BULLETIN
ANL/ACEA-101

**LECTURE BACKGROUND NOTES ON
TRANSIENT SODIUM BOILING AND
VOIDING IN FAST REACTORS**

by

D. Okrent and H. K. Fauske

ARGONNE CENTER FOR EDUCATIONAL AFFAIRS

MASTER

January 1972

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LECTURE BACKGROUND NOTES ON TRANSIENT SODIUM BOILING AND VOIDING IN FAST REACTORS*

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*These notes were prepared while one author (D. O.) spent a year as Visiting Professor at The University of Arizona in fulfillment of the AUA Distinguished Appointment Award

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PREFACE

The objective of these "lecture background notes" is to facilitate the preparation of a lecture series on sodium boiling and voiding phenomena relevant to fast reactor safety as part of a course to be given by a professor not actively doing research in this field or even generally knowledgeable in fast reactor technology. A knowledge is assumed of the fundamentals of transport phenomena in fluids (as typified by Bird, Stewart, and Lightfoot, Transport Phenomena, John Wiley and Sons) and of heat transfer (as typified by Eckert and Drake, McGraw Hill). Also, for some phenomena, reference may be made to generally available books or journal articles for details of derivations, etc.

The objective is to discuss and describe the important physical phenomena, as they are understood today, but not to enter into any treatment of the methods employed in numerical solution of the sets of equations which arise with each set of assumptions and approximations.

Since this is a rapidly developing field, some of the ideas presented may later prove to be invalid. In other areas, phenomena may be identified but no meaningful theoretical treatment is available. Indications are given where appropriate of the probable lines of future development in the subject.

These notes may also be useful as a set of working notes for the student. They have been prepared with a rather modest amount of editing. It is assumed that if they are useful, revisions will be appropriate on a relatively frequent basis.

1. SYNOPSIS

Sodium boiling and voiding is an important feature of many postulated transients and accidents in fast power reactors, and a basic understanding of the mechanisms and processes important to this phenomenon is needed in order to develop an analytic, predictive capability. The low system pressure of sodium-cooled fast reactors and the consequent large difference between specific volume of vapor and liquid, coupled with the high sodium thermal conductivity and the possible significant superheat which may be applicable in the absence of entrained gas bubbles, lead sodium to a tendency toward unstable boiling and rapid channel voiding as compared to the relatively stable boiling which is readily achieved with water at pressures associated with thermal reactors.

Whenever the coolant flow through a heated channel is too low to remove the heat in the liquid phase, boiling will occur. The first bubble which nucleates will contain vapor at a higher pressure than the static pressure in the surrounding liquid because of the superheat. This pressure increase will tend to deactivate any other potential nucleation sites in the neighborhood of the first bubble. Because of the unstable conditions with sodium, the liquid coolant begins to be ejected from the channel by the expanding vapor bubble. Only a liquid layer (or film) still resides on the channel walls in contact with the bubble.

When the bubble reaches cooler zones of the channel, condensation of vapor will begin. When condensation overrides the vapor production in the hot zones of the channel, the temperature and pressure of the vapor bubble will decrease. This leads to deceleration and reversal of flow of the ejected liquid above the bubble. (The detailed flow pattern of the liquid below the bubble will depend on a balance of forces due to system pressure and bubble expansion.) At the same time, the static pressure in the liquid

is decreased, and if the temperature of this liquid is sufficiently high, another bubble will nucleate. The first bubble may be totally expelled from the hot part of the channel or may condense completely.

This sequence of events may be repeated until the heated part of the channel is completely filled by a single vapor bubble, unless an important change occurs in contributing phenomena, e.g., local or gross loss of the film on the channel wall, rupture of the clad, and escape of fission product gas, and possibly hot fuel. Prior to such a change, flow oscillations may be expected to continue; upon reentry of the liquid, the surface area available for condensation is reduced until vaporization again overrides condensation.

Matters which bear particularly on the mode and rate of sodium voiding include the following:

- a) the degree of superheat (local and bulk)
- b) the mode of vapor bubble formation and growth and the consequent form of the liquid and vapor fluids (also the processes of condensation and bubble collapse), i.e., flow regime and flow pattern
- c) the driving forces behind vapor bubble expansion, particularly the modes of energy transfer from the vapor-liquid interfaces, and whether voiding is due to boiling (intact geometry), fission gas release, or molten-fuel-coolant interaction.

The degree of sodium superheat in alkali metals has been found to vary widely in experiments performed under differing conditions, and there has been only partial success in explaining these results. The presence of gas bubbles clearly leads to small or negligible superheat by providing nucleation sites. In the absence of gas bubbles in the liquid, nucleation at those surface cavities which have not previously been fully wetted by the sodium is favored by many workers as the dominant mechanism. Larger cavities

are more readily deactivated as nucleation sites, and the radius of remaining smaller cavities is dependent on a balance of the forces (liquid pressure) tending to drive sodium into the cavities and the forces (surface tension and pressure of vapor and trapped gas) resisting sodium entry. The smaller the remaining active sites, the higher the superheat which will later be required during some transient to nucleate an unstable bubble.

Many other parameters and processes appear to possibly influence the degree of superheat including coolant velocity, heat flux, impurities in the coolant, aging of the surface, etc. As one example of the limited applicability of the cavity deactivation concept, questions have been raised concerning the loss of original surface cavity configuration upon exposure to coolant flow and heat flux.

The concept involving formation and growth of large individual sodium vapor bubbles, separated by slugs of liquid coolant filling the channel, has received more acceptance than the concept involving the formation of a nearly homogeneous, two-phase mixture of liquid and small vapor bubbles, at least for channels, or groups of channels, which have an approximately isothermal temperature dependence in a direction transverse to normal flow. The film of liquid left on the channel contributes importantly to the driving force for bubble expansion because of the large film area compared to the interface area between vapor and liquid at the bubble ends. In the cases of fission gas release and molten fuel-coolant interaction the single-bubble concept has been used to describe the voiding process. In the fuel-coolant interaction process, the rate of mixing, fragmentation, and mode of heat transfer are important considerations.

For subassemblies having sizeable power (and temperature) gradients, and/or where superheat is modest, the large, single-vapor bubble concept may run into difficulty. Also, for long bubbles, vapor flow and pressure

gradients within the bubble itself appear to be significant for a description of the course of events.

This set of lecture background notes includes the following:

1. Introductory remarks on fast reactor safety, which are intended to provide some perspective on the role played by sodium boiling.
2. A discussion of superheat which reviews the experimental data and nucleation models with emphasis on the pressure-temperature history effect on radius of active cavity sites, including the role played by inert gas.
3. A discussion of the growth and collapse of spherical bubbles.
4. A historical description of the development of computer codes to describe voiding and a detailed description of the analytical formulation of typical models for calculating voiding due to boiling, fission gas release, and molten fuel-coolant interaction.

2. INTRODUCTION

2.1 General Design Considerations for Fast Power Reactors

Sodium-cooled, fast power reactors employ a low-pressure coolant, which will not flash in the event of a primary system rupture; also, the primary system is frequently double-walled or its equivalent.¹⁻³ Safety considerations, therefore, have not been centered on the assumption of a gross loss of coolant to the same extent as for water-cooled reactors; however, the matter requires serious consideration in each design. The presence of a small prompt neutron lifetime, coupled with the potential for large reactivity insertion rates from coherent collapse of the core (and from large-scale voiding of the core sodium in larger reactors) has frequently led to the choice of reactivity accident as the accident of greatest severity. A lesser emphasis has been placed on the sodium-air reaction; the primary systems are usually surrounded by an oxygen-depleted atmosphere. Considerable attention is being given currently to questions of local (subassembly scale) faults, means of detecting such faults, and the possibility of propagation from subassembly to subassembly of fuel failure and/or sodium boiling.

The typical large fast reactor under construction or active design today employs mixed oxide (Pu, U)O₂ fuel, helium-bonded to stainless-steel clad jackets having a diameter of about 1/4 in. The fuel elements are close-packed into subassemblies which in turn are close-packed into a core having an average composition of about 35% fuel, 45% sodium, and 20% steel. The control and safety rods are made of tantalum and/or boron carbide. Axial and radial blankets of natural or depleted uranium oxide, about a foot thick, surround the cylindrical core whose diameter exceeds its height by a factor of two or three.

The fuel is about 20% plutonium, and the smeared density of fuel over the jacket cross section is about 85%. Current fuel elements vent the fission product gases to a large, contained plenum above or below the core section within each fuel element, although reactor designs for future reactors consider venting the fission product gases to the coolant with a delay time ranging from hours to days.

A typical current peak oxide fuel-element rating is 15-16 kW/ft. A typical peak power density is 600-700 kW/l of core. Advanced oxide fuel elements, and other potential fuel elements such as helium or sodium-bonded, mixed carbides, may have much higher ratings. Some approximate characteristics of LMFBRs and large pressurized water reactors are compared in Table 2-1.

Between the radioactive primary system sodium (^{24}Na plus corrosion products plus fission products escaping from vented or failed fuel, etc.) and the steam generators is an intermediate sodium heat exchanger and secondary sodium system. Typical average core exit coolant temperatures are 1000-1100°F; the core ΔT is about 300°F. Primary system pumps are usually mechanical. Three to six primary system loops are usually considered, although as low as two has been proposed.

A major early design choice lies with primary system layout: the loop system⁴, in which coolant is piped from the reactor vessel to the pump and intermediate exchanger, which are located in inert atmosphere cells; or the pool system^{1,3} in which the entire primary system lies submerged in a large pool of primary system sodium, which is contained in a large vessel. Examples of loop and pool systems are illustrated in Figs. 2-1 and 2-2.

Control rods have utilized top entry in all reactors built thus far. Flow is up through the core, facilitating thermal convection cooling on a loss-of-pumping power.

TABLE 2-1 COMPARISON OF PARAMETERS OF LIQUID-METAL-COOLED FAST REACTORS (LMFBR) AND PRESSURIZED-WATER REACTOR (PWR)

	LMFBR		PWR
	Oxide (Aronstein 1968a)	Sodium- bonded carbide (Noyes 1968a)	Oxide
Fuel element dia (in.)	0.25	0.35-0.4	0.42
Peak fuel-element rating (kW/ft)	15-17	~45	17-19
Average specific power [kW/kg (U + Pu)]	150	110-150	30
Power peaking factor	1.6	1.5	3
<u>Reactivity requirements for operation</u>			
Controlled reactivity changes (Δk)		<u>LMFBR</u>	<u>PWR</u>
Fuel burnup		0.01	0.10
Power defect		0.005	0.025
Manoeuvring and margin		<u>0.005</u>	<u>0.035</u>
	Total:	0.02	0.16
Effective delayed neutron fraction		0.0036	0.0052

Fig. 2-1. A Loop-type Primary System

Fig. 2-2. A Pool-type Primary System

2.2 Some Nuclear Considerations

It is generally accepted that large fast power reactors can be designed to have a substantially negative Doppler coefficient of reactivity. The fertile ^{238}U (or thorium) has been measured and calculated to provide significant negative reactivity effects upon heating.⁵⁻⁸ Furthermore, theory and experiment indicate that the reactivity contribution from the fissile isotopes should be quite small.^{7,9-11} Other things being equal, the reactor having a higher concentration of ^{238}U or a higher proportion of low-energy neutron flux in the region from 500 to several thousand volts will have a larger Doppler effect. Oxide fuel provides a somewhat softer neutron energy spectrum than does carbide. Small amounts of moderator can be added to either reactor type (or to a metal-fueled reactor) to augment the Doppler coefficient or to help maintain its value in the absence of sodium. However, softening the neutron energy spectrum also leads to an increase in parasitic capture in ^{239}Pu and in fission products, and the breeding ratio drops somewhat.

For super-prompt-critical transients, the time for heat transfer between particles of PuO_2 and UO_2 in mechanically mixed oxides can introduce significant delays in the Doppler coefficient for larger particle sizes. Pitterle¹² found that if 0.1-msec transients can be taken as representative, fuel segregation does not appear to have a substantial effect on nuclear bursts for particles of diameter less than 20 or 30 μm in good thermal contact. Fischer¹³ concluded that 75 μm represents an acceptable oxide particle diameter. Thus, the determination on acceptable particle size can vary with reactor design and assumed accident conditions.

The calculated explosive energy rises rapidly with decreasing Doppler effect for $T \, dk/dT \approx 0.002-0.003$; loss of sodium can move the

Doppler effect into this range for reactors having an enrichment ratio of $\text{Pu}/\text{Pu} + \text{U} \approx 0.25$ or 0.3 .

The Doppler coefficient, dk_D/dT , varies approximately as the inverse of the absolute temperature over the temperature range of major interest, although for high temperatures the coefficient will fall off more rapidly.^{7,14}

Current estimates of the accuracy of prediction of the Doppler effect for a specific power reactor design are generally around 25%.^{9,10,15-17} Reviews of the current knowledge of the Doppler effect can be found in various references.^{6,7,9,10}

Large fast reactors, particularly those fueled with ^{239}Pu plus ^{238}U , are susceptible to an appreciably positive reactivity effect upon the loss or expulsion of sodium. This reactivity arises mainly from the hardening of the neutron spectrum when sodium atoms are expelled. When the spectrum hardens, the ratio of capture to fission in plutonium decreases, the fast fission yield in ^{238}U and in ^{240}Pu increases, and the capture rate of ^{238}U and of fission products decreases more rapidly than does the fission cross section of ^{239}Pu . All these effects add reactivity.

A compensating factor lies in the increase in neutron leakage with the loss of sodium. However, neutron leakage tends to become less important as reactor sizes increase. The positive reactivity component from spectral hardening tends to dominate, and it becomes increasingly difficult to design a reactor having a net negative sodium-void reactivity effect without introducing some compromises in other aspects of reactor performance.

The change in reactivity with loss of sodium has a strong spatial dependence. The negative leakage component is very small near the core center; hence, for a reactor having an overall zero sodium density coefficient of reactivity there will be a strong positive component from the central regions of the core that is balanced by a negative contribution

from the portions near the boundary. If expulsion of sodium were to begin at the center of the core, a reactivity gain could occur initially in a reactor where a total loss of sodium would lead to a loss of reactivity.

The reactivity effect attributed to sodium density changes caused by the temperature changes in normal reactor startup and operation are small compared with the accompanying Doppler reactivity effects. The reactivity gain due to large-scale sodium voiding in a large reactor can exceed the Doppler effect substantially. Voiding a central subassembly should add less than 10 or 15¢; voiding an outermost subassembly should lose reactivity.

Because the reactivity change associated with loss of sodium is the net reactivity effect of opposing phenomena, because many of the cross sections required for accurate prediction are quite uncertain, and because definitive measurements in representative critical experiments remain to be made, considerable error is to be expected in predictions of sodium void effects; large differences occur among comparable predictions by different physics groups.¹⁸⁻²⁰

Reviews of recent analytical developments on the sodium coefficient can be found in Refs. 6, 10, 21, and 22, among others.

The Doppler effect, sodium void effect, and some other performance characteristics of a range of reactors having idealized spherical or infinite-slab geometry and equilibrium core and blanket compositions are given in Table 2-2.²³

The Doppler effect, $T dk/dT$ at operating temperature was estimated to be in the range of -0.005 to -0.008 for large oxide- or carbide-fueled reactors, when deliberate spectral softening is not used. Somewhat higher fuel enrichments, or an increase in parasitic capture in the core could lead to somewhat smaller values. Reactor No. 11 illustrates the increase in Doppler effect upon adding 7.5% BeO to reactor No. 3. Reactor No. 12

illustrates the large reduction in Doppler effect in the fuel if a strongly absorbing structural material like niobium is used in reactor 11. (The reduction in Doppler effect and breeding ratio and the increase in positive sodium-void effect would be appropriately smaller if, say, only 8% of the core structural material were a strong absorber like niobium).

If all the core sodium were removed, hardening of the neutron energy spectrum would reduce the Doppler effect of reactors like No. 3 or 8 by one fourth to one half.²⁴ Some discrepancies currently exist between experiment and theory in this regard.²⁵

Sodium coefficients and the sodium-void reactivity effect are strong functions of reactor size, composition, and geometry. Reactors 2 to 4 illustrate the oxide case, whereas reactors 1, 3, and 5 show the effect of core size at fixed composition in spherical geometry (which simulates a cylinder of $L/D = 1$). The reactivity effects tabulated correspond to uniform removal of 40% of the core sodium and to removal of 40% of the core sodium from only those inner portions of the core that contribute a positive reactivity effect. Loss of sodium from the blanket generally reduces reactivity in increasing amounts as the volume fraction of sodium increases, especially as the blanket thickness decreases. Reactivity losses ranging from 0.5 to several percent Δk are possible on complete sodium loss from the blanket.^{26,27}

A large-volume core having one dimension deliberately foreshortened (e.g., a flat cylinder) will have more neutron leakage, a higher fuel enrichment and critical mass, and a sodium-void effect more characteristic of a small spherical reactor. However, an infinite slab does have a somewhat more positive sodium-void effect than a sphere of identical composition, as is illustrated in Table 2-2. The practical reactor of the same composition should have a sodium-void effect between that of a sphere and an infinite

TABLE 2-2 NUCLEAR PERFORMANCE OF PLUTONIUM-²³⁸U OXIDE- AND CARBIDE-FUELED FAST REACTORS

Reactor No.	Spherical core volume (litres)	Fuel type ^b	Core volume fractions fuel/cladding/Na	Critical mass, kg of fissile isotopes	Ratio of ²³⁹ Pu + ²⁴¹ Pu to ²³⁸ U + ²⁴⁰ Pu	Breeding ratio ^c (BR)	Core conversion ratio	Na-void effect, %Δk ^d				% core volume with positive Na coefficient (sphere)	Doppler effect T $\frac{dk}{dT}$
								Sphere		Slab			
								Uniform core removal	Worst case	Uniform core removal	Worst case		
1	2000	O	30/20/50	699	0.163	1.30	0.72	+0.05		+0.60			0.005
2	3000	O	15/15/70	777	0.261	1.18	0.46	-1.13	+0.10	-0.07	+0.22	13	0.005
3	3000	O	30/20/50	965	0.148	1.29	0.79	+0.34	+0.55	+0.78	+0.84	46	0.006
4	3000	O	45/20/35	1141	0.113	1.41	1.03	+0.53	+0.59	+0.72	+0.87	61	0.007
5	5000	O	30/20/50	1473	0.134	1.29	0.88	+0.70	+0.81	+1.03	+1.08	57	0.008
6	1000	C	30/20/50	463	0.153	1.47	0.72	-0.14		+0.48	+0.55		0.005
7	2000	C	15/15/70	644	0.226	1.35	0.50	-1.03	+0.11	+0.09		13	0.005
8	2000	C	30/20/50	792	0.128	1.46	0.87	+0.31	+0.50	+0.77	+0.82	46	0.006
9	2000	C	45/20/35	937	0.098	1.57	1.12	+0.49		+0.67			0.007
10	3000	C	30/20/50	1104	0.118	1.46	0.95	+0.58		+0.95			0.007
11	3000	O	27.5/17.5/ (BeO) 47.5	936	0.158	1.18	0.77	+0.14	+0.36			40	0.011
12	3000	O	27.5/17.5/ (BeO) 47.5 (Nb)	1376	0.251	0.85	0.48	+1.0	+1.03			75	0.001

^a Reactors have 18-in.-thick blankets and 6-in.-thick reflectors. For spherical reactors, blanket composition is 45% fertile material, 20% steel, and 35% sodium. For slabs, blanket volume fractions are same as for core. Oxide core has oxide blanket, etc. In equilibrium reactor, fission products equivalent to 5% burn-up replace core fuel. Isotopic composition of plutonium estimated, assuming "Mix core and blanket Pu, cell excess". ²⁴⁰Pu/²³⁹Pu = 0.15 for metal, 0.2 for carbide, and 0.25 for oxide. Blanket fuel is 2% plutonium. Reflector is 60% steel and 40% sodium.

^b Fuel type: O, Oxide; C, carbide.

^c Breeding ratio equals fissile isotopes produced divided by fissile isotopes used up in entire reactor.

^d The value %Δk corresponds to uniform removal of 40% of sodium originally present in region under consideration. For "worst case", removal was only from central zone having positive contribution.

slab. For sphere or slab, in large sizes, there may be a large, positive, central sodium-void effect.

Large reactors are also subject to a large radial maximum-to-average power ratio unless gradations in enrichment or fuel concentration are used. Two or three zones of enrichment are usually employed. This flattening of the power and the flux leads to more positive sodium-void effects.

2.3 Some Accident Considerations

A direct consequence of the short prompt neutron lifetime in fast reactors is the fact that a single super-prompt-critical burst will take its course before mechanically-produced motions of control mechanisms, triggered at the onset of the burst, can be large enough to produce any significant effect on the outcome of the burst.* During such bursts, there is also insufficient time for conduction of heat from fuel to coolant in the usual heterogeneous power reactor unless the sodium is contacted by fuel in the form of vapor or very small particles.

If the fuel in solid form is tending to expand, the motion will probably be so rapid that inertial effects, resisting the motion, become significant.

The motion may lag the energy buildup, and appreciable stresses may appear. If fuel displacement is to be the phenomenon that loses reactivity and terminates the burst, very high pressures may result before sufficient expansion can take place.

A second important characteristic of fast reactors is that they gain reactivity when the fuel is rearranged in a more dense configuration, displacing coolant. In the typical fast power reactor design, the volume fraction of the core that normally is occupied by the fuel material ranges

* The power rises as $\exp [\alpha t] = \exp [(\Delta k / \ell) t]$ where Δk is measured above prompt critical and ℓ is the prompt neutron lifetime. If $\ell = 5 \times 10^{-7}$ for a fast power reactor, and $\Delta k = 0.0005$, the power rises by a factor of ten every millisecond. For a thermal reactor having a lifetime, say 40 times larger, $\Delta k = 0.02$ would result in the same rate of increase of power.

between 20 and 40%. The critical mass is usually several times greater than that which would result for a reflected sphere whose core was pure fuel material at its normal density. Thus, large amounts of excess reactivity are potentially available from any phenomenon which moves the fuel into geometric arrangements of higher average fuel density. However, the short prompt lifetime limits to a fraction of a dollar the amount of reactivity which can be inserted by the typical phenomena considered before temperature rises sharply and the burst is terminated. Rather than the total amount of excess reactivity available, it is the rate of reactivity insertion at prompt critical which primarily determines the course of events. Increased fuel density might occur due to inward bowing of the fuel. It might happen if the fuel material melted and settled within the fuel-pin jackets. It might result upon escape of molten fuel from the fuel jackets and reassembly into a more dense configuration, possibly near the core center due to pressures generated within the fuel pins, or possibly near the bottom of the core under the action of gravity.

Reactivity might also be gained by voiding of sodium, particularly from the central regions of the reactor. However, except for controlled experiments, the manner in which an actual power reactor may become super-prompt-critical is difficult to define realistically. The power coefficient is expected to be negative with a negative prompt component due to the Doppler effect. Total amounts of reactivity required to operate are relatively small. The reactivity addition rates normally available via control mechanisms are usually kept quite low. Instrumentation should observe excess power levels or short reactor periods, while the reactor is well below prompt critical. A signal should then initiate action which terminates the event.

2.3.1 Sodium Voiding

The rate of reactivity change from gross sodium voiding will depend on the pattern of voiding as well as the reactivity worth. In the postulated accident resulting from a loss-of-coolant flow without scram, boiling initiates near the top of the core. The blanket voids first, followed by voiding downward through the core. Reactivity is first lost, then gained. Depending on core design, particularly as it affects coherence of voiding between subassemblies, reactivity insertion rates ranging from 15 to 65\$/sec have been ascribed to this source.^{2,28-30} However, the role of fuel failure in voiding has not been allowed for in these analyses; the place of failure, the pressures generated from fuel-coolant interaction, and the possibility of fuel movement are some of the additional considerations that would be involved.

If boiling is postulated to be caused by a limited magnitude, superprompt-critical excursion which is turned around by the Doppler effect, boiling may initiate at or near the center of the core.²³ In the limiting case of a core with perfectly flattened power and completely uniform flow, boiling and voiding could be coherent across the core, and an initiating reactivity ramp of only 20¢/sec could, in principle, trigger a secondary reactivity insertion rate of 50 to 100\$/sec (or more, if fuel failure is assumed to mix molten fuel and sodium at the center of each subassembly simultaneously).

Even if boiling does not begin at the core center, a moderate reactivity ramp of 1\$/sec, if not terminated by a scram, can lead to sizeable reactivity insertion rates. The possible course of such a transient (ignoring fuel-failure effects) is illustrated in Figs. 2-3 through 2-10.²⁹ The maximum reactivity ramp due to sodium voiding was about 20\$/sec in this example. However, fuel-failure threshold temperatures may have been reached before

Fig. 2-3. Coolant Temperature Profiles at Various Times during 1\$/sec Reactivity Excursion in Large, Oxide Fueled Reactor

Fig. 2-4. Vapor Void Fraction Profiles at Various Times

Fig. 2-5. Mass Flow Rate Profiles at Various Times

Fig. 2-6. Average Mass Flow Rate versus Time

Fig. 2-7. Pressure Profiles at Various Times

Fig. 2-8. Fuel Centerline Temperatures at Various Times

Fig. 2-9. Reactor Power Level versus Time

Fig. 2-10. Feedback Reactivities versus Time

boiling. Also, the voiding model used suffered from several deficiencies, as will be discussed below. However, the pattern of such an accident can be grasped qualitatively from this example.

Voiding of a single subassembly should add only several cents; hence, large numbers of subassemblies must be involved to add a dollar. It is theoretically possible, however, for voiding to occur when the reactor is close to prompt critical, and less than a dollar is then required to induce a major excursion in power.

Large gas bubbles in the coolant might void several subassemblies coherently; however, a very large bubble would be required to add a dollar.

2.4 Introduction to Sodium Boiling

Typical sodium-cooled fast reactor designs employ a coolant temperature rise through the core of 50 to 200°C and a bulk exit coolant temperature of about 600°C. Hence, a considerable margin exists to the sodium boiling temperature range of 930°C to 1030°C, corresponding to a core pressure of 1 1/2 to 3 atm. The ratio of heat production to coolant flow may have to exceed the design value by more than three for boiling to occur. However, under postulated abnormal operating or accident conditions, such as local or gross flow blockage in a subassembly, an uncontrolled rise in core power, or a loss-of-coolant flow without prompt reactor shutdown, boiling temperatures may be attained.

The boiling process or voiding of the sodium is of interest in studies of abnormal reactor behavior for several reasons:

- 1) Due to the change in state, the overall heat transfer effectiveness may be sharply reduced, leading to a rapid temperature rise of clad and fuel, clad failure, and fuel melting.

- 2) Rapid reactivity changes (positive or negative) may occur due to voiding, oscillation, or reentry of liquid sodium.

3) Pressure surges due to boiling or subsequent condensation of the sodium vapor bubbles, or due to physical interactions between liquid sodium and molten fuel, may damage the core structure or produce reactivity changes by inducing rapid motion of the fuel.

The boiling process can generally be divided into three areas. The first concerns sodium boiling and motion with relatively intact reactor coolant channels. The second concerns fission gas release and the third, sometimes termed "violent boiling or fuel-coolant interaction," concerns vapor production and pressure generation when molten fuel material encounters liquid sodium. These notes will discuss all three modes of voiding.

Among the phenomena which require consideration in the study and analysis of channel voiding and reentry are:

- 1) bubble nucleation and sodium superheat
- 2) bubble growth (and collapse)
- 3) possible forms of two-phase flow
- 4) mechanisms of mass and heat transport during voiding, collapse, and reentry
- 5) whether voiding is due to boiling, fission gas release, or fuel-coolant interaction.

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3. SODIUM SUPERHEAT

3.1 Comparison of Na and H₂O Under Boiling Conditions

While the occurrence of spontaneous vaporization is possible, extensive experimentation has shown that initiation of nucleation (void formation) in engineering systems occur from contact with a heated surface or gas bubbles in the liquid. The postulated mechanism for surface nucleation is that boiling is initiated from microcavities present on the surfaces that contain small bubbles of gas and vapor. The bubble growth is accomplished against the liquid pressure and the surface tension forces by an elevation of the pressure of the vapor in the cavities. Since the vapor and liquid across the bubble surface must be in thermal equilibrium, the temperature must be higher than the saturation temperature of the liquid such that $P_{\text{vapor}} - P_{\text{liquid}} = \frac{2\sigma}{r}$. The liquid superheat is defined as $T_{\text{wall}} - T_{\text{saturation}}$ at the initiation of nucleation.

In close-packed fuel systems such as LMFBRs, the consequences of void formation are severe in that flow starvation as well as positive reactivity insertion can result. This is due to a combination of factors including sodium properties and exit core pressures. A key uncertainty associated with boiling in sodium systems is the degree of superheat necessary to initiate nucleation. In clean experiments with liquid metal systems, superheats of the order of 10-900°F have been observed for a variety of liquid metals, surface finishes and other variables. The large spread in data is indicative of the fact that many variables do affect superheat in ways which are only partly understood at this time. Figure 3-1 presented by Fauske¹ demonstrates the state of art as of the International Conference on the Safety of Fast Reactors at Aix-en-Provence, France, September 1967. Such high superheats, while they inhibit the initiation of nucleation, result in

Fig. 3-la. Superheat Data Presented at the Aix Conference

Fig. 3-lb. Superheat Data Published Previous to the Aix Conference

a large degree of energy storage and hence a large amount of vaporization when nucleation is finally initiated.

However, in reactor systems using commercial grade sodium it is anticipated that significantly lower superheats will be required for nucleation. This behavior may result from the large variety of surface conditions present in the engineering system or from suspended particulate matter or gas bubbles in the sodium which serve as nucleation sites. Consequently, it may be anticipated but must be demonstrated that superheats more comparable to those in water systems (for water 1-20°F superheat at 2000-500 psia) may result in sodium, although theoretically under identical surface conditions, superheats for sodium systems are about an order of magnitude greater than for water systems. Figure 3-2 illustrates this order of magnitude difference in superheats. The higher results for sodium are primarily due to the higher boiling point, higher surface tension and lower vapor density of sodium in the low pressure region (see Table 3-1). However, as will be discussed, the governing mechanisms for sodium versus water boiling require careful consideration of the effective size of the active cavity and the cavity inert gas content. Therefore, the general conclusion of Fig. 3-2 may not be true for real reactor sodium systems versus real reactor water systems. However, the consequences of boiling initiation at a given degree of superheat in sodium is considerably more severe than for high pressure water systems, as is discussed below.

1. Fraction of liquid superheated and subcooled. As a result of the low conductivity of water, due to the very steep temperature profile near the wall (characteristic of high Prandtl Number fluids ≥ 0.7) only a small portion of the water near the wall is superheated and the remainder subcooled. For high conductivity, (Prandtl Number is .004-.005) the temperature profile is much less steep at the wall and in the bulk fluid and a much larger fraction

Fig. 3-2. Superheat Values for Sodium and Water for a Given Cavity Size

TABLE 3-1 RELATIVE PARAMETERS OF WATER AT 2250 PSI AND NA AT 1 ATMOSPHERE

	H ₂ O	Na
k, thermal conductivity		
k, film coefficient (BTU/hr-ft ² -°F)	3000	30,000
h _{fg} , latent heat of vaporization (BTU/lb)	414	1890
C _p , specific heat (BTU/lb°F)	1.9	0.3
v _l , specific volume of liquid (ft ³ /lb)	.027	.0216
v _v , specific volume of vapor (ft ³ /lb)	.152	65
T _{sat} , boiling point (°F)	652	1632
σ, surface tension (lb/ft)	.00045	0.008

of the liquid can generally be superheated. The more shallow temperature profile and the small film temperature drop for Na, which are illustrated in Fig. 3-3, allow the superheat energy which can be stored in a sodium system to exceed that of water reactor systems.

2. Ratio of latent heat of vaporization to specific heat. The quenching or condensation of vapor formed by nucleation is accomplished by the subcooled portion of the liquid. The effectiveness of this condensation is enhanced by a low ratio of latent heat of vaporization to specific heat. For water at 2250 psia this ratio h_{fg}/C_p is relatively low at

$$\frac{414 \text{ BTU/lb}}{1.9 \text{ BTU/lb}^\circ\text{F}} = 220, \text{ compared to the very high value of}$$

$$\frac{1890 \text{ BTU/lb}}{.3 \text{ BTU/lb}^\circ\text{F}} = 6000 \text{ for sodium at 1 atm}$$

3. Ratio of specific liquid volume to specific vapor. The consequences of voiding with respect to flow blockage become more severe as the volume of the vapor formed increases. Specifically the higher the ratio of $\frac{v_v}{v_l}$, the more severe the consequences of voiding. For water at 2250 psia the ratio is relatively low at $\frac{.1520}{.0271} = 5.6$, whereas for sodium at approximately atmospheric pressure the ratio is relatively high, $\frac{65.0}{.0216} = 3000$. Note that a similarly high ratio exists for water at low pressure $\frac{8.515}{.01724} = 495$ at 50 psia and consequently water systems operated at low pressure are susceptible also to severe consequences if voiding should occur.

Taken together, these factors indicate the following behavior (water at pressure approximately 500-2250 psia, sodium at approximately 10-100 psia) as the heat flux is increased. In a water system, nucleation will be initiated

Fig. 3-3. Comparative Energy Storage in Water and Sodium Reactor Channels

at low superheats and gradually be increased until bulk boiling takes over. The heat flux limit will generally be determined by critical heat flux or fuel centerline melt. On the other hand for sodium, as the heat flux is increased, the wall temperature will increase and a large amount of energy may be stored by the superheated portion of the liquid. At some point nucleation will be initiated, causing a considerable fraction of the sodium to vaporize that cannot be condensed. Due to the large associated increase in volume, this vaporization can cause rapid voiding of the fuel channel.

It should be noted that a decrease in sodium flow rate can yield the same result as the postulated increase in heat flux. In fact, local voiding would probably occur in a real system due to local flow decrease which can result from minor variations in channel geometry (bowing, etc.). This situation is aggravated by a close-packed design which is not favorable to coolant mixing.

With regard to reactor performance, in addition to uncertainties in structural and mechanically induced effects due to this sudden generation of vapor, severe thermal problems can occur due to flow choking. The incidence of boiling yields a large increased pressure drop due to item 3. In low pressure liquid systems, the coolant flow characteristic is thus multivalued as shown on Fig. 3-4. From this figure, it is seen that if the coolant flow rate falls such that the system enters the negative sloping region, a deteriorating situation exists, leading to flow reduction to point 4, which essentially leads to a heat transfer crisis.

3.2 Review of Experimental Results Through 1967

The experimental results on sodium superheat up to late 1967, including the many experiments reported at the International Conference on Fast Reactor Safety held at Aix-en-Provence led to a very great range of apparently possible superheat, as was previously illustrated in Fig. 3-1.

Fig. 3-4. Flow Characteristics for Sodium-cooled Cores

We quote from Fauske's review of the Aix Conference¹ as follows:

"Four papers dealt directly with new experimental data on superheat of potassium and sodium.²⁻⁵ These data (Fig. 3-1a) were particularly noteworthy by their large variation, 9 to 1480°F, as compared to the variation in previously published data, 0 to 500°F (Fig. 3-1b). Surprisingly, there was little (in some cases^{2,3} zero) discussion of the older data.

Influential Variables

"Extrinsic variables that can affect superheat behavior include the heat flux, heating surface, fluid velocity, pressure, and pressure-temperature history, aging, purity, radiation, heating method...and perhaps many more.

Heating Method

"Investigators who have used direct heating^{2,3} (joule heating in the liquid metal itself) report superheats considerably higher than do those who have used indirect heating methods^{4,5} (conduction heating through the coolant-channel wall). The latter are more representative of the type of heating that takes place in an actual reactor coolant channel. Considering the direct-heating studies of Ref. 2 and taking into account the resistivities and thermal capacities of steel and sodium, it appears that approximately two-thirds of the thermal energy supplied was dissipated in the sodium and that one-third was stored in the wall. Perhaps direct heating causes homogeneous nucleation (and, therefore, higher superheats) rather than the wall nucleation that occurs with indirect heating. [And perhaps not; Fig. 3-1b shows earlier Argonne National Laboratory (ANL) data⁶ obtained via direct heating where observed superheat values are relatively small.] It should also be noted that, quite possibly, variables other than the heating method may have caused the differing results, as will be seen later.

"The importance of indirect heating is emphasized by the results of experiments in which subcooled nucleate boiling occurred.⁷ (Negative bulk

superheat could be an important consideration in the analysis of transient expulsion.) Measurements of the actual wall superheat present during these experiments would be most welcome and are being obtained in experiments now in progress.⁷

Surface Condition

"French superheat data² spread from about 20 to 200°C, values that they consider compatible with the condition of the surface of the experimental tube used. The importance of surface roughnesses is indicated in Eq. 3-17 by the variable r_a , the cavity radius.

"Thorough studies of the relation between surfaces and incipient boiling have been published,⁸⁻¹⁰ and they clearly show the effect of roughness. With as-received surfaces (commercially smooth), measured potassium superheats have ranged from 500°F at 1500°F (saturation temperature) down to 230°F at 1770°F, but, when eight 0.006-in. dia holes were drilled 0.040 in. deep in the same surface, the superheat was reduced⁹ about 40 fold. These data agree reasonably well with predictions from an expression similar to Eq. 3-20 when $\cos \theta_r$ is equal to 1.* However, it is important to note that Ref. 9 indicates some doubt as to whether the data illustrated in Fig. 3-1b represent incipient superheat measurements or stable nucleate boiling superheats. In a more recent paper,¹⁰ it appears that these data represent the behavior of the superheat existing in the stable boiling mode and that the incipient superheat is more closely related to the saturation temperature than to the cavity size or surface condition.

Aging

"In the recent French experiments,² the mean value of the superheats reached for a given pressure appears to increase slowly with time. This observation is in agreement with the Ispra data³ and indicates that the

* With the partial pressure $P_A^* \approx 0$.

larger nucleation sites are eliminated as boiling goes on, i.e., the larger nucleation sites get washed out and completely wetted with sodium; thus, smaller cavities are activated, and--according to Eq. 3-20--superheat increases. However, earlier experimental data⁶ show the opposite trend, as noted in Fig. 3-5. Aging effects were not reported by the experimenters who used conduction heating techniques.^{4,5}

"The possible increase in superheat with time appears to be an important consideration in further studies on superheat. During the life of a fast reactor, the coolant could conceivably clean the heating surface to a large degree, leaving smaller and smaller cavities available for nucleation.

Heat Flux

"Differing results were reported at the Aix Conference concerning heat flux. Heating power sometimes appears to be an unimportant parameter,^{2,3} but Refs. 4 and 5 indicated a definite heat-flux dependency. Reference 4 reports a decrease in superheat with increasing heat flux--Ref. 5 an increase--as shown in Fig. 3-6. Also indicated in Fig. 3-6 are data from a Russian paper¹¹ published in 1966. Quoting this paper: "The tendency of the nucleate boiling to change to natural convection can be noticed at low heat fluxes. In this case, the value of the liquid superheat sharply increases." In a more recent study,¹⁰ there was no sizeable effect of heat flux upon incipient superheat. Obviously, the possible heat-flux dependency deserves further study.

Velocity

"In recent Ispra experiments, measured superheats were generally lower under forced or natural circulation than under stagnant conditions, for the same mockup and filling,³ but no quantitative velocity dependency was reported. However, this velocity relation agrees with earlier data⁶ where relatively low superheat values were observed during forced circulation.

Fig. 3-5. Superheat Values Evidently Can Change as Experiments Age.
In fast reactors the aging effect may be important.

Fig. 3-6. Heat Flux May Influence Superheat as Indicated by These Values for Sodium

Fig. 3-7. A Velocity Effect is Apparent in These Data for Bulk Superheat in Sodium at 5 psia

More recent data⁷ show a definite velocity effect (Fig. 3-7). These data are for bulk superheats and could differ significantly from results with wall superheat.

Pressure and Pressure-temperature History

"Both pressure and pressure-temperature history were shown by ANL to be important variables.⁵ The pressure effect--increasing superheat with decreasing pressure according to Eq. 3-20 and Fig. 3-2--is also apparent (but to a lesser degree) in the French data.² Earlier published data^{6,19} also clearly show the expected pressure effect on the incipient superheats (Fig. 3-8). A simplified analysis¹² appeared to predict the effects of pressure-temperature history reasonably well,⁵ and may explain part of the variation found in published data where this variable is not accounted for.

Instability

"The characteristic instability of boiling sodium--intermittent bubbling--was noted only at low heat fluxes and pressures.^{4,5} Recent observations are in agreement with previous work,^{11,13,14} where analyses have shown that stability increases with increasing heat flux and pressure. Two new papers^{10,15} also discuss the instability of alkali-metal boiling. An instability map that indicates pressure levels and heat fluxes necessary to maintain stable boiling for a particular surface is given in Ref. 5.

Purity and Gas Content

"The extremely high values of superheat reported in the Ispra paper³ were attributed primarily to a more efficient purification of the liquid metals before testing. System cleanliness undoubtedly is very important. However, in most experiments reported, neither the actual purity of the liquid metal nor the dissolved gas content has been very well known. Better agreement among various laboratories will be achieved when the systems and the fluids in them are better defined.

Fig. 3-8. Incipient-boiling Superheats in Sodium Show a Definite Pressure (or temperature) Effect

Radiation

"The effects of nuclear radiation on incipient boiling were discussed in a British paper at the Aix Conference.¹⁶ Several different types of radiation were considered: electrons, protons, alpha particles, photons, neutrons, fission fragments, and alpha recoils. It was concluded to be highly unlikely that, at saturation pressures near 1 atm, any nuclear radiation encountered would induce nucleation of vapor bubbles in bulk sodium before boiling would occur at surface cavities. However, a number of assumptions were made in these theoretical calculations; experimental verification seems in order."

Much additional experimental work has been performed, and to some extent, the range of anticipated superheat in a reactor system has been narrowed down as will be discussed later in sections 3.6 and 3.7. But first, some discussion of the theoretical ideas concerning nucleation of boiling will be given.

3.3 Homogeneous Nucleation Theory

The potential sources of bubble nucleation include the following:

- 1) Homogeneous nucleation due to the spontaneous formation of critical size nuclei containing no foreign matter and disallowing the possibility of nucleation at the container walls.
- 2) Heterogeneous nucleation within the liquid due to suspended microscopic particles that contain permanent gas pockets, or more particularly, due to suspended small bubbles of inert gas.
- 3) Heating at the walls with surface crevices or cavities containing gas and/or vapor.

We may use Schlelendahl's discussion¹⁷ to illustrate that homogeneous nucleation would lead to much higher superheats than are observed and, hence, cannot be the dominating process.

"According to Ref. 18, the probability for the appearance of a spherical critical nucleus of radius r is approximately

$$N_k = N_L e^{-\Delta F_g / kT} \quad (3-1)$$

where ΔF_g is the free nucleation energy

$$\Delta F_g = \frac{4\pi}{3} \sigma r_k^2 \quad (3-2)$$

and N_L is the modular density of the pure material, i.e., the Loschmidt number. This yields the critical radius

$$r_k = \sqrt{\frac{3kT}{4\pi\sigma} \ln \frac{N_L}{N_k}} \quad (3-3)$$

"On the other hand, we have from the steady-state condition for the bubble internal pressure

$$p_k - p_f = \frac{2\sigma}{r_k} \quad (3-4)$$

"If the saturated vapor curve is known and we assume the vapor is in thermodynamic equilibrium with the liquid, we can calculate the maximum superheat from Eq. 3-3 and 3-4, for a number N_k of nuclei per mole at pressure p_f .

"Figure 3-9 shows the calculated temperature T at which bubble formation due to spontaneous nucleation is to be expected. Figure 3-9 also gives the region covered by the experimental results of various authors.

"It is clear that this theoretically calculated superheat depends very slightly on the selected nucleus density, and that all measurements are far below the theoretical values."

3.4 Nucleation at Gas Bubbles

The conditions of equilibrium at a curved interface yield relations which are useful in predicting nucleation conditions in coolant channel geometries, which, of course, are not in equilibrium.

First, consider a liquid in contact with a bubble of its own vapor. For curved interfaces, mechanical equilibrium requires that surface tension forces balance the pressure difference between vapor and liquid

$$P_V - P_\ell = \sigma \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \quad (3-5)$$

where r_1 and r_2 are the principle radii of curvature. For a spherical surface, $r_1 = r_2$, and

$$P_V - P_\ell = \frac{2\sigma}{r} \quad (3-6)$$

For a small bubble surrounded by a liquid at uniform temperature, thermodynamic equilibrium then requires that

$$T_V = T_\ell \quad (3-7)$$

Fig. 3-9. Sodium-boiling Delay

If the liquid pressure, P_ℓ is defined as $P_\ell = P_{\text{saturation}}$ (or P_{sat}), corresponding to some saturation temperature, T_{sat} , the liquid will be superheated by an increment $T_V - T_{\text{sat}}$ in order to maintain mechanical equilibrium with a bubble of radius r .

For a fixed system pressure, the relation of the vapor and liquid states for various stable bubble radii is illustrated in Fig. 3-10. Note that as $T = T_V = T_\ell$ increases, the difference between the vapor pressure and the system pressure increases, thereby permitting smaller and smaller bubbles to be stable. Conversely, if the temperature T_V exceeds that point at which an existing bubble is mechanically stable, the internal pressures will exceed the restraining surface tension and liquid pressures, and the bubble will grow, leading to reduced restraining forces from surface tension and continued bubble growth, assuming that mass transfer of vapor from the liquid continues to maintain P_V at the level corresponding to the temperature.

One may derive approximate expressions for the superheat, using the Clapeyron equation

$$\frac{dP}{dT} = \frac{h_{fg}}{v_{fg}T} \quad (3-8)$$

Assuming $\frac{dP}{dT} \sim \text{constant}$, one has

$$\frac{\Delta P}{\Delta T} = \frac{h_{fg}}{v_{fg}T} \quad (3-9)$$

Then, if bubble radius

Fig. 3-10. Relation between Vapor and Surrounding Liquid States for Various Stable Bubble Radii

$$r = \frac{2\sigma}{P_V - P_\ell} = \frac{2\sigma}{\Delta P} = \frac{2\sigma}{\Delta T} \cdot \frac{v_{fg} \cdot T}{h_{fg}} \quad (3-10)$$

Take $\Delta T = T_{\text{vap}} - T_{\text{sat}}$

and $T \approx T_{\text{sat}}$

Then

$$T_V - T_{\text{sat}} \approx \frac{2\sigma}{r} \frac{v_{fg}}{h_{fg}} T_{\text{sat}} \quad (3-11)$$

If a gas exists in the bubble, in addition to vapor, mechanical equilibrium considerations lead to the equation

$$P_g + P_V - P_\ell = \frac{2\sigma}{r} \quad (3-12)$$

or

$$P_V - P_\ell = \frac{2\sigma}{r} - P_g \quad (3-13)$$

and the amount of liquid superheat required to be in equilibrium with a bubble of radius r will be reduced accordingly.

The expression for superheat becomes

$$T_V - T_{\text{sat}} = \left(\frac{2\sigma}{r} - P_g \right) \frac{T_{\text{sat}} v_{fg}}{h_{fg}} \quad (3-14)$$

Various other approximate treatments of the Clapeyron equation lead to similar relationships for the approximate relationships between superheat and bubble radius.

A rough estimate of the superheat required as a function of radius of initial gas bubble (at system pressure) is given in Table 3-2. As can be seen, for bubbles of radius of only 10^{-3} in., the superheat becomes diminishingly small.*

3.5 Models for Surface Nucleation

3.5.1 Hsu's Criterion for Wall Cavity Nucleation¹⁹

"One of the more widely accepted models for predicting incipient-boiling superheat is that introduced by Hsu.²⁰ This model states that for a given cavity of radius r_c to produce vapor bubbles, a necessary condition is that the temperature in the fluid thermal layer at a distance r_c from the heater surface be greater than or equal to the temperature required by the equilibrium equation,

$$T = T_{\text{sat}} + \frac{2\sigma}{r} \frac{T_{\text{sat}}}{h_{fg} P_v} \quad (3-15)$$

Curve A in Fig. 3-11 represents a plot of this equation for the case of saturated boiling [i.e., $T(y = \infty) = T_g$]. With net heat flux, there would be a temperature profile in the fluid layer as illustrated by curves B or C.

The model then requires that this temperature curve be tangent to or intersect with curve A in order to produce bubble ebullition. Hsu²⁰ estimates the temperature profile by the transient conduction equation while Davis and Anderson²¹ utilize the steady-state conduction equation. In the latter case,

* In section 3.7 below, the possibility of bubble formation as a function of reactor system design and the effect on potential superheat is discussed.

TABLE 3-2

APPROXIMATE SUPERHEAT REQUIRED AS A FUNCTION OF RADIUS
OF INITIAL VAPOR BUBBLE AT ATMOSPHERIC PRESSURE

radius, r , in.	superheat, ΔT , °F
$3 \cdot 10^{-2}$	0.6
$3 \cdot 10^{-3}$	6
$3 \cdot 10^{-4}$	60
$3 \cdot 10^{-5}$	600

Fig. 3-11. Hsu's Criterion for Bubble Nucleation in Ordinary Fluids

the minimum wall temperature necessary for vaporization at a given heat flux is $T_{w,m}$, as indicated by the tangent curve B, where curve B is given by

$$T = T_w - qy/k \quad (3-16)$$

This method also predicts the radius of the cavity, $r_{c,m}$, which would initiate boiling at a given heat flux and wall superheat (see Fig. 3-11). Another necessary condition of course is that cavities of this dimension be present and that at least some of them be nonflooded, i.e., contain a vapor pocket.

In the case of saturated water at 1 atm pressure, with a q of 100,000 BTU/hr-ft², the model predicts that the wall temperature, $T_{w,m}$, must exceed 226°F (14°F superheat). The corresponding $r_{c,m}$ would be 3×10^{-4} in., sufficiently small to have a good probability of entrapping vapor. On a heating surface of normal "as-received" finish, cavities of such dimensions would be plentiful so that incipient boiling would be likely to occur as predicted.

In the case of liquid metals, however, the situation becomes more complicated. Because of the higher thermal conductivities, the temperature profile at a given heat flux tends to be much less steep for liquid metals. This leads to intersections of curves A and B at much greater values of $r_{c,m}$. Thus, for potassium at 1 atm and $q = 100,000$ BTU/hr-ft², $r_{c,m}$ would $>4 \times 10^{-3}$ in., with a corresponding wall superheat of 4°F. These larger cavities are much more likely to be flooded, especially with well wetting fluids such as the alkali metals. It then becomes questionable whether there would exist any nonflooded cavities of radius $r_{c,m}$ to initiate boiling at the predicted superheat. This possible lack of potential sites seems to

be born out by experimental evidence (see Fig. 3-1) which indicates incipient superheats much higher than predicted by the foregoing criterion. Apparently, larger cavities become flooded to some extent and only smaller cavities remain as potential sites--thus, requiring a superheat higher than the minimum superheat predicted by Hsu's criterion. Therefore, in the case of liquid metals (especially the alkali metals), it may be said that Hsu's criterion imposes a necessary condition, but does not define a sufficient or limiting condition for establishing incipient-boiling superheat. Assuming Hsu's criterion to be satisfied, a further criterion is needed for predicting the dimension of the largest unflooded cavity--the largest existing potential site. This dimension would then set the real limit on the superheat required for initial vapor generation."

3.5.2 Pressure-temperature History Model for Cavity Deactivation

Holtz¹² suggested that the operating history of an experimental apparatus prior to an incipient boiling test and the physical condition of the liquid and heating surface determine the size range of surface cavities that are not completely flooded with liquid and thus are available as possible nucleation sites. The model maybe summarized as follows:

1. For a cavity to be a potential nucleation site, it must not be completely filled at time of initial charging. That is, at initial filling, the liquid bridges the cavity at some cavity radius, r_0 , trapping vapor and gas in the cavity.

2. During preboiling operations, the liquid interface may be caused to move deeper into the cavity by changes in the local vapor pressure and local static pressure. For any given condition the position of the liquid interface is determined by a balance of pressure and surface forces.

3. For a given cavity, the minimum wetted radius (r_a), corresponding to the deepest penetration of the liquid into the cavity, determines the critical bubble radius for subsequent bubble nucleation.

4. The superheat for incipient boiling is determined by the excess vapor pressure (above static pressure) required to generate a bubble nucleus of the critical radius, as determined by a balance of pressure and surface forces at the boiling temperature and pressure.

Figure 3-12 illustrates that the depth of liquid penetration into a surface cavity will increase as the liquid pressure is increased, while a temperature decrease, which causes the vapor pressure to decrease and the surface tension to increase, may result in either an increase or decrease in penetration depth. If we consider the initial steady-state conditions prior to maximum penetration as T_{Li} , P_{Li} , P_{Ai} , and note that the inert gas can either leave or enter the cavity,* the final penetration depth will be determined by the sequence of changes in T_{Li} , P_{Li} , and P_{Ai} .

Consider the bubble equilibrium equation at the initial conditions

$$P_{Li} - P_V(T_{Li}) - P_{Ai} = \frac{2\sigma(T_{Li})|\cos \theta_a(T_{Li})|}{r_{ai}} \quad (3-17)$$

If the liquid temperature is reduced from T_{Li} to T_L' , the vapor pressure in the cavity P_V and the surface tension σ will decrease, while the contact angle θ_a and the inert gas partial pressure P_A will increase, the latter due to the decrease in gas solubility in the liquid. The new equilibrium condition will be given by

* Holtz's first model included only vapor pressure effects. Later workers^{19,22} included the effects of inert gas.

Fig. 3-12. Surface Cavity during Liquid Penetration

$$P_{Li} - P_V(T_L') - P_{Ai} = \frac{2\sigma(T_L') |\cos \theta_a(T_L')|}{r_{ai}} \quad (3-18)$$

If the gas pressure were increased, instead, and the rate of wetting in surface cavities is fast relative to the rate of diffusion of inert gas in the liquid, the minimum wetted radius can be calculated from the relation

$$r_{a2} = \frac{2\sigma(T_{Li}) |\cos \theta_a(T_{Li})|}{P_L' - P_V(T_{Li}) - P_{Ai}} \quad (3-19)$$

Now, consider the incipient-nucleation conditions of the wall and liquid temperature are increased. The cavity pressure will increase, causing the radius of curvature of the vapor-liquid interface to change sign, as illustrated in Fig. 3-13. When conditions are reached such that the following relation is satisfied

$$P_V(T_w) + P_A^* - P_L = \frac{2\sigma(T_w) |\cos \theta_r(T_w)|}{r_{a2}} \quad (3-20)$$

nucleation and bubble growth will occur, where r_{a2} is calculated from 3-19 (assuming that was the previous history).

Combining Eq. 3-19 and 3-20 yields

$$P_V(T_w) - P_L = [P_L' - P_V(T_{Li}) - P_{Ai}] \frac{\sigma(T_w) \cos \theta_r(T_w)}{\sigma(T_{Li}) \cos \theta_a(T_{Lw})} - P_A^* \quad (3-21)$$

where $P_V(T_w) - P_L$ is the pressure excess required for nucleation. The superheat requirement is then

Fig. 3-13. Surface Cavity at Incipient Nucleation

$$\Delta T = T_w - T_{sat} \quad (3-22)$$

with T_{sat} corresponding to P_L .

The pressure-temperature-history model has met with partial success in explaining experimental results. The model has continued to undergo evolution. In a recent modification, Holtz et al.²³ attempted to allow for the diffusion of inert gas in and out of unwetted surface cavities and for the effect of the cover gas system on superheats. The following assumptions were made to examine the question of superheat in the absence of gas bubbles:

- 1) The system has been operating at a steady-state condition for a sufficiently long time (this length of time is probably of the order of a day).
- 2) No inert gas bubbles come out of the liquid solution as the sodium flows around the primary loop; hence, the concentration of the inert gas in the liquid solution is constant as the liquid sodium is pumped around the system.
- 3) The solution at the sodium-inert gas interface is assumed saturated with the inert cover gas.

Since the sodium at the inert gas interface is assumed saturated with inert gas, the concentration of inert gas in the liquid solution can be expressed by Henry's Law.

$$C = K(T_{gb})P_{gb} \quad (3-23)$$

Since it is assumed that no gas is removed from the liquid during the circulation of the sodium around the primary loop, the concentration of inert gas in the sodium at all points in the system is the same as that expressed by

Eq. 3-23. However, the system has been operating for sufficiently long time; hence, the partial pressure of inert gas in the surface cavities will be in equilibrium with the concentration of inert gas dissolved in the liquid sodium.

$$C = K[T_L(X)]P(X) \quad (3-24)$$

Combining Eqs. 3-23 and 3-24 yields the partial pressure of inert gas in the surface cavities of the core zone as a function of temperature and axial location.

$$P(X) = \frac{K(T_{gb})P_{gb}}{K[T_L(X)]} \quad (3-25)$$

Combining Eq. 3-25 with the equation obtained from the pressure temperature modeling technique for determining the maximum penetration of the liquid vapor interface into a surface cavity, the minimum radius of curvature of the liquid-vapor interface inside the surface cavities can be expressed as a function of core location.

$$R(X) = \frac{2\sigma[T_L(X)]}{P_L(X) - P_V[T_L(X)] - \left\{ \frac{K(T_{gb})}{K[T_L(X)]} \right\} P_{gb}} \quad (3-26)$$

Now, employing Eqs. 3-25 and 3-26 and the equation for the prediction of the incipient-boiling conditions from the pressure-temperature-history model, the wall temperature at the inception of boiling can be determined as a function of core position by solving the implicit equation

$$P_V[T_w(X)] + \frac{K(T_{gb})}{K[T_w(X)]} P_{gb} - P_L(X) = \frac{2\sigma[T_w(X)]}{R(X)} \quad (3-27)$$

In order to use Eqs. 3-26 and 3-27 to predict the incipient-boiling conditions during various specified reactor accidents as a function of core length, the following must be known or specified: system geometry, steady-state pressures and temperatures, thermophysical properties of the working fluid, Henry's Law constant as a function of temperature, and the type of accident.

3.5.3 Other Models

Chen¹⁹ modified the original pressure-temperature history model by assuming that the partial pressure of inert gas, present in the surface cavities during filling of the system with the liquid metal, remains constant. Other assumptions included (1) a cylindrical cavity, (2) perfect nonwetting at fill (contact angle of 180°) and (3) perfect wetting at boiling (contact angle of 0°).

Dwyer²² has also modified the original pressure-temperature history model by attempting to account for both the change in the inert-gas partial pressure in the surface cavities and the fact that the radius of curvature of the nonwetted liquid entering the surface cavities may be different than the radius of curvature of the wetted liquid. Dwyer has suggested an R/R' ratio of 0.75.

The results of both Collier and Kosky²⁴ and Singer and Holtz²⁵ have demonstrated that the partial pressure of inert gas in the surface cavities can change significantly with both time and temperature. They show that the assumptions as to the partial pressure of inert gas in the surface cavities in the above models are oversimplifications of the actual physical picture.

The use of a reentrant cavity was postulated by Bankoff²⁶ (see Fig. 3.14). The minimum radius of curvature upon penetration of liquid under prepressurization will exist when the meniscus forms a hemisphere at the sharp corner at position 1. Similarly, upon subsequent heating, the bubble will be forced from the cavity once the meniscus is driven past the hemispherical shape at position 2. This enables the derivation of the same equations previously discussed for the pressure-temperature history model without the assumptions of a switch of contact angle from the nonwetted condition to a wetted condition.

3.6 Experimental Results Since 1967 and Their Correlation with Theory

Although the above general equations representing the pressure-temperature history model are relatively simple, it has been difficult to obtain experimental data which are capable of either verifying, disproving, or refining the basic model. Accurate experimental data are difficult to obtain because it is extremely difficult to assess the contact angle and the inert gas partial pressure, both during the establishment of a history and at incipient nucleation.

3.6.1 Pool Boiling Data

Schultheiss²⁷ has conducted extensive experimental superheat tests with sodium in a system which employed the vapor pressure of sodium as the method of applying pressure to the system. Artificial surface cavities of 0.2 and 0.4 mm were used. During establishing of the system, the system was isothermal. For this case, one can write for the man-made cavity, assuming nonwetting (basic assumption of the pressure-temperature history model in the case of a cylindrical cavity; $\theta > 90^\circ$)

$$\left(P_V'(T) + \rho H \right) - P_V(T) = \frac{2\sigma(T)}{R'} \quad (3-28)$$

Fig. 3-14. Penetration of Liquid into Reentrant Cavity

But since the system was isothermal, $P_V'(T) = P_V(T)$, the above equation can be simplified to

$$R' = \frac{2\sigma(T)}{\rho H} \quad (3-29)$$

The 10 cm head of sodium above the artificial surface cavities resulted in only a pressure difference of about 0.006 atm and the condition that $R' > 0.4 \text{ mm} > 0.2 \text{ m}$. To conduct a superheat run from these predetermined conditions, the sodium near the artificial cavity was heated with a hot finger until nucleation occurred. Since R' was greater than the largest available nucleation site, nucleation was forced to occur from the largest available surface cavity (i.e., the artificial surface cavity). The incipient superheats measured during these tests are in good agreement with the predictions of the pressure-temperature model when it is assumed that nucleation occurred from the artificial surface cavities as illustrated in Fig. 3-15. On the other hand, Schultheiss has suggested that these artificial cavities are totally filled with sodium and that nucleation occurs from a nonwetting region at the bottom of the artificial surface cavities. However, the fact that Schultheiss was unable to obtain large superheats, as measured by other experimenters, adds support to the contention that the artificial cavities were not filled with liquid, but remained unflooded nucleation sites despite approximately six months of operation, i.e., the contact angle (advancing) may possibly be greater than 90° for a sodium-vapor stainless-steel system.²⁸

In the experiments of Schultheiss, the pressure-temperature history was not severe enough to force the liquid interface past the mouth of the

Fig. 3-15. Incipient Boiling Superheat of Sodium at Artificial Cylindrical Cavities of 0.4 mm and 0.2 mm Diameter

artificial surface cavities, i.e., the height of the liquid column was not sufficiently high.

In order to vary the deactivation pressure in a gas-free system, Holtz²⁹ built an experiment so that the pressure could be controlled by employing an AC, three-phase, annular-linear induction pump. The apparatus is illustrated in Fig. 3-16.

The following experimental results were obtained from these tests.

(a) A large incipient-boiling superheat can result in the boiling surface losing a previous established pressure-temperature history as a result of hysteresis effect. The high velocity during expulsion from the heated zone results in breaking up the liquid film and activation of larger cavities. Hence, subsequent boiling superheats from these surface cavities will be determined by the new pressure-temperature history to which the boiling surface is exposed.

It was experimentally observed that when the boiling surface lost a pressure-temperature history, it was not possible to regain that history (i.e., boil at the higher superheat) unless the same pressure and temperature conditions were applied for at least 20 min.

(b) If the pressure-temperature history is reestablished for at least 20 min prior to each test run, the incipient-boiling superheats show good agreement with the pressure-temperature model if a ratio for the receding to advancing radii of curvature of about 0.34 is employed. Also, by reestablishing the history for a minimum of 20 min, the scatter of the superheat data was significantly reduced.

Figure 3-17 illustrates the pressure-temperature history effect by showing a plot of incipient superheat vs. deactivation pressure (i.e., pressure experienced by the system for at least 20 min prior to conducting the test run. The data points represent superheats for boiling at 1.24 psia

Fig. 3-16. Schematic Diagram Illustrating Experimental Apparatus

Fig. 3-17. Plot of Incipient Boiling Superheat versus Deactivation Pressure

for various deactivation pressures. The theoretical curve was obtained from the pressure-temperature-history model by employing a R/R' ratio of 0.342.

(c) The incipient-boiling superheat appears to be independent of the approach to boiling. Data obtained by approaching boiling both by increasing the temperature at constant pressure and by decreasing the pressure at constant temperature are in agreement. This is illustrated in Fig. 3-18 which represents the measured superheats obtained by decreasing the system pressure at nearly constant temperature by solid circles, and those obtained by increasing the temperature at constant pressure by open circles.

The agreement between the above experimental data and the pressure-temperature-history model indicate that perhaps the inert gas partial pressure in the surface cavities both during the establishing of a pressure-temperature history and during incipient nucleation may be the cause for much of the scatter in most of the superheat data illustrated in Fig. 3-1. Also, these tests have demonstrated that the history can be lost following a superheat test run. Hence, in an experimental test apparatus employing an inert cover gas, it would appear necessary to calculate a new history after each test run in order to make a valid comparison between experiment and theory. Knowledge of the inert gas partial pressure in the surface cavities both during the establishing of a history and during boiling inception is a complex problem; certainly, the assumption that $P_g = 0$ is not valid in systems which employ an inert cover gas.

3.6.2. Forced Convection Data

The validity of applying pool-boiling data to reactor systems may be questioned since in several recent publications³⁰⁻³³ in which the incipient-boiling superheat of liquid metals during forced convection has been reported, it has been observed that an increase in the liquid velocity results in a decrease in the wall superheat while an increase in the wall heat flux

Fig. 3-18. Experimental Data and Theoretical Predictions
for 0.34 atm, 538°C History

results in an increase in the wall superheat. Several mechanisms have been proposed which attempt to explain the decrease of the incipient superheat with increasing velocity, among which are the entrainment of gas bubbles in the flowing liquid and turbulent fluctuations at the heated wall. However, the available experimental data do not conclusively prove that the liquid velocity per se is the cause of the apparently decreasing incipient wall superheat. An alternate interpretation of the reported data in which the velocity of the liquid and the wall heat flux are unimportant in determining the actual nucleation conditions has been suggested.³⁴

In the above forced convection liquid-metal boiling experiments which are interpreted as showing a velocity effect upon incipient superheat, it is assumed that the maximum observed liquid superheat is in fact that superheat required for incipient nucleation. This usually is associated with conditions at or just downstream of the outlet of the heated section. This is a necessary assumption, since only liquid temperature measurements were made. No measurements of the location of incipient nucleation were made. In reality, boiling inception may occur anywhere in the heated section where the liquid is superheated. If inception of boiling is assumed to occur at some fixed location ("favorite cavity") other than the end of the heated zone at a small superheat independent of liquid velocity, then the observed flow rate effect upon the maximum superheat can be explained without considering turbulence effects on nucleation phenomena. This is illustrated by noting that the data discussed in Refs. 30 and 31 can be represented by a model of the form, $Re \cdot \Delta T = \text{constant}$ or $U \cdot \Delta T = \text{constant}$, as shown in Figs. 3-19 and 3-20. The above interpretation of the velocity effect is in agreement with a recent analytical study of the turbulence effect upon superheat³⁵ where it is found that the predicted velocities are almost an order of magnitude greater than the experimental ones for a given percentage

Fig. 3-19. Data of Chen at $P_s = 1.04$ atm Compared to the Correlation of Pezzilli and the Favorite Cavity Hypothesis

Fig. 3-20. Data of Fornari Compared to the Correlations of Pezzilli and the Favorite Cavity Hypothesis

decrease in wall superheat at incipient boiling. Additional support for the "favorite cavity" hypothesis was obtained in a recent experimental study³⁶ where the usual velocity effect was noted when the data were correlated in terms of exit superheats, but disappeared when the variation in location of inception was accounted for (see Fig. 3-21).

The influence of pressure-temperature history was investigated by taking several boiling runs at 2 psia, imposing a history of 10 psia and 1600°F, and repeating these runs. It was found that the first run after the new history withstood a large superheat. However, the incipient superheats for succeeding runs quickly decayed to values comparable to the data taken before the imposition of the above history. In general, the behavior was similar to that found in pool-boiling studies.

In Refs. 32 and 33, a marked increase in superheat is also reported with increasing heat flux. It is apparent from the authors interpretation of measured liquid temperatures that it was also assumed that the location of incipient boiling was at the end of the heated zone. Again, for this case if it is assumed that incipient boiling occurs at a small superheat at some fixed location other than the end of the heated zone, the data can also be represented by a model of the form $Q/\Delta T = \text{constant}$ as shown in Fig. 3-22. In view of these observations, one may conclude that experiments reported in Refs. 30 through 33 have not proven or unambiguously demonstrated a velocity or heat flux effect on incipient superheat.

3.7 General Discussion of Status of Superheat, Application to Reactor Systems

From the previous discussion it can be concluded that the deactivation model for incipient superheat can be applied to reactor systems with sufficient confidence to be useful in overall fast reactor safety analysis. It is possible that large cavities may remain active for a long time, indicating

Fig. 3-21. Incipient Boiling Superheats in a Forced-convection Sodium System

Fig. 3-22. Interpretation of the Observed Heat Flux Effect upon the Incipient Superheat

that nonwetting conditions do exist for sodium-stainless-steel systems. In such cases, the coolant history plays a major role in determining the maximum size cavity available for surface nucleation. Furthermore, velocity and heat flux effects have not conclusively been proven to affect incipient superheat in forced convection systems. Therefore, at least for the time being, the theory and data derived from pool-boiling experience may be applied to forced convection systems.

However, in applying the pressure-temperature-history model to reactor systems, the necessity for analyzing the effect of inert gas transport, as well as the fact that favorite cavities may exist in the system should be emphasized. The latter effect is not presently well understood and is not accounted for in any nucleation model.

In view of these observations, the following conclusions were drawn for the pot- and loop-type designs²³ (see Fig. 3-23):

1. The probability of inert-gas bubbles being present in pot-type design is small,* so nucleation most likely will take place from surface cavities. The partial pressure of inert gas in these cavities is calculated (Eq. 3-25) to be relatively small compared to the system pressure (for this type of system, the temperature at the free surface represent the lowest temperature in the system, and therefore low gas concentration), which results in calculated superheats of about 100°C during a loss-of-flow accident. However, in view of the possibility of favorite cavities being present in the core, voiding initiation may actually take place within the core (as much as 1/3 upstream from the core exit), at essentially no

* Among the considerations included are the absence of vented fuel and cover gas in the pot, and that the pump is located well below the free surface (no mechanical entrainment). For further discussion on gas bubble entrainment in reactor systems and its effect upon incipient superheat, see Ref. 23.

(A) POT-TYPE REACTOR

(B) LOOP TYPE REACTOR

Fig. 3-23. Schematic Diagrams of Pot-type and Loop-type Reactor Systems

superheat. In view of the current state of art, it is recommended that sensitivity studies be carried out to determine what situation leads to the worst case in terms of reactivity insertion, i.e., voiding initiation at the exit of the core at 100°C or voiding initiation within the core at essentially no superheat.

2. The probability of inert gas bubbles being present in loop-type design is considerably higher since gas may come out of solution in the heat exchanger;* however, if gas bubbles are not present, the partial pressure of inert gas in the surface cavities will be high (in this case the free surface temperature represents essentially the highest temperature in the system); in either case, little or no superheating may be required to initiate boiling in the loop-type design during a loss-of-flow accident, with voiding initiation taking place at the core exit. Additional experimental proof for the above conclusions are needed. In particular, the problem of wettability and long term operation need further considerations.

* If nucleation sites are present in the heat exchanger and the coolant is nearly saturated with cover gas, gas will most likely come out of solution since the solubility decreases with decreasing temperature. Furthermore, the gas entrainment problem may be quite different for the loop-type than the pot-type design.

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NOMENCLATURE

C	Molar concentration
C_p	Specific heat of liquid at constant pressure
h_{fg}	Latent heat of vaporization
k	Liquid thermal conductivity or Boltzman constant
K(T)	Constant in Henry's Law
P	Pressure
P_A^*	Partial pressure of inert gas at incipient nucleation
P_L'	Pressure of liquid at maximum cavity-penetration conditions
q	Heat flux
R	Bubble radius
r_a	Radius of advancing surface cavity
r	Radius of curvature of vapor-liquid interface in cavity at maximum liquid penetration
T	Temperature
T_L'	Temperature of liquid at maximum cavity-penetration conditions
v	Specific volume
x	Channel position
y	Distance

Greek Letters

θ	Contact angle of liquid
σ	Surface tension

Subscripts

A	Inert gas
a	Advancing conditions
ap	Gas in plenum
e	Exit
eq	Equivalent

i	Initial conditions
L, l	Liquid
nuc	Nucleation conditions
p	Plenum
r	Receding conditions
sat	Saturation
ss	Steady-state conditions
V	Vapor
w	Wall
1	Conditions at maximum cavity penetration (temperature decrease)
2	Conditions at maximum cavity penetration (pressure increase)

Superscripts

*	Conditions after step-change transient
'	Conditions due to temperature decrease

4. BUBBLE GROWTH AND COLLAPSE

4.1 Formation of Individual Large Bubbles

While the form of the vapor bubble during voiding is far from spherical during most of the process of voiding, it is of practical interest to study the behavior of the growth process of small spherical bubbles partly because it provides insight into why (or when) a single large bubble rather than very many small bubbles is reasonable and partly because some of the important physical processes are the same as for the large, nonspherical bubbles.

Bubble growth can begin when the superheat is adequate to lead to an imbalance between the pressure of vapor and gas within a nucleating cavity and the surface tension and local external pressure, that is

$$P_V + P_g - p_l > \frac{2\sigma}{R_{cr}} \quad (4-1)$$

As the bubble radius increases, the surface tension force is reduced, and the bubble continues to grow, driven by the available energy of superheat. As the bubble expands, energy and mass are imparted to it across the liquid-vapor interface; its motion is resisted by the inertial forces of the liquid which must be accelerated to allow for bubble growth. As growth continues, the heat transfer within the liquid to the bubble-liquid interface becomes increasingly important in determining the strength of the source of energy and mass to the bubble.

The bubble growth can be roughly divided into three time periods, or phases, as follows:

<u>Phase</u>	<u>Dominant Phenomena</u>	<u>Velocity of Liquid-Vapor Interfaces</u>
Acceleration phase	liquid inertia	increasing
Inertia-controlled phase	liquid inertia	constant
Heat diffusion controlled phase	heat conduction in liquid	decreasing

Following Schlechtendahl,¹ one can first use an approximate treatment to argue that individual large bubbles occur preferentially, instead of a homogeneous two-phase mixture if

1) formation of the first bubble from a critical bubble nucleus makes other nearly critical nuclei in the environs subcritical for a sufficient time,

2) during most of its growth the bubble remains in contact with liquid whose temperature is above the saturation value at system pressure.

Considering a single spherical bubble in an incompressible inviscous liquid of infinite extent, the equation of motion for the spherically symmetrical domain of the liquid is

$$\frac{\rho_l}{g_c} \left(\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} \right) = - \frac{\partial P}{\partial r} \quad (4-2)$$

Integrating the equation of continuity

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 u_r \right) = 0 \quad (4-3)$$

from the radius of bubble interface R to r , where $r > R$, gives the radial liquid velocity u_r in terms of the interface velocity:

$$\dot{R} = \frac{dR}{dt}$$

(4-4)

$$u = \dot{R} \left(\frac{R}{r} \right)^2$$

Substituting Eq. 4-4 into 4-2 gives

$$\frac{\rho_l}{g_c} \left[\frac{2R\dot{R}^2 + R^2\ddot{R}}{r^2} - \frac{2R\dot{R}^4}{r^5} \right] = - \frac{\partial P}{\partial r} \quad (4-5)$$

which upon integration from r to infinity

$$\frac{P - P_\infty}{\rho_l} = \frac{R^2\ddot{R} + 2R\dot{R}^2}{r} - \frac{R^4\dot{R}^2}{2r^4} \quad (4-6)$$

where P is the liquid pressure at radius r

R = bubble radius

P_∞ = system pressure far from the bubble

The initial acceleration $\ddot{R}$ dies away quickly, and the phase interface velocity remains nearly constant as long as the inertia effects outweigh heat conduction effects (cooling by vaporization).

For small R , we can then write

$$\frac{P - P_\infty}{\rho_l} = \dot{R}^2 \left(\frac{2R}{r} - \frac{R^4}{2r^4} \right) \quad (4-7)$$

and, for the phase interface ($r = R$)

$$\frac{P_V - P_\infty}{\rho_l} = \frac{3}{2} \dot{R}^2 \quad (4-8)$$

From Eqs. 4-8 and 4-9, we can get

$$P - P_\infty = (P_V - P_\infty) \frac{2R}{3r} \left(2 - \frac{R^3}{2r^3} \right) \quad (4-9)$$

A bubble nucleus at a distance r from the point of the first bubble that was also nearly critical becomes subcritical because of the pressure rise which is given roughly by Eq. 4-9.

Using the Clapeyron-Clausius equation to relate the additional superheat needed to counter ΔP , and assuming that coolant temperature was uniform and rising at a constant rate, $\dot{T}$, due to the imposed transient, Schlechtendahl derived the relation

$$r > \left\{ \frac{2}{3} \left(2 - \frac{R^3}{2r^3} \right) \right\} \frac{\Delta T_s}{\dot{T}} \dot{R} \quad (4-10)$$

for another bubble to form, where ΔT_s is the original superheat. Table 4-1 gives some results obtained with Eq. 4-10.

To place the results of Table 4-1 in perspective, Schlechtendahl estimated that cooling failure, without scram, might lead to coolant temperature rise rates of ~ 300 deg/sec or less.

The analysis above assumed a uniform temperature in the sodium. As discussed in section 3.1, the high thermal conductivity of sodium leads to a relatively flat temperature profile in the bulk coolant. Thus, for a typical wire-wrapped oxide fuel element having a local linear heat rating of

TABLE 4-1 MINIMUM DISTANCE OF BUBBLE FORMATION, CM

$\dot{T}$ $\frac{\text{deg}}{\text{sec}}$	0.5	1	5	10	50	100
10	2	22	62	700	2000	22,000
50	0.4	4.4	12.4	140	400	4,400
100	0.2	2.2	6.2	70	200	2,200
500	0.04	0.4	1.2	14	40	440
1000	0.02	0.2	0.6	7	20	220

10 kW/ft, one might expect a temperature differential across the coolant of about 25°C. Thus, for an initial wall superheat of 30°C, the bulk coolant superheat would still exceed this differential, after allowing for the film drop, and the bubble would grow into superheated coolant everywhere. However, for lesser superheats, bubble collapse might tend to occur, permitting the nucleation of other bubbles in the environs.

The radial power gradients across subassemblies, at the core edge in particular, might lead to a situation where considerable bulk superheat existed at the inner edge, while the coolant was still subcooled at the outer edge. This would bring into play radical changes in bubble environment with bubble expansion, and the simple analyses above would not be directly applicable.

4.2 Analysis of Bubble Growth and Collapse (We quote Theofanous)²

4.2.1 Statement of the Problem

"We consider a spherical vapor bubble in dynamic and thermal equilibrium with an infinite sea of stationary, incompressible liquid at time $t = 0$.

$$T_V = T_L = T_\infty, \quad P_L = P_\infty$$

and

$$P_V = P_V^* = P_L + \frac{2\sigma}{R_{cr}}.$$

The origin of a spherical coordinate system employed coincides with the center of the bubble. Body forces, such as gravity, will be neglected, and the imposed pressure field at infinity will be spherically symmetric.

The process, therefore, of bubble growth or collapse, will remain spherically symmetric throughout, and the center of the bubble will always coincide with the origin of the coordinate system. Then, r (the distance from the center) and t (time) will be the two appropriate independent variables for this problem. Figure 4-1 represents the conceptual model.

The system, as described above, is in a state of superheat, inasmuch as $P_L < P_V$ and the vapor is saturated. A small perturbation will be enough to remove it from this unstable equilibrium state and a spontaneous process will guide it towards the thermodynamic equilibrium state which is stable under the given conditions. In this fashion, an equilibrium bubble can grow in a constant pressure field, using, for example, a small step decrease in P_∞ at $t = 0$. Also, growth occurs with a decreasing pressure field where P_∞ is a prescribed (decreasing) function of time. On the other hand, for collapse, (starting again with the equilibrium bubble) a monotonically increasing pressure field is required. Also, a step change may be used in the liquid temperature at $t = 0$ to a value less than the one corresponding to the equilibrium saturation temperature at P_∞ . In all cases, heat, mass, and momentum transfer between the two phases are coupled to determine the overall behavior of the system."

4.2.2 Interphase Transfer

"The removed-from-equilibrium (superheated or subcooled) vapor-liquid system, will spontaneously exchange mass between its phases to establish a state of minimum free energy. Thus, vaporization will occur in a superheated system, and condensation in a subcooled system. Energy will be exchanged between the phases partly by means of the net mass transfer and partly by means of a mechanism similar to thermal conduction or radiation. A mechanical interaction between the phases will take place mainly through pressure forces."

Fig. 4-1. Sketch of System. (T_v, P_v, R_v, R_L , and T_L are functions of time t . P_∞ may be constant or time dependent.)

4.2.2.1 Rate of Mass Transfer

"Consider the plane interface between a liquid and its vapor. Even the plane problem is so complicated and confused that, at least at this point, there is no need to take into account any corrections for the curved interface. Furthermore, the radius of curvature is much larger than the mean intermolecular distance for problems of present interest. Let us use temperature and pressure to define the thermodynamic state of the vapor and the liquid at points close to the interface. (As to be seen later, the appropriate pressure for the liquid will be the saturation equilibrium pressure at the interface equilibrium temperature.) We seek an expression relating the net mass flux at the interface to the thermodynamic states of the two phases at the neighborhood of their contact.

It should be emphasized that purposely the above description of the problem is left somewhat vague. We hope thus to draw the necessary caution of the reader on the lack of knowledge concerning evaporating or condensing surfaces. A literature survey indicated that a substantial amount of work has been done by many investigators in various fields and specializations. The results, nevertheless, as is generally recognized, are neither definitive nor even systematic. The theoretical formulations for the overall phase change problem traditionally have utilized the kinetic theory approach. The interface was treated as a mathematical surface of discontinuity for the various thermodynamic variables. The vapor state was assumed uniform up to the interface while the liquid state at the interface was supposed to be well defined and known. In this conceptual framework, one can assume a distribution function for the vapor molecules (usually Maxwellian, with a minor correction for the effect of macroscopic motion) and evaluate the rate at which they impinge on the liquid surface.

$$W_- = \sqrt{\frac{M}{2\pi R}} \frac{P_V}{\sqrt{T_V}} \quad (4-11)$$

At equilibrium, then, the same number rate of liquid molecules vaporize and a similar formula, therefore, should hold for the vaporization rate.

Now

$$T_V = T_L \text{ and } P_V = P_L^* .$$

As a consequence, it is assumed that the same expression will give the rate of liquid molecules vaporizing under any conditions, if it is evaluated at the conditions of the liquid at the interface.

$$W_+ = \sqrt{\frac{M}{2\pi R}} \frac{P_L^*}{\sqrt{T_L}} \quad (4-12)$$

The net rate of mass transfer across the interface can then be calculated.

$$W = W_+ - W_- \quad (4-13)$$

In general, such a calculation was found to disagree with experimental evidence. A coefficient of vaporization or condensation, c , (the two names are used indiscriminantly) was then introduced as the probability that a molecule impinging on the liquid surface will condense. If, then, we call W_+ the molecular flux, as given by kinetic theory, towards the vapor, and W_- the one towards the liquid, we have

cW_- , rate that molecules condense on the liquid surface

$(1 - c)W_-$, rate that molecules reflect from the liquid surface

cW_+ , rate that molecules vaporize from the liquid surface.

The last holds since at equilibrium, $W_+ = W_-$, and rate of molecules vaporizing = $W_- - (1 - c)W_- = cW_- = cW_+$. Under nonequilibrium conditions, therefore, the net flux W is:

$$W = cW_+ + (1 - c)W_- - W_- = c(W_+ - W_-) \quad (4-14)$$

(where W is taken positive for vaporization, i.e., $W_+ > W_-$). The problem, thus, is transposed now to the one of evaluating c . The theoretical predictions for c are in an evolving stage, and the experimental results have created confusion because of the contradictions reported. A difficulty in the theoretical treatments stems from the repetitive and rigid approach used in formulating the problem. In particular, one notes the posteriori and rather unnatural method of introducing the vaporization or condensation coefficient in the rate equations. It should be expected that a more fundamental approach to molecular dynamics, involving force fields and interactions, would not only give information on the vaporization coefficient, but would help in understanding the behavior of the vapor dose to the interface, in understanding the "limits" of a vaporizing liquid surface, and in understanding the effects of the molecular nature of substances and the intermolecular structure of vaporizing (or condensing) liquid surfaces. The main difficulty in the experimental work, besides the ones imposed from the theoretical formulation, is in measuring the interface temperatures. Other problems involve the use of extremely pure substances inasmuch as minute contamination of the interface may have a decisive effect on the measured values of c , with a tendency always to decrease it. Experimental values for c cover very wide

ranges. For example, for water, c is reported from $\sim 10^{-2}$ up to 1.00. For liquid metals, a dependence on pressure was found through condensing experiments with Hg and Na. So, values of c as low as $5 \cdot 10^{-2}$ have been reported at atmospheric conditions. Information is given in reviews by Paul,³ Hickman,⁴ Knacke and Stanski,⁵ and an extensive literature survey in the interesting work of D. Wilhelm.⁶ Johnstone and Smith⁷ offer an explanation for the divergence in the experimental results. They say that the difference in the reported values lies in the length of exposure time used in each set of experiments. They claim that short exposure times--method of jets--give $c = 1$, while long exposure times give values as low as $2 \cdot 10^{-2}$ (for water). In other words, they claim that c is not a constant nor even just a function of the state as was believed before, but it is also a function of time!! This behavior is believed to be due to a finite relaxation time for the appropriate orientation of interfacial molecules. As it becomes evident from this brief discussion, there is still much to be learned before sufficient understanding of the possible nonequilibrium effects may be claimed. This area, although intimately related to the main goals of this work, is in itself a major topic and not the subject of the investigation. We briefly outline the basic ideas and mention some of the weaknesses.

The Knudsen equation (4-14) is utilized

$$W = c \sqrt{\frac{M}{2\pi R}} \left\{ \frac{P_L^*}{\sqrt{T_L}} - \frac{P_V}{\sqrt{T_V}} \right\} \quad (4-15)$$

and c is treated as a parameter. The flux will be reduced to zero only when $T_L = T_V$ and $P_L^* = P_V$, corresponding to a thermodynamic equilibrium condition. A constant value of c is used (covering parametrically the range

of values observed experimentally) throughout each process, but a time dependence of c as Johnstone and Smith⁷ suggest could easily be accommodated in our solutions. Furthermore, c may even be used to account for traces of impurities common in real engineering systems. One of the objectives of this work is to determine the effect of c on the solution, find conditions where this effect becomes significant, and finally infer its value from comparisons with appropriate experimental data obtained with vapor bubbles. In accordance with the model, the value $c = 1$ is the largest possible. As we will see in the applications, the behavior of a system with $c = 1$ is quite close to the behavior of a system for which thermodynamic equilibrium at the interface is assumed. The value of c that really reduces P_V to P_L^* and T_V to T_L , W still being finite (i.e., phase change under equilibrium conditions) is $c = \infty$."

4.2.2.2 Rate of Energy Transfer

"The energy transfer between the phases can be described in terms of the mass molecular fluxes and the internal energy per unit mass of the molecules transferred. Since there is a temperature discontinuity, energy is also exchanged between liquid and vapor without net mass transfer. The microscopic mechanism of energy exchange between molecules of vapor and the liquid surface is yet to be resolved. We assume that the "reflected" molecules transfer to the liquid all their internal energy, and they remove from the liquid an amount corresponding to the state of the liquid surface.

We have accordingly:

$$U_{V \rightarrow L} = W - \hat{U}_V \Big|_{T_V} \quad (4-16)$$

for the rate of internal energy transfer from the vapor to the liquid.

Similarly, the rate of internal energy transfer from the liquid to the vapor is:

$$U_{L \rightarrow V} = cW_+ \hat{U}_V|_{T_L} + (1 - c)W_- \hat{U}_V|_{T_V} \quad (4-17)$$

The net rate of transfer with the direction L→V considered as positive is

$$\begin{aligned} \Delta U_{L \rightarrow V} &= cW_+ \hat{U}_V|_{T_L} + (1 - c)W_- \hat{U}_V|_{T_V} - W_- \hat{U}_V|_{T_V} \\ &= cW_+ \hat{U}_V|_{T_L} - cW_- \hat{U}_V|_{T_V} \\ &= -W \hat{U}_V|_{T_L} + cW_- (\hat{U}_V|_{T_L} - \hat{U}_V|_{T_V}) \\ &= W \hat{U}_V|_{T_V} + cW_+ (\hat{U}_V|_{T_L} - \hat{U}_V|_{T_V}) \end{aligned} \quad (4-18)$$

where the contributions from the two mechanisms of energy transfer can easily be identified in the last two forms given for $\Delta U_{L \rightarrow V}$.

4.2.2.3 Rate of Momentum Transfer

"At the interface, the jump of $\frac{2\sigma}{R_V}$ over the normal stresses provides the coupling relationship for the rate of momentum exchange between the phases. The correction of this equilibrium jump for the dilational deformation of the interface during the motion, as noted by

Scriven,⁸ is not included in the present formulation. The radial component of the stress tensor at the liquid side of the interface is

$$\begin{aligned}
 \Pi_{rrL}(R_V, t) &= P_L(R_V, t) - \mu \left\{ 2 \frac{\partial u_r}{\partial r} - \frac{2}{3} (\nabla \cdot \underline{u}) \right\} \Big|_{R_V} \\
 &= P_L(R_V, t) + \mu \left(\frac{4}{r^3} \epsilon R_V^2 \dot{R}_V \right) \Big|_{R_V} \\
 &= P_L(R_V, t) + \dot{R}_V \frac{4\mu\epsilon}{R_V}
 \end{aligned} \tag{4-19}$$

Eqs. 4-23 and 4-26 have been utilized. Similarly on the vapor side, neglecting the viscous contribution, we have

$$\Pi_{rrV}(R_V, t) = P_V(R_V, t) \tag{4-20}$$

Now, since

$$\Pi_{rrL}(R_V, t) = \Pi_{rrV}(R_V, t) - \frac{2\sigma}{R_V} \tag{4-21}$$

$$\therefore P_V(R_V, t) = P_L(R_V, t) + \frac{2\sigma}{R_V} + \frac{4\mu\epsilon}{R_V} \dot{R}_V \tag{4-22}$$

The surface tension contribution is important only at the initial stages of growth or the final stages of collapse. The viscous term is in most cases quite small but is retained for the sake of generality and completeness."

4.2.3 The Conservation Equations

"As to be mentioned later in detail, the two major assumptions on the pressure-time behavior of the vapor made by previous investigators are the following:

- (a) $P_V(r,t) = P_V(t) = P_\infty = \text{constant}$ -- Heat transfer in liquid controlling.
- (b) $P_V(r,t) = P_V(t) = P_V(0) = \text{constant}$ -- Inertia of the liquid controlling.

In general, $P_V(t)$ will follow neither of the two extremes. Particularly this will not be the case for a time-dependent pressure at infinity. On the other hand, the actual magnitude of $P_V(t)$ determines the radius-time dependence through the equation of motion for the bubble wall. The main effort here is to present a scheme which will estimate $P_V(t)$ reasonably well. This is accomplished by means of the conservation equations of continuum mechanics for each phase, and their coupling at the interface as discussed in the previous section on interphase transport. The equations for both liquid and vapor are presented and simplified appropriately in the following sections. The solutions thus obtained yield detailed information on all parameters characterizing the bubble at any moment throughout the process."

4.2.3.1 Continuity for the Liquid

"In this study, the liquid is considered incompressible.

$$(\nabla \cdot \underline{u}) = 0 \quad (4-23)$$

As to be seen in the applications for the growth case, this assumption is well justified in that the attained velocities are well below the sonic velocity in the liquid. Under extreme conditions, such as a violent collapse, compressibility effects do become important near the last stages.

The integrated equation of continuity becomes

$$r^2 u_r(r, t) = R_V^2 u_r(R_V, t) = f(t) = \text{a function of time only.} \quad (4-24)$$

A mass balance between vaporizing liquid and bubble vapor content change is used to obtain a relation between u_r and $\dot{R}_V$.

$$\rho_L \left\{ \dot{R}_V - u_r(R_V, t) \right\} = \frac{1}{3\dot{R}_V^2} \frac{d}{dt} \left\{ \rho_V \dot{R}_V^3 \right\} = \rho_V \dot{R}_V \quad (4-25)$$

with ρ_V assumed to be uniform in r and constant. In relating u_r and $\dot{R}_V$, this assumption is adequate; however, for the vapor continuity, ρ_V is permitted to vary.

Defining

$$\varepsilon = \frac{\rho_L - \rho_V}{\rho_L}$$

we obtain

$$u_r(R_V, t) = \frac{\rho_L - \rho_V}{\rho_L} \dot{R}_V = \varepsilon \dot{R}_V \quad (4-26)$$

Except at high pressures, ρ_V is several orders of magnitude smaller than ρ_L and the $\varepsilon \approx 1$:

Returning now to (4-24) we have for $f(t)$,

$$f(t) = \epsilon R_V^2 \dot{R}_V \sim R_V^2 \dot{R}_V \quad (4-27)$$

4.2.3.2 Motion for the Liquid

"The only nontrivial component of the Navier-Stokes equations is the radial component. The viscous terms vanish identically for the present case as was the first shown by Poritsky.⁹

$$\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} = -\frac{1}{\rho_L} \frac{\partial P}{\partial r} \quad (4-28)$$

But from (4-24)

$$\frac{\partial u_r}{\partial t} = \frac{\partial}{\partial t} \left(\frac{f(t)}{r^2} \right) = \frac{1}{r^2} \frac{df(t)}{dt} \quad (4-29)$$

and Eq. (4-28) becomes

$$\frac{1}{r^2} \frac{df}{dt} + \frac{1}{2} \frac{\partial}{\partial r} (u_r^2) = -\frac{1}{\rho_L} \frac{\partial P}{\partial r} \quad (4-30)$$

Integrating from R_V to some arbitrary radius, R_{ref} , we have

$$\begin{aligned} - \left\{ \frac{1}{R_{ref}} - \frac{1}{R_V} \right\} \frac{df}{dt} + \frac{1}{2} \left\{ u_r^2(R_{ref}, t) - u_r^2(R_V, t) \right\} &= -\frac{1}{\rho_L} \\ \left\{ P_L(R_{ref}, t) - P_L(R_V, t) \right\} & \end{aligned} \quad (4-31)$$

but $R_{\text{ref}}^2 u_r(R_{\text{ref}}, t) = R_V^2 u_r(R_V, t)$. Hence,

$$-\left\{\frac{1}{R_{\text{ref}}} - \frac{1}{R_V}\right\} \frac{df}{dt} + \frac{1}{2} \left\{\left(\frac{R_V}{R_{\text{ref}}}\right)^4 - 1\right\} u_r^2(R_V, t) = -\frac{1}{\rho_L} \left\{P_L(R_{\text{ref}}, t) - P_L(R_V, t)\right\} \quad (4-32)$$

This form of the equation is appropriate in judging dimensions of dynamic interaction between bubbles and surroundings. Note that for R_{ref} even quite close to R_V , $\left(\frac{R_V}{R_{\text{ref}}}\right)^4 \ll 1$, while R_{ref} should be taken at least $10 R_V$ to assure $\frac{1}{R_{\text{ref}}} \ll \frac{1}{R_V}$. In practical applications where R_V is small, the infinite system approximation (i.e., take $R_{\text{ref}} \rightarrow \infty$) is valid and the equation reduces to

$$\frac{1}{R_V} \frac{df}{dt} - \frac{1}{2} u_r^2(R_V, t) = -\frac{1}{\rho_L} \left\{P_{\infty}(t) - P_L(R_V, t)\right\} \quad (4-33)$$

Using (4-27)

$$\frac{df}{dt} = \epsilon R_V^2 \ddot{R}_V + 2\epsilon R_V \dot{R}_V^2 \quad (4-34)$$

in (4-33), we obtain

$$\epsilon R_V \ddot{R}_V + 2\epsilon \dot{R}_V^2 - \frac{1}{2} \epsilon^2 \dot{R}_V^2 = -\frac{1}{\rho_L} \left\{P_{\infty}(t) - P_L(R_V, t)\right\} \quad (4-35)$$

Finally, substituting for $P_L(R_V, t)$ from (4-23)

$$R_V \ddot{R}_V + (2 - \frac{1}{2} \epsilon) \dot{R}_V^2 = - \frac{1}{\epsilon \rho_L} \left\{ P_\infty(t) - P_V(R_V, t) + \frac{2\sigma}{R_V} + \frac{4\mu\epsilon}{R_V} \dot{R}_V \right\} \quad (4-36)$$

This equation, known as extended Rayleigh equation, governs the bubble wall motion. The driving force is $\left\{ P_\infty(t) - P_V(R_V, t) \right\}$. For the applications in this study, ϵ was taken unity and, therefore,

$$R_V \ddot{R}_V + \frac{3}{2} \dot{R}_V^2 = - \frac{1}{\rho_L} \left\{ P_\infty(t) - P_V(R_V, t) \right\} - \frac{2\gamma}{R_V} - \frac{4\eta}{R_V} \dot{R}_V \quad (4-37)$$

4.2.3.3 Energy for the Liquid

"The energy balance written for an infinitesimal convected shell of the liquid, is

$$\rho_L \hat{c}_L \frac{DT}{Dt} = - (\underline{\nabla} \cdot \underline{q}) - T \left(\frac{\partial P}{\partial T} \right)_{\hat{V}} (\underline{\nabla} \cdot \underline{u}) - (\underline{\tau} : \underline{\nabla} \underline{u}) \quad (4-38)$$

which for an incompressible liquid ($\underline{\nabla} \cdot \underline{u} = 0$) with no heat generation by viscous dissipation ($\underline{\tau} : \underline{\nabla} \underline{u} = 0$) becomes (in spherical coordinates-constant physical properties)

$$\rho_L \hat{c}_L \left\{ \frac{\partial T}{\partial t} + u_r \frac{\partial T}{\partial r} \right\} = k_L \frac{1}{r^2} \frac{\partial}{\partial r} \left\{ r^2 \frac{\partial T}{\partial r} \right\} \quad (4-39)$$

or

$$\frac{\partial T}{\partial t} + \frac{R_V^2 \dot{R}_V}{r^2} \frac{\partial T}{\partial r} = \alpha \frac{1}{r^2} \frac{\partial}{\partial r} \left\{ r^2 \frac{\partial T}{\partial r} \right\} \quad (4-40)$$

with $\alpha = \frac{k_L}{\rho_L c_L}$ and u_r from (4-25) and (4-27). This equation poses an extremely interesting case because of the complication introduced by the convective term. When the boundary conditions are steady, a similarity transformation exists^{8,10} which reduces the problem to a tractable form. For the general case, we modify appropriately an integral method.^{11,12} We assume that a finite thermal boundary layer can be defined as $\delta(t) = R_L(t) - R_V(t)$, such that

$$(a) \text{ at } r = R_L(t), T(R_L, t) = T_\infty \text{ and } \frac{\partial T}{\partial r}(R_L, t) = 0$$

$$(b) \text{ at } r = R_V(t), T(R_V, t) = T_L(t) = \text{temperature of liquid at the interface}$$

A quadratic temperature profile is assumed, and a form consistent with the above requirements is

$$T(r, t) = T_\infty + (T_L - T_\infty) \left[\frac{R_L - r}{R_L - R_V} \right]^2, \quad R_V \leq r \leq R_L \quad (4-41)$$

$$T(r, t) = T_\infty \quad R_L < r$$

The variables, $R_L(t)$ and $T_L(t)$, are determined from the requirements that

(a) the energy balance is satisfied in its integrated form over the volume of the liquid; and

(b) the continuity of the heat flux at the interface is satisfied.

Multiplying (4-40) by the volume element $r^2 dr$ and integrating from R_V to ∞ , and noting that the gradients vanish for $r \geq R_L$, we obtain

$$\int_{R_V}^{R_L} \frac{\partial T}{\partial t} r^2 dr + R_V^2 \dot{R}_V \int_{R_V}^{R_L} \frac{\partial T}{\partial r} dr = \alpha \int_{R_V}^{R_L} \frac{\partial}{\partial r} \left\{ r^2 \frac{\partial T}{\partial r} \right\} dr$$

After substituting for $T(r,t)$, using (4-41), and performing the indicated integrations, we find

$$\frac{d}{dt} \left\{ (T_L - T_\infty) (R_L - R_V) (R_L^2 + 3R_L R_V + 6R_V^2) \right\} = 60 \alpha R_V^2 \frac{T_L - T_\infty}{R_L - R_V} \quad (4-42)$$

The continuity of the heat flux at the interface is expressed with

$$hW = k_L \left. \frac{\partial T}{\partial r} \right|_{R_V} = -2 k_L \frac{T_L - T_\infty}{R_L - R_V} \quad (4-43)$$

The heat supplied to (or removed from) the surface through the liquid is assumed to equal the heat absorbed (or given up) as latent heat of vaporization (or condensation). All other forms of heat transfer between vapor and the interface are neglected since they are small compared to the large value of h . The signs in (4-43) are taken consistently with the positive direction postulated for W . Equation (4-43) when differentiated w.r.t. time gives the second differential equation required for the determination of the heat transfer problem in the liquid phase:

$$-\frac{h}{2k_L} \frac{dW}{dt} = \frac{d}{dt} \left\{ \frac{T_L - T_\infty}{R_L - R_V} \right\} \quad (4-44)$$

4.2.3.4 Continuity for Vapor

"Uniformity of pressure in the vapor bubble is assumed

$$P_V(r,t) = P_V(R_V,t) = P_V(t) \quad r \leq R_V, t \geq 0 \quad (4-45)$$

Furthermore, uniformity of temperature is also assumed.

$$T_V(r, t) = T_V(R_V, t) = T_V(t) \quad r \leq R_V, \quad t \geq 0 \quad (4-46)$$

To include the heat transfer in the vapor phase in the present formulation, in order to test its importance in the overall process, would be an extremely difficult task. On the other hand, order-of-magnitude estimations can be misleading due to the complexity of the actual situation. We, nevertheless, recognize that the bulk of the vapor will be uniform with $P_V = P_V(t)$, and $T_V = T_V(t)$. These values of pressure and temperature are the appropriate ones to be used in mass and energy balances for the vapor. It is also recognized that a sharp temperature gradient in the vapor will exist close to the bubble wall (within a few mean free paths). As mentioned previously, very little is known about this region. We use, therefore, the bulk vapor properties $T_V(t)$, $P_V(t)$ in the expression for the rate of phase change W . Any gradients in the vapor will be absorbed in the vaporization coefficient c decreasing its "appropriate" value. More detail in the transport processes in the vapor phase should be warranted, only when the vapor-liquid interface and the meaning of c are better understood.

The vapor bubble, hence, is viewed as an open macroscopic system exchanging mass and energy with its surroundings. A mass balance is

$$\frac{1}{3R_V^2} \frac{d}{dt} \left\{ \rho_V R_V^3 \right\} = \frac{k_V}{3R_V^2} \frac{d}{dt} \left\{ R_V^3 \frac{P_V}{T_V} \right\} = W \quad (4-47)$$

Ideal gas behavior for the vapor is assumed. Expanding,

$$\frac{1}{R_V} \frac{dR_V}{dt} + \frac{1}{3P_V} \frac{dP_V}{dt} - \frac{1}{3T_V} \frac{dT_V}{dt} = \frac{WT_V}{k_V R_V P_V} \quad (4-48)$$

4.2.3.5 Momentum for the Vapor

"The assumption of uniform pressure reduces the momentum equation to a trivial form, and hence, no use is made of this balance.

4.2.3.6 Energy for the Vapor

"In a total energy balance over the vapor bubble, kinetic, potential, and surface energy contributions are neglected in comparison to internal energy terms. The pdV work is included. The amount of heat transferred without net mass transfer, included in (4-18) for the total internal energy transfer $\Delta \hat{u}_{L \rightarrow V}$, is also utilized. The macroscopic balance takes the form:

$$\frac{d}{dt} \left\{ \frac{1}{3} \rho_V R_V^3 \hat{u}_V \Big|_{T_V} \right\} = R_V^2 \Delta \hat{u}_{L \rightarrow V} - P_V R_V^2 \dot{R}_V \quad (4-49)$$

or by (4-18)

$$\begin{aligned} \frac{d}{dt} \left\{ \frac{1}{3} \rho_V R_V^3 \hat{u}_V \Big|_{T_V} \right\} &= R_V^2 W \hat{u}_V \Big|_{T_V} + R_V^2 cW_+ \left\{ \hat{u}_V \Big|_{T_L} - \hat{u}_V \Big|_{T_V} \right\} \\ &\quad - P_V R_V^2 \dot{R}_V \end{aligned} \quad (4-50)$$

Using (4-47) for W and expanding,

$$\rho_V R_V \frac{d\hat{u}_V}{dt} = -3P_V \frac{dR_V}{dt} + 3cW_+ \left\{ \hat{u}_V \Big|_{T_L} - \hat{u}_V \Big|_{T_V} \right\} \quad (4-51)$$

For an ideal gas,

$$\frac{d\hat{U}_V}{dt} = \hat{c}_V \frac{dT_V}{dt}, \quad \hat{U}_V|_{T_i} = \hat{c}_V(T_i - T_o) \quad (4-52)$$

where T_o is an arbitrary reference temperature. So, finally, (4-51)

becomes

$$\hat{c}_V k_V \frac{R_V}{T_V} \frac{dT_V}{dt} = -3 \frac{dR_V}{dt} + 3 c_W \frac{\hat{c}_V}{P_V} \left\{ T_L - T_V \right\} \quad (4-53)$$

4.2.4 Initial Conditions

"Equations 4-37, 42, 44, 48, and 53 provide a complete system of ordinary differential equations in the five unknown functions of time, R_V , T_V , P_V , R_L , and T_L . Once their initial values are given and the pressure at infinity $P_\infty(t)$ prescribed, the equations can be integrated to obtain the evolution of the system in time. A fourth-order Runge-Kutta procedure was used.

The initial conditions utilized are

- (a) Growth at constant or decreasing pressure field.

$$T_V(0) = T_L(0) = T_\infty$$

$$P_V(0) = P_V^*(0) = P_L(0) + \frac{2\sigma}{R_{cr}} = P_\infty(0) + \frac{2\sigma}{R_{cr}}$$

$$R_L(0) = R_V(0) = R_{cr}$$

$$\text{Superheat} = \Delta T_s(t) = T_\infty - T_\infty^*(t).$$

- (b) Collapse in increasing pressure field. (Method I)

$$T_V(0) = T_L(0) = T_\infty$$

$$P_V(0) = P_V^*(0) = P_L(0) + \frac{2\sigma}{R_{cr}} = P_\infty(0) + \frac{2\sigma}{R_{cr}}$$

$$R_V(0) = R_L(0) = R_{cr}$$

$$\text{Subcooling} = \Delta T_s(t) = T_{\infty}^*(t) - T_{\infty}.$$

(c) Collapse in a constant pressure field (Method II).

$$P_V(0) = P_L(0) + \frac{2\sigma}{R_{cr}} = P_{\infty} + \frac{2\sigma}{R_{cr}}$$

$$T_V(0) = T_V^*(0)$$

$$T_{\infty} = T_L(0) = T_{\infty}^* - \Delta T_s$$

$$\text{Subcooling} = \Delta T_s = T_{\infty}^* - T_{\infty}$$

$$R_{L_1}(0) = R_V(0) = R_{cr}.$$

4.2.5 Method of Solution

"In this section, the parallelism between the various experimental methods and the corresponding numerical procedures adopted to approximate them are outlined."

4.2.5.1 Growth

"As previously noted, we always start from a superheated system at its unstable equilibrium state. Growth at constant pressure (P_{∞}) can be initiated by a small perturbation in any of the system variables. Numerically, it was found convenient to make a step decrease (ΔP_{∞}) of the pressure at infinity of the order of $\frac{\Delta P_{\infty}}{P_{\infty}} \sim 10^{-6}$. The solution was found to be independent of the magnitude of this perturbation as is to be expected. This method would correspond to the experimental procedure of Hooper and Abdelmessih.¹³ Growth in a decreasing pressure field was numerically simulated by imposing the appropriate $P_{\infty}(t)$. This method would correspond to the experimental procedure of Hewitt and Parker¹⁴ for some of their growth experiments."

4.2.5.2 Collapse

"For the collapse case, we start from a reasonably large bubble. Therefore, $\frac{2\sigma}{R_V} \ll P_V$ and $P_V(0) \simeq P_{\infty}(0)$. Collapse in an increasing pressure field can easily be simulated by imposing the appropriate $P_{\infty}(t)$,

which, thus, soon overcomes $P_v(0)$ and produces bubble collapse. For a given subcooling, the pressure rises to a value P_f , such that $\Delta T_s = T_\infty^* - T_\infty$, and remains constant thereafter. This method would correspond to the experimental procedure of Florschuetz and Chao,¹⁵ which we call Method I. In Method II, we consider a system at the unstable equilibrium point and suppose at $t = 0^+$ a step decrease is made in the liquid temperature by an amount corresponding to the required subcooling. The bubble collapses under constant pressure field conditions. This method corresponds to the limiting case of the experimental procedure of Hewitt and Parker¹⁴ which involved injection of superheated vapor in a subcooled liquid. The limit of the actual process is extremely difficult to achieve experimentally, and uncertainties are thus involved with respect to the actual initial conditions."

4.2.6 Results of Bubble Growth Calculations

Theofanous et al.¹⁶ presented the results of calculations using the following conditions:

Sodium case

Constant pressure $P_\infty = 1 \text{ atm}$

Superheat $\Delta T_g = 151.5^\circ\text{K}$; $R_o = 10^{-4} \text{ cm}$

Water case

Constant pressure $P_\infty = 10^{-1} \text{ atm}$

$\Delta T_g = 2.73^\circ\text{K}$; $R_o = 10^{-2} \text{ cm}$

Theoretical solutions

Rayleigh solution (inertia controlling).

Asymptotic solution (heat conduction controlling), Scriven solution.

Solutions of Theofanous et al.

Equilibrium solution ($T_V = T_L$, $P_V = P_L^*$, vapor always saturated at liquid interface temperature). Constant density solution and given value for c . Solutions for $c = 1$, 10^{-1} , and 10^{-2} with variable density. (Values for c less than one indicate interfacial mass transfer resistance. The equilibrium solution is equivalent to a value of $c \rightarrow \infty$.)

We may quote from Ref. 16 as follows:

"Figures 4-2 and 4-3 present a thorough comparison of theoretical solutions for specific cases of practical interest. We make the following observations:

1. The restrictions imposed upon the growth rates by the combination of inertia and heat conduction are readily ascertained. The nonequilibrium condition imposes further restrictions. Only a relatively large radii is the asymptotic (heat conduction) solution approached.

2. The vaporization coefficient, c , affects the growth rate significantly, with marked decreases in growth rate accompanying decreasing coefficients. The family of curves obtained for various coefficients provide a converging sequence as a function of c with the clearly defined limit being the equilibrium solution as $c \rightarrow \infty$. The limit was obtained in an independent manner.

3. At low superheats, negligible error is introduced by assuming a constant vapor density, $\bar{\rho}_V$. However, at high superheats, the differences in solutions for the variable density and constant density models are significant. The dotted lines in Fig. 4-3 represent the constant density solution and are to be compared with the solid curves given for $c = 1$ and $c = 10^{-2}$. The constant density solutions, not shown in Fig. 4-3 would overlap the solid curves.

Fig. 4-2. The Growth Behavior of a Sodium Vapor Bubble in Highly Superheated Sodium at Atmospheric Pressure

Fig. 4-3. The Growth Behavior of a Steam Bubble in Superheated Water at Low Pressure and Small Superheat

4. The $\dot{R}_V$ vs. R_V/R_O curves are flattened with increasing k_L and decreasing vaporization coefficient, c . For $c = 10^{-2}$, an almost constant value of $\dot{R}$ is attained.

5. For this sodium case, with its high superheat, the Rayleigh solution yields a reasonable approximation to the equilibrium solution up to $R_V/R_O = 1000$. As expected, the high thermal conductivity of the sodium diminishes the heat transfer limitations for growth.

6. On the other hand, for the sodium case, the relative radius at which the actual solution approaches the asymptotic solution would never be reached in a practical application.

In Figs. 4-4 to 4-6, the complete solution of the system (of the five basic quantities) is presented as a function of time for the sodium case. Note that the interface liquid temperature, T_L , is always higher (considerably) than the liquid saturation temperature at infinity, T_∞^* ; the vapor pressure, P_V , is always higher than the liquid pressure at infinity, P_∞ , and the vapor is subcooled. The subcooling is illustrated in terms of $(P_V - P_V^*)$. With subcooling in the vapor, the possibility of bulk vapor nucleation is recognized, but has not been included in the model. Reference 17 indicates that a substantial subcooling without nucleation may be possible. The driving force for the bubble growth is indicated as $(P_V - \bar{P}_\infty)$."

4.2.7 Results of Bubble Collapse Calculations

In a companion paper to Ref. 16, Theofanous et al.,¹⁸ used the same basic set of equations to examine nonequilibrium bubble collapse of sodium vapor in moderately high and highly subcooled liquid. In the calculation they produced subcooling either by imposing an increasing pressure profile at infinity (Method I) or by imposing a step decrease in liquid

Fig. 4-4. Growth Curves for the Sodium Case

Fig. 4-5. The Variation of T_v , T_L , and Boundary Layer Thickness with Time for the Sodium Case

Fig. 4-6. The Variation of P_v , P_v^* with Time for the Sodium Case

temperature. The conditions of the calculations are as follows and correspond to a Jacob No. = 24.2:

Method I

Initial pressure	$P_{\infty} = 0.8 \text{ atm}$
Linear pressure increase ($\tau = 1 \text{ msec}$)	$P_f = 1 \text{ atm} \rightarrow T_s = 24.3^\circ\text{K}$
Initial temperature	$T_l = T_v = 1130^\circ\text{K}$
Initial radius	$R_v = 0.676 \text{ cm}$

Method II

Initial pressure	$P_{\infty} = 1 \text{ atm}$ and remains constant
Initial liquid temperature	$T_L = 1130^\circ\text{K}$
Initial vapor temperature	$T_V = 1154.3^\circ\text{K}$
	$\left. \begin{array}{l} T_L = 1130^\circ\text{K} \\ T_V = 1154.3^\circ\text{K} \end{array} \right\} \Delta T_s = 24.3^\circ\text{K}$
Initial radius	$R_o = 0.676 \text{ cm}$

Additional evaluations were made to substantiate the conclusions but are not illustrated in these notes.

Theofanous et al.¹⁸ discussed their results, as follows:

4.2.7.1 The Finite Rate of Phase Change

"The three values of c chosen ($1, 10^{-1}, 10^{-2}$) cover the range of acceptable values for the vaporization coefficient and were also illustrated in the growth problem.¹⁶ As to be expected, at lower subcoolings the solutions obtained are practically independent of the value for c for the range given. For these cases, the heat transfer in the liquid and, to a smaller extent, inertia are the controlling mechanisms for the collapse of the vapor bubbles.

"As the subcooling increases, the nonequilibrium effect for the phase change becomes increasingly important. This is clearly demonstrated in Figs. 4-7 to 4-11 by the three distinctly different solutions obtained for

the three values of c . In particular, note that for the example chosen, the value of $c = 1$ leads to a rebounding bubble for both Methods I and II, whereas for $c = 10^{-1}$ and 10^{-2} , a monotonic mode of collapse is exhibited. As illustrated in Figs. 4-9 to 4-11, the rebounding bubbles develop large pressure and temperature peaks at points just before the rebound.

"For the case of $c = 10^{-1}$, the peaks degenerate into simple maxima. The bubble wall history (R_V vs. t) as given by purely inertia (Rayleigh) or heat transfer are plotted for comparison in Figs. 4-7 and 4-8. The heat transfer solution can only be compared with the Method II solution and is done so in Fig. 4-7.

"The inertia solution obtained for both Methods I and II (for Method I by numerical integration of the Rayleigh equation), as shown in Figs. 4-7 and 4-7, compares well at the early stages of collapse with the $c = 1$ case. This fact can be deduced by examination of Fig. 4-11. For $c = 1$, the pressure of the vapor remains almost constant for the major portion of the collapse and equal to the saturation pressure at T_∞ .

"The saturation pressure corresponding to the vapor temperature is compared with the actual vapor pressure in Fig. 4-11. We see that the assumption of saturation conditions is not warranted and that $P_V^* > P_V$.

4.2.7.2 The Pressures Developed in the Liquid During Collapse²

"The theoretical results show that when a bubble collapse, large pressures can develop in the vapor, provided the collapse remains spherically symmetrical and the subcooling is large. On the other hand, one can calculate the pressure field established in the liquid surrounding the bubble as follows.

From (4-24) and (4-27),

$$u_r(r, t) = \frac{1}{r^2} \dot{R}_V R_V^2 \quad (4-55)$$

Fig. 4-7. Comparison of Theory (Method II) with Inertia and Heat Transfer Solutions for a Collapsing Sodium Bubble. $\Delta T_s = 24.3^\circ\text{K}$, $P_\infty = P_L(0) = 1.01 \cdot 10^6$ dynes/sq cm, $N_{ja} = 24.4$.

Fig. 4-8. Comparison of Theory (Method I) with Inertia Solution for a Collapsing Sodium Bubble in a Transient Pressure Field. ΔT_s (final) = 24.3°K, $P_f = 1.01 \cdot 10^6$ dynes/sq cm, $P_L(0) = 0.807 \cdot 10^5$ dynes/sq cm.

Fig. 4-9. The Temperature of the Vapor and the Temperature of the Liquid at the Interphase as Function of Time Based upon Method II. $\Delta T_s = 24.3^{\circ}\text{K}$, $P_{\infty} = P_L(0) = 1.01 \cdot 10^6$ dynes/sq cm.

Fig. 4-10. The Temperature of the Vapor and the Temperature of the Liquid at the Interphase as Function of Time Based upon Method I. $\Delta T_s = 24.3^\circ\text{K}$, $P_f = 1.01 \cdot 10^6$ dynes/sq cm.

Fig. 4-11. The Pressure of the Vapor and the Saturation Pressure Corresponding to the Temperature of the Vapor as Function of Time for Various Cases Considered. $\Delta T_s = 24.3^\circ\text{K}$ (II means Method II, I means Method I).

$$\frac{\partial u_r}{\partial t} = \frac{1}{r^2} R_V^2 \ddot{R}_V + \frac{2}{r^2} R_V \dot{R}_V^2 \quad (4-56)$$

$$\frac{\partial u_r}{\partial r} = - \frac{2}{r^3} R_V^2 \dot{R}_V \quad (4-57)$$

and Eq. 4-29 becomes

$$\left[R_V^2 \ddot{R}_V + 2 R_V \dot{R}_V^2 \right] \frac{dr}{r^2} - 2 R_V^4 \dot{R}_V^2 \frac{dr}{r^5} = - \frac{1}{\rho_L} dP \quad (4-58)$$

valid at each time instant $t = t'$.

Integrating (4-58) from ∞ to some radius $R \geq R_V$, where the pressure, $P(R, t')$, is required and rearranging, we obtain

$$P(R, t') = P_\infty + \rho_L \left\{ R_V \left(\frac{R_V}{R} \right) \ddot{R}_r + \left[2 \left(\frac{R_V}{R} \right) - \frac{1}{2} \left(\frac{R_V}{R} \right)^4 \right] \dot{R}_V^2 \right\} \quad (4-59)$$

Using the quantities R_V , $\dot{R}_V$, $\ddot{R}_V$ for any time t' at the bubble wall, one may evaluate the pressure for any point $R \geq R_V$ in the liquid from (4-59).

Note in particular that the condition

$$\frac{\partial}{\partial R} P(R, t') = 0$$

gives

$$\left(\frac{R}{R_V} \right)^3 = \frac{2 \dot{R}_V^2}{2 \dot{R}_V^2 + R_V \ddot{R}_V}$$

which indicates that as long as the liquid accelerates ($\ddot{R}_V < 0$), there is a pressure maximum in the liquid. This maximum moves towards the bubble wall and it reaches it at the point where $\ddot{R}_V = 0$. An illustration is presented of the magnitude of these pressures. The specific calculation was performed for the case of a sodium bubble collapsing according to Method I, utilizing $\tau = 4$ msec, $\Delta T_s = 150^\circ\text{K}$, $P_f = 1$ atm, and $R_0 = .676$ cm. Figure 4-12 presents the pressure distribution in the liquid for the time $t' = 1.65$ msec after the start of the collapse. It is, thus, shown that large pressures, even larger than those in the vapor, are generated in the liquid, too!

Fig. 4-12. Pressure Distribution in the Liquid for $t = 1.65$ msec. $\Delta T_s = 150^\circ\text{K}$, $P_f = 1$ atm, $R_0 = 0.676$ cm, $\tau = 4$ msec (Method I) Sodium.

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NOMENCLATURE

c_L	Heat capacity of the liquid
c_V	Heat capacity of the vapor at constant volume
c	Vaporization or condensation coefficient
c'	Velocity of sound
h	Heat of vaporization or condensation
H	Enthalpy
k_L	Thermal conductivity of liquid
k_V	$= \frac{M}{R}$
M	Molecular weight of vapor
P	Pressure; P_V = pressure in vapor, P_L = pressure in liquid at R_V , P_∞ = pressure in liquid at infinity, P_V^* = saturation pressure at $T = T_V$, P^* = saturation pressure of $T = T_\infty$, P_L^* = saturation pressure at $T = T_L$.
q	Conduction heat flux vector
r	Radial coordinate
R	Gas constant
R_V	Radius of vapor bubble; $\dot{R}_V = \frac{dR_V}{dt}$, $\ddot{R}_V = \frac{d^2R_V}{dt^2}$
$R_L - R_V$	$\delta(t)$ Thermal boundary layer thickness in liquid
t	Time
T	Temperature; T_V = temperature in vapor, T_L = temperature of liquid at vapor-liquid interphase, T_∞ = temperature of liquid at infinity, T_∞^* = saturation temperature at $P = P_\infty$
u_r	Radial liquid velocity, $\underline{u}$ = velocity vector
U_V / T_i	Internal energy of vapor at temperature T_i
$\Delta U_{L \rightarrow V}$	Rate of internal energy transfer, between liquid and vapor. (The direction liquid $\rightarrow$ vapor taken positive)
W	Net mass flux ($W > 0$ for vaporization)
W_-	Mass rate of molecules impinging the liquid surface as given from kinetic theory
W_+	Mass rate of molecules vaporizing from liquid surface as given from kinetic theory

Greek Letters

α	Thermal diffusivity
β	Coefficient in asymptotic solution
γ	$= \frac{\sigma}{\rho}$
ζ	$= \frac{R}{R_o \rho_L - \rho_V}$
ϵ	$= \frac{\rho_L - \rho_V}{\rho_L}$
μ	Viscosity
η	$= \frac{\mu}{P}$
π_{rr}	Radial component of stress tensor
π	$= 3.14 \dots$
ρ	Density
$\bar{\rho}_V$	Vapor density corresponding to saturation conditions at $T = T_\infty$
σ	Surface tension
$\underline{\tau}$	Shear stress tensor
τ	Pressure rise time for collapse Method I

Subscripts

V	Vapor
L	Liquid
o	Initial condition
cr	Critical value
ref	Reference

Superscripts

*	Saturation condition
^	Property per unit mass

5. SODIUM VOIDING CALCULATIONS

5.1 Introduction

The business of attempting a description of the processes of boiling, bubble growth, voiding, flow oscillation, and coolant reentry in sodium-cooled reactors has been an extremely active one for the past six years. Considerable strides in our understanding and description of the processes have been made, both experimentally and theoretically--but more remains to be learned.

In this limited discussion of the subject, we will write down the basic equations and examine some of the kinds of approximations made to facilitate their solution. We shall review in a semihistorical way the development of specific codes, and thereby see the process of evolution which has taken place in the physical models. And we will look at some examples of calculational results.

The objective of the calculational methods discussed below is, in the most general terms, to describe sodium behavior under transient boiling conditions. More specifically, it is to describe the place and time of boiling initiation, the way boiling progresses, the rate at which sodium is voided from different regions, the type of flow which ensues after any initial voiding, and the temperature conditions of the clad and fuel, since clad failure and the escape of fission product gases or hot fuel will affect the sodium flow as well as the entire course of the transient.

Included among the qualitative considerations which enter such a calculation are the following:

- a) What kind of two-phase flow?

BUBBLY
HOMOGENEOUS

ANNULAR

FOG

SLUG

MULTI-SLUG

We shall see that current calculational models are slug-flow or multi-slug flow representation (although questions can be raised concerning its applicability with low superheat and/or a considerable power gradient across a subassembly).

b) What is the nature of the liquid film left on the clad when a large bubble is formed? How thick is it? How long and under what conditions does

it maintain its integrity? Does rapid vaporization or high velocity vapor flow cause film breakup? How and to what degree is a film restored when liquid again flows by a previously voided surface?

- c) How to allow for energy and mass transfer at liquid-vapor interfaces?
- d) How to allow for bubble condensation, either in subcooled liquid or at "cool" surfaces?
- e) How to allow for regions of flow discontinuity?
- f) What are the dominant phenomena on liquid reentry and what are the flow patterns?
- g) What is the correct amount of superheat?
- h) How to allow for power, temperature, and flow gradients in a subassembly?

5.2 Some History of Code Development

5.2.1 TRANSFUGUE and BURP

One of the first efforts to describe transient sodium boiling with a code was by Noyes et al.¹ in the code TRANSFUGUE. This code solved the equations of continuity, conservation of momentum, and conservation of energy in the following form:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v}{\partial z} = 0 \quad (5-1)$$

$$\rho \frac{\partial v}{\partial t} + \rho v \frac{\partial v}{\partial z} + g\rho + \frac{f\rho v^2}{2D_c} + \frac{\partial P}{\partial z} = 0 \quad (5-2)$$

$$\rho \frac{\partial H}{\partial t} + \rho v \frac{\partial H}{\partial z} = \gamma \phi(z, t) \quad (5-3)$$

The sodium was treated as having homogeneous two-phase flow (without slip) beyond the point where boiling began, with

$$\rho = \frac{\rho_g \rho_l}{\rho_g (1 - x) + \rho_l x} \quad (5-4)$$

Thermodynamic equilibrium between vapor and liquid with no superheat was assumed, and

$$x = \text{the quality} = \frac{H - H_f}{H_{fg}} \quad (5-5)$$

Because of numerical instabilities in the solution, the following additional assumptions were made:

- 1) the vapor density was held constant independent of pressure,
- 2) the saturation temperature was held constant everywhere in the channel.

These artificial restrictions allow the vapor pressure (which produces the voiding) to be incompatible with the actual wall temperatures. The conservation of energy has been decoupled from the conservation of momentum and vapor pressures which are too high result, leading to excessively large voiding rates.

Noyes¹ pointed out this flaw in TRANSFUGUE. He also observed that the flow was unlikely to remain or perhaps to form a two-phase mixture of liquid and vapor bubbles (partly because the pressurization of the fluid would suppress the nucleation and growth of other bubbles), and that the usual picture of steady-state boiling of water was not applicable for transient boiling of sodium.

Thus, for steady-state boiling of water, one has

Noyes postulated that transient boiling of sodium would have the following picture:

Noyes¹ also described a fairly simple code, BURP, which encompassed these ideas:

- 1) the void spreads axially from a single starting point,
- 2) pressure in the void region equals the highest vapor pressure of the boundary liquid (at the ends of the bubble, but not on the channel walls, which were assumed dry),
- 3) self-pressurization suppresses boiling away from the single, growing void region which is 100% vapor, and
- 4) superheat was not treated.

BURP pointed the way of the future more than did TRANSFUGUE.

5.2.2 MacFarlane and SAS1A^{2,3}

MacFarlane's model treated the two-phase flow as equilibrium without superheat, but assumed annular flow and allowed for vapor slip with respect to the liquid. His code also allowed the pressure and vapor density to vary along the flow channel. However, due to instabilities in the original formulation of the numerical solution, he also decoupled the equations of conservation of energy and momentum, and permitted the vapor pressures not to be compatible with the wall temperature, so that pressure buildup could exceed that actually available from the saturation temperature.

Some of the analytical details of this formulation, as used in the code SAS1A, are discussed in section 5.3.1 below.

TRANSFUGUE and SAS1A ignored superheat effects and have, therefore, been labeled "equilibrium models" in some reviews.⁴

The next generation of codes not only allowed for superheat, but included the picture of a single large bubble with slug flow of liquid, or several large bubbles, separated by liquid slugs, wall to wall.

5.2.3 VOID⁵

The first code to try to account for superheat was VOID in 1966. This code considered consecutive slabs of vapor and liquid with heat transfer between liquid and vapor according to an empirical relationship.

5.2.4 BLOW^{6,7}

Experimental observations⁸⁻¹¹ tended to confirm that void growth occurs as a single large bubble, but also that some heat removal capability remains (at least for a while) in the region of the wall surrounding the vapor, suggesting strongly that a liquid film remained on the wall--a film which could absorb considerable heat from the metal wall by the process of vaporization--and simultaneously supply vapor to the expanding bubble.

Also, while the film remains intact, rapid overheating, weakening and failure of the metal clad wall--burnout--should not occur.

The first code to incorporate all these effects was the BLOW code.⁶ It allows for mass and energy transfer between vapor and liquid-slug interface, as well as from the film, and was able to predict oscillatory flow in qualitative agreement with experimental results.

5.2.5 SLUG¹²

SLUG is similar to BLOW, except that it allows for heat transfer in the liquid slugs, which BLOW neglects. Some analytical details of the formulation of SLUG are discussed in section 5.3.2 below.

5.3 Description of Detailed Analytical Models

5.3.1 Coolant Dynamics Module in SAS1A³

The two-phase flow model of MacFarlane² was incorporated into the accident analysis code SAS1A with some modifications 1) to allow boiling to begin anywhere in the channel, not only at the exit, and 2) to change from two-phase flow to a relatively simple slug flow model when the "limitations of the two-phase flow model were exceeded."

SAS1A employs a cylindricized concept of the coolant flow channel, as is illustrated in Fig. 5-1 and utilizes a sophisticated fuel element heat transfer code, which includes some improvements and new features over ARGUS¹³ and FORE.¹⁴ In addition to a new differencing scheme, SAS1A takes the following into account:

- 1) dimensional changes due to thermal expansion and phase change
- 2) latent heat absorption due to phase change
- 3) variable bond-gap size and the associated thermal resistance at the fuel-clad interface
- 4) temperature dependence of fuel thermal conductivity and heat capacity

The description of the coolant dynamics (or voiding) model is quoted from Ref. 3 as follows:

"The main function of the coolant dynamics module in SAS1A is to provide a reasonable detailed profile of vapor voiding, based on the single-channel model shown in Fig. 5-1. The calculated temporal and spatial void distribution, along with input void-reactivity coefficients, is then used to obtain the coolant-voiding component of feedback reactivity, which is input to the Point Kinetics Module. Coupling of the coolant dynamics and Heat Transfer Modules arises naturally via the heat flux at the cladding surface.

"Two different models are used for different parts of the boiling expulsion. A slip-flow, two-phase model is used for the initial portion and up to a short time after flow reversal (ejection of coolant from the channel inlet) when a reasonably large void fraction (>0.90) exists. Since the slip-flow model is less meaningful from a physical standpoint when the vapor-void fraction is approaching 1.0 and coolant is being expelled from both ends of the channel, a switch is then made to a simple slug model to continue the voiding calculation. Throughout all the coolant dynamics calculations, the

Fig. 5-1. Single Fuel Pin, Coolant Channel and Structure

coolant channel geometry is assumed to remain intact, that is, there are presently no models for considering vapor blanketing of the fuel rods and subsequent interaction of molten fuel (produced by clad melting or other mechanisms, e.g., failure of clad due to excessive pin internal pressure) with the coolant.

5.3.1.1 Sodium-void Model

"In the slip-flow model used in the treatment of the two-phase fluid, it is assumed that the vapor and liquid each move at a separate characteristic velocity. Since the vapor velocity is higher, it "slips" past the liquid. The following assumptions are made in the formulation of the voiding model:

1) No subcooled boiling or liquid superheat occurs; vapor is formed only when the local bulk coolant temperature exceeds the local saturation temperature.

2) There is thermodynamic equilibrium between phases, and liquid and vapor temperatures equal the local saturation temperature after boiling starts.

Heat transfer in the boiling region is treated in a simple manner wherein the heat-transfer coefficient is calculated by assuming that the liquid film stays in contact with the channel wall; hence, cooling of the cladding is maintained during boiling and expulsion of coolant.

5.3.1.2 Hydrodynamics Equations for Two-phase Flow

The conservation equations for the two-phase mixture are:

Continuity

$$\frac{\partial}{\partial t} [\alpha \rho_g + (1 - \alpha) \rho_l] + \frac{\partial G}{\partial z} = 0 ; \quad (5-6)$$

Momentum

$$\frac{\partial G}{\partial t} + \frac{\partial}{\partial z} \left[\alpha \frac{G_g^2}{\rho_g} + (1 - \alpha) \frac{G_l^2}{\rho_l} \right] + \frac{\partial p}{\partial z} + g[\alpha \rho_g + (1 - \alpha) \rho_l] + \left(\frac{\partial p}{\partial z} \right)_{fr} = 0 ; \quad (5-7)$$

Energy

$$\frac{\partial}{\partial t} [\alpha \rho_g H_g + (1 - \alpha) \rho_l H_l] + \frac{\partial}{\partial z} G[x H_g + (1 - x) H_l] = \gamma \phi(z, t) , \quad (5-8)$$

where

α = volume fraction of vapor;

x = vapor quality;

ρ_g = vapor density;

ρ_l = liquid density;

$G = \alpha G_g + (1 - \alpha) G_l$ = total mass flow rate;

$\left(\frac{\partial p}{\partial z} \right)_{fr}$ = frictional pressure gradient;

H_g = vapor enthalpy;

H_l = liquid enthalpy;

γ = ratio of channel perimeter to cross-sectional area;

p = pressure;

$\phi(z, t)$ = heat flux at channel wall;

g = acceleration of gravity.

The continuity and energy equations (Eqs. 5-6 and 5-8) are put in a more useful form for numerical solution by the following manipulations. Performing the differentiation indicated in the first term of Eq. 5-6 yields

$$(\rho_g - \rho_l) \frac{\partial \alpha}{\partial t} + \frac{\partial G}{\partial z} = 0 \quad (5-9)$$

if the terms $\alpha \partial \rho_g / \partial t$ and $(1 - \alpha) \partial \rho_l / \partial t$ are neglected. This is permissible since these terms are small with respect to $(\rho_g - \rho_l) \partial \alpha / \partial t$ in a boiling sodium system. Similarly, the energy equation can be written as

$$(\rho_g H_g - \rho_l H_l) \frac{\partial \alpha}{\partial t} + \frac{\partial G}{\partial z} (x\lambda + H_l) + G\lambda \frac{\partial x}{\partial z} + G \frac{\partial H_l}{\partial z} = \gamma \phi(z, t), \quad (5-10)$$

where $\lambda = \text{heat of vaporization} = H_g - H_l$. In the derivation of Eq. 5-10, it is assumed that the terms $\alpha \frac{\partial}{\partial t} (\rho_g H_g)$ and $(1 - \alpha) \frac{\partial}{\partial t} (\rho_l H_l)$ can also be neglected (a reasonable assumption for a boiling sodium system). Combining Eqs. 5-9 and 5-10 gives

$$[\lambda \rho_g + \lambda x \rho_l - \lambda x \rho_g] \frac{\partial \alpha}{\partial t} + G\lambda \frac{\partial x}{\partial z} + G \frac{\partial H_l}{\partial z} = \gamma \phi(z, t).$$

Since $\rho_g \ll \rho_l$ this can be approximated as

$$(\rho_g + \rho_l x) \frac{\partial \alpha}{\partial t} + G \frac{\partial x}{\partial z} + \frac{G}{\lambda} \frac{\partial H_l}{\partial z} = \frac{\gamma \phi(z, t)}{\lambda}. \quad (5-11)$$

Momentum Eq. 5-7 is rewritten in the following way. The second term, representing the momentum pressure gradient, can be written in terms of the total flow rate as

$$\left[\frac{\partial p}{\partial z} \right]_{\text{mom}} = \frac{\partial}{\partial z} \left[\alpha \frac{G^2}{\rho_g} + (1 - \alpha) \frac{G^2}{\rho_l} \right] = \frac{\partial}{\partial z} \left\{ \frac{G^2}{\rho_l} \left[\frac{\beta x^2}{\alpha} + \frac{(1 - x)^2}{1 - \alpha} \right] \right\},$$

where

$$\beta = \rho_l / \rho_g .$$

Also, the static-head term can be written in terms of β as

$$\left[\frac{\partial p}{\partial z} \right]_{\text{stat}} = g[\alpha \rho_g + (1 - \alpha) \rho_l] = g \rho_l [(\alpha/\beta) - \alpha + 1] .$$

Further, the friction gradient is

$$\left[\frac{\partial p}{\partial z} \right]_{\text{fr}} = \frac{f_{\text{sp}} G^2}{2 \rho_l D_h} (1 - x)^{2-a} \phi_l^2 ,$$

where

$$\phi_l^2 = \text{two-phase friction multiplier} = f_1(X_{\text{tt}});$$

$$X_{\text{tt}} = \text{Lockhart-Martinelli flow parameter (see Eq. B-21);}$$

$$a = 0.2;$$

$$f_{\text{sp}} = \text{single-phase friction factor;}$$

$$D_h = \text{channel equivalent diameter.}$$

Thus, the momentum equation becomes

$$\frac{\partial G}{\partial t} + \frac{\partial}{\partial z} \left\{ \frac{G^2}{\rho_l} \left[\frac{\beta x^2}{\alpha} + \frac{(1 - x)^2}{1 - \alpha} \right] \right\} + g \rho_l [(\alpha/\beta) - \alpha + 1]$$

(5-12)

$$+ \frac{f_{\text{sp}} G^2}{2 \rho_l D_h} (1 - x)^{2-a} \phi_l^2 + \frac{\partial p}{\partial z} = 0 .$$

"The most rigorous solution to Eqs. 5-9, 5-11, and 5-12 would be given by the "sectionalized compressible model" as described by Meyer.¹⁵ However, a numerical solution using this model requires a knowledge of the maximum signal, or sonic, velocity in the fluid. Since the sonic velocity in two-phase sodium cannot be accurately predicted and the computer solution is very much slower, sonic effects were omitted and the momentum equation was used in the so-called "momentum integral" form.* Hence, integrating Eq. 5-12 over the length of the channel gives

$$\int_0^L \frac{\partial G}{\partial t} dz + \frac{G^2}{\rho_\ell} v' \Big|_0^L + g \int_0^L \rho_\ell [(\alpha/\beta) - \alpha + 1] dz$$

$$+ \int_0^L \frac{f_{sp} G^2 (1-x)^{2-a}}{2\rho_\ell D_h} \phi_\ell^2 dz + p \Big|_{P_o}^{P_N} = 0, \quad (5-13)$$

where

$$v' = \frac{\beta x^2}{\alpha} + \frac{(1-x)^2}{1-\alpha};$$

L = channel length;

P_o = channel pressure just inside the inlet;

P_N = channel pressure just inside the exit.

In this form the transfer of momentum across the control-volume boundaries (ends of the reactor channel) is treated explicitly while the details of momentum transfer within the channel are suppressed. In contrast to the work of Meyer and others using the momentum integral model, however, the fluid

* See Appendix 5-2.

properties used in evaluating the various integrals in Eq. 5-13 are pressure-dependent, rather than being evaluated at a constant reference pressure.**

In order to include cases where boiling starts at an arbitrary point in the channel, the momentum integral (Eq. 5-13) is broken into two parts as follows:

Lower nonboiling leg and boiling portions

$$\int_0^{z_{ub}} \frac{\partial G}{\partial t} dz + \frac{G^2}{\rho_\ell} v' \Big|_0^{z_{ub}} + g \int_0^{z_{ub}} \rho_\ell [(\alpha/\beta) - \alpha + 1] dz$$

$$+ \int_0^{z_{ub}} \frac{f_{sp} G^2 (1-x)^{2-a}}{2\rho_\ell D_h} \phi_\ell^2 dz + p \Big|_{P_0}^{P_{z_{ub}}} = 0, \quad (5-14)$$

where

$$z_{ub} = z_{ub}(t) = \text{length of lower nonboiling leg at time } t.$$

Upper nonboiling leg

$$\int_{z_{ub}}^L \frac{\partial G}{\partial t} dz + \frac{G^2}{\rho_\ell} \Big|_{z_{ub}}^L + g \int_{z_{ub}}^L \rho_\ell dz + \int_{z_{ub}}^L \frac{f_{sp} G}{2 D_h} dz$$

$$+ p \Big|_{P_{z_{ub}}}^{P_N} = 0. \quad (5-15)$$

** To achieve stability in the numerical solution of voiding in SAS1A, the motion of the boiling interface down the channel, rather than being determined by the saturation temperature corresponding to the local pressure, is determined by a fixed saturation temperature distribution derived from the one-phase pressure distribution. As pointed out earlier, the SAS1A solution allows the local pressure to be incompatible with the local temperature, as does TRANSFUGUE.

Since the mass flow rate and liquid properties are approximately constant with position in the upper nonboiling leg, integration of Eq. 5-15 gives

$$P_{z_{ub}} = \frac{\partial G}{\partial t} (L - z_{ub}) + g\rho_l (L - z_{ub}) + \frac{f_{sp}}{2\rho_l D_h} G^2 (L - z_{ub}) + P_N. \quad (5-16)$$

In this manner the upstream single-phase liquid leg and the two-phase boiling portion of the channel are treated using one momentum integral (Eq. 5-14), and the downstream liquid leg being expelled from the channel exit is treated by a second integral (Eq. 5-15). The boundary condition for the downstream pressure $P_{z_{ub}}$ for Eq. 5-14 is time varying as determined by Eq. 5-16. Thus, the rate of change of average channel velocity is given as the difference between the imposed pressure drop in the channel and the various integrals in Eq. 5-14.

5.3.1.3 Equations for Single-phase Flow

"The conservation equations for the single-phase fluid in the subcooled portion of the channel are readily obtained from the two-phase Eqs. 5-5 and 5-8 by setting $\alpha = 0$. They are:

Continuity

$$\frac{\partial \rho}{\partial t} + \frac{\partial G}{\partial z} = 0 ; \quad (5-17)$$

Momentum

$$\frac{\partial G}{\partial t} + \frac{\partial}{\partial z} \left(\frac{G^2}{\rho} \right) + \frac{\partial p}{\partial z} + g\rho + \left(\frac{\partial p}{\partial z} \right)_{fr} = 0 ; \quad (5-18)$$

Energy

$$\frac{\partial}{\partial t} (\rho H) + \frac{\partial}{\partial z} (GH) = \gamma \phi(z, t) . \quad (5-19)$$

Combining Eqs. 5-17 and 5-19 gives the energy equation in the form

$$\rho \frac{\partial H}{\partial t} + G \frac{\partial H}{\partial z} = \gamma \phi(z, t) ; \quad (5-20)$$

since the liquid phase is assumed to be incompressible, the continuity equation reduces to $\partial G / \partial z = 0$.

5.3.1.4 Boundary Conditions and Equation of State for Two-phase Fluid

"The heat flux at the fuel-rod surface as a function of axial position and time is given in the usual way by an equation of the form

$$\phi(z, t) = h(G, T_c) [T_{NE'} - T_{NC}] , \quad (5-21)$$

where

$T_{NE'}$ = surface temperature of the fuel cladding;

T_{NC} = bulk coolant temperature.

The convection film coefficient is calculated by the expression

$$\frac{hD_h}{k} = C_1 \left[\frac{D_h G C_\ell}{k} \right]^{C_2} + C_3 , \quad (5-22)$$

which is the form of correlation used for calculation of convection heat-transfer coefficients for fluids of low Prandtl number (such as the liquid metals). Here

D_h = channel equivalent diameter;

k = coolant thermal conductivity;

G = coolant mass flow rate;

C_ℓ = coolant heat capacity;

C_1, C_2, C_3 = constants depending on particular correlation;

$h = h(G, T_c)$ = convection film coefficient.

The additional boundary conditions required to complete the problem formulation are:

a) Temperatures at $t = 0$ in the coolant and fuel, calculated from the steady-state forms of the appropriate equations.

b) Coolant temperature at the channel inlet as a function of time, assumed constant in calculations (except after flow reversal).

c) Channel outlet pressure as a function of time.

In computing the exit pressure loss, we assumed that none of the potential pressure increase is recovered and that there is a further loss of one velocity head due to the fuel bundle and fittings, etc. The exit pressure-loss term is given by

$$P_N - P_{ex} = \frac{G_N |G_N|}{2\rho_N}, \quad (5-23)$$

where

P_{ex} = exit plenum pressure.

d. Channel inlet pressure as a function of time determined by the inlet pressure loss, given by

$$P_I - P_o = \frac{G_o |G_o|}{2\rho_o}, \quad (5-24)$$

where

P_I = inlet plenum pressure.

This is the single-phase pressure loss at the channel inlet for an area ratio of 0.5 (corresponds to a core coolant fraction of 0.5) according to data given by Lottes.¹⁶ The channel driving pressure head ($P_I - P_{ex}$) is either constant (power transient) or exponentially decreasing (flow-coastdown transient) in time. An alternative means of specifying a flow-reduction transient in SASIA is by a tabular input of flow rate versus time.

It is necessary to have an expression which gives the vapor volume fraction, α , in a two-phase flow system. The experimental vapor fraction is most frequently correlated by an expression of the form

$$\alpha = f_2(X_{tt}) , \quad (5-25)$$

where

$$X_{tt} = \left(\frac{1-x}{x} \right)^{0.9} \left(\frac{\rho_g}{\rho_l} \right)^{0.5} \left(\frac{\mu_l}{\mu_g} \right)^{1.1} ; \quad (5-26)$$

μ_g = vapor viscosity;

μ_l = liquid viscosity;

and the quantity X_{tt} is the Lockhart-Martinelli two-phase flow parameter.¹⁷

The correlation represented by Eq. 5-25 defines the vapor-liquid slip-velocity ratio since the slip ratio is implicit within it. Equation 5-25 and the assumption that the channel is completely filled with the fluid mixture, i.e.,

$$\alpha_g + \alpha_l = 1.0 ,$$

fix the fraction of the channel cross section occupied by the liquid. Thus, Eq. 5-25 is used to relate the variables x and α in Eq. 5-11. The particular $f_2(X_{tt})$ built into the code at present is

$$\alpha = 1.0 - \frac{1.0}{\sqrt{1.0 + \frac{21.0}{X_{tt}}}} .$$

5.3.1.5 Slug-expulsion Model

"In general, the core is only partially voided at the time the two-phase boiling calculation breaks down (i.e., the limitation of the model is exceeded) and, in many cases, significant rates of reactivity due to coolant voiding are being introduced at this point in time. Therefore, a different model was devised to continue the voiding beyond this point. One of the most straightforward and direct ways of doing this is to continue the voiding calculation by means of a so-called slug-ejection model. The basic assumption in developing the model is that there is a differential pressure across the liquid slug which tends to move it out of the core. This driving pressure for ejection is generated by the coolant vapor pressure. The saturation pressure used in the model corresponds to a temperature midway between those of the coolant and the clad. Accordingly, the driving forces are given by the difference between the saturation pressure and the pressure at the end of the channel.

The equation which describes the motion is derived by writing a force balance on the slug, which includes inertial and frictional terms. Considering a force balance (neglecting gravitational forces and spatial acceleration terms) on the slug, one obtains from Eq. 5-18,

$$\frac{\partial(\rho V)}{\partial t} + \frac{\partial p}{\partial z} + \left(\frac{\partial p}{\partial z} \right)_{fr} = 0 .$$

At any instant

$$\frac{dp}{dz} = \frac{P_e - P_s}{L(t)} ,$$

where

P_s = saturation pressure;

P_e = channel exit pressure;

$L(t)$ = slug length as function of time;

V = slug velocity;

ρ = liquid density;

t = time.

Since we assume incompressible flow in the slug, $\partial\rho/\partial t = 0$, which gives

$$\frac{dv}{dt} = - \frac{1}{\rho} \frac{dp}{dz} - \frac{1}{\rho} \frac{fv^2\rho^2}{2\rho D_h} . \quad (5-27)$$

Substituting the above equation into Eq. 5-27 along with

$$L(t) = L_o - \int_{t_o}^t v(t') dt'$$

gives

$$\frac{dv}{dt} = \frac{P_s - P_e}{\rho \left[L_o - \int_{t_o}^t v(t') dt' \right]} - \frac{fv^2}{2D_h} , \quad (5-28)$$

where

D_h = channel hydraulic diameter;

f = friction factor = $A(Re)^b$;

Re = Reynolds number = $D_h v \rho / \mu$;

μ = liquid viscosity.

Thus, Eq. 5-28 describes the motion of the liquid slug as a function of time, and its solution gives the propagation rate of the voiding pattern in the coolant channel. Equation 5-28 is solved in SAS1A using the Runge-Kutta method. The integral in the equation is evaluated using the trapezoidal rule. Boundary values required for solution of the problem are

L_0 = initial length of liquid slug

and

v_0 = initial velocity of slug.

These values are obtained from the two-phase calculation at the time the switch over to the calculation for slug expulsion is made.

5.3.2 SLUG¹²

The description of SLUG is quoted from Ref. 12 as follows:

"Basically the approach taken is to calculate the heat transfer from the fuel pin to the coolant until at some point in space and time the coolant reaches a certain temperature, corresponding to a specified amount of superheat. At this point inception of boiling occurs. The stages of spherical bubble growth are approximated by a thin bubble assumed to occupy the entire coolant channel area, except for the liquid film remaining at the clad wall. The coupled solution to the energy and hydrodynamic equations of the coolant,

and the heat transfer equations of the fuel pin are then continuously solved during the voiding process.

5.3.2.1 Heat Transfer Model

"Since the principal area under investigation is the hydrodynamics of slug expulsion and not a heat transfer model for the fuel pin, a simple radial heat flow model is employed for the pin neglecting axial heat flow and angular variations of heat conduction. The channel is divided into a number of axial segments, the heat generation rate within each segment being constant. The radial temperature distributions are calculated for each of the sections in succession following the direction of the coolant flow. The steady-state equations are solved directly, while the finite difference method is used to obtain the transient solution. The thermophysical properties of the fuel, bond, and clad are taken to be constant functions of temperature and space. The properties of the coolant are also taken to be constant up to the time of boiling, however, during expulsion the sodium properties are calculated functions of temperature.

Consider the transient resulting from an instantaneous loss of flow for the channel configuration as shown in Fig. 5-2. The calculation procedure is as follows:

- a) The steady-state rise in temperature of the coolant as it flows from axial node $j-1$ to j is calculated as:

$$T_j^m = T_{j-1}^m + \frac{\dot{Q}_{j-1} V_f}{wc_p} \quad (5-29)$$

where V_f is the fuel volume between the axial nodes $j-1$ and j .

The temperature drop across the fuel element is found to be

Fig. 5-2. Reactor Channel Geometry and Power Profile Used for the Calculations

$$T_j^i = \frac{\dot{Q}_j}{4k_f} (a^2 - r^2) + \frac{\dot{Q}_j a^2}{2} \left[\frac{1}{h_{bd} a} + \frac{1}{k_{cl}} \ln \frac{c}{b} + \frac{1}{h_{cl} c} \right] + T_j^m \quad (5-30)$$

- b) The partial differential equation for transient radial heat conduction through a cylindrical fuel element is written as

$$\rho c_p \frac{\partial T}{\partial t} = k \left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right] + \dot{Q} \quad (5-31)$$

where k , ρ , and c_p are considered to be constants with respect to r and t . This equation is solved by use of finite difference techniques for a typical interior point within the fuel. The heat transmission through the bond, clad, and wall film is found by use of the thermal-circuit method. The transient calculation for the coolant is found by solving Eq. 5-32

$$\rho c_p V_m \frac{\partial T_m}{\partial t} = \dot{q} - \rho c_p V_m \dot{x} \frac{\partial T_m}{\partial x} \quad (5-32)$$

where V_m is the volume of the coolant, $\dot{q}$ is the rate of heat addition to the coolant, and $\dot{x}$ is the coolant velocity and equal to zero for a stagnant condition. The transient heat transfer calculation proceeds until the superheat criterion is satisfied, initiating bubble nucleation.

5.3.2.2 Spherical Bubble Growth Approximation

At the instant of bubble nucleation a thin vapor space which occupies the entire channel cross section is assumed. The vapor space is bounded by the upper and lower liquid slug. It is assumed that the pressure and temperature of the vapor are in equilibrium with the surrounding

liquid. Knowing the vapor temperature, the pressure and other thermophysical properties of the sodium¹⁸ are determined.

5.3.2.3 Momentum Considerations

With this estimate of the vapor pressure, the microscopic momentum equation can now be integrated by a fourth-order Runge-Kutta routine to obtain the position x , the velocity $\dot{x}$, and the acceleration $\ddot{x}$, of both the upper and lower vapor-liquid interfaces. The forces acting on the liquid slugs during expulsion are due to gravity, pressure differences, frictional drag, and expansion losses at the channel-plenum junction. The resistance coefficient for expansion is calculated as

$$K_e = \left[1 - \frac{D_e^2}{D_p^2} \right]^2 \quad (5-33)$$

and the frictional drag is estimated from the Blasius relation, where the friction factor is given by

$$f = 0.316 \text{ Re}^{-0.25} \quad (5-34)$$

The resulting force balance on the upper slug is given by

$$\sum F = (P_V - P_p) - \frac{f\rho(L_1 - x)\dot{x}^2}{2 \cdot D_e} - \frac{\rho K_e \dot{x}^2}{2} - (L_1 + L_2 - x)\rho g \quad (5-35)$$

The inertia of the column is given in terms of the acceleration of the interfaces

$$\sum M\ddot{x} = \left(L_1 + \frac{D_e^2}{D_p^2} L_2 \right) \rho \ddot{x} \quad (5-36)$$

The acceleration of the interface is obtained by equating Eqs. 5-35 and 5-36.

$$\ddot{x} = \frac{(P_V - P_p) - \frac{f\rho(L_1 - x)\dot{x}^2}{2 \cdot D_e} - \frac{\rho K_e \dot{x}^2}{2} - (L_1 + L_2 - x)\rho g}{\left(L_1 + \frac{D_e^2}{D_p^2} L_2 \right) \rho} \quad (5-37)$$

A similar equation is obtained for the bottom slug.

5.3.2.4 Energy Considerations

Having determined the interface position from the momentum equation, the energy equation for the vapor is then solved. The rate of vapor generation is proportional to the heat transferred by conduction across the slug interfaces and the wall film. Thus, the energy equation, neglecting sensible heat, is written as

$$kA \left. \frac{dT(\xi_1, t)}{d\xi_1} \right|_{\xi_1} + kA \left. \frac{dT(\xi_2, t)}{d\xi_2} \right|_{\xi_2} + Q' - \rho c_p \sigma S \frac{dT_{fm}}{dt} = A \frac{d}{dt} (\rho_V \lambda \Delta x) \quad (5-38)$$

The temperatures $T(\xi_1, t)$, $T(\xi_2, t)$, and $T_{fm}(t)$ are in equilibrium with the vapor temperature. On the left hand side of Eq. 5-38, the first two terms define the heat transfer across the slug interfaces, Q' is the heat transferred across the wall film, which is equal to

$$Q'(t) = h \cdot C \int_{x=\xi_1}^{x=\xi_2} [T_{cl}(x, t) - T_V(t)] dx \quad (5-39)$$

(vaporization)	$h = k(T)/\sigma$
(condensation)*	$h = 1.50 \text{ cal/cm}^2\text{sec}^\circ\text{C}$
(film thickness)	$\sigma = 0.015 \text{ cm}$

The fourth term takes into account the energy storage in the wall film layer.

In order to evaluate the temperature gradients at the liquid-vapor interfaces, the heat conduction equation for the upper and lower slugs must be solved. For the upper slug, neglecting radial temperature gradients, this equation is

$$\frac{\partial T}{\partial t}(\xi, t) = \alpha \frac{\partial^2 T}{\partial \xi^2} + \frac{Q(\xi, t)}{\rho_l c_p} \quad (5-40)$$

where $Q(\xi_1, t)$ is the equivalent volumetric heat input rate from the cladding to the liquid slug, at the interface position which is given by:

$$Q(\xi_1, t) = \frac{h}{D_e} [T_{cl}(\xi_1, t) - T_m(\xi_1, t)] \quad (5-41)$$

The initial and boundary conditions are:

$$T(\xi_1, 0) = T(\xi_1)$$

$$T(0, t) = \phi(t)$$

$$T_1(\infty, t) < \infty$$

The solution for the interface temperature gradient is found by Laplace

Transform methods:²⁰

$$\left. \frac{dT(\xi_1, t)}{d\xi_1} \right|_{\xi_1=0} = - \int_0^t \frac{1}{\sqrt{\pi\alpha(t-\tau)}} \frac{d\phi(\tau)}{d\tau} d\tau - \frac{\phi(0)}{\sqrt{\pi\alpha\tau}}$$

* Due to the lack of transient information, the condensation coefficient in these calculations corresponds to typical values obtained experimentally for steady film condensation of sodium vapor.¹⁹

$$\begin{aligned}
& + \int_0^\infty \int_0^t \frac{\xi_1}{2k\sqrt{\pi\alpha(t-\tau)^3}} \exp\left[\frac{-\xi_1^2}{4\alpha(t-\tau)}\right] Q(\xi_1, \tau) d\tau d\xi_1 \\
& + \int_0^\infty \frac{T(\xi_1, 0)}{2\alpha} \frac{\xi_1}{\sqrt{\pi\alpha t^3}} \exp\left[\frac{-\xi_1^2}{4\alpha t}\right] d\xi_1
\end{aligned} \tag{5-42}$$

The right hand terms give the effect of the interface boundary conditions, the volumetric heat generation, and the initial condition respectively. $\phi(t)$ is the equilibrium temperature of the liquid-vapor interface which is determined by the vapor pressure. $\phi(t + \Delta t)$ is guessed for each time increment and couples the energy and momentum equations. Knowing the temperature gradients at the interfaces, Eq. 5-38 is solved for the new length of the vapor slug. This is compared with the displacement calculated from the momentum Eq. 5-37. If these do not agree within a specified error tolerance, a new value of $\phi(t + \Delta t)$ is assumed, and the iteration continued until the error criterion is satisfied. The calculation then proceeds to the next time step.

The variation in wall film thickness due to evaporation and condensation is calculated by assuming that the energy transfer at the clad surface equals the latent heat of vaporization plus the sensible heat storage within the film layer. The effects of the shear force generated by the gas flow over the film layer and surface-tension gradients due to temperature fluctuation at the interface were neglected. Thus, the change in film thickness is calculated from the following equation:

$$Q' = \rho_l c_p \sigma S \frac{dT_{fm}}{dt} + \rho_l \lambda S \frac{d\sigma}{dt} \tag{5-43}$$

where Q' is defined by Eq. 5-39.

5.4 Representative Results with Sodium Voiding Codes

A typical voiding pattern obtained with the two-phase annular flow model of SAS1A has already been illustrated in Fig. 2-4. In this section we will illustrate some parametric calculations based upon the slug model.

5.4.1 Effect of Wall Film

Figure 5-3 illustrates the rate of voiding with and without the presence of liquid film on the wall. It is clear that the vapor contribution from the residual film dominates that from the slug interfaces. This effect holds true throughout the expulsion, except during the initial stages of vapor growth. Figure 5-4 shows that when a dry wall is considered the initial superheat pressure quickly drops to the system pressure of 3 atm. With a wet wall there is an initial pressure buildup while film vaporization is occurring in the core region due to the heat generation in the fuel, followed by a depressurization due to partial vapor condensation in the blanket region.

5.4.2 Effect of Power Density and Superheat

The effects of both power density and superheat, when wall film is considered, are shown in Figs. 5-5 and 5-6. For a low power density superheat is the overriding factor which governs the expulsion rate, while for the higher power density the effect of superheat is not as important, although still significant, since the total voiding time for 20°C is about double that for 150°C. In the case of 150°C superheat, the magnitude of the power density is not very significant, because of the large amount of energy storage in both the coolant and fuel pin.

In the case of both low superheat and power density, reentry occurred in the blanket region for several reasons. The rates of condensation and vaporization are governed by the temperature gradient between the clad surface and vapor. Although the heat transfer coefficient used for

Fig. 5-3. Voided Channel Height versus Expulsion Time, Illustrating the Effect of the Presence and Absence of Liquid Wall Film

Fig. 5-4. Vapor Pressure within the Bubble versus Expulsion Time, Illustrating the Effect of Wall Film on the Pressure History

Fig. 5-5. Voided Channel Height versus Expulsion Time, Showing the Effect of Both Superheat and Power Density

Fig. 5-6. Vapor Pressure versus Expulsion Time, Illustrating the Effects of Both Superheat and Power Density of the Pressure History within the Vapor Bubble

vaporization is about four times that for condensation, the positive gradient in the core region is much smaller in magnitude than the negative gradient in the blanket region. The fact that the cladding temperature in the blanket region is much lower than the saturation temperature of the vapor, results from the specified condition of stagnant coolant during the period from the initiation of the transient up to inception of boiling. Shortly after the interfaces enter the blanket region, the rate of condensation on the cold walls causes the vapor pressure to fall below the system pressure (Fig. 5-6), resulting in reentry of the upper and lower slugs. As the power density is increased, film vaporization in the core region overrides the rate of condensation in the blanket, resulting in complete expulsion. Expected pressure generation due to complete vapor collapse was not considered in these calculations, as well as vapor condensation in the upper and lower plenum after complete expulsion has occurred.

5.4.3 Effect of Fuel Pin Capacity

In order to demonstrate the fuel pin heat capacity effect on the expulsion rate, the heat transfer dynamics of the fuel pin are decoupled from the expulsion model. In both cases, the axial power profile is the same. However, in the decoupled case the wall heat flux is time independent and taken to be

$$q_j = \frac{\dot{Q}_{jDf}^2}{4D_{cl}}$$

The results are summarized in Figs. 5-7 and 5-8. As would be expected, the expulsion rate is faster when the heat capacity of the fuel is neglected; in this case the voiding rate is about twice as fast. As shown in Fig. 5-8, the pressure buildup with a time-independent wall surface heat flux is

Fig. 5-7. Voided Channel Height versus Expulsion Time, Illustrating the Effect of the Heat Capacity of the Fuel Pin on the Rate of Expulsion

Fig. 5-8. Vapor Pressure versus Expulsion Time, Illustrating the Effect of the Heat Capacity of the Fuel Pin on the Pressure History within the Vapor Bubble

extremely rapid, since heat is continuously being transferred into the vapor space, resulting in film vaporization and a rapid pressure rise. In this case, condensation can only occur at the upper and lower slug interfaces.

When the heat transfer within the fuel pin is coupled to the expulsion model the pressure cannot rise as rapidly, since a large fraction of the heat generation goes into energy storage of the clad and fuel material.

5.4.4 Discussion of Voiding Calculations

Some interesting physical features of the sodium expulsion process are indicated by these calculations. The growth of the vapor slug is dominated by heat conduction from the liquid slugs in the early growth period ($Z/D_e \ll 1$) and by heat conduction from the exposed walls, in the late growth period ($Z/D_e \gg 1$), provided, in the later case, that the exposed wall is covered by an adhering film of residual liquid. The nature of the expulsion and reentry will therefore depend strongly on whether or not liquid film is present. Chugging will most likely occur for an accident condition of a sudden loss of flow if there exists a long blanket or plenum region, since the cladding temperature in these regions will then be below the saturation temperature. Reentry may occur if the heated portion of the coolant channel is short and/or the heat flux is relatively low. For typical LMFBR geometries and heat fluxes reentry may take place if film vaporization in the core region should cease due to film breakup or dryout. However, upon contact between sodium coolant and uncovered hot clad, sufficient vaporization is likely to occur to prevent any deep penetration of the coolant into the active fuel zone.

Superheat is also an important parameter, which affects both the time to initiate boiling and the rate of voiding. The superheat effect is most pronounced for low heat fluxes and at the beginning of the voiding process.

For large superheats the power density appears to be of little significance, while for lower superheats the power density effects the rate of voiding.

Essentially the same model as that presented here has been used in the study of the isothermal depressurization of Freon 113 in a 0.72 cm dia glass tube.²¹ For this case, it was thought desirable to take into account the variation in the thermal boundary layer of the wall film, which depends upon the age of the liquid surface as a function of axial distance.

Figure 5-9, taken from this work, shows the experimental interface position as a function of time for a superheat of 109°F, compared to the theoretical solution computed in the manner described above. It will be noted that the agreement is quite good over the entire expulsion history. Similar agreement was obtained with several different superheats.

It is rather interesting that this model agrees well with experimental data, even during the early stages of ejection, despite the assumption of a disc of vapor filling the tube cross section. This early growth period accounts for an appreciable portion of the total ejection time. An alternate approach, which has been proposed recently by Schlechtendahl⁷ employs a spherical bubble geometry in the early growth stages. In fact, the bubble initiating at the tube wall quickly assumes an intermediate shape between a disc and a sphere.

The ability of the model to predict the effective delay time before rapid expulsion commences is encouraging, and implies that the assumption of a slug geometry, even during the early stages of vapor growth, approximates the surface-volume relationship of the vapor region satisfactorily, at all times.

Fig. 5-9. Voided Channel Height versus Time for Slug Expulsion of Freon-113 at Constant Plenum Pressure

5.5 Recent Developments in Voiding Codes

5.5.1 Modifications of the Slug Code

Voiding models discussed so far have been based upon the slug and annular flow models, with the former flow regime (BLOW and SLUG) being the most popular. While these slug models appear to describe the growth mechanics of individual bubbles in the early voiding stage, the amplitude and frequency of chugging as well as dryout are not correctly predicted. These discrepancies occur primarily as a result of not accounting for the induced motion in the liquid film adhering to the heated surface. As will be shown later, this effect may result in thinning of the liquid film with subsequent early dryout and little or no evaporation cooling from the film for certain flow conditions. This may explain the local dryout spots produced as early as 20 msec after boiling initiation in the complete flow blockage tests reported in Ref. 22. Furthermore, the single-bubble slug ejection coolant boiling model of Cronenberg and Fauske (also similar to the model of Schlechtendahl) has been coupled with the fuel-pin heat transfer module in the SAS fast reactor accident analysis code.²³ Since boiling was assumed to take place rather than flashing,* the single-bubble model was modified by allowing a new vapor bubble to form in the liquid coolant at any point where the liquid temperature exceeds the local time-dependent saturation temperature by more than the original superheat criterion. Up to ten bubbles are treated simultaneously.

A more serious problem with the original model as pointed out in Ref. 24 is the assumption that the pressure within a vapor bubble is uniform. Simple calculations of vapor production rates and the resulting vapor

* In the case of a pump coastdown accident, flashing may occur rather than boiling in view of the flow and pressure conditions existing at the time of vapor formation. If this is the case, the single-bubble concept as originally proposed may be adequate.

velocities indicate that if a vapor bubble voids an appreciable fraction of the core, then large axial pressure gradients must exist within the bubble.

Figure 5-10 shows some results²³ obtained with the modified model for a flow coastdown case with 30°C of superheat and with no pressure gradients in the vapor.

More recently the slug-ejection coolant-voiding model used in the SAS code has been modified to account for axial variations in the pressure in a vapor bubble, and results have been obtained using this new capability.²⁵ The model now accounts for pressure gradients due to friction and due to vapor acceleration caused by vaporization and condensation, but neglects the induced motion in the liquid film adhering to the fuel pin.

Some results obtained for a flow-coastdown accident with failure to scram in the FFTF reactor are shown in Fig. 5-11; two cases are shown. The cases are identical, except that a superheat of 30°C was assumed for one case, and 70°C for the other.

In the 30°C-superheat case, the results for the initial stages of voiding were similar to the results obtained with no vapor-pressure gradients (see Fig. 5-10). A number of vapor bubbles were formed near the top of the core, and then were swept upward into the cooler regions above the core, where they collapsed.

After the initial stages of voiding, the vapor-pressure gradients caused significant changes in the voiding history. The pressure at the lower liquid-vapor interface in the core region was higher than the pressure at the upper interface in the cooler fission-gas plenum, so the lower interface moved downward through the core more rapidly and voided more of the core, whereas the upper interface moved upward more slowly. After the core region was voided, the vapor-pressure gradient was sufficient to retard reentry of

Fig. 5-10. Sodium-voiding Dynamics during Initial Phase.
Coastdown conditions, no fission product gas.

Fig. 5-11. Sodium-void Expulsion as Calculated by SAS2A Code for 30° and 70°C Superheats and for Coastdown Accident Conditions

the lower liquid slug until film dryout occurred in much of the core and decreased the vapor source.

These results indicate that vapor-pressure gradients have a significant effect on the consequences of a flow-coastdown accident. The vapor-pressure gradients result in a more rapid insertion of voiding reactivity, and they inhibit reentry into the core region and rewetting of the cladding.

In the above treatment, it is assumed that the overall properties of multiple-bubble slug flow can be predicted if one can describe the dynamics of a typical individual bubble. This is done by considering a unit cell consisting of one bubble and part of the liquid slugs of each side of it as shown in Fig. 5-12. In the case of no pressure gradient, the treatment of the individual bubble is identical to that outlined for the SLUG model. In the current version of SAS2A, the initial liquid film thickness is treated only as a parameter, and the induced motion in the liquid film as a result of vapor velocity gradient is neglected, i.e., two-phase flow (or annular flow) is not accounted for. Recent developments in these areas are discussed below.

5.5.2 Liquid Film Thickness

The film of liquid left on the channel wall contributes importantly to the driving force for bubble expansion. The thickness of this film will also determine the initial void fraction required for the annular flow analysis as well as to a great extent determine the dryout time of this film. Recent data²⁶ of liquid-film thickness measurements in steady and transient slug flow are shown in Fig. 5-13 and 5-14, respectively. The steady-state data (Fig. 5-13) obtained in round tubes at velocities up to 10 m/sec suggest that the liquid film left behind is characterized by a liquid fraction of 0.15, or that the ratio of the gas bubble velocity to average liquid velocity is a constant. This value is approximately the ratio of the maximum

Fig. 5-12. Illustration of Slug Flow Pattern and Model Approximation

Fig. 5-13. Comparison of Bubble Velocity with Average Liquid Velocity for Single-bubble Slug Ejection; Zero Acceleration

Fig. 5-14. Liquid Fraction Data in an Air-water System for Transient Slug-voiding in an Annulus with and without Wire Wrap; Constant Acceleration

to average velocity for liquid in turbulent flow in round tubes and would imply that the slug bubble is traveling at a velocity characteristic of the centerline velocity in the liquid ahead as suggested in Ref. 27:

$$1 - \alpha = 1 - \left(\frac{u_{ave}}{u_{max}} \right)_{\text{turbulent flow}} \quad (5-44)$$

The transient data obtained for voiding in a 19-pin test section, a 7-pin section with and without wire wrap, and an annulus also with and without wire wrap, however, show that the liquid fraction or film thickness varies both with velocity, u , and acceleration a as follows:

$$1 - \alpha \sim C \sqrt{u/a} \quad (5-45)$$

For these data, taken with air and water in transparent glass sections, the acceleration of the liquid remained constant while the velocity continuously increased for each event. The data shown in Fig. 5-14 for the annulus also illustrate the effect of the wire wrap in increasing the amount of liquid left behind in the test section. Data for the 7- and 19-pin sections appear quite similar to the data shown in Fig. 5-14, with the exception that the difference between wrapped and unwrapped sections is not as large. The above observations suggest that larger size bundles would not show marked differences in liquid fraction from those illustrated in Fig. 5-14.

For voiding conditions in which acceleration is essentially zero, the film thickness or liquid fraction criteria given by Eq. 5-44 should apply. For voiding under conditions for which acceleration is important, it appears that calculations might well be started with an initial liquid fraction of

0.15 to 0.2 with liquid fraction subsequently varying with $\sqrt{u/a}$ as shown in Fig. 5-14. The latter dependence is in agreement with recent theoretical developments.²⁸

5.5.3 Annular Flow Regime

The initial value of the void fraction, α , required to start the two-phase annular flow calculations, is determined by the thickness of the liquid film left behind during the slug flow regime, as discussed earlier. The subsequent void propagation and variation in the liquid-film thickness as a result of vapor shear may be obtained by solving simultaneously the well known conservation equations written for equilibrium two-phase separated flow (continuity, momentum and energy of both phases, and momentum of vapor phase)²⁹ together with the constitutive equations describing the increased wall- and interface-shear stresses at the onset of annular flow.

5.5.4 Transition from Slug to Annular Flow

The assumption of uniform pressure within the bubble as the vapor velocity becomes large is not valid, requiring consideration of annular flow between the liquid slugs. The development of axial pressure and temperature gradients within the bubble will affect the motion of the liquid slugs, as well as the behavior of the liquid film. The liquid-film thickness variation will not only depend upon evaporation and condensation fluxes, but also upon the drag forces exerted by the vapor on the liquid film. The onset of cocurrent annular two-phase flow between the liquid slugs can be taken to occur when the local vapor flux exceeds the critical flooding velocity,²⁹

$$u_g > \rho_g^{1/2} [gD(\rho_f - \rho_g)]^{1/2} \quad (5-46)$$

The flooding results from a sudden instability and formation of waves in the film several times its own thickness in amplitude. This results in

a large increase in the pressure gradient and induced motion in the liquid film. For typical LMFBR conditions, the critical flooding velocity is in the order of 10 m/sec. From a simple energy balance written for the vapor,

$$u_g \sim \frac{qA_H}{\rho_g h_{fg} A} \quad (5-47)$$

it can be seen that this velocity may be locally exceeded relatively early in the process.

5.5.5 The Occurrence of Dryout

Since existing voiding models are generally based upon the slug flow regime, they do not consider the possibility of induced motion in the liquid film adhering to the fuel pins. Consequently, the time for dryout of this liquid film is simply based upon the rate of radial liquid transport, $W_{l,r}$, from a stationary film as a result of evaporation

$$W_{l,r} = \frac{qA_H}{h_{fg}} \quad (5-48)$$

For typical LMFBR conditions and an initial liquid fraction of 0.15,²⁶ this results in dryout 0.2 to 0.3 sec following local voiding, which is not in agreement with dryout data reported in Ref. 22.

As a result of the axial vapor velocity gradient existing at the time of flooding, the induced motion in the liquid film is nonuniform and, therefore, gives rise to a net axial transport of mass locally in the liquid film, $W_{l,a}$ as illustrated in Fig. 5-15. An approximate estimate of $W_{l,a}$ can be obtained by assuming quasi-steady incompressible flow and neglecting gravity and specifying the appropriate constitutive equations describing

Fig. 5-15. Simplified Dryout Model following Onset of Flooding

the increased wall and interface shear stresses following the onset of flooding:

$$\text{Momentum of both phases: } \frac{dP}{dz} + \frac{4}{D} \tau_w = 0 \quad (5-49)$$

$$\text{Momentum of vapor phase: } \alpha \frac{dP}{dz} + \frac{4\sqrt{\alpha}}{D} \tau_i = 0 \quad (5-50)$$

$$\text{Wall shear stress: } \tau_w = 1/2 C_{f\ell} \rho_\ell u_\ell^2 \quad (5-51)$$

$$\text{Interface shear stress: } \tau_i = 1/2 C_g [1 + 75 (1 - \alpha)] \rho_g u_g^2 \quad (5-52)$$

Solutions of the above equations together with Eq. 5-47 for LMFBF conditions and an initial liquid fraction of 0.15 leads to the following conclusions:³⁰ (1) the rate of axial mass transport $W_{\ell,a}$, can be at least an order of magnitude larger than the radial mass transport, $W_{\ell,r}$, (2) local dryout spots may, therefore, occur as early as 20 to 30 msec after local voiding, in agreement with data reported in Ref. 22, and (3) current voiding models should, therefore, be extended to include two-phase wavy annular flow between the liquid slugs.

The validity of using Eqs. 5-49, 50, 51, and 52 for calculating void fractions in steady annular flow of liquid metal-vapor mixtures have been demonstrated.³¹ Predicted values of liquid volume fraction as well as predicted two-phase frictional pressure drops³² based on these void fractions are compared with experimental data in Figs. 5-16 and 5-17, respectively. The validity of assuming incompressible flow in estimating the void fraction has also been demonstrated analytically by including the compressible terms

Fig. 5-16. Experimental Liquid Volume Fraction Comparison for Potassium and Sodium with Solutions from Eqs. 5-49, 50, 51, and 52

Fig. 5-17. Experimental Two-phase Friction Pressure Drop Comparison for Potassium and Sodium with Solutions from Eqs. 5-49, 50, 51, and 52

in the momentum equations.³³ However, the quasi-steady-state approximation needs to be compared to the transient solution of the complete conservation equations.

5.6 Other Modes of Voiding; Fission Gas Release and Fuel-coolant Interaction

If failure of the clad occurs prior to coolant boiling, coolant voiding may occur as a result of either fission-gas release or molten fuel-coolant interaction. Depending on the type of accident leading to failure of the fuel pin, only fission gas or molten fuel (possibly with fission gas) is ejected from the rupture. The release of fission gas only may result from a transient involving loss of cooling (e.g., loss of electrical power, pump seizure, flow blockage) in which the clad temperature rapidly increases while the fuel temperature is not significantly affected and no molten fuel is formed. The release of molten fuel may result from accidental reactivity insertions or loading and enrichment errors that lead to an overpower condition causing a significant amount of molten fuel to be present before clad failure and coolant boiling. As in the case of coolant boiling, the reactivity feedback due to coolant voiding as a result of either fission gas release or fuel-coolant interaction must be predicted. Some early attempts at modeling the above modes of voiding based upon extensions of the SLUG model developed for coolant boiling will be discussed below.

5.6.1 Voiding due to Fission Gas Release

Analyses of coolant voiding due to fission gas release^{34,35} have generally been made by assuming that the ejected gas forms a single cylindrical bubble, which expands isothermally, thereby expelling the coolant from the subassembly in a similar manner to that of sodium boiling. The validity of the single-bubble model approach to fission gas voiding depends largely on the rate of fission gas release from the fuel pins. From two-phase flow regime studies,³⁶ it would appear necessary that the volumetric flow rate

for the gas is at least greater than that for the liquid. Thus, a tentative criterion for the validity of the slug model to describe voiding due to fission gas release may be expressed as³⁵

$$\sum_{i=1}^k Q_i(0)/\rho_{bo} \geq U_o A_c \quad (5-53)$$

The gas release rate, on the other hand, depends on many variables: the fission gas pressure inside the rod (which depends on the burnup attained by the fuel up to the moment of failure), the size and location of the breach in the cladding, fuel configuration within the rod, number of pin failures, and coherency of failures. However, little or no data are available on the flow resistances, e.g., the nature of void and cracks in the fuel region. In this section, only the equations describing the gas flow rate for cases where the internal resistance to the gas flow is negligible will be discussed; this assumption appears to be valid if the failure is located in the gas plenum of the fuel pin or in its close vicinity.

"Let* the pressure, density, and temperature of the gas in the gas plenum at any time be P_g , ρ_g , T_g , and the corresponding values of these parameters in the gas bubble be P_b , ρ_b , and T_b . The application of conservation laws for a multiple-pin failure gives the following relations: The law of mass conservation for gas plenum

$$\frac{dV\rho_{gi}}{dt} = -Q_i I(t - t_i), \quad (5-54)$$

and bubble

* This material is taken from Ref. 35.

$$\frac{dA_c(\Delta x)\rho_b}{dt} = \sum_{i=1}^N Q_i I(t - t_i) ; \quad (5-55)$$

the law of energy conservation for gas plenum

$$\frac{d\rho_{gi} V C_v T_{gi}}{dt} = - Q_i C_p T_{gi} I(t - t_i) , \quad (5-56)$$

and bubble

$$\frac{d\rho_b A_c(\Delta x) C_v T_b}{dt} + \frac{1}{J} P_b \frac{dA_c(\Delta x)}{dt} = \sum_{i=1}^N Q_i C_p T_{gi} I(t - t_i) + h\Delta TS, \quad (5-57)$$

where in writing down above equations, it is assumed that the gas is ideal. In Eq. 5-56, it is implied that expansion of the gas in the gas plenum is adiabatic. In the above set of equations, subscript i refers to the pin i that ruptures at time $= t_i$; N is the total number of pins ruptured; Q_i is the mass discharge rate from pin i ; A_c is the effective flow area of the assembly; C_v and C_p are the specific heats of gas at constant volume and constant pressure, respectively; h is the film heat transfer coefficient between the bubble and the surroundings; ΔT is the temperature difference between bubble and the surroundings; S is the effective surface area; Δx is the bubble length as shown in Fig. 5-12; $I(t - t_i)$ is the unit step function defined as

$$I(t - t_i) = \begin{cases} 0 & \text{for } t < t_i \\ 1 & \text{for } t \geq t_i \end{cases} , \quad (5-58)$$

where rupture time, t_i has the following specifications:

$$0 \leq t_i < t_r, \quad (5-59)$$

where t_r is the initial transit time of the coolant for a single pass through the assembly. It may be noted here that in the present state of the art, the exact failure mechanism for fuel elements is not yet known and, therefore, it becomes necessary to assume initiating occurrences that may lead to failure propagation. Consequently, depending upon the size of assembly and the initial coolant flow rate, one must assume the minimum number of pins ruptured and their rupture times so that the single-bubble regime is established (see 5-53).

The rate Q_i of gas flow of plenum i into bubble is, if the process of discharge is assumed quasi-stationary, equal to³⁷

$$Q_i = C_{di} A_{oi} \rho_{gi} \left(\frac{P_b}{P_{gi}} \right)^{1/\gamma} \left\{ \frac{2\gamma}{\gamma - 1} \left[1 - \left(\frac{P_b}{P_{gi}} \right)^{\frac{\gamma-1}{\gamma}} \right] RT_{gi} g_c \right\}^{1/2} \quad (5-60)$$

for

$$\frac{P_b}{P_{gi}} \geq \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma-1}},$$

and

$$Q_i = C_{di} A_{oi} \rho_{gi} \left(\frac{2}{\gamma + 1} \right)^{\frac{1}{\gamma-1}} \left(\frac{2\gamma}{\gamma + 1} RT_{gi} g_c \right)^{1/2} \quad (5-61)$$

for

$$\frac{P_b}{P_{gi}} \leq \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma - 1}},$$

where

A_{oi} = area of breach in pin i

C_{di} = coefficient of discharge

R = gas content

$$\gamma = C_p / C_v.$$

In the first case (Eq. 5-60) the flow is subcritical and in the second (Eq. 5-61) it is critical."

For a gas plenum pressure of ~1000 psia and system pressure say 100 psia, the ratio $P_{bo}/P_{go} \approx 0.1$ is well below the critical pressure ratio (≈ 0.48); therefore, for most liquid-metal breeder reactors it is valid to assume that the initial gas discharge through any finite size breach in the fission plenum region is critical. In view of the above assumption, the use of Eq. 5-61 in Eq. 5-53 gives

$$\frac{\rho_{go}}{\rho_{bo}} \left(\frac{2}{\gamma + 1} \right)^{\frac{1}{\gamma - 1}} \left(g_c R T_{go} \frac{2\gamma}{\gamma + 1} \right)^{1/2} \sum_{i=1}^k C_{di} A_{oi} \geq U_o A_c. \quad (5-62)$$

By replacing ρ_{go} by P_{go}/RT_{go} and ρ_{bo} by P_{bo}/RT_{bo} and assuming that temperature of the gas in the plenum is equal to temperature of the coolant outside the plenum, Eq. 5-62 gives the following general criterion for conditions for which the single-bubble model is valid:

$$\sum_{i=1}^k C_{di} A_{oi} \geq \beta \frac{P_{bo}}{P_{go}} \frac{U_o A_c}{(RT_{bo} g_c)^{1/2}}, \quad (5-63)$$

where

$$\frac{1}{\beta} = \left(\frac{2}{\gamma + 1} \right)^{\frac{1}{\gamma-1}} \left(\frac{2\gamma}{\gamma + 1} \right)^{1/2}.$$

If all breached pins have the same breach size, A_o , and the same coefficient of discharge, C_d , Eq. 5-63 simplifies to

$$A_o \geq \frac{\beta}{k C_d} \frac{P_{bo}}{P_{go}} \frac{U_o A_c}{(RT_{bo} g_c)^{1/2}}, \quad (5-64)$$

which, for a circular breach becomes

$$d \geq \left(\frac{4}{\pi} \frac{\beta}{k C_d} \frac{P_{bo}}{P_{go}} \frac{U_o A_c}{(RT_{bo} g_c)^{1/2}} \right)^{1/2}. \quad (5-65)$$

Having obtained the criterion for establishing the applicability of the single-bubble model to fission gas release, the dynamic solution of Eqs. 5-56 and 5-57 and Eqs. 5-33 through 5-37 (see section 5.3.2.3) and Eqs. 5-60 and 5-61 (depending upon whether the flow through the breach is subcritical or critical) can be obtained for various breach sizes and number of pins failed. Typical results³⁸ of such computations for an FFTF driver subassembly are displayed in Fig. 5-18, where the clad breach size is assumed to be so large that its effect on the gas release rate may be

Fig. 5-18. Displacement of Upper and Lower Gas-Liquid Interfaces, following Rapid Gas Release in an FFTF Driver Fuel Subassembly, as a Function of Time, with Number of Fuel Pins Rupturing Simultaneously as Parameter

neglected.* The calculation of the flow transients was terminated when the upper interface reached the subassembly outlet. From Fig. 5-18 it follows that the flow transients initiated by near-simultaneous failure of more than ~90 pins, will under the above assumptions of the model result in gas blanketing of the fuel pins for time periods >0.25 sec, which potentially could lead to failure of some or all of the remaining pins. Furthermore, it may be seen from Fig. 5-18 that, if the number of pins that fail near-simultaneously exceeds ~170, there exists a definite possibility for the gas to reach the subassembly inlet nozzle and to enter into adjacent sub-assemblies. This could have important reactivity implications for the case of a local core accident.

As mentioned previously, the calculations presented here are based on conservative assumptions (in particular as regards neglecting the flow resistance inside the fuel pins due to the presence of reflector and the fuel), and represent thus an upper bound for the assumed clad rupture location. Therefore, considerably more analysis and data pertaining to fuel failure mechanisms, fuel configuration and impedance to gas flow are required.

5.6.2 Voiding due to Molten Fuel-coolant Interaction

Rather limited theoretical work has been done on the modeling of the sodium UO_2 -Na interaction process. Two basically different approaches have been taken; the thermodynamic models^{40,41} which estimate the maximum work available from the interaction and the transient models which estimate the rate processes involved.⁴²⁻⁴⁵ The latter models, however, are parametric in nature, and pressure generation and volume expansion (voiding rate) depend generally on variables like: fuel ejection rate, mixing time,

* In performing these calculations, it should be noted that the three dimensional flow effects³⁹ in the outlet and inlet plenums have been taken into account.

fragmentation process and particle size, the volumes of liquid coolant, condensable and noncondensable gases present in the mixing zone, the size of the mixing zone and the rate and mechanism of heat transfer between fuel and coolant. Insufficient data are available to describe these variables with any degree of accuracy.

The fact that there have been a number of parametric models of the fuel-coolant interaction problem is due, in part, to the various accident situations which must be considered. For example, the basic mechanism of interaction for fuel pin failure may be quite different than the more severe partial- (or whole-) core meltdown problem. Indeed, the latter case may lead to what could be termed a "vapor explosion" initiated by an entrapment of coolant by molten fuel.

In view of the framework of these notes, only the problem of limited quantities of molten fuel in coolant will be discussed, such as would occur in the case of fuel failure prior to coolant boiling. This situation may arise as a consequence of localized power transients, where it has been demonstrated by MacFarlane et al.² that reactivity increases of 50 cents/sec may lead to fuel melting and element failure before coolant boiling occurs. As in the case of fission gas release, the current understanding of voiding due to fuel-coolant interaction can be demonstrated by a simple extension of the SLUG model developed for sodium boiling by including fuel particles* within the vapor bubble.

* While the rate of molten fuel ejection from failed fuel pin(s) will not be discussed, it is considered to be an important variable in the evaluation of the voiding history. Some limited analytical work on this subject is presented in Ref. 46. In the above illustration, the amount of fuel interaction with the coolant in a given time, will be considered a parameter.

5.6.2.1 Extension of the SLUG Boiling Model⁴⁴

Consider the mixing volume, shown in Fig. 5-19 as the active region for fuel-coolant interaction. This mixing zone expands against an upper and lower liquid plug, in a manner similar to that outlined in section 5.3.2. The rate of heat addition to or from the sodium vapor and liquid (wall film and liquid associated with fuel particles), within the interaction zone, is defined by the heat transport across the fuel particles, the upper and lower slug interfaces, and along the surface on which liquid wall film adheres. Thus, the macroscopic energy balance is written as⁴⁴

$$\begin{aligned}
 & k_{\text{Na}} A \left. \frac{dT(\xi_1, t)}{d\xi_1} \right|_{\xi_1=0} + k_{\text{Na}} A \left. \frac{dT(\xi_2, t)}{d\xi_2} \right|_{\xi_2=0} + n_s k_f S \left. \frac{dT_s(r, t)}{dr} \right|_{r=a} \\
 & + h_{\text{fm}} C \int_{x=\xi_2}^{x=\xi_1} \left(T_{\text{li}}(x, t) - \phi(t) \right) dx A \Delta x \frac{dP(t)}{dt} = \frac{\partial (\bar{h}_v m_v)}{\partial t} + \frac{\partial (\bar{h}_l m_l)}{\partial t}
 \end{aligned} \tag{5-66}$$

This equation reduces to that developed for the SLUG boiling model (Eq. 5-38), by neglecting the heat transport from the fuel particles and the pressure work term, in addition to making the simplifying assumption that the liquid-vapor mass remains constant within a time increment and that the sensible heat of the vapor is negligible compared to the latent heat.

To obtain a solution for the gradient at the surface of the fuel particles, the assumption is made that the liquid and vapor are in equilibrium and that any surface resistance is neglected.* Thus, $T_s(r = a, t)$ are equal

* The process of intermittent contact between fuel particles and coolant in the mixing zone may give rise to a sufficiently high time averaged heat transfer coefficient such that the heat transfer is controlled by the particle side.

Fig. 5-19. Model for Fuel-Coolant Interaction.

to $\phi(t)$. Therefore, applying the condition of symmetry and a constant initial temperature distribution equal to the fuel melting point (2800°C), the solution to the heat equation, assuming concentric isothermal surfaces within a sphere whose properties are constant, results in a surface gradient of the form⁴⁴

$$\left. \frac{dT_s(r,t)}{dr} \right|_{r=a} = \frac{2\pi}{a^3} \sum_{n=1}^{\infty} n(-1)^n \exp \left[-\alpha_f n^2 \pi^2 t / a^2 \right] \left\{ -T_o \frac{a^2}{n\pi} (-1)^n - \alpha_f \pi n (-1)^n \int_0^t \phi(\tau) e^{\alpha_f n^2 \pi^2 \tau / a^2} d\tau \right\} \quad (5-67)$$

Heat transfer cutoff from the fuel particles is assumed to occur when the liquid mass associated with the fuel particles ($m_{\ell,s}$) has been completely vaporized. The decrease in this liquid is calculated as

$$\frac{dm_{\ell,s}(t)}{dt} = \frac{1}{\lambda} \left[k_f S n_s \left. \frac{dT_s(r,t)}{dr} \right|_{r=a} - m_{\ell,s}(t) c_p \frac{d\phi(t)}{dt} \right] \quad (5-68)$$

where the second term on the right hand side of Eq. 5-68 refers to the sensible heat storage in the liquid associated with the fuel. A similar equation is written for the liquid wall film. Equation 5-66 can be solved for the change in vapor mass once the rates of heat transfer from each source and the change in liquid mass have been determined. The volumetric rate of expansion of the interaction zone is then calculated as

$$A \frac{d(\Delta x)}{dt} = \frac{1}{\rho_{\ell}(t)} \frac{dm_{\ell}}{dt} + \frac{1}{\rho_v(t)} \frac{dm_v}{dt} \quad (5-69)$$

By specifying the quantities in the mixing zone (i.e., volumes of fuel, coolant and vapor as well as fuel particle size), a solution for the pressure in bubble and corresponding voiding rate, is obtained in a similar manner as discussed in section 5.3.

In the case the vapor volume in the mixing zone is zero, the equation of state for the liquid must be used to determine the expansion rate.

$$\frac{dV}{dt} = -\beta_T V \frac{dP}{dt} + \alpha_P V \frac{dT}{dt} \quad (5-70)$$

In the above equation, β_T is the isothermal compressibility, α_P is the thermal expansion coefficient, and dT/dt is the temperature rise of the sodium mixing region and determined from Eq. 5-67.

5.6.2.2 Parametric Calculations; Effect of Initial Vapor in the Mixing Zone and Condensation

Some of the parametric calculations performed by Cronenberg et al.⁴⁴ and which they compared with data from a single-pin failure experiment⁴⁷ carried out in the TREAT facility, are summarized below. The initial conditions for each calculation are listed in Table 5-1. The volume fractions represent the percentage of the mixing volume (taken to be 2.47 cm³, based upon fuel-failure data from the single-pin experiment) taken up by each component, while the bracketed percentage corresponds to that fraction of molten fuel in the failed section of the pin which is assumed to interact with the sodium.

If the fragmentation process of fuel into small particles is due to transition boiling,⁴⁴ then one should expect a certain amount of vapor to be present in the mixing zone prior to rapid expulsion. As shown in Fig. 5-20 (examples A and B) the effect of initial vapor presence on the voiding and

TABLE 5-1

Initial Conditions for Calculational Simulation of H-2 Experiment

Mixing zone height (L)	10.0 cm
Mixing zone volume (V)	2.47 cm ³

Problem Label	Initial Conditions	
A	$d_s = 500\mu$	$V_f = 10\% (33\%)$
	$n_s = 3776$	$V_l = 90\%$
	$h_c = 0.00$	$V_v = 0\%$
B	$d_s = 500\mu$	$V_f = 10\% (33\%)$
	$n_s = 3776$	$V_l = 40\%$
	$h_c = 0.00$	$V_v = 50\%$
C	$d_s = 500\mu$	$V_f = 10\% (33\%)$
	$n_s = 3776$	$V_l = 40\%$
	$h_c = 1.5$	$V_v = 50\%$

pressurization rates is appreciable, especially during the early stages of expansion. Not only is there a pressure reduction of approximately an order of magnitude, but also a considerable peaking delay. For purely liquid-phase expansion two competing effects occur. These are thermally induced liquid expansion counteracted by the inertia* of the constraining plugs. During the early period, before vaporization is initiated, all the heat released by the fuel goes into sensible heat addition to the liquid which results in significant thermal expansion, thus pressurization. As the transient heating rate decreases with time and the momentum of the plugs increases, the pressure relaxes until it falls below that of saturation, thus initiating vaporization (~ 0.25 ms later). If, however, vapor is initially present in the interaction zone, the heating and pressurization processes are completely altered. Unlike the previous case, a significant portion of the heat released by the fuel is given up as latent heat of vaporization, thus the thermal expansion of the liquid component is reduced. To a large extent, both bulk condensation and vapor cushioning reduce any pressurization.

The effect of condensation is also illustrated in Fig. 5-20 (examples B and C). The difference between an adiabatic ($h_c = 0$) and diabatic ($h_c = 1.5$ cal/sec-cm²°C) film layer along the walls of the test vessel and intact portions of the failed pin is quite significant. As illustrated, the inclusion of condensation in the analysis results in a significant reduction in the pressure in the mixing zone, thus the expulsion rate. While fair agreement is noted between calculations and experiment with initial conditions corresponding to example C, it should be noted that other combinations of values required for the unknown parameters in the model may possibly give similar agreement with the experimental voiding curve. However, continued

* In the early phase of the pressurization, acoustic effects should be accounted for.⁴³ However, these can be shown to have little influence on the rate of voiding.

Fig. 5-20. Effect of Initial Vapor Presence and Film Condensation on Voiding and Pressure Histories

interpretation of fuel-failure experiments through parametric models of the type illustrated above, should lead to better understanding and characterization of the required input parameters.

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NOMENCLATURE

a	Outer radius of fuel or acceleration
A	Cross-sectional area of coolant channel
A_H	Heated area
A_o	Area of breach
A_{oi}	Area of breach in pin i
b	Inner radius of clad
c	Outer radius of clad or friction factor
C	Clad outer circumference
C_d	Coefficient of discharge
c_p	Specific heat of gas at constant pressure; also, specific heat of liquid
C_v	Specific heat of gas at constant volume
C_{di}	Coefficient of discharge for breach in pin i
D_e	Equivalent channel diameter
D_p	Equivalent plenum diameter
f	Blasius coefficient of friction
g	Acceleration due to gravity
h	Film heat transfer coefficient between bubble and surroundings; also, heat transfer coefficient or enthalpy
h_{fg}	Latent heat of vaporization
$I(t-t_i)$	Unit step function $\begin{matrix} 0 & \text{for } t < t_i \\ 1 & \text{for } t \geq t_i \end{matrix}$
J	Mechanical equivalent of heat
k	Number of pins that fail at $t = 0$; also, thermal conductivity
K_e	Resistance coefficient for expansion
L_1	Height of fuel pin
L_2	Height of upper plenum
n_s	Number of fuel particles
N	Number of pins ruptured

P	Pressure
P_b	Pressure in gas bubble at any time, t
P_{bo}	Pressure in gas bubble at $t = 0$
P_{gi}	Pressure in gas plenum i at any time, t
P_{go}	Pressure in gas plenum at $t = 0$
$\dot{q}$	Rate of heat addition to coolant
$\dot{Q}$	Volumetric heat generation rate
Q_i	Mass rate of discharge of gas from plenum i
Q'	Heat transfer across wall film
r	Radius
R	Gas constant
R_e	Reynolds number
S	Heat transfer surface area between bubble and surroundings; also, surface area of wall film or particle
t	Time
t_i	Rupture time of a pin i
τ_r	Initial transit time of coolant for a single pass through fuel assembly
T	Temperature
T_b	Temperature of gas in gas bubble
T_{gi}	Temperature of gas in gas plenum at any time, t
T_{go}	Temperature of gas in gas plenum at $t = 0$
u	Velocity
U_o	Initial coolant velocity in fuel subassembly
V	Volume; also, volume of gas plenum
V_b	Volume of gas bubble at any time, t
V_{bo}	Volume of gas bubble at $t = 0$
w	Mass flow rate
x	Distance from channel inlet; also, axial coordinate

$\dot{x}$	Velocity
$\ddot{x}$	Acceleration
Z	Length of wall film

Greek Letters

α	Thermal diffusivity of liquid or void fraction
λ	Heat of vaporization
γ	Latent heat of vaporization; also, ratio of specific heat at constant pressure to specific heat at constant volume
ρ	Density; also, density of coolant
ρ_b	Density of gas in bubble at any time, t
ρ_{b0}	Density of gas in bubble at $t = 0$
ρ_{gi}	Density of gas in gas plenum i at any time, t
ρ_{go}	Density of gas in gas plenum at $t = 0$
ξ_1, ξ_2	Lagrangian distance coordinates with origins in slug interfaces
$\phi(t)$	Slug liquid-vapor interface temperature
σ	Film thickness
ΔT	Temperature difference between bubble and surroundings

Subscripts

bd	Bond
cl	Clad
f	Fuel
fm	Film
i	Radial node in fuel
j	Axial node
l	Liquid coolant
m	Coolant
p	Plenum

APPENDIX 5-1

Derivation of Equations of Conservation of Mass, Momentum, and EnergyThe Basic Equations of Conservation of Mass, Momentum, and Energy

First note $\frac{\partial}{\partial t}$ is the partial derivative with respect to time, holding x, y, z constant. It is the change with time at a fixed position in space. $\frac{d}{dt}$ is total derivative

$$= \frac{\partial}{\partial t} + \frac{\partial}{\partial x} \frac{dx}{dt} + \frac{\partial}{\partial y} \frac{dy}{dt} + \frac{\partial}{\partial z} \frac{dz}{dt} \quad (5-1-1)$$

If we are moving with fluid at velocity of fluid

$$\frac{d}{dt} = \frac{\partial}{\partial t} + v_x \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial y} + v_z \frac{\partial}{\partial z} \quad (5-1-2)$$

Equation of Continuity

Consider a region of volume $\Delta x \Delta y \Delta z$ fixed in a space, through which fluid is flowing

$$\text{Rate of mass accumulation} = \left\{ \begin{array}{l} \text{rate of} \\ \text{mass in} \end{array} \right\} - \left\{ \begin{array}{l} \text{rate of} \\ \text{mass out} \end{array} \right\} \quad (5-1-3)$$

$$\Delta x \Delta y \Delta z \frac{\partial \rho}{\partial t} = \Delta y \Delta z \left[(\rho v_x)_x - (\rho v_x)_{x+\Delta x} \right] + \text{terms for other faces} \quad (5-1-4)$$

Divide by $\Delta x \Delta y \Delta z$ and taking limit, we get

$$\frac{\partial \rho}{\partial t} = - \frac{\partial (\rho v_x)}{\partial x} - \frac{\partial (\rho v_y)}{\partial y} - \frac{\partial (\rho v_z)}{\partial z} \quad (5-1-5)$$

or

$$\frac{\partial \rho}{\partial t} = - (\nabla \cdot \rho \vec{v}) \quad (\text{Eq. of continuity or conservation of mass}) \quad (5-1-6)$$

If we perform differentiation on RHS

$$\frac{\partial \rho}{\partial t} = \left[\rho \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) \right] - \left[v_x \frac{\partial \rho}{\partial x} + v_y \frac{\partial \rho}{\partial x} + v_y \frac{\partial \rho}{\partial y} + v_x \frac{\partial \rho}{\partial z} \right] \quad (5-1-7)$$

Transpose second term on RHS to LHS, giving

$$\frac{d\rho}{dt} = - \rho (\nabla \cdot \vec{v}) \quad (\text{This is another way of expression the same law}) \quad (5-1-8)$$

Equation of Motion - or Conservation of Momentum

$$\left\{ \begin{array}{l} \text{rate of momentum} \\ \text{accumulation} \end{array} \right\} = \left\{ \begin{array}{l} \text{rate of} \\ \text{momentum in} \end{array} \right\} - \left\{ \begin{array}{l} \text{rate of} \\ \text{momentum out} \end{array} \right\} + \left\{ \begin{array}{l} \text{sum of} \\ \text{forces} \\ \text{acting} \\ \text{on system} \end{array} \right\} \quad (5-1-9)$$

Consider only momentum in x direction, temporarily

Rate of Momentum in =
through x face

$$\rho v_x v_x|_x \Delta y \Delta z$$

Rate of Momentum out =
through x + Δx face

$$\rho v_x v_x|_{x+\Delta x} \Delta y \Delta z$$

Rate of Momentum in x =
direction in through
y face

$$(v_z \Delta x \Delta z) \rho v_x|_y$$

Rate of Momentum in x =
direction out through
y + Δy

$$(v_y \Delta x \Delta z) \rho v_x|_{y+\Delta y}$$

Similar expressions occur for z face

So, expression for net convective flow in of momentum in x direction is

$$\Delta y \Delta z (\rho v_x v_x|_x - \rho v_x v_x|_{x+\Delta x}) + \Delta x \Delta z (\rho v_y v_x|_y - \rho v_y v_x|_{y+\Delta y})$$

$$+ \Delta x \Delta y (\rho v_z v_x|_z - \rho v_z v_x|_{z+\Delta z})$$

Similarly there can be viscous forces arising at all six surfaces due to molecular transport phenomena in a fluid

If τ_{xx} is the normal stress on the x surface ($\Delta y \Delta z$), τ_{yx} is the shear stress on the y surface ($\Delta x \Delta z$) in the x direction, etc. we obtain

$$\begin{aligned} & \Delta y \Delta z \left(\tau_{xx} \Big|_x - \tau_{xx} \Big|_{x+\Delta x} \right) + \Delta x \Delta z \left(\tau_{yx} \Big|_y - \tau_{yx} \Big|_{y+\Delta y} \right) \\ & + \Delta x \Delta y \left(\tau_{zx} \Big|_z - \tau_{zx} \Big|_{z+\Delta z} \right) \end{aligned}$$

The contribution from hydrostatic pressure and gravity to the x momentum will be

$$\Delta y \Delta z \left(P \Big|_x - P \Big|_{x+\Delta x} \right) + \Delta x \Delta y \Delta z$$

Finally, the rate of accumulation of x momentum is

$$\Delta x \Delta y \Delta z \frac{\partial (\rho v_x)}{\partial t}$$

Substitute these in the equation, divide by $\Delta x \Delta y \Delta z$ and go to limit, getting

$$\frac{\partial (\rho v_x)}{\partial t} = - \left(\frac{\partial}{\partial x} \rho v_x v_x + \frac{\partial}{\partial y} \rho v_y v_x + \frac{\partial}{\partial z} \rho v_z v_x \right) \quad (5-1-10)$$

$$-\left(\frac{\partial}{\partial x} \tau_{xx} + \frac{\partial}{\partial y} \tau_{yx} + \frac{\partial}{\partial z} \tau_{zx}\right) - \frac{\partial p}{\partial x} + \rho g_x \quad (5-1-10)$$

Or in three dimensions

$$\frac{\partial \rho \vec{v}}{\partial t} = -[\nabla \cdot \rho \vec{v} \vec{v}] - \nabla p - [\nabla \cdot \tau] + \rho \vec{g} \quad (5-1-11)$$

Similarly as with equation of continuity, one can obtain

$$\rho \frac{d\vec{v}}{dt} = -\nabla p - [\nabla \cdot \tau] + \rho \vec{g} \quad (5-1-12)$$

in terms of total derivative.

Energy Conservation

$$\left\{ \begin{array}{l} \text{Rate of accumulation} \\ \text{of internal and} \\ \text{kinetic energy} \end{array} \right\} = \left\{ \begin{array}{l} \text{rate of internal and} \\ \text{kinetic energy in by} \\ \text{convection} \end{array} \right\} - \left\{ \begin{array}{l} \text{rate of internal} \\ \text{and kinetic energy} \\ \text{out by convection} \end{array} \right\} \quad (5-1-13)$$

$$+ \left\{ \begin{array}{l} \text{net rate of} \\ \text{heat addition} \\ \text{by conduction} \end{array} \right\} - \left\{ \begin{array}{l} \text{net rate of work} \\ \text{done by system on} \\ \text{surroundings} \end{array} \right\}$$

Internal energy means energy associated with random translational and internal motions of molecules

$$\text{Accumulation} = \Delta x \Delta y \Delta z \frac{\partial}{\partial t} (\rho u + \frac{1}{2} \rho v^2)$$

Net rate of convection in =

$$\Delta y \Delta z \left\{ v_x \left(\rho u + \frac{1}{2} \rho v^2 \right) \Big|_x - v_x \left(\rho u + \frac{1}{2} \rho v^2 \right) \Big|_{x+\Delta x} \right\} + \text{terms in } y \text{ and } z$$

Net rate of energy input by conduction is

$$\Delta y \Delta z \left\{ q_x \Big|_x - q_x \Big|_{x+\Delta x} \right\} + \text{terms in } y \text{ and } z$$

Rate of work done on surrounding

a) gravity (force $\times$ velocity in direction of force)

$$- \rho \Delta x \Delta y \Delta z (v_x g_x + v_y g_y + v_z g_z)$$

b) static pressure

$$\Delta y \Delta z \left\{ (\rho v_x) \Big|_{x+\Delta x} - (\rho v_x) \Big|_x \right\} + \text{-----}$$

c) viscous forces

$$\Delta y \Delta z \left(\tau_{xx} v_x + \tau_{xy} v_y + \tau_{xz} v_z \right) \Big|_{x+\Delta x} - (\quad) \Big|_x$$

$$+ \Delta x \Delta z \left(\tau_{yx} v_x + \tau_{yy} v_y + \tau_{yz} v_z \right) \Big|_{y+\Delta y} - (\quad) \Big|_y$$

etc.

Divide by $\Delta x \Delta y \Delta z$ and going to limit gives

$$\frac{\partial}{\partial t} \rho \left(U + \frac{1}{2} v^2 \right) = - \underbrace{\left(\nabla \cdot \rho \vec{v} \left(U + \frac{1}{2} v^2 \right) \right)}_{\text{convection}} - \underbrace{(\nabla \cdot \vec{q})}_{\text{conduction}} \quad (5-1-14)$$

$$+ \rho (\vec{v} \cdot \vec{g}) - (\nabla \cdot \rho \vec{v}) - (\nabla \cdot [\tau \cdot \vec{v}])$$

word done work done work done by
 by gravity by pressure viscous forces

If we define mass flow $\vec{G} = \rho \vec{v}$ and consider single-phase flow in a vertical channel of fixed area, neglecting radial variations, we can rewrite Eq.

5-1-6 as

$$\frac{\partial \rho}{\partial t} + \frac{\partial G}{\partial z} = 0 \quad (5-1-15)$$

Eq. 5-1-11 becomes

$$\frac{\partial G}{\partial t} + \frac{\partial}{\partial z} (G^2 / \rho) = - \frac{\partial p}{\partial z} - \rho g - \frac{\partial \tau}{\partial z} \quad (5-1-16)$$

where $\frac{\partial \tau}{\partial z}$ is the friction pressure gradient and is frequently represented as

$$\frac{(f/\rho) |G| G}{2D_h} \text{ in single-phase flow}$$

If we recall $e = H - \frac{P}{\rho} + \frac{1}{2} v^2$, we can rewrite Eq. 5-1-14 in the form

$$\frac{\partial}{\partial t} (\rho H) = - (\nabla \cdot \rho \vec{v} H) - (\nabla \cdot \vec{q}) - (\tau : \nabla \vec{v}) + \frac{D\rho}{Dt} \quad (5-1-17)$$

Using the definition of G and if we neglect the contributions of friction and of pressure change to the conservation of energy compared to the large heat flux under sodium boiling conditions,* we can, for a vertical channel of fixed area, rewrite Eq. 5-1-17 as

$$\frac{\partial}{\partial t} (\rho H) + \frac{\partial}{\partial z} (GH) = \gamma \phi(z, t) \quad (5-1-18)$$

where ϕ = heat flux at channel wall and γ = ratio of channel perimeter to cross-sectional area, or using Eq. 5-1-15

$$\rho \frac{\partial H}{\partial t} + G \frac{\partial H}{\partial z} = \gamma \phi(z, t) \quad (5-1-19)$$

The laws of conservation of mass, momentum, and energy for an arbitrarily shaped, but fixed volume V with a surface S can also be written in integral form

$$\frac{\partial}{\partial t} \left[\int_V \rho dV \right] + \int_S \rho \vec{u} \cdot d\vec{S} = 0$$

$$\frac{\partial}{\partial t} \left[\int_V (\rho \vec{u}) dV \right] + \int_S (\rho \vec{u}) \cdot \vec{u} d\vec{S} = \int_S (\vec{p}) \cdot d\vec{S} + \int_V P \vec{\psi} dV \quad (5-1-21)$$

$$\frac{\partial}{\partial t} \left[\int_V (\rho e) dV \right] + \int_S (\rho e) \vec{u} \cdot d\vec{S} = - \int_S \vec{\psi} \cdot d\vec{S} + \int_V q''' dV \quad (5-1-22)$$

$$+ \int_S (\vec{u} \cdot \vec{p}) dS + \int_V P \vec{u} \cdot \vec{\psi} dV$$

* This assumption may not always be suitable.

where

ρ = density

$\mathbf{u}$ = velocity

p = pressure

e = internal energy/unit mass (including kinetic energy)

$\vec{\psi}$ = body force/unit mass

$\vec{\phi}$ = heat flux vector

q''' = volumetric internal heat generation

Appendix 5-2

Some Analytic Approximations for Solution of the Conservation Equations

Meyer discussed four numerical methods (or hydrodynamic models) of solving single- or two-phase flow without slip in a one-dimensional channel. He wrote the laws of conservation of mass, momentum, and energy in the following form:

$$\frac{\partial \rho}{\partial t} + \frac{\partial G}{\partial z} = 0 \quad (5-2-1)$$

$$\frac{\partial G}{\partial t} + \frac{\partial}{\partial z} (G^2/\rho) = - \frac{\partial p}{\partial z} - \frac{(f/\rho) |G| G}{2D_h} - \rho g \quad (5-2-2)$$

$$\rho \frac{\partial H}{\partial t} + G \frac{\partial H}{\partial z} = (\phi/l_1) + \left[\frac{\partial p}{\partial t} + (G/\rho) \frac{\partial p}{\partial z} \right] + \frac{(f/\rho) |G| G^2}{2\rho D_h} \quad (5-2-3)$$

He assumed the existence of a differentiable equation state

$$\rho = \rho(H, p) \quad (5-2-4)$$

and an empirical relationship to specify the friction factor, f :

$$f = f(H, P, G, \phi). \quad (5-2-5)$$

1. Sectionalized Compressible Model

This is the most direct and detailed representation of channel behavior. The model is based on a direct numerical solution of multiple point (or sectionalized) difference equation approximations to the conservation laws.

In order to make this difference representation explicit in the variables G , p , and H , Meyer rewrote Eq. 5-2-1 as

$$R_h \frac{\partial H}{\partial t} + R_p \frac{\partial p}{\partial t} + \frac{\partial G}{\partial z} = 0 \quad (5-2-6)$$

where

$$R_h = \left. \frac{\partial p}{\partial H} \right|_{p=\text{const.}} \quad \text{and} \quad R_p = \left. \frac{\partial p}{\partial p} \right|_{H=\text{const.}} \quad (5-2-7)$$

Then, Eq. 5-2-3 and 5-2-6 can be combined to yield one equation with only $\partial p/\partial t$ and a second with only $\partial H/\partial t$ as a time derivative:

$$\begin{aligned} (\rho/c^2) \frac{\partial p}{\partial t} + \rho \frac{\partial G}{\partial z} + (R_h G/\rho) \frac{\partial p}{\partial z} - (R_h G) \frac{\partial H}{\partial z} = \\ - R_H \left[(\phi/\ell_1) + \frac{(f/\rho) |G| G^2}{2\rho D_h} \right] \end{aligned} \quad (5-2-8)$$

$$\begin{aligned} (\rho/c^2) \frac{\partial H}{\partial t} + \frac{\partial G}{\partial z} - (R_p G/\rho) \frac{\partial p}{\partial z} + (R_p G) \frac{\partial H}{\partial z} = \\ R_p \left[(\phi/\ell_1) + \frac{(f/\rho) |G| G^2}{2\rho D_h} \right] \end{aligned} \quad (5-2-9)$$

where c , the isentropic sonic velocity is given by

$$c = 1/\sqrt{R_p + (R_h/\rho)}$$

Obviously, this method is capable of describing the detailed motion of acoustic waves as well as other phenomena.

The major difficulty with the compressible model is one of numerical stability. The required time-step size is on the order of the time for a sonic wave to pass through one space step [$\Delta t \leq \Delta z / (c + |v|)$], and the computer-time requirements can be very large.

2. Momentum Integral Model

In order to permit an increase in time-step, Meyer eliminated sonic effects from the physical model. He did this by assuming that density was a function of enthalpy only

$$\rho = \rho(H, p^*) \quad (5-2-11)$$

with p^* a constant reference pressure.

Meyer also simplified Eq. 5-2-3 by neglecting the effects on enthalpy rise rates of dissipation and pressure changes with space and time:

$$\rho(\partial H / \partial t) + G(\partial H / \partial z) = \phi / \ell_1 \quad (5-2-12)$$

In the sectionalized compressible model, local fluid accelerations can be affected by pressure changes without immediate acceleration of particles at other positions in the channel. However, since the revised equation of state (5-2-11) does not permit compression with increases in local pressure, Meyer's model was analogous to the use of a rigid body in mechanics problems. That is, pressure and velocity disturbances are propagated at infinite velocity. An increase in pressure drop tends to accelerate fluid particles at all positions. Meyer, therefore, integrated the momentum equation

(5-2-2) over channel length, and used it to represent only the behavior of the average mass velocity, $\bar{G}$:*

$$\frac{d\bar{G}}{dz} = \frac{1}{L} (\Delta p - F) \quad (5-2-13)$$

where

$$\bar{G} = (1/L) \int_0^L G \, dz \quad (5-2-14)$$

and the total resistance to fluid flow, F , is given by

$$F = (G_n^2 / \rho_n) - (G_o^2 / \rho_o) + \int_0^L \frac{(f/\rho) |G| G}{2D_h} dz + \int_0^L \rho g dz \quad (5-2-15)$$

The combination of the mass and energy equations (5-2-1 and 5-2-12), however, describe local fluid heating rates, and, therefore, also yield local rates of fluid expansion. That is, from Eq. 5-2-1 and 5-2-11,

$$\left(\frac{\partial G}{\partial z} \right) = - \left(\frac{d\rho}{dH} \right) \left(\frac{\partial H}{\partial t} \right) \quad (5-2-16)$$

and by combination with Eq. 5-2-12

$$\frac{\partial G}{\partial z} = - \frac{1}{\rho} \left(\frac{d\rho}{dH} \right) \left[\left(\frac{\phi}{\ell_1} \right) - G \frac{\partial H}{\partial z} \right] \quad (5-2-17)$$

* Note that others (see MacFarlane and SAS1A in section 5.3.1) have used the momentum integral form with different assumptions concerning the equation of state.

The momentum integral model is limited by numerical stability considerations to time-steps on the order of the time for fluid to be transported through one mesh increment $\left(\Delta t \leq \frac{\Delta z}{|v|}\right)$.

3. Single Mass Velocity Model

Further computational simplification is obtained, if the effect of fluid expansion can be neglected in handling the mass velocity profile. That is, if $\partial P / \partial t$ is small, then by Eq. 1, $\frac{\partial G}{\partial z} \approx 0$. The single mass velocity model is based on the use of a mass velocity which is a function of time only and not of position.

$$G = G(t) \quad (5-2-18)$$

(Clearly, this model cannot be applied to situations of flow reversal, as are encountered in sodium voiding. This model was designed for other possible applications.)

4. Channel Integral Model

In this case, rather than integrating only the conservation of momentum equation over the entire channel, all three conservation equations are solved in an integrated manner.

(Again, this model is not applicable to calculations of sodium voiding, and is mentioned only to define the model name.)