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ROCKET ENGINE OPERATIONS — NUCLEAR

REPORT NO. RN-S-0036

TO

AEC-NASA SPACE NUCLEAR PROPULSION OFFICE

NON-LINEAR OSCILLATIONS OF REACTOR CORES (U)

NERVA PROGRAM

CONTRACT SNP-1

JANUARY 1964

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14

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ABSTRACT

This report presents analyses of some possible modes of non-linear vibration in which parts of the ROVER reactor core might separate. These analyses include investigations of (1) non-linear effects of separation of the core from its support blocks, (2) reactivity and power fluctuations which might occur during one possible non-linear mode of core motion, and (3) possible coupling of nuclear and mechanical modes of core motion.

This report is a reprint of a memorandum submitted to REON by A. Stanley Thompson of the VARNA Corporation, Danville, California and part-time consultant to REON. Publication of this report should not necessarily be construed as REON concurrence and approval with all aspects of Mr. Thompson's analysis and conclusions.

for W. D. Stinnett
REON Technical Systems Manager

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INTRODUCTION

The core of the ROVER reactor is composed of a bundle of fuel elements, about 52 in. long, suspended from longitudinal support rods, and restrained against separation of its parts by both longitudinal and radial springs. In this memorandum a report is made of analyses of some possible modes of non-linear vibration in which parts of the core separate. Based on the work which was carried out this writer believes it to be likely that, under vibratory conditions reasonably to be expected, the present springs cannot restrain the core adequately against separation of its parts. Such separation is probably intolerable because, under the non-linear mechanical vibratory conditions which then exist, vibrations can be maintained with a whole spectrum of frequencies. It also appears that in such separation both the mechanical and neutronic operating conditions are altered in a direction to make more possible a nuclear power input to maintain vibration. The separation under vibratory conditions could be expected to lead to breakage of the sort which has been observed due to the propagation of shock waves in the material.

It is recommended that the design be altered, by the use of stronger spring forces and by the control of the direction of pressure drop in the reactor coolant in such a way that the components of the core will not separate under vibratory conditions which may exist. This writer believes that, to accomplish such an integral packaging of the ROVER core will require that

1. Radial pressure gradients should be everywhere directed inward.
2. The axial force exerted, under full power conditions, by the combination of longitudinal springs and axial pressure drop through the core should probably be of the order of 100 times the weight of the core (100 g). Accomplishing this objective will probably require the design of a system of support rods which can operate under a smaller stress than that on the present system.

It is further recommended that any revised NERVA reactor should be examined as a system, analytically as well as experimentally, to determine possible coupled modes of vibration and their stability.

The non-linear analyses presented in this memorandum are approximations based on an attempt to obtain simplified descriptions of possible modes of motion of a complicated core. To obtain reliable results, and to determine the stability of the modes of vibration investigated, much more work would be necessary. It is thought, however, that it is more important at present to alter the design so that these non-linear modes cannot exist. It may be that a longer-range analysis of a more general nature should be carried out to determine whether such non-linear modes may constitute a reactor hazard under unexpected operating conditions for segmented reactors of the type considered.

ANALYSIS

The following analyses are composed of three sections.

In the first section an investigation is made of the non-linear effects of separation of the reactor core from its support blocks during vibration on the natural frequency of vibration.

In the second section an investigation is made of the reactivity changes, and the resulting power fluctuations, neglecting delayed neutrons, which occur during one possible non-linear mode of motion of the segmented reactor core, assuming that no coupling of nuclear and mechanical effects occurs whereby energy generated by the nuclear process becomes available to augment the mechanical vibration.

Consideration is given in the third section to a mechanism by which a non-linear mode of motion such as that considered in the second section may result in a coupling of the nuclear and mechanical modes of motion, and possibly in instability of the amplitude of the resulting oscillatory mode.

Non-linear Motion of Reactor Core on Longitudinal Supports

The reactor core is considered to be mounted on longitudinal supports rods as shown in Figure 1. The support rods are assumed to act in tension as flexible supports for the core of mass, M , having an overall axial spring constant, k . The reactor is held against a system

Figure 1

of support blocks by a force, F , which is assumed to be due to gas pressure, gravity, and relatively soft constraining springs. For this system, assuming that the structure remains integral, natural frequencies of various modes of vibration were calculated, ranging from about 25 to 75 cps. The problem to be considered in this section is the effect on frequency of longitudinal motions of the core so large that the reactor overcomes the restraining force, F , sufficiently to bounce clear of the support blocks on the ends of the support rods. This situation is idealized as shown in Figure 2.

Figure 2

Here x_1 represents the motion of the mass, M , against the spring, k , while x_2 represents its motion after it has bounced free of the support rods.

The equation governing the motion of the mass in region (1) is

$$M \frac{d^2 x_1}{dt^2} + k x_1 + F = 0 \quad (1)$$

and in the region (2)

$$M \frac{d^2 x_2}{dt^2} + F = 0 \quad (2)$$

The simultaneous solution of these two equations gives the following relationships

$$\frac{x_2}{x_1} = - \frac{(\omega_{T_2})^2}{8} \frac{\cos \frac{\omega_{T_1}}{2}}{1 - \cos \frac{\omega_{T_1}}{2}} \quad (3)$$

and

$$\omega T_2 = -2 \frac{1 - \cos \omega T_1}{\sin \omega T_1} \quad (4)$$

where

X_1 is the maximum amplitude of motion in region (1)

X_2 is the maximum amplitude of motion in region (2)

T_1 is the cyclic period of motion in region (1)

T_2 is the cyclic period of motion in region (2)

and

ω is the frequency of the linear system in which the mass does not leave the support blocks, given by

$$\omega^2 = \frac{k}{M} \quad (5)$$

Equations (3) and (4) are solved numerically for values of X_2/X_1 and ωT_2 as functions of assumed values of ωT_1 . The period of the non-linear vibration, T , is

$$\omega T = T_1 + T_2 \quad (6)$$

The result is shown approximately in Figure 3. It can be seen that the period of vibration lengthens as the amplitude of free motion increases, so that, for successively lower frequencies of vibration larger amplitudes of vibration exist at which stable oscillations can be maintained.

It is apparent that the natural frequencies of vibration of the system are a priori determinate only if the core is restrained to motion in contact with the springs. This requires that the force, F , be large enough to overcome the separation forces. The force required is given in terms of the expected amplitude of vibration by

$$F = \frac{k X_1}{2} \quad (7)$$

In the present case this vibratory amplitude can be interpreted in terms of the maximum tensile stress in the support rods, given by

$$\sigma = \frac{X_1}{\lambda} E \quad (8)$$

Comparing Equations (7) and (8) it can be seen that the dynamic stress for the system designed just barely to operate within the limits set by incipient separation from the support blocks is twice the static stress under the load, F. If

$$\sigma = 50,000 \text{ psi}$$

$$E = 30,000,000 \text{ psi}$$

$$l = 52 \text{ in.}$$

then

$$X_1 = 0.087 \text{ in.}$$

and

$$F = 74,000 \text{ lb}$$

DISCUSSION

It would appear reasonable that a significant fraction of the longitudinal force, F, should be provided by means other than gaseous pressure drop. Among other reasons, the gaseous forces are subject to dynamic amplification from flow disturbances, nuclear power perturbations, etc. Also, at part flow conditions the available gaseous forces are reduced markedly, leaving the core dependent on external constraints to keep it from separating.

The gaseous force on the present core, at full flow conditions, is about 125,000 lb. Letting this force be one-half the total would require

$$F = 250,000 \text{ lb}$$

To carry this force, keeping the system integral under the conditions analyzed, would require more cross-section in support rods, by a factor of about 3.4. It appears that a much stronger support system should be provided to give adequate assurance against separation under vibratory conditions.

Non-linear Reactivity Changes

The purpose of this section is to investigate the effects on reactivity, and on reactor power, of motions of a segmented reactor core in which the parts are not in continual contact. The situation is idealized as shown in Figure 4. The mass, M, here

PERIOD VS AMPLITUDE

Figure 3

Figure 4

represents some one segment of the reactor core which is attached to a spring of stiffness, k . The mass is assumed to bounce away, in a time short compared with its total cyclic period, from contact with a relatively rigid wall which is located at a distance, x_0 , from the neutral position of the mass-spring system. It is also assumed that the reactivity of the reactor core is a linearly decreasing function of the distance, x . Thus, if the control mechanism is slower than necessary to follow the details of the oscillation, it will merely take a position in which it has increased the average reactivity just sufficiently to compensate for the averaged decrease in density during the oscillatory separation. If the control mechanism is faster it may follow the details of the oscillation, or, in the worst case, may couple with the motion adversely, so as to augment the oscillation in power.

The equation governing the motion of the mass is

$$\frac{1}{\omega^2} \frac{d^2 x}{dt^2} + x = 0 \quad (9)$$

where

$$\omega^2 = \frac{k}{M} \quad (10)$$

The solution to this equation, subject to appropriate boundary condition, is

$$x = x_{\max} \sin \omega T \quad (11)$$

and the period of oscillation is

$$T = \frac{1}{\omega} \left[\pi - 2 \sin^{-1} \left(\frac{x_0}{x_{\max}} \right) \right] \quad (12)$$

Changes in reactor power are described by

$$\frac{d \ln (P/P_0)}{dt} = \lambda(\delta - \beta - \alpha x) + \beta \frac{P_0}{P} \quad (13)$$

where

P is the reactor power

P_0 is the average reactor power

λ is the mean neutron life cycle frequency between fissions

δ is the excess reactivity

β is the delayed neutron fraction

α is the displacement coefficient of reactivity

It has been assumed in Equation (13) that the period, T , is short compared with the delayed neutron fraction. Neglecting, for convenience, the delayed neutrons, and substituting the value of x from Equation (11)

$$\frac{d \ln (P/P_0)}{dt} = \lambda(\delta - \alpha x_{\max} \sin \omega T) \quad (14)$$

The solution to this equation gives

$$\ln \frac{P_{\max}}{P_{\min}} = \frac{\lambda x_{\max}}{\omega} \left[(2\omega t' - \pi) \sin \omega t' + 2 \cos \omega t' \right] \quad (15)$$

where t' is the time in the cycle at which the power is maximum, given by

$$\sin \omega t' = \frac{\delta}{\alpha x_{\max}} \quad (16)$$

The excess reactivity, for cyclic conditions, is required to be

$$\delta = \alpha x_{\max} \frac{\cos \omega t_0 + \cos \frac{\omega t_0}{2}}{\pi - \frac{\omega t_0}{2}}$$

A case of interest is where $x_0 \ll x_{\max}$, so that $t_0 \sim 0$. This implies that when the system is at rest the spring barely contacts the mass, M. Then $\omega T = \pi$ and the ratio of maximum to minimum power in the cycle is found to be

$$\ln \left(\frac{P_{\max}}{P_{\min}} \right) = 0.133 \lambda \alpha x_{\max} T$$

if

$$\lambda = 0.56 \times 10^5 \text{ per sec}$$

$$T = 0.05 \text{ sec}$$

$$\pi x_{\max} = 0.007 (\text{\$1})$$

Then $P_{\max}/P_{\min} = 10$. A rough plot of the power as a function of time is shown in Figure 5.

Discussion

Alternate modes of motion of the segmented reactor core can be imagined, some of which will give greater, some less, power excursions than the one shown here. The amplitude of the power oscillations will be decreased by the effect of the delayed neutrons. The control system itself may affect the oscillation. Finally it is possible that there may be coupling between the mechanical oscillation and the mechanism by which nuclear power is generated. For the complicated non-linear system needed to represent the separated segments of the real reactor, the analysis of these effects becomes more complicated. However, it appears that the important point is to design the system so that no separation of the segments of the reactor may occur

Non-linear Nuclear Oscillations

A neutron-induced natural frequency for a nuclear reactor is given by*

$$f = \frac{1}{2\pi} \sqrt{\frac{\alpha T \lambda P_0}{c_p \gamma V}} \quad (18)$$

*"Study of Reactor Kinetics," A. Stanley Thompson, American Society of Mechanical Engineers, Paper No. 62-WA-218, Nov. 1962.

Thermal Power from Nuclear Reactors, Thompson and Rodgers, John Wiley and Sons, Inc., New York, 1956, p. 123.

The Physical Theory of Neutron Chain Reactors, Weinberg and Wigner, The University of Chicago Press, 1958, p. 607.

Figure 5

where αT is the temperature coefficient of reactivity, the other terms being as previously described. If the ratio of frequency of this neutron-induced vibration to the mechanical natural frequency of any part of the reactor which controls a significant amount of reactivity approaches the numerical value, one-half, serious coupling of these two modes of vibration can occur. The magnitude of the resulting oscillations may then be unstable. It appears that, so long as the present reactor is maintained as an integral unit, the nuclear frequency so described is very low, and no such coupling is likely. Let us consider the case of the non-linear oscillation from the last section, in which separation of the components of the reactor core occurs. For this case Equation (18) can be written.

$$f = \frac{1}{2\pi} \sqrt{\frac{\delta'}{\Delta T} \frac{P_0}{c_p \gamma V}} \quad (19)$$

where δ' is the total change of reactivity when the core separates, and ΔT is the temperature rise in the reactor required to cause this separation. It is difficult to estimate the value of ΔT . However, it appears that, with the present design, changes in radial pressure distribution of the order of 1 psi could result in substantial radial motion against the restraining springs under certain flow conditions. If the flow rate is maintained constant during an oscillation, then the pressure drop through the core should be about proportional to the temperature of the fluid flowing. If the temperature of the fluid follows that of the wall, then

$$\Delta T \sim \frac{\Delta p}{p} T$$

where T is the average core temperature and p is the pressure drop through the core. Taking the average core temperature as 1000°R and the pressure drop as 40 psi, then the order of magnitude of temperature change ΔT required to cause the core to separate during a transient might be expected to be

$$\Delta T = \frac{1 \times 1000}{40} = 25 \text{ R}$$

Let

$$= 0.5 \times 10^5 \text{ per sec}$$

$$\delta' = 0.007 (\$1)$$

$$P_0 = 80 \text{ Mw}$$

$$c_p = 0.125 \text{ Btu per lb} \cdot \text{R}$$

$$\gamma V = 2300 \text{ lb}$$

[REDACTED]

Then, by Equation (19)

$$f = 10 \text{ cps}$$

Discussion

The effect of delayed neutrons is to reduce the magnitude of the power oscillations shown in Figure 5, perhaps by a factor of two or more.* The amplitude of the oscillation will also depend on the details of the core oscillation. Whether the mechanical oscillations will couple to an important extent with the neutron-induced oscillations in the circumstances existing in the present ROVER core is not known. It is apparent however that the tendency in this direction is increased by the non-linear effects on both excursions of reactivity thereby made available. It appears desirable to do whatever is possible to avoid such non-linear oscillations.

* Thermal Power from Nuclear Reactors, Thompson and Rodgers, John Wiley and Sons, Inc., New York, 1956, p. 103.