**NEAR FIELDS: INHERENT COMPUTATIONAL DIFFICULTIES
IN MOMENT-METHOD FORMULATIONS**

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MASTER

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NEAR FIELDS: INHERENT COMPUTATIONAL DIFFICULTIES IN MOMENT-METHOD FORMULATIONS

Abstract

Tangential and radial near fields, linear charge densities, and current distributions are calculated with seven thin-wire modeling programs using varying basis and weight functions and numerical treatments. The test case geometry is a half-wave $\lambda/4$ linear dipole with $\Omega = 15$. Each program is explained briefly and compared with similar codes. The results are presented graphically, and some basic observations are made.

1. Introduction

A thorough comparison of many of the thin-wire integral equation codes now in use for electromagnetics has been made by Miller, et al.¹ Various theoretical aspects were reviewed and conclusions concerning error criteria, convergence rates, and current distributions were presented therein. In this report, we will discuss a number of these codes, and some others as well, in slightly greater detail. We will focus upon the inherent effects of the current basis functions and boundary constraints upon the tangential and radial near fields, linear charge densities, and current distributions. After discussing the essential features of each code, data will be presented for both the scattering and radiation problems and then concluding observations concerning these data will be made.

Thin-Wire Integral Equation Codes

General Formulation

Assuming the reader is familiar with numerical electromagnetics, the following discussion will only summarize the highlights. The Pocklington integral equation

$$\hat{s} \cdot E^{inc}(s) = -\frac{1}{4\pi i \omega \epsilon} \int_{C(r)} ds' \hat{l}(s') \left\{ -\frac{\partial^2}{\partial s \partial s'} + k^2 \hat{s}' \cdot \hat{s} \right\} \frac{e^{-ikR}}{R}, \quad (1)$$

where $C(r)$ is the range of integration over the wires, $\hat{s}(r)$ is the unit vector tangent to the conductor and parallel to its axis at the observation point r , $\hat{s}'(r')$ is the axially aligned unit vector at the source point r' , and R is the distance from the source point to the surface of the conductor at the observation point,

is the general electromagnetic equation of interest here, and the basic approach to its solutions in the method-of-moments.

Denoting any linear operator equation as

$$Ll = g, \quad (2)$$

with L a linear operator, l an unknown response, and g a known forcing function, we begin by noting that the method-of-moments, in effect, projects the space of g onto another space and enforces the equality between the projections.² Therefore, by taking the inner product of Eq. (2) with a suitable set of independent weight functions $\{W_m\}$ that define the projection space, we obtain an equation for the equality of the projections

$$\langle W_n, Ll \rangle = \langle W_n, g \rangle \quad n = 1, 2, \dots, N. \quad (3)$$

It may be necessary to expand l in a set of basis functions $\{f_m\}$, where

$$l = \sum_{m=1}^M a_m f_m \quad (4)$$

so that Eq. (3) becomes

$$\sum_{m=1}^M a_m \langle W_n, Lf_m \rangle = \langle W_n, g \rangle \quad n = 1, 2, \dots, N. \quad (5)$$

Equation (5) can be written in matrix notation as

$$[A] \{A\} = [G], \quad (6)$$

where

$$W_{nm} = \langle W_n | f_m \rangle,$$

$$A_m = a_m,$$

and

$$G_n = \langle W_n | g \rangle$$

so that the solution is simply

$$[A] = [W]^{-1} [G]. \quad (7)$$

The numerical solutions presented here will all be based upon a formulation leading to a matrix equation of the form of Eq. (7); the differences will be in the choices of $\{f_m\}$ and $\{W_n\}$.

The weight functions that are most widely used fall into one of two classes. The first class leads to a collocation solution by choosing

$$W_n = \delta(x - x_n) \quad n = 1, 2, \dots, N, \quad (8)$$

thus sampling the operator equation at a discrete set of points $\{x_n\}$. The other class of weight functions leads to a Galerkin type formulation

$$W_n = f_n \quad n = 1, 2, \dots, N \quad (9)$$

so that

$$\sum_{m=1}^M a_m \langle f_n | f_m \rangle = \langle f_n | g \rangle \quad n = 1, 2, \dots, N. \quad (10)$$

Both the basis functions used in the description of the unknown and the corresponding weight functions are quite important in determining the characteristics of a numerical technique. The basis functions and boundary constraints of the test codes are listed below.

The codes used here are mostly LLL versions and modifications of the WF-MBA/LLL program, also known as AMP or BRACK, which was generated at MBAssociates. The basic code uses subsectional collocation, a three-term sinusoidal basis (constant, sine, and cosine), and the Pocklington integral equation. The other LLL codes vary in the basis functions and current constraints used, but are otherwise essentially the same. The Ohio State code of Richmond³ was also included for comparison, for this code uses the Galerkin method and overlapping sinusoidal weight and basis functions.

Test Codes

The major features of each code are listed along with the LLL computer catalogue name.

- Three-term sinusoidal with current extrapolation:

Code: WF-MBA/LLL

Current basis: $I_1(s) = A_1 + B_1 \sin K(s - s_1) + C_1 \cos k(s - s_1)$.

Current boundary constraint: extrapolation to match current amplitudes at adjacent segment centers.

- Three-term sinusoidal with junction slope and amplitude matching:

Code: WF-MBA/LLL2A

Current basis: $I_i(s) = A_i + B_i \sin k(s - s_i) + C_i \cos k(s - s_i)$

Current boundary constraint: interpolation to actual segment ends for current amplitude and slope continuity.

NOTE: There are two source models: (a) constant tangential field and (b) current slope discontinuity.

- Two-term sinusoidal:

Code: WF-MBA/LLL2B

Current basis: $I_i(s) = B_i \sin k(s - s_i) + C_i \cos k(s - s_i)$

Current boundary constraint: interpolation to actual segment ends for current amplitude continuity.

- Two-term sine and constant:

Code: MF-MBA/LLL2C

Current basis: $I_i(s) = A_i + B_i \sin k(s - s_i)$

Current boundary constraint: interpolation to actual segment ends for current amplitude continuity.

- Pulses:

Code: WF-MBA/LLL2D

Current basis: $I_i = A_i$

Current boundary constraint: none.

- Finite difference:

Code: WF-MBA/LLL2E

Current basis: $I_i = A_i$.

NOTE: Differential operator in Pocklington integral equation replaced by center finite differences.

- Galerkin's method (OSU), piecewise sinusoidal basis, piecewise overlapping sinusoidal weights

Code: WF-OSU

Current basis $I_i = [A_i^{(+)} \sin \{k(s_{i+})\} + A_i^{(-)} \sin \{k(s_{i-})\}] / \sin(kd_i)$,

where $s_{i+} = s - s_i + d_i/2$,

$s_{i-} = s_i - s + d_i/2$,

and

$d_i = \text{length of segment } i$

Current boundary constraint: current amplitude continuity at junctions.

A detailed discussion of the LLL programs is provided in Appendix A while a discussion of the OSU program is given by Richmond.³

Expected Behavior

There is a fundamental difference between collocation and Galerkin's method. The total tangential electric field on a perfect conductor $E_{\text{tan}} = E_{\text{tan}}^{\text{inc}} + E_{\text{tan}}^{\text{scat}}$ must be zero. This means $E_{\text{tan}}^{\text{scat}}$ must be the negative of $E_{\text{tan}}^{\text{inc}}$ in all cases and hence must be zero away from the source for the antenna mode, while being equal and opposite to the incident field (1 V/m in this case) for the scattering case. Collocation, with its delta-weighted integral, can enforce precisely the boundary condition at each sample point but can ensure nothing about the values between points. Galerkin's method will not necessarily have precise enforcement anywhere because this scheme enforces the boundary condition in an integrated sense through its particular weighting integral.

The behavior of the fields determined by the computed current distribution is dependent upon the character of the overall current distributions as represented by the appropriate union of the basis functions. We will now use an intuitive approach to explain the graphical data presented later.

Considering the Pocklington integral in Eq. (1), the integral may also be written as (for a straight wire)

$$\hat{s} \cdot E^{\text{inc}}(s) = -\frac{1}{4\pi i \omega \epsilon} \int_L I(s') (G''(s, s') + k^2 G(s, s')) ds',$$

where $G(s, s')$ is the thin wire approximation to the free space Green's function. A detailed examination of the behavior of $G(s, s')$ near $s = s'$ indicates that the major contribution to the integral is the result of the $G''(s, s')$ term. Qualitatively, $G(s, s')$ exhibits behavior that is quite similar to that of a delta function, for it is very sharply peaked for s near s' . Its first and second derivatives also display a behavior that is similar to that of a delta function's first (doublet) and second (triplet) derivatives as shown:

Therefore, we may expect the integral to show a similar derivative sifting quality as an integral of the product of a function and the corresponding delta function derivative. Hence,

$$\int_L I(s') G''(s, s') ds' \approx I'(s).$$

With this sifting quality, the field integral involving the current is quite dependent upon the continuity of the current slope and amplitude. Behavior of the integral caused by a discontinuity in current amplitude is illustrated using a general step function to describe an amplitude discontinuity.

We would expect the fields to be characterized by doublet behavior whenever current amplitude discontinuities exist. Similarly, slope discontinuities can also be described by a general piecewise linear function.

We would expect singlet behavior as a result of current slope discontinuities. It is clear that many codes calculate currents that have one or both of these discontinuities. The magnitude of these discontinuities, and hence the variations of the fields, is dependent upon the current bases, the interpolation equations, and the required relationships (e.g., extrapolation to segment centers of WF-MBA/LLL1). For a proper choice of bases and interpolation schemes, the effects can be reduced to a minimum and the physical parameters made to vary smoothly.

Numerical Evaluations

Test Geometry

The test case geometry is a half-wave, 1-m length (L) linear dipole with $\Omega \approx 15$ [$\Omega = 2 \ln(L/a)$] and divided into 20 segments for computational purposes. The test runs were performed for center excitation of the dipole as an antenna and for plane-wave broadside incidence as a scatterer. The antenna modes with their distribution of sources are depicted below. All of the codes except the OSU and current slope discontinuity, have a 1/2-V source located on each of the antenna's two center segments, i.e., segments 10 and 11,

with the source being the tangential electric field strength at the centers of the driven segments. The OSU and slope discontinuity codes have their 1-V sources placed at the junction of the two center segments, i.e., between segments 10 and 11,

with the source in this case arising from a specified discontinuity of the current slope (WF-MBA/LLL2A (b)) or from the integration of the assumed delta-function electric field (WF-OSU). See Miller and Deadrick for more details.⁴

The scattering mode is depicted as

A 1-V/m plane wave is incident broadside to the dipole.

Near Field Plots

A discussion of the near field and plotting program is contained in Appendix B. The graphical data are obtained by calculating the appropriate parameter values along the wire structure in the dipole's near field, which is one-wire radius from the segment axis, i.e., the wire surface. The abscissa is the normalized distance along the structure from its end at the left toward its center at the right. The field, current, and charge units are MKS. The scales on corresponding plots are the same for all codes. Consequently, some plots are occasionally off scale. Following the near field figures, the input admittance and impedance values are presented in Table 1 for the antennas and Table 2 for the scatterers.

Discussion of Results

- Three-term sinusoidal with current extrapolation:

Inherent in the WF-MBA/LLL1 code's current extrapolation scheme are small current amplitude discontinuities at segment junctions. Consequently, the tangential fields show doublet behavior. The discontinuities caused by the source disappear for the scattering mode case. The amplitude discontinuities also affect the radial electric fields, for these fields follow the behavior of the first spatial derivative of the current and are singlet in this case (Figs. 1 and 2).

- Three-term sinusoidal with junction slope and amplitude matching:

Because the current slopes and amplitudes are matched explicitly at segment junctions in the WF-MBA/LLL2A code case, the fields calculated are much smoother than in the above extrapolation method. End effects are still present, but they are of a much smaller magnitude (Figs. 3, 4, and 5). The different behavior of the two sources may be seen in the log plots of the tangential field in Figs. 3 and 4.

- Two-term sinusoidal:

The interpolation scheme in the WF-MBA/LLL2B code matches only current amplitude at segment ends. Consequently, the current slopes can be quite discontinuous at segment ends so that the tangential fields exhibit a singlet behavior. The current distribution itself is extremely nonphysical and emphasizes that a piecewise sinusoidal basis is not suited for use with point-matched weights when using the Pocklington equation (Figs. 6 and 7).

- Two-term sine and constant:

The WF-MBA/LLL2C code is nearly identical to the two-term sinusoidal code discussed above with the only difference being the basis. Significantly, the general behavior is very similar and is only different in scale. This demonstrates the need for both sine and cosine terms to be used with the constant term if point matching is to be generally successful in the Pocklington equation (Figs. 8 and 9).

- Pulses:

The WF-MBA/LLL2D code results in E_{θ} show the effects of current amplitude discontinuities. The current distribution is exactly the step function behavior used earlier to discuss this effect. The tangential fields are doublet, and the radial fields are singlet. Since the change is not defined for this basis except in an extended sense, it is not shown.² (See Figs. 10 and 11.)

- Finite difference:

The WF-MBA/LLL2E code employs a finite difference form of the Pocklington integral equation whereby the differential operator is moved outside the integral and approximated in a finite difference sense. The second derivative is expressed in central difference form at the observation point with a variable difference interval used in its computation. Three results are shown for each case obtained from difference intervals of 2Δ , Δ , and $\Delta/15$.

In the case where the interval equals Δ , the fields are very similar to those computed by the three-term sinusoidal with junction slope and amplitude matching. For the interval chosen to be $\Delta/15$, the central finite differences approximate the differential operator well, and the fields are similar to those calculated by the pulses code.

The conclusions to be drawn from these results are: (1) the most appropriate difference interval is Δ , i.e., $g''(x, x') \approx [g(x + \Delta, x') + g(x - \Delta, x') - 2g(x, x)] / \Delta^2$; (2) the finite difference approach using a pulse current basis is numerically closely equivalent to the analytically differentiated kernel using the three-term current basis and edge matchings. (See Figs. 12 through 17.)

- Sinusoidal basis and sinusoidal weights:

The interpolation scheme used by the sinusoidal basis and sinusoidal weights of the WF-OSU code ensures current amplitude, but not current slope, continuity. This is expected to produce a singlet behavior for the tangential electric fields, which is the case. The radial fields also nominally display the step-like behavior that is expected. However, Galerkin's method involves a weighted integral over the involving boundary conditions, which somewhat modifies the resulting field behavior. (See Figs. 18 and 19.)

- A "smoothed" current basis:

It has been suggested that the near field variations that result from discontinuous bases might be decreased by some appropriate smoothing of the calculated current distribution. Some results pertaining to this question are given in Figs. 20 and 21 for the pulse and three-term current bases originally considered in Figs. 1 and 10.

In both cases, the original treatment resulted in current amplitude discontinuities at segment junctions. The smoothing was subsequently accomplished by developing a current representation from a three-term edge-matched basis whose sampled values at the segment centers were equated to the previously calculated results. This produced a current distribution that passed through the points determined by the original computation, but which was now continuous in amplitude and slope along the entire antenna. This achieved a smoothing of the initial current representation. The near fields are indeed smoother in nature as can be seen by comparing Fig. 20 with 10 and Fig. 21 with 1. However, no substantial improvement in the boundary condition matching has been achieved. Indeed, on an rms basis, it appears the smoothed fields may not satisfy the tangential field requirement very well. It is not obvious that such a smoothing procedure renders any more than an aesthetic improvement in the calculated results.

Conclusions

We have presented near-field calculation results to demonstrate the sensitivity of these quantities to the integral equation and the numerical treatment used in its solution. All the techniques considered are found to exhibit numerically generated artifacts in the behavior of the near fields associated with the junctions of the segments used to model the straight wire. The two principal points to emphasize from this study are:

- boundary condition matching of the tangential field is generally improved as the smoothness of the current basis function is increased at the segment junctions, and
- field points used in a point-matching calculation should be located to avoid the anomalous behavior associated with the junction near fields.

Fig. 1. Three-term sinusoidal with current extrapolation in the antenna mode (WF-MBA/LLL1).

Fig. 1. (cont.)

Fig. 2. Three-term sinusoidal with current extrapolation in the scattering mode (WF-MBA/ILL1).

Fig. 2. (cont.)

Fig. 3. Three-term sinusoidal with junction slope and amplitude matching in the antenna mode and tangential field source case. (WF-MBA/LLL2A).

Fig. 3. (cont.)

Fig. 4. Three-term sinusoidal with junction slope and amplitude matching in the antenna mode and slope discontinuity source cases (WF-MBA/1.1.1.2A).

Fig. 4. (cont.)

Fig. 5. Three-term sinusoidal with junction slope and amplitude matching in the scattering mode (WF-MBA/LL12A).

Fig. 5. (cont.)

Fig. 6. Two-term sinusoidal in the antenna mode (WF-MBA/LLL2B).

Fig. 6. (cont.)

Fig. 7. Two-term sinusoidal in the scattering mode (WF-MBA/LLL2B).

Fig. 7. (cont.)

Fig. 8. Two-term sine and constant in the antenna mode (WF-MBA/LLL2C).

Fig. 8. (cont.)

Fig. 9 Two-term sine and constant in the scattering mode (WF-MBA/LLL2C).

Fig. 9. (cont.)

Fig. 10. Pulses in the antenna mode (WF-MBA/LLL2D).

Fig. 10. (cont.)

Fig. 11. Pulses in the scattering mode (WF-MBA/LLL2D).

Fig. 11. (cont.)

Fig. 12. Finite difference in the antenna mode and difference interval of 2Δ (WF-MBA/LLL2E).

Fig. 12. (cont.)

Fig. 13. Finite difference in the antenna mode and difference interval of Δ (WF-MBA/LLL2E).

Fig. 13. (cont.)

Fig. 14. Finite differences in the antenna mode and difference interval of $\Delta/15$ (WF-MBA/LL12E).

Fig. 14. (cont.)

Fig. 15. Finite difference in the scattering mode and difference interval of 2Δ (WF-MBA/LLL2E).

Fig. 15. (cont.)

Fig. 16. Finite difference in the scattering mode and difference interval of Δ segment (WF-MBA/LLL2E).

Fig. 16. (cont.)

Fig. 17. Finite difference in the scattering mode and difference interval of $\Delta/15$ (WF-MBA/LLL2E).

Fig. 17. (cont.)

Fig. 18. Sinusoidal basis, sinusoidal weights in the antenna mode (WF-OSU).

Fig. 18. (cont.)

Fig. 19. Sinusoidal basis, sinusoidal weights in the scattering mode (WF-OSU).

Fig. 19. (cont.)

Fig. 20. Smoothed pulses in the antenna mode (modified WF-MBA/LLL2D).

Fig. 20. (cont.)

Fig. 21. Smoothed three-term sinusoidal with current extrapolation (modified WF-MBA/LLL1).

Fig. 21. (cont.)

Table 1 – Input admittances and impedances calculated for the half wave, $\Omega = 15$ dipole antenna test case.

Code	Y_{in} (Millimhos)	Z_{in} (Ohms)
WF-MBA/LLL1	9.52 - j5.26	80.5 + j44.5
WF-MBA/LLL2A (a)	9.34 - j5.33	80.8 + j46.2
WF-MBA/LLL2A (b)	8.67 - j4.76	88.6 + j48.7
WF-MBA/LLL2B	7.01 - j4.67	98.8 + j65.8
WF-MBA/LLL2C	0.55 - j1.41	241 + j614
WF-MBA/LLL2D	0.87 - j2.11	166 + j405
WF-MBA/LLL2E (2 Δ)	2.66 - j3.98	116 + j174
WF-MBA/LLL2E (Δ)	5.40 - j5.45	91.8 + j92.6
WF-MBA/LLL2E ($\Delta/15$)	0.96 - j2.28	157 + j372
WF-OSU	9.50 - j5.40	79.6 + j45.2

Table 2 – Transfer admittances calculated for the half wave, $|\Omega| = 15$ dipole scattering test case.

Code	Y_{in} (Millimhos)
WF-MBA/LLL1	6.22 - j3.79
WF-MBA/LLL2A	6.11 - j3.84
WF-MBA/LLL2B	4.88 - j3.99
WF-MBA/LLL2C	0.36 - j1.76
WF-MBA/LLL2D	0.58 - j2.15
WF-MBA/LLL2E (2 Δ)	1.89 - j3.39
WF-MBA/LLL2E (Δ)	3.67 - j4.11
WF-MBA/LLL2E ($\Delta/15$)	6.42 - j2.25
WF-OSU	6.20 - j3.85

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Appendix A-Test Codes

The AMP Program

Although AMP (WF-MBA/LLL1) and its solution technique have been described in some detail elsewhere, the basic interpolation scheme and boundary constraints should be reexamined briefly.^{1,4-6}

The AMP program uses the Pocklington integral equation for the current distributions on a wire structure. The numerical solution method can be described as collocation with a three-term sinusoidal interpolation basis and delta weight functions. (All of the LLL derivatives use the Pocklington equation and delta-function weighting, differing only with respect to current interpolation and basis functions.) The AMP current function is the three-term sinusoidal basis with A, B, and C coefficients:

$$\begin{aligned} I(s) &= \sum_{i=1}^N \epsilon_i \{ A_i + B_i \sin k(s - s_i) + C_i \cos k(s - s_i) \} \\ &= \sum_{i=1}^N \epsilon_i I_i(s), \end{aligned} \quad (1)$$

where

$$\epsilon_i = \begin{cases} 1 & \text{s on segment } i \\ 0 & \text{otherwise} \end{cases}$$

It is easily seen that for a structure with N-field sample points, i.e., N segments, 3N constants are needed to specify the current on each segment of the structure. Because the field sampling at the segment centers supplies N of the 3N unknowns, 2N more constraints are needed. It is not necessary to solve the integral equation itself for the extra unknowns, for two of the three constants for each segment may be obtained by requiring the current on adjacent segments to satisfy some appropriate constraints. In the AMP program the current on an observation segment is extrapolated and made to match the current values on the two or more adjacent segments.⁷

For a two-wire junction, using Eq. (1)⁵

$$A_i + C_i = I_i \{ B_i \sin k(s_i - s_i) \equiv 0 \}, \quad (2a)$$

$$A_i - B_i s_{i-1} + C_i k_{i-1} = I_{i-1} = A_{i-1} + C_{i-1}, \quad (2b)$$

and

$$A_i + B_i s_{i+1} + C_i k_{i+1} = I_{i+1} = A_{i+1} + C_{i+1}, \quad (2c)$$

where

$$\frac{s}{k} \left| \begin{matrix} i \pm 1 \end{matrix} \right. = \frac{\sin}{\cos} \left[k(d_i \pm 1 + d_i)/2 \right],$$

and

$$d_i = \text{length of segment } i.$$

The 3 by 3 system of Eq. (2) may be solved for the current function constants A_i , B_i , and C_i in terms of I_{i-1} , I_i , and I_{i+1} to obtain

$$A_i = D_i^{-1} \{ [-s_{i-1} k_{i+1} - s_{i+1} k_{i-1}] I_i + s_{i+1} I_{i-1} + s_{i-1} I_{i+1} \} , \quad (3a)$$

$$B_i + D_i^{-1} \{ [k_{i-1} - k_{i+1}] I_i + [k_{i+1} - 1] I_{i-1} - [k_{i-1} - 1] I_{i+1} \} , \quad (3b)$$

and

$$C_i = D_i^{-1} \{ [s_{i-1} + s_{i+1}] I_i - s_{i+1} I_{i-1} - s_{i-1} I_{i+1} \} , \quad (3c)$$

where

$$D_i = s_{i+1} [1 - k_{i-1}] + s_{i-1} [1 - k_{i+1}] .$$

Thus the current on segment i can be written in terms of the three center current values I_{i-1} , I_i , and I_{i+1} :

$$\begin{aligned} I_i(s) = & D^{-1} \{ [s_{i+1} [1 - \cos k(s - s_i)] + \sin k(s - s_i)(k_{i+1} - 1)] I_{i-1} \\ & + [\sin k(s - s_i) [k_{i-1} - k_{i+1}] + \cos k(s - s_i) [s_{i+1} + s_{i-1}]] \\ & - s_{i+1} k_{0-1} - s_{i-1} k_{i+1}] I_i + [s_{i-1} [1 - \cos k(s - s_i)] \\ & + \sin k(s - s_i) [1 - k_{i-1}]] I_{i+1} \} \\ = & X_i(s) I_{i-1} + Y_i(s) I_i + Z_i(s) I_{i+1} . \end{aligned} \quad (3d)$$

The system of equations that results from the collocation solution (i.e., identical field and current match points) to the integral equation has the N -segment center currents as unknowns.

As a complete discussion of the calculation of the impedance matrix elements is too involved to be repeated here, it should suffice to say that the N -field samples that constrain the N unknowns are calculated in exactly the same fashion as the tangential and radial near fields whose treatment is explained in Appendix B. A contribution to the field at the observation point caused by each term of the current function of the other $N-1$ segments, as well as its own, is calculated. The only difference is that the matrix elements are computed for unknown A 's, B 's and C 's in a linear system while the near fields are subsequently computed for a known set of A 's, B 's, and C 's.

The main characteristics of the AMP code are:

- three-term sinusoidal current basis, and
- current extrapolation to match current amplitudes at adjacent segment centers.

The GIANT (WF-MBA/LLL2) program was originally conceived as an expanded impedance matrix version of AMP. The name GIANT is a mnemonic device derived from Giant ANTenna program. Since the matrix was expanded to 3N by 3N to accommodate AMP's three-term current basis explicitly, different boundary constraints, i.e., interpolation schemes, could be easily implemented. Therefore, the program's flexibility, also allowed new basis functions to be treated, generating the subset of programs now to be discussed.

- Three-term sinusoidal with junction slope and amplitude matching.

The MKIIIC version of the GIANT program (WF-MBA/LLL2A) evolved as a larger but more stable and accurate version of AMP. The impedance matrix was expanded from N by N to 3N by 3N, as the linear system of equations expanded to the order of 3N. In the expanded impedance matrix, the first N rows were field sample equations computed at the centers of each of the structure's N segments. As the current basis is handled explicitly, 3N columns are necessary for the equations to occupy the top N by 3N portion of the matrix. The remaining 2N equations (the lower 2N by 3N portion of the matrix) are current-interpolation boundary-constraint equations. For simple wire junctions, current amplitudes for adjacent segments are matched precisely at the observation segment's minus reference end, i.e., the junction of the two segments. Current slopes (i.e., current derivatives or "charge") are matched exactly at the observation segment's positive reference end. The equations are entered a pair at a time in the impedance matrix. The amplitude and slope equations are:

$$I_i - I_{i-1} = A_i - B_i \sin \frac{kd_i}{2} + C_i \cos \frac{kd_i}{2} - (A_{i-1} + B_{i-1} \sin \frac{kd_{i-1}}{2} + C_{i-1} \cos \frac{kd_{i-1}}{2}) = 0, \quad (4)$$

and

$$\frac{I_i}{\partial(ks)} - \frac{I_{i+1}}{\partial(ks)} = B_i \cos \frac{kd_i}{2} - C_i \sin \frac{kd_i}{2} - (B_{i+1} \cos \frac{kd_{i+1}}{2} + C_{i+1} \sin \frac{kd_{i+1}}{2}) = 0. \quad (5)$$

As in the AMP program, if the observation segment is not connected to another segment and ends in free space, the current is constrained to be zero at the wire's end. Although this version has a special charge redistribution technique for multiple junctions, it will not be discussed here.

The MKIIIC program can be run in the antenna mode for two different types of sources. The first type of excitation is the same as AMP, a model in which the tangential field is a constant for the entire length of the center source segment(s). The second type is a bicone antenna model in which the source, a delta function of charge, was introduced as a current slope discontinuity at the junction of the segments bounding the source. The details of the source models will be discussed in a future report.⁸

In this discussion the essential characteristics of the GIANT - MKIIIC program are:

- expanded 3N by 3N impedance matrix to handle explicitly each term of the three-term current function,
- current interpolation at the actual segment ends, i.e., constraining current amplitude and slope continuity at the segment junctions, and
- two different source models, constant tangential field and bicone antenna current slope discontinuity.

Two-term sinusoidal. The two-term sinusoidal program (WF-MBA/LLL2B) is the first of the GIANT derivatives. In it the impedance matrix, and hence the linear system, is 2N by 2N because of the two-term current function

$$I(s) = \sum_{i=1}^N \epsilon_i \left\{ B_i \sin k(s - s_i) + C_i \cos k(s - s_i) \right\}. \quad (6)$$

After the N-field sample equations for the N segments are obtained exactly as in AMP, they are added together to result in N-1 equations that occupy the upper N-1 by 2N portion of the matrix. The remaining N+1 equations (rows) are current constraint equations of which the Nth and the 2Nth rows are free end or zero current amplitude equations. The intervening N-1 equations are simple current amplitude matching equations at segment ends. The current interpolation equation is:

$$I_i - I_{i+1} = B_i \sin k \frac{d_i}{2} + C_i \cos k \frac{d_i}{2} - (-B_{i+1} \sin k \frac{d_{i+1}}{2} + C_{i+1} \cos k \frac{d_{i+1}}{2}) = 0. \quad (7)$$

The important characteristics of the two-term sinusoidal program are:

- two-term, purely sinusoidal current function,
- current interpolation at segment ends constraining current amplitude continuity,
- pairwise addition of field sampling equations, and
- 2N by 2N impedance matrix.

Sine and Constant. The sine and constant program (WF-MBA/LLL2C) is virtually identical with the two-term sinusoidal code, the only difference being that the cosine basis term has been replaced by a constant term. The current function and interpolation equations are

$$I(s) = \sum_{i=1}^N \epsilon_i \left\{ A_i + B_i \sin k(s - s_i) \right\}, \quad (8)$$

and

$$I_i - I_{i+1} = A_i + B_i \sin k \frac{d_i}{2} - (A_{i+1} + B_{i+1} \sin k \frac{d_{i+1}}{2}) = 0. \quad (9)$$

The important characteristics of the sine and constant program are:

- two-term current function consisting of a constant term and sinusoidal term,
- current interpolation at segment ends constraining current amplitude continuity,
- pairwise addition of field sampling equations, and
- 2N by 2N impedance matrix.

Pulse program. The pulse program (WF-MBA/LLL2D) is the simplest of the GIANT derivatives, consisting of N by N system of equations that are the N-field samples at each of the N-segment centers caused by a pulse current basis. The currents are assumed to be constant over the entire segment and no attempt is made to impose current interpolation or physical end conditions. The current function is

$$I(s) = \sum_{i=1}^N e_i A_i \quad (10)$$

The important characteristics of the pulse code are:

- single, constant term current function,
- no current interpolation equations or end conditions, and
- N by N impedance matrix.

Finite differences. Essentially a pulse basis, N by N matrix program, the finite difference code (WF-MBA/LLL2E) differs from the pulse code above in that the exact differentials in the integral equation

$$\hat{s} \cdot \vec{E}^{inc}(s) \Big|_{s=s_i} = -\frac{1}{4\pi i \omega \epsilon} \int_L ds' I(s') \left(\frac{\partial^2}{\partial s^2} + k^2 \hat{s} \cdot \hat{s}' \right) \frac{e^{-ikR}}{R} \Big|_{s=s_i}$$

$$i = 1, 2, \dots, N \quad (11)$$

are replaced by central finite differences. The interval over which these finite differences are made varies, and several results from the varying parameter are shown.

Important characteristics of the finite difference program are

- Green's function derivatives evaluated in a finite difference sense,
- constant term current function, and
- N by N impedance matrix.

Appendix B-Near Field Program

Though embodying many modifications, the near field program grew essentially out of two subroutines in the original AMP program.⁶ Basically the program computes the values of the tangential and radial electric fields, linear charge densities, and currents at a sufficiently large number of points along the wire structure so that plots of these values appear continuous. In this way, the near field behavior of these parameters may be inspected and compared not only for their inherent interest, but also to compare changes in computational modeling. The structure geometries, excitation frequency, and the current coefficients (A's, B's and C's) from the aforementioned programs are input parameters. Along the portion of the structure for which near field information was desired, each segment was sampled at a predetermined number of points, 75 for the results presented here. At each point, the fields, charges, and currents were computed and later entered into a storage array as output. All of these values were computed at one-wire radius from the center of the wire, i.e., at the "surface" of the wire.

The electric field at a point in space caused by each structure segment is calculated assuming a three-term current function

$$\frac{I_z(s)}{\gamma} = 1 + \sin k z + \cos k z. \quad (12)$$

Each segment is considered to be located on the origin of a local cylindrical system with the point at which the field is computed being (ρ', ϕ', z') . The geometry for a filament of length d is shown below:

For a sine or cosine current distribution, the field can be written in closed form as in Eq. (6), and the ρ and z components for a current $I_0 \begin{Bmatrix} \sin k z \\ \cos k z \end{Bmatrix}$ being:⁹

$$\begin{aligned} E_z(\rho', z') = & \frac{I_0}{\lambda} i \frac{\eta}{2} \left[\frac{e^{-ikr_2}}{kr_2} \begin{Bmatrix} \cos k d/2 \\ -\sin k d/2 \end{Bmatrix} - \frac{e^{-ikr_1}}{kr_1} \begin{Bmatrix} \cos k d/2 \\ \sin k d/2 \end{Bmatrix} \right. \\ & - \left(i + \frac{1}{kr_2} \right) \frac{e^{-ikr_2}}{(kr_2)^2} \left(kz' - k \frac{d}{2} \right) \begin{Bmatrix} \sin k d/2 \\ \cos k d/2 \end{Bmatrix} \\ & \left. - \left(i + \frac{1}{kr_1} \right) \frac{e^{-ikr_1}}{(kr_1)^2} (kz' + k d/2) \begin{Bmatrix} -\sin k d/2 \\ \cos k d/2 \end{Bmatrix} \right] \quad (13) \end{aligned}$$

and

$$\begin{aligned}
 E_{\rho}(\rho', z') = & \frac{I_0}{\lambda} (-i \frac{\eta}{2}) \frac{1}{k\rho'} \left[(kz' - k \frac{d}{2}) \frac{e^{-ikr_2}}{kr_2} \left\{ \cos k \frac{d}{2} \right\} \right. \\
 & - (kz' + k \frac{d}{2}) \frac{e^{-ikr_1}}{kr_1} \left\{ \begin{array}{l} \cos k \frac{d}{2} \\ \sin k \frac{d}{2} \end{array} \right\} + \frac{e^{-ikr_2}}{kr_2} \left\{ \begin{array}{l} \sin k \frac{d}{2} \\ \cos k \frac{d}{2} \end{array} \right\} \\
 & - (kz' - k \frac{d}{2})^2 (i + \frac{1}{kr_2}) \frac{e^{-ikr_2}}{(kr_2)^2} \left\{ \begin{array}{l} \sin k \frac{d}{2} \\ \cos k \frac{d}{2} \end{array} \right\} \\
 & \left. - \frac{e^{-ikr_1}}{kr_1} \left\{ \begin{array}{l} -\sin k \frac{d}{2} \\ \cos k \frac{d}{2} \end{array} \right\} + (kz' + k \frac{d}{2})^2 (i + \frac{1}{kr_1}) \frac{e^{-ikr_1}}{(kr_1)^2} \left\{ \begin{array}{l} -\sin k \frac{d}{2} \\ \cos k \frac{d}{2} \end{array} \right\} \right] . \quad (14)
 \end{aligned}$$

The expression for the field of a constant current distribution involves an integral of $\exp(-ikr)/r$ that must be evaluated numerically. The field components caused by a current I_0 are

$$E_{\rho}(\rho', z') = \frac{I_0}{\lambda} (-i \frac{\eta}{2}) (k\rho') \left[(i + \frac{1}{kr_2}) \frac{e^{-ikr_1}}{(kr_1)^2} - (i + \frac{1}{kr_1}) \frac{e^{-ikr_2}}{(kr_2)^2} \right] , \quad (15)$$

and

$$\begin{aligned}
 E_z(\rho', z') = & \frac{I_0}{\lambda} (-i \frac{\eta}{2}) \left[\int_{-\frac{d}{2}}^{\frac{d}{2}} \frac{e^{-ikr}}{r} dz + (i + \frac{1}{kr_2}) (kz' - k \frac{d}{2}) \frac{e^{-ikr_2}}{(kr_2)^2} \right. \\
 & \left. - (i + \frac{1}{kr_1}) (kz' + k \frac{d}{2}) \frac{e^{-ikr_1}}{(kr_1)^2} \right] . \quad (16)
 \end{aligned}$$

Once these field components have been calculated (with $I_0/\lambda = 1$ in the program), they are scaled to reflect the actual current component amplitudes that have been calculated, where the actual current is given by

$$I_1(s) = A_1 + B_1 \sin k(s-s') + C_1 \cos k(s-s') \quad (17)$$

because the A's, B's, and C's are known quantities.

The electric field components calculated by Eqs. (13) to (16) are centered and aligned in the cylindrical coordinate system of the source segment. These components are resolved into the x, y, and z cartesian coordinates of the structure, and the contributions from each segment to the field at the observation point are summed to yield the total field.

The linear charge distributions are calculated in a straightforward manner. Calculating the linear charge density as a spatial derivative of the time integral of the current gives

$$Q(s) = -j \left\{ B_i \cos k(s-s') - C_i \sin k(s-s') \right\} . \quad (18)$$

As many of the program do not have the full three-term current function, appropriate A, B, or C coefficients are set identically equal to zero as is necessary in the field, charge, and current equations.