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DATE: 3/28/49

TID-5050

TO: I. Karlan

FROM: J. Chernick

SUBJECT: Initial Experiments

on the Brookhaven Reactor. -- VII

TID-5050

III. ANALYSIS OF THE CLINTON CRITICAL EXPERIMENT

In the present report, we compute the fundamental pile constants of the Clinton reactor from their critical experiment (P.P.R. Vol. V, Chap. 23). These constants include the critical size, multiplication factor, migration area and critical laplacian. In addition, we have used their data to obtain the spontaneous neutron yield of natural uranium. We get 14.0 neut./kg sec. which agrees well with the value of 14.2 neut./kg sec. determined by Fermi from an exponential experiment (C-26). The accuracy of our determination is estimated at 10%.

The critical size can be determined from the Clinton data with good accuracy. (Even better results would have been obtained if they had used more loadings in the neighborhood of criticality). We get 217.80 ± 0.04 cm for the actual loading radius by extrapolating from below critical and 218.40 ± 0.05 cm by extrapolating from above critical. These values are close to those cited in the P.P.R. The difference of 0.6 cm must be attributed to some small systematic error; possibly to the necessity of correcting in an arbitrary manner for the presence of a small absorber placed in the pile at the first loading above criticality.

For the remaining constants, M^2 , Δ_c , $k_p - 1$, it is easy to obtain agreement within 10% of the theoretically predicted values. Better agreement is perhaps too much to expect at present due to various systematic

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errors. Among the latter are:

1. The conversion of pile period to excess k.

The latter figure has changed by 15% since the publication of the Clinton data. In the analysis of a critical experiment the accuracy of the determination of M^2 , the migration area, is no better than the accuracy of this conversion factor.

2. The conversion of observed activities to thermal flux.

Our determination of the spontaneous neutron yield of uranium is no better than this figure, or within 10% of the correct value.

3. The reflector savings.

The latter is intrinsically difficult to determine from the Oak Ridge data, the reason being that the reflector savings is always small compared to the loading radius. Our attempt to obtain this parameter showed that the data was not sensitive to changes in the reflector savings of 20 cm or more. The reflector savings is, however, required in the determination of the critical Laplacian. We can hope to improve on the Clinton critical experiment by running transverses at various loadings and extrapolating for the augmented pile radius and hence for the reflector savings.

4. Other systematic errors.

Here we can lump such variables as temperature and barometric effects, unknown foil weights and cadmium ratios, irregularities of the loading, etc. An example of the latter was the non-loading of three central channels in the Clinton pile. We can hope to avoid some of these systematic errors by keeping a record of temperature and pressure, weighing the foils etc.

Theoretical predictions

From data published in the Chicago Handbook (Chap. IV,E) we find the following values for the fundamental constants of the Clinton pile.

$$f = 0.890$$

$$f_{ad} = 0.004$$

$$f_{mod} = 0.106$$

$$p = 0.886$$

$$\epsilon = 1.029$$

$$\eta = 1.315$$

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From these data

$$k_{\infty} = \eta_{\infty} pf = 1.067$$

The value of L , the diffusion length, assumed for the AGOT graphite of the pile is 50.2 cm, density 1.6. The average age of fission neutrons in graphite at indium resonance energy (1.44 ev) is taken as 334 cm² and θ_0 the age at thermal energy is obtained by adding an increment of 53 cm². The latter increment is admittedly uncertain in view of possible crystalline effects at low energies. The value of θ_0 is thus taken at 387 cm².

To obtain L_0 , the diffusion length of thermal neutrons in the pile, we have $L_0^2 = f_{\text{mod}} L^2$. However, one must correct L^2 for the increased temperature of pile neutrons and the presence of empty cylindrical channels. The corrected expressions are

$$L_{\perp}^2 = f_{\text{mod}} L^2 \sqrt{\frac{T}{300}} \left[1 + \phi \left(1 + \frac{a}{\lambda_{\text{th}}} \right) \right]$$

$$L_{\parallel}^2 = f_{\text{mod}} L^2 \sqrt{\frac{T}{300}} \left[1 + \phi \left(1 + \frac{2a}{\lambda_{\text{th}}} \right) \right]$$

where $L_{\perp}$, $L_{\parallel}$ are the pile diffusion lengths respectively perpendicular to and parallel to the channels, $T = 370^{\circ}$ is the absolute temperature of the pile neutrons, $\phi = 0.034$ is the fraction of pile volume taken up by empty channels, $a = 2.1$ is the channel radius and $\lambda_{\text{th}} = 2.7$ is the transport mean free path for thermal neutrons.

The neutron age is likewise increased by the presence of channels and also by the presence of uranium since the latter is much less efficient as a slowing down medium. On the other hand the age is decreased by inelastic scattering in the uranium rods. The corrected age for a pile is found by taking

$$\theta_{\perp} = \left[387 (1-P) + 231 P \right] \left[1 + V_0/V_1 \right] \left[1 + \phi \left(1 + \frac{a}{\lambda_f} \right) \right]$$

$$\theta_{\parallel} = \left[387 (1-P) + 231 P \right] \left[1 + V_0/V_1 \right] \left[1 + \phi \left(1 + \frac{2a}{\lambda_f} \right) \right]$$

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where $P = 0.15$ is the probability of a fast neutron making an inelastic collision in a metal rod of radius 1.4 cm, $V_0/V_1 = 0.015$ is the ratio of the volume of metal to that of graphite and $\lambda_f = 3.5$ cm is the mean free path of a fast neutron.

Insertion of the data gives

$$\begin{array}{llll} L_{\perp}^2 = 314.6 & \Theta_{\perp} = 389.1 & M_{\perp}^2 = 703.7 & \Delta_{\perp} = -95.2 \times 10^{-6} \\ L_{11}^2 = 322.4 & \Theta_{11} = 396.7 & M_{11}^2 = 719.1 & \Delta_{11} = -93.2 \times 10^{-6} \end{array}$$

As average values over the pile, then

$$L_0^2 = 317 \text{ cm}^2, \Theta_0 = 392 \text{ cm}^2.$$

The fundamental constants predicted for the Clinton pile are therefore:

$$M^2 = 710 \text{ cm}^2, \Delta_c = -94 \times 10^{-6} \text{ cm}^{-2}, k_{\infty} = 1.067.$$

Experimental Findings

In the analysis of the experimental data, the Plutonium Project Report takes the reflector savings at 42.5 cm. With this value they get

$$M^2 = 630 \text{ cm}^2, \Delta = -1.045 \times 10^{-6}, k_{\infty} = 1.066$$

For a conversion factor, 1 inhour was taken as equivalent to 3×10^{-5} in k. If the more recent figure of 2.6×10^{-5} is used, we get

$$M^2 = 546 \text{ cm}^2, \Delta = -1.045 \times 10^{-6}, k_{\infty} = 1.057$$

It is clear that most of the trouble lies in taking too low a value for the reflector savings.

Re-evaluation of the Clinton Data

We shall re-evaluate the Clinton data by using a more reasonable value for the reflector savings. Before doing so, it is of interest to try to obtain the reflector savings directly from the data together with a precision estimate.

Before criticality, the formula to be fitted is

$$(1) \quad A_s = c \left[\left(\frac{1}{r_l + \delta} \right)^2 - \left(\frac{1}{r_c + \delta} \right)^2 \right]^{-1}$$

where A_s is the observed activity, r_l the "observed" loading radius, r_c

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the critical loading radius, δ the reflector savings, and c a proportionality constant which can be related to the spontaneous neutron yield of uranium. The three constants, c , δ and r_c , are to be obtained on the basis of 5 points. Equation (1) is deliberately left in non-linear form so that the resulting least square solution will weight most the points near criticality where the activities are largest. The least square technique has been fully described in B.N.L. Log # C-820. Taking $c = .0073$, $\delta = 42.5$ cm and $r_c = 217.4$ cm for an approximate set of solutions and linearizing equation (1) by Taylor series expansion we get

$$c = 0.0103 \pm 0.0022$$

$$\delta = 11 \pm 23 \text{ cm}$$

$$r_c = 217.8 \pm 0.4$$

for our least square solution.

These results show that

1. The reflector savings can not be determined from the Clinton data with any precision whatever.
2. In the complete absence of information on reflector savings, the critical radius is determined to a precision of about 0.2%.
3. Under the same conditions, the proportionality constant c is correct to about 20%. With a better estimate of the reflector savings the precision can of course be greatly improved.

It was therefore thought advisable to take the theoretical reflector savings and combine this with the experimental data. The diffusion length of the Clinton graphite is about 50 cm. The reflector savings according to equation (8) of B.N.L. Log # C-2853 is then 53 cm. There are theoretical uncertainties in the latter figure but it is certainly to be preferred to the "experimental" value of 42.5 cm given in P.P.R. Vol. V.

We can now write equation (1) as

$$(1A) \quad A_s = c \left[\frac{1}{(r_l + 53)^2} - \frac{1}{(r_c + 53)^2} \right]$$

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and determine the constants c and r_c by least squares. We get

$$c = 0.006453 \pm 0.000110$$

$$r_c = 217.54 \pm 0.12$$

the internal consistency of the data now resulting in a standard error in the critical loading radius of 1 part in 2000 and an error of 2% in the proportionality constant. This takes no account, of course, of systematic errors.

Harmonic Corrections

One systematic error that can be eliminated is that due to the non-linearity of the curves of $1/A_g$ against $1/R^2$ for small loading radii. We can refine our results by applying harmonic corrections to the observed activities. The corrections can be obtained from our formula (B.N.L. Log # C-2853)

$$n = \sum_m \frac{4}{m\pi B_m^2} \left[1 - \frac{1}{I_0(RB_m)} \right]$$

The following table gives the required harmonic corrections:

Observed Activity	Harmonic Correction	Corrected Activities
620	1.081	670
1180	1.047	1235
2918	1.019	2973
6344	1.012	6420
10558	1.007	10632

Repetition of our previous work now yields

$$c = 0.006762 \pm 0.000036$$

$$r_c = 217.80 \pm 0.04$$

The increased precision of the fit is reflected in smaller standard errors.

The value of applying harmonic corrections to the data is thus confirmed.

The critical radius corresponds to a loading of 360.9 channels as against

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360 channels given in P.P.R.

Spontaneous Neutron Yield

By use of the final equation of B.N.L. Log # C-2853, we can correlate the proportional constant c which we have just obtained with the natural neutron yield of uranium. We have

$$Q = cK \left(\frac{\lambda_p M^2}{3pL^2} \right) \left(\frac{\pi j_1^3 J_1(j_1)}{8} \right) = 0.9451 cK \left(\frac{\lambda_p M^2}{pL^2} \right)$$

where K is the factor required to convert bare indium activity to thermal flux. Taking $\lambda_p = 2.71$, $p = 0.886$, $M^2 = 687$, $L^2 = 317$, $c = .00676$, we find

$$Q = 0.0424 K \text{ neut/cm}^3$$

To estimate K , we use the formula

$$nv = 0.102 A_s \left(1 - \frac{1.07}{C.R.} \right)$$

for standard indium foils. We do not know the cadmium ratio for the Clinton pile, but taking this as 6, we get

$$K = 0.0837$$

and

$$Q = 3.55 \times 10^{-3} \text{ neut/cm}^3 \text{ sec.}$$

From the lattice constants and the fact that there were 65 4" slugs per channel, we can obtain the density of natural uranium in the pile.

We find

$$\text{density} = 18.9 \left[\frac{65 \times 1/3}{24} \right] \left[\frac{\pi(1.4)^2}{(20.32)^2} \right] \times 10^{-3} = 0.2544 \times 10^{-3} \text{ kg/cm}^3.$$

Hence we get 14.0 neut/kg sec with an estimated accuracy of 10%. Our ignorance of the cadmium ratio introduces an error of about 2%. The possible error in the ratio L^2/M^2 is likewise a few percent. The chief uncertainties lie in the conversion of activity to flux and in the experimental value of the proportionality constant which, as we have seen, depends upon the assumed reflector savings and may be subject to other systematic errors.

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Reduction of Data Above Criticality

The basic data from loadings above critical are the observed pile periods and loading radii. For a thermal pile

$$k_{\infty} = 1 + M^2 \beta_c^2,$$
$$\beta_c^2 = \frac{T^2}{H^2} + \frac{j_1^2}{R_c^2}$$

with H = the augmented pile height and R_c = the augmented pile radius.

Also, if the effective multiplication constant of the pile, k_{eff} , is close to unity

$$k_{\text{eff}} = k_{\infty} - M^2 \beta^2,$$
$$\beta^2 = \frac{T^2}{H^2} + \frac{j_1^2}{R^2}$$

Combining these relations we have for the excess k of the pile, k_{ex} :

$$k_{\text{ex}} = k_{\text{eff}} - 1 = M^2 (\beta_c^2 - \beta^2) = M^2 j_1^2 \left(\frac{1}{R_c^2} - \frac{1}{R^2} \right)$$

On the other hand k_{ex} is connected with the pile period T through the formulas

$$I(\text{in hours}) = \frac{54}{T} + \frac{20.7}{T+0.62} + \frac{207}{T+22} + \frac{543}{T+6.5} + \frac{2070}{T+31.7} + \frac{802}{T+80.3}$$

$$k_{\text{ex}} = I(2.6 \times 10^{-5})$$

We can therefore write

$$2) \quad \frac{1}{I} \left(\text{or } \frac{1}{k_{\text{ex}}} \right) = C \left(\frac{1}{R_c^2} - \frac{1}{R^2} \right) - 1$$

and determine the constants C and R_c from the data. There is a complete analogy between equation (2) and equation (1) which holds before critical. In the form in which (2) has been set up, the points near critical which correspond to the longest periods will receive the most weight. The observational data are tabulated below:

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Order #	# Filled Channels	Loading Radius	Augmented Radius	I	$\frac{1}{I}$
7	369	220.22	273.22	27.19	3.68×10^{-2}
8	373	221.41	274.41	43.95	2.28
9	377	222.60	275.60	62.1	1.61
10	381	223.78	276.78	81.0	1.23
11	385	224.95	277.95	93.7	1.07
12	389	226.11	279.11	110	0.91

In setting up this table we had to correct for the small absorber worth 3.87 inhours introduced in the 7th loading. The reflector savings is taken at 53 cm as before. The least square solution yields

$$R_c = 271.40 \pm 0.05, r_c = 218.40 \pm 0.05$$

$$C = (0.6653 \pm 0.023) \times 10^{-8},$$

$$M^2 j_1^2 = \frac{2.6 \times 10^3}{0.6653},$$

and

$$M^2 = 676, \beta_c^2 = 0.970 \times 10^{-4}, k_\infty = 1.066$$

Correction For Unloaded Central Channels

J. Levin has brought to our attention a later report on the Clinton critical experiment (CP-1300). In this report, an attempt is made to correct for the 3 unloaded central channels by means of a calibration which showed that each channel was equivalent to 4 peripheral channels. For the small range of loading radii above critical, it should therefore be possible to correct the actual pile loading to a theoretically uniform loading by subtracting 9 channels from the loading. This rough correction corresponds to a decrease in pile area and hence in pile radius. We thus expect that the Laplacian will be somewhat increased. We find

$$M^2 = 645, \beta_c^2 = 0.987 \times 10^{-4}, k_\infty = 1.064.$$

Modification of the Theoretical Predictions

The migration area of 710 cm² predicted by the Chicago Handbook is

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based on a value of the neutron age in graphite at indium resonance energy of 334 cm^2 . However Fermi, Marshall and Marshall (CP-1084) measured the age of fission neutrons in graphite at 1.44 ev and found a value of $(17.8)^2 \text{ cm}^2$ reduced to a density of 1.6. They also calculated the same range on the basis of the Block Staub spectrum of fission neutrons and obtained $\theta_{(1.44 \text{ ev})} = (17.15)^2$. More recently Hill, Roberts and Mc Gammon (ORNL 187) in a series of careful measurements found $\theta = 310.6 \text{ cm}^2$. * If we take 311 cm^2 as the correct age we get $\theta_0 = 364$ for graphite and $\theta_0 = 370$ for the Clinton pile. The value of the migration area is then reduced from 709 cm^2 to 687 cm^2 .

Again the diffusion length of 50.2 cm accepted for the Clinton AGOT graphite may be low by about 1 cm due to their cadmium sheet technique of determining graphite quality. A 2% error in the diffusion length will result in a 4% correction to both f_{mod} and L^2 , the migration area remaining practically unchanged. However this changes f_{mod} from 0.106 to 0.102, f from 0.890 to 0.894 and hence k_{∞} from 1.067 to 1.072.

Summary

In collecting our results, we have first the following theoretical predictions for the Clinton Pile:

1. $L = 50.2, M^2 = 687: k_{\infty} = 1.067, \Delta_c = -97.5 \times 10^{-6}$
2. $L = 51.2, M^2 = 687: k_{\infty} = 1.072, \Delta_c = -105 \times 10^{-6}$

From the P.P.R. we get as experimental findings, corresponding to a reflector savings of 42.5 cm, and according as the factor 3×10^{-5} or 2.6×10^{-5} is used to convert inhours to excess k :

3. $(3 \times 10^{-5}): M^2 = 630, k_{\infty} = 1.066, \Delta_c = -104.5 \times 10^{-6}$
4. $(2.6 \times 10^{-5}): M^2 = 546, k_{\infty} = 1.066, \Delta_c = -104.5$

From our own analysis, using a reflector savings of 53 cm, a conversion factor of 2.6×10^{-5} and according as we do or do not correct for the non-uniformity of loading:

* All these values indicate that -10- the Chicago estimate is too high by about 20 cm^2 .

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5. (no loading correction): $M^2 = 676$, $k_{\infty} = 1.066$, $\Delta_c = -97.0 \times 10^{-6}$
6. (9 channel correction): $M^2 = 645$, $k_{\infty} = 1.064$, $\Delta_c = -98.7 \times 10^{-6}$

The low value of M^2 given by 4 indicates that the estimate of 42.5 cm for the reflector savings is much too low. There is reasonable agreement between 1 and either 5 or 6. However, our conjecture that the graphite quality of the Clinton pile was underestimated (2) leads to a value of k_{∞} in apparent disagreement with the experimental results.

For our best present estimate we have therefore

Theory (L = 50.2 cm): $M^2 = 687$, $k_{\infty} = 1.067$, $\Delta_c = -97.5 \times 10^{-6} \text{ cm}^{-2}$
Experiment (L = 53 cm): $M^2 = 645$, $k_{\infty} = 1.064$, $\Delta_c = -98.7 \times 10^{-6} \text{ cm}^{-2}$
with agreement well within 10%.

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