg q$, along with the provision that the nucleus not change its spin, yields

$$k(\hat{z} \times \vec{b}_0) \approx k_1(\hat{z} \times \vec{b}_1) + k_2(\hat{z} \times \vec{b}') + k_3(\hat{z} \times \vec{b}'') ,$$

$$\vec{b}_0 \approx \frac{k_1 \vec{b}_1 + k_2 \vec{b}_2 + k_3 \vec{b}_3}{k} . \quad (25)$$

A straightforward extension of Glauber formalism is

$$\begin{aligned}
\mathcal{M}_{3\pi C} &= \int d^2 b_1 d^2 b_2 d^2 b_3 e^{i(\vec{q}_1 \cdot \vec{b}_1 + \vec{q}_2 \cdot \vec{b}_2 + \vec{q}_3 \cdot \vec{b}_3)} \tilde{\mathcal{M}}_{3\pi C}(b_1, b_2, b_3) \\
\tilde{\mathcal{M}}_{3\pi C} &= \sum_{j=1}^A \int \left(\prod_{k=1}^A d^3 r_k \right) |u(\vec{r}_1, \dots, \vec{r}_A)|^2 \tilde{\mathcal{M}}_{\pi\pi}(b_3) \tilde{\mathcal{R}}(b_2) \tilde{\mathcal{M}}_{\pi N}(\vec{b}_1 - \vec{s}_j) \\
&\quad \prod_{z_i < z_j} [1 - \gamma(\vec{b}_0 - \vec{s}_i)] \prod_{z_\ell > z_j} [1 - \gamma(\vec{b}_1 - \vec{s}_\ell)] [1 - \gamma(\vec{b}' - \vec{s}_\ell)] [1 - \gamma(\vec{b}'' - \vec{s}_\ell)] , \\
\vec{b}_2 &\equiv \vec{b}' - \vec{b}_1 , \\
\vec{b}_3 &\equiv \vec{b}'' - \vec{b}' ,
\end{aligned} \tag{26}$$

where $\vec{q}_i$ is the momentum transfer conjugate to $\vec{b}_i$, and $(\tilde{\mathcal{M}}_{\pi\pi}, \tilde{\mathcal{R}}, \tilde{\mathcal{M}}_{\pi N})$ are Fourier transforms of $(\mathcal{M}_{\pi\pi}, \mathcal{R}, \mathcal{M}_{\pi N})$. In particular,

$$\begin{aligned}
\tilde{\mathcal{M}}_{\pi N}(b) &= \frac{1}{(2\pi)^2} \int \mathcal{M}_{\pi N}(q) e^{-i\vec{q} \cdot \vec{b}} d^2 q \\
&= 4i P_{\pi N}^{\text{CM}} M_{\pi N} \gamma(b) .
\end{aligned} \tag{27}$$

In order to evaluate (26), one uses (20) and (2)-(5) to find

$$\begin{aligned}
&\int \gamma(\vec{b}_1 - \vec{s}) \gamma(\vec{b}_1 + \vec{b}_2 - \vec{s}) \rho(\vec{s}, z) d^2 s dz \\
&\approx \int \gamma(\vec{b}_1 - \vec{s}) \gamma(\vec{b}_1 + \vec{b}_2 - \vec{s}) d^2 s \int \rho(\vec{b}_1 + \frac{\vec{b}_2}{2}, z) dz \\
&\approx \frac{\sigma'}{16\pi a} e^{-\frac{b_2^2}{4a}} \frac{T(\vec{b}_1 + \frac{\vec{b}_2}{2})}{A} ,
\end{aligned} \tag{28}$$

Dropping small terms involving the product of three γ factors for the outgoing pions, one finds

$$\begin{aligned}
F_{3\pi C} &= \frac{i\sqrt{2}}{(2\pi)^4 2^3} \sqrt{\frac{P_{3\pi}}{P_\pi}} \sum_{\text{diagrams}} \left(\frac{P_{\pi N}^{\text{CM}} M_{\pi N}}{M_N} \right) \int d^2 b_1 d^2 b_2 d^2 b_3 e^{i(\vec{q}_1 \cdot \vec{b}_1 + \vec{q}_2 \cdot \vec{b}_2 + \vec{q}_3 \cdot \vec{b}_3)} \\
&\quad \tilde{W}(b_1, b_2, b_3) ,
\end{aligned} \tag{29}$$

where

$$\tilde{W} = \tilde{\mathcal{M}}_{\pi}(b_3) \tilde{\mathcal{R}}(b_2) \frac{e^{-z_1 \frac{\sigma' T(b_1)}{2}} - e^{-z_2 \frac{\sigma' T(b_1)}{2}}}{z_2 - z_1},$$

and

$$z_1 = \frac{T(b_0)}{T(b_1)}$$

$$z_2 = 1 + \frac{T(\bar{b}_1 + \bar{b}_2)}{T(b_1)} + \frac{T(\bar{b}_1 + \bar{b}_2 + \bar{b}_3)}{T(b_1)} - \frac{\sigma}{8\pi a} \left[\begin{aligned} & \frac{T(\bar{b}_1 + \frac{\bar{b}_2}{2})}{T(b_1)} e^{-\frac{b_2^2}{4a}} \\ & + \frac{T(\bar{b}_1 + \bar{b}_2 + \frac{\bar{b}_3}{2})}{T(b_1)} e^{-\frac{b_3^2}{4a}} \\ & + \frac{T(\bar{b}_1 + \frac{\bar{b}_1 + \bar{b}_3}{2})}{T(b_1)} e^{-\frac{|\bar{b}_2 + \bar{b}_3|^2}{4a}} \end{aligned} \right].$$

This expression reduces to (22) in the large nucleus, small γ limit. Its calculation, however, is a formidable problem. Since $M_{\pi\pi}$ is small, $\mathcal{M}_{\pi\pi}$ is dominated by resonance structure, and both $\tilde{\mathcal{M}}_{\pi\pi}$ and $\tilde{\mathcal{R}}$ must be considered functions of more than one variable. The integrand of (29) is thus a function of all the kinematic variables describing 3π production. Furthermore, there does not seem to be any simple way to extract a quantity which can be identified with Δ . No effort has been made to date to evaluate (29).

V. THE PRODUCTION OF $C_{4.44}^*$

Glauber theory can be used to calculate $\pi C \rightarrow \pi C^*$ and $\pi C \rightarrow 3\pi C^*$ by making the substitution in (1) and (8)

$$|u(\vec{r}_1, \dots, \vec{r}_A)|^2 = u_f^*(\vec{r}_1, \dots, \vec{r}_A) u_i(\vec{r}_1, \dots, \vec{r}_A). \quad (30)$$

The problem now becomes one of finding a suitable form for $u_f^* u_i$. The simplest approach is to use an effective one-particle excitation for the excited state of the nucleus. For $A=2$, and denoting the excited orbital by $\bar{\varphi}$,

$$\begin{aligned} u_i &= \sqrt{\frac{1}{2!}} [\varphi_1(\vec{r}_1) \varphi_2(\vec{r}_2) - \varphi_1(\vec{r}_2) \varphi_2(\vec{r}_1)], \\ u_f &= \sqrt{\frac{1}{2!}} [\bar{\varphi}_1(\vec{r}_1) \varphi_2(\vec{r}_2) - \bar{\varphi}_1(\vec{r}_2) \varphi_2(\vec{r}_1)], \\ u_f^* u_i &= \frac{1}{2!} [\bar{\varphi}_1^*(\vec{r}_1) \varphi_1(\vec{r}_1) \varphi_2^*(\vec{r}_2) \varphi_2(\vec{r}_2) - \bar{\varphi}_1^*(\vec{r}_1) \varphi_1(\vec{r}_2) \varphi_2^*(\vec{r}_2) \varphi_2(\vec{r}_1) \\ &\quad + (\vec{r}_1 \leftrightarrow \vec{r}_2)], \\ &= \frac{1}{2!} [\bar{\rho}_1(\vec{r}_1) \rho_2(\vec{r}_2) + \bar{\rho}_1(\vec{r}_2) \rho_2(\vec{r}_1) + (\text{correlation terms})]. \end{aligned} \quad (31)$$

This is easily generalized to

$$u_f^*(\vec{r}_1, \dots, \vec{r}_A) u_i(\vec{r}_1, \dots, \vec{r}_A) \approx \frac{1}{N} \sum_{i=1}^N \bar{\rho}(\vec{r}_i) \prod_{\ell \neq i} \rho(\vec{r}_\ell). \quad (32)$$

As in (3), correlation terms have been dropped. This is expected to be a rather poor description of $C_{4.44}^*$, which is well known to be a state with collective properties, and should be considered a parametrization rather than an actual excitation mechanism.

Electron scattering data is often used to determine $\bar{\rho}$. In the case of $eC \rightarrow eC^*$, the weakness of the electromagnetic interaction insures that one particle operators dominate, and matrix elements with multiparticle correlation terms are down by powers of α . The multiple scattering of an incident pion, however, is significant. In fact, in (7) and (22) the entire multiple scattering series has been summed. It thus seems more reasonable, if one wishes to calculate $\pi C \rightarrow 3\pi C^*$, to determine $\bar{\rho}$ from $\pi C \rightarrow \pi C^*$ than from $eC \rightarrow eC^*$. Using

$$\begin{aligned}
 1 &= \int \rho(\vec{r}) d^3 r , \\
 0 &= \int \bar{\rho}(\vec{r}) d^3 r , \\
 T(b) &\equiv A \int \rho(\vec{b}, z) dz , \\
 \bar{T}_M(\vec{b}) &\equiv A \int \bar{\rho}_M(\vec{b}, z) dz ,
 \end{aligned} \tag{33}$$

with M labeling the spin projection of the $J=2$ final state of the nucleus, one finds by summation of the Glauber series,

$$\begin{aligned}
 F_{\pi C^*}^M &= \frac{ik}{2\pi} \int d^2 b \left(\prod_k d^3 r_k \right) e^{i\vec{q} \cdot \vec{b}} \left[\frac{1}{A} \sum_{i=1}^A \bar{\rho}_M(\vec{r}_i) \prod_{\ell \neq i} \rho(\vec{r}_\ell) \right] \\
 &\quad \left\{ 1 - \prod_{m=1}^A [1 - \Gamma(\vec{b} - \vec{s}_m)] \right\} \\
 &= \frac{ik}{2\pi} \int d^2 b e^{i\vec{q} \cdot \vec{b}} \left(\frac{e^{-\frac{\sigma' T(b)}{2}}}{1 - \frac{\sigma' T(b)}{2A}} \right) \frac{\sigma' \bar{T}_M(\vec{b})}{2A} .
 \end{aligned} \tag{34}$$

This formula is essentially the same as that of Ravenhall and Schult.^{13/}

The Glauber amplitude for $\pi C \rightarrow 3\pi C^*$ is, then,

$$F_{3\pi C^*}^M = \frac{ik}{2\pi} \sum_{j=1}^A \int d^2b \left(\prod_{k=1}^A d^3r_k \right) e^{i\vec{q} \cdot \vec{b}} \left[\frac{1}{A} \sum_{m=1}^A \rho_M(\vec{r}_m) \prod_{n \neq m} \rho(r_n) \right] \\ \Gamma_{\pi, 3\pi}(\vec{b} - \vec{s}_j) \prod_{z_i < z_j} [1 - \gamma(\vec{b} - \vec{s}_i)] \prod_{z_\ell > z_j} [1 - \Gamma_{3\pi, 3\pi}(\vec{b} - \vec{s}_\ell)] \quad (35)$$

Of the A nucleons, there are now two of special interest: there is the excited nucleon and the nucleon on which 3π production occurs. If the excited nucleon is on the beam side of the production site, it sees one pion, if to the downstream side, three pions. Of course, the excited nucleon may in fact be the production nucleon. So the sums in (35) will have three contributions.

Using the same approximations as in the derivation of (22), and with

$$x \equiv 1 - \frac{\sigma' T(b)}{2A}, \\ \bar{x} \equiv 1 - \frac{\sigma' \bar{T}_M(\vec{b})}{2A}, \\ y \equiv 1 - \Delta \frac{\sigma' T(b)}{2A}, \quad (36)$$

the scattering amplitude becomes

$$F_{3\pi C^*}^M = \frac{i\sqrt{2}}{(2\pi)^4 2^3} \sqrt{\frac{p_{3\pi}}{p_\pi}} \sum_{\text{diagrams}} \left(\frac{p_{\pi N}^{CM} M_{\pi N}}{M_N} \right) \mathcal{M}_{\pi\pi\pi} \mathcal{R} \bar{Z}_M(t), \quad (37)$$

where

$$\bar{Z}_M(-q^2) = \int d^2b e^{i\vec{q} \cdot \vec{b}} \frac{1}{A} \left[-(1-x)(1-\bar{x}) \sum_{j=1}^{A-1} j (x^{A-1-j} y^{j-1} + z^{j-1} y^{A-1-j}) \right. \\ \left. + (1-\bar{x}) \sum_{j=1}^A x^{A-j} y^{j-1} \right].$$

Summing the series, and using (6), one finds

$$\bar{Z}_M(-q^2) = \int d^2b e^{i\vec{q} \cdot \vec{b}} \frac{\sigma' T_M(\vec{b})}{2A} \left(\frac{1}{\Delta-1} \right) \left[\frac{e^{-\Delta \frac{\sigma' T(b)}{2}}}{1 - \Delta \frac{\sigma' T(b)}{2A}} - \frac{e^{-\frac{\sigma' T(b)}{2}}}{1 - \frac{\sigma' T(b)}{2A}} \right. \\ \left. + \frac{e^{-\frac{\sigma' T(b)}{2}} - e^{-\Delta \frac{\sigma' T(b)}{2}}}{\frac{\sigma' T(b)}{2}} \right] \quad (38)$$

The cross section for $3\pi C^*$ production is given by

$$d\sigma = \frac{1}{(2\pi)^8 2^6} \sum_{M=-2}^2 \left| \sum_{\text{diagrams}} \left(\frac{p_{\pi N}^{CM} p_{\pi N}^M}{p_{\pi N}^M} \right) \mathcal{M}_{\pi\pi\pi} \mathcal{R} \bar{Z}_M(t) \right|^2 d^8\tau. \quad (39)$$

To calculate $\bar{T}_M(b)$, the excitation form factor may be parametrized by

$$\bar{\rho}_M(\vec{r}) = \frac{2C}{\sqrt{5} \pi R^5} r^2 e^{-\frac{r^2}{R^2}} Y_{2M}(\theta, \varphi), \quad (40)$$

where the angles are referred to the recoil direction of the nucleus ($\hat{x}$), which is perpendicular to the beam axis ($\hat{z}$) in the Glauber approximation. If Cartesian combinations of the $\bar{\rho}_M$ are made

$$\begin{aligned} \bar{\rho}_0 &= \bar{\rho}_0 \sim \frac{1}{\sqrt{6}} (2x^2 - y^2 - z^2) \\ \bar{\rho}_{1+} &= \frac{1}{\sqrt{2}} (\bar{\rho}_1 + \bar{\rho}_{-1}) \sim -\sqrt{2} z x \\ \bar{\rho}_{1-} &= \frac{1}{\sqrt{2}} (\bar{\rho}_1 - \bar{\rho}_{-1}) \sim -\sqrt{2} y x \\ \bar{\rho}_{2+} &= \frac{1}{\sqrt{2}} (\bar{\rho}_2 + \bar{\rho}_{-2}) \sim \frac{1}{\sqrt{2}} (z^2 - y^2) \\ \bar{\rho}_{2-} &= \frac{1}{\sqrt{2}} (\bar{\rho}_2 - \bar{\rho}_{-2}) \sim \sqrt{2} y z \end{aligned} \quad (41)$$

then, by symmetry, $\bar{\rho}_{1-}$ and $\bar{\rho}_{2-}$ give zero contribution when the y integration is done. The z integration will kill the $\bar{\rho}_{1+}$ term in the $\pi C \rightarrow \pi C^*$ case. This argument is not quite valid for $\pi C \rightarrow 3\pi C^*$, since there is an asymmetry between incident and produced systems. This will affect the term in $\bar{Z}$ corresponding to (excited nucleon) = (production nucleon). A small $\bar{\rho}_{1+}$ term is also expected to arise from $q_z \neq 0$. Neither of these effects will be considered in this paper, and the $\bar{\rho}_{1+}$ term will not be calculated.

VI. CALCULATION OF $\pi^- C \rightarrow \pi^+ \pi^- \pi^- C_{4.44}^*$

Experimental $\pi C \rightarrow \pi C^*$ data at $4.5 \text{ GeV}/c^{14/}$ and $3 \text{ GeV}/c^{15/}$ were analyzed using (34) and (40). The shape of the experimental t distribution is well reproduced by $\bar{R} = 1.77 \text{ fm}$, and the normalization gives $C = 3.19 \pm .02$. Theoretical curves for $\pi C \rightarrow \pi C^*$ are shown in Fig. 6. The experimental data will appear in Ref. 14. Equation (38) is now used to calculate $\bar{Z}_0$ and $\bar{Z}_{2+}$, and the results are shown in Fig. 7. The $2+$ contribution to $\pi C \rightarrow 3\pi C^*$ is greater than it is in $\pi C \rightarrow \pi C^*$, when compared to the $M=0$ contribution.

The next step is to do a Monte Carlo calculation of (39) at incident pion momentum $6 \text{ GeV}/c$, in order to compare with the experimental results of Ref. 4. Unfortunately, the low pion momentum adversely affects the validity of two approximations that have been made throughout this research. First, a significant fraction of events have $M_{\pi C}$ in the resonance region, $M_{\pi N} \lesssim 1.5 \text{ GeV}$, and the approximation that $\gamma(b)$ is energy independent underestimates the interaction in this region. The effect may be estimated by comparing, in I, the results of FIT 1, which uses experimental $\pi C \rightarrow \pi C^*$ data for the resonance region, and FIT 3, which uses an energy independent πC interaction. FIT 1 yields a $3\pi C^*$ cross section 15% higher than FIT 3, and the calculation performed here may be expected to be too small by a similar amount. This effect is negligible at $15.1 \text{ GeV}/c$ incident momentum.

The second approximation that is in doubt is the identification $q^2 \rightarrow q_{\perp}^2 \rightarrow -t$, which is implied in (22) and (37). At low energies $q_{\parallel}$ is significant, and the tacit assumption that $\vec{q} \cdot \vec{r} = \vec{q} \cdot \vec{b}$ is not valid. One might naively think of identifying $q^2 \rightarrow q_{\perp}^2 \rightarrow -t' = -(t - t_{\min})$, but t distributions result which are in violent

disagreement with experiment. The best way to solve this problem would be to re-derive the entire theory, as is done in Ref. 7. However, it is not at all clear, if one were to do this, how one would make a statement about 3π absorption which is equivalent to (9). Examination of the curves in Ref. 7 indicates that if one uses the infinite energy ($q_{\parallel} = 0$) expressions, less error is produced by taking $\bar{Z}(q^2) = \bar{Z}(-t)$ rather than $\bar{Z}(q^2) = \bar{Z}(-t')$. This procedure will, at 6 GeV/c, presumably give cross sections that are a few percent too small. Again, the effect should be negligible for $\pi C \rightarrow 3\pi C$ at 15.1 GeV/c.

The Monte Carlo calculation was performed, generating a total of 26,377 events with $.8 < M_{3\pi} < 1.8$ GeV. Figure 8 shows $d\sigma/dM_{3\pi}$, along with the experimental results from Ref. 4. This experimental data has been corrected for spectrometer acceptance. The shape of the $M_{3\pi}$ spectrum is consistent with experiment, and is virtually identical with the $M_{3\pi}$ spectra in I. The total cross section for $3\pi C^*$ production in this theory is 58 μb , which is 44% of the FIT 3 result in I. This is consistent with the absorption estimate in I. The normalization of the experimental data is not available at the time of writing.

The shape of the experimental t distribution is analyzed in some detail in Ref. 4. To this end, the normalization of the theoretical distribution in $q_{\perp}^2$ is fit to the data presented in Ref. 4, and the results shown in Fig. 9. The confidence level of the fit is $\sim 10^{-9}$. If the last three data bins ($q_{\perp}^2 > .09 (\text{GeV}/c)^2$) are dropped, the confidence level of the fit rises to .16. However, it should be remembered that the $q_{\perp}^2$ acceptance of the spectrometer has not been folded into the experimental data. (In Ref. 4 the acceptance is not divided into the data, but is multiplied into the forms to which the data is fit.) Since the acceptance is presumably a decreasing function of $q_{\perp}^2$, its inclusion would probably decrease the confidence level of the fit. It is entirely possible that the agreement would be improved if the $q_{\parallel} \neq 0$ effect were handled more realistically.

CONCLUSION

The stated purpose of this research has been fulfilled, in that part of the discrepancy between theory and experiment has been removed for $\pi C \rightarrow 3\pi C$. It is conceivable that the remaining discrepancy might disappear if the multi-impact parameter approach suggested in Section IV were used. The calculation of $\pi C \rightarrow 3\pi C^*$ is a definite improvement over that in I, as the absorption of the 3π system is calculated according to an explicit model, rather than estimated. The same problems as encountered in the reaction $\pi C \rightarrow 3\pi C$ should also affect the $3\pi C^*$ calculation, as well as the finite energy effects. However, when the experimental cross section for $3\pi C^*$ production becomes available, this calculation should aid an evaluation of the validity of the C^* excitation parametrization used in Section VI.

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FIGURE CAPTIONS

- Fig. 1 Diagrams calculated in Deck model.
- Fig. 2 $|Z(t)|^2$ for $\pi^- C \rightarrow \pi^+ \pi^- \pi^- C$. a) $\Delta=3$, most naive theory. b) $\Delta=2.17-.14i$, theory of Section II. c) $\Delta=1.68-.06i$, 3π absorbed as πp . d) $\Delta=1.1$, approximates experiment.
- Fig. 3 3π mass distribution for $\pi^- C \rightarrow \pi^+ \pi^- \pi^- C$. Experimental curve is from Ref. 2. The theoretical curve has been multiplied by 3 to make the peaks coincide.
- Fig. 4 Distribution in momentum transfer $t' = t - t_{\min}$ for $\pi^- C \rightarrow \pi^+ \pi^- \pi^- C$. a) Coherent cross section, $\frac{d\sigma_C}{dt'} = \frac{d\sigma_{TOT}}{dt'} - \frac{d\sigma_I}{dt'}$. The solid curve refers to the theory of Section II; the broken curve allows the 3π to be absorbed as πp . b) Incoherent cross section, $\frac{d\sigma_I}{dt'}$. Experimental curves from Ref. 1.
- Fig. 5 Definition of various impact parameters with respect to an arbitrary reference axis. The top two pions are from the upper ($\pi\pi$) vertex, the lowest pion from the lower (πN) vertex.
- Fig. 6 Calculated cross section for $\pi C \rightarrow \pi C_{4.44}^*$. M is the spin state of the C^* .
- Fig. 7 $|\bar{Z}_M(q^2)|^2$ for $\pi^- C \rightarrow \pi^+ \pi^- \pi^- C_{4.44}^*$. M is the spin state of the C^* .
- Fig. 8 3π mass spectrum for $\pi^- C \rightarrow \pi^+ \pi^- \pi^- C_{4.44}^*$. Experimental curve is from Ref. 4.
- Fig. 9 Distribution in $q_{\perp}^2$ for $\pi^- C \rightarrow \pi^+ \pi^- \pi^- C_{4.44}^*$. The normalization of the theoretical curve has been fit to the experimental data. Experimental curve from Ref. 4.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 8

Fig. 9