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S0131 — AN IBM-650 CODE TO SOLVE PRESSURE AND THERMAL STRESS PROBLEMS IN CORE SUBASSEMBLY PLATES

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S0131 — AN IBM-650 CODE TO SOLVE PRESSURE AND THERMAL STRESS PROBLEMS IN CORE SUBASSEMBLY PLATES

C. M. Friedrich

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In the design of nuclear reactor core subassemblies, a stress analysis is often made of a subassembly whose plates differ in geometry, in reactor coolant pressure loading, and in temperature distribution. Bettis code S0131 was developed for the IBM-650 computer to assess the adequacy of the subassembly design and to assist in determining acceptable manufacturing tolerances. The output of the code gives redundant weld reactions, deformations, and stresses in the individual plates.

S0131—AN IBM-650 CODE TO SOLVE PRESSURE AND THERMAL STRESS PROBLEMS IN CORE SUBASSEMBLY PLATES

C. M. Friedrich

INTRODUCTION

The two main causes of cross section stresses and deflections in the plates of a rectangular subassembly in a nuclear reactor core are (1) thermal stresses produced by heat generation in the plates containing uranium and (2) pressure stresses which arise when the fluid pressures in the channels between plates and outside the subassembly have different values. Various IBM-650 codes, using the Wolontis symbolic program, have been developed at Bettis to calculate the stresses and deflections.

Code S0102* is based on the "beam on an elastic foundation" theory and is suitable for dimensional parametric survey studies before detailed stress analysis is required.

Code W0040** is based on finite difference theory and is restricted to subassemblies with identical fuel plates. It gives the same results as code S0131 when applicable.

Code S9013† determines the moment distribution in a Vierendeel truss subjected to external loads and is therefore not suited to the problem of determining pressure and temperature stresses.

Code S0131 is particularly designed for a subassembly whose fuel plates differ in geometry as well as in pressure loading and temperature distribution. The differences can occur by design, by manufacturing tolerances, and by non-uniform thermal neutron flux distribution. The geometry is restricted to I-beam shaped clad fuel plates welded together to form two side plates as shown in Fig. 1. Dimensions, pressure loading, and temperature distribution are assumed to be symmetrical about the two centerlines and are assumed not to vary axially in the neighborhood of the cross sections

* "Stress Solution for Plate Type Seed Subassemblies," by F. V. Naugle, Bettis Plant

** "Subassembly Pressure and Temperature Stress," by N. S. Small and V. Davis, Bettis Plant

† "Moments in a Vierendeel Truss," by A. Martenson, Bettis Plant

Fig. 1 Typical Cross Section of I-Beam Shaped Clad Fuel

under consideration. The plane cross sections are assumed to remain plane and parallel to each other with no net force on the cross sections in the axial direction.

The method of solution in code S0131 consists of the following steps:

- 1) Consider a portion of the subassembly between two plane cross sections a unit distant apart in the axial direction perpendicular to the cross section.
- 2) Cut the cross section at the side plate welds and consider the weld reactions to be external loads acting on the I-beam shaped plates on either side of each weld cut.
- 3) Because of symmetry, only half of each I-beam need be considered. Each half is a T-beam as shown in Fig. 2. Consider the T-beam to be composed of three simple uniform flexible beams connected to a rigid element as shown in Fig. 3.

Fig. 2 Geometry of a Parallel Plate

Fig. 3 Free Body Elements of a Parallel Plate

- 4) The deflection equations and stress equations for each uniform beam, as developed in the following section, are used to determine the stresses and deflections of each uniform beam in terms of weld reactions, pressure loading, and temperature distribution.
- 5) The deflections for the three uniform beam elements of each T-beam are combined to give the weld deflections of each T-beam in terms of weld reactions, pressure loading, and temperature distribution, as described under the heading "Parallel Plate."
- 6) The weld deflections of adjoining T-beams are equated to give relationships that determine the weld reactions in terms of pressure loading and temperature distribution, as described under the heading "Matrix Equations."
- 7) Once the weld reactions are known, the equations of step 4 can be used to give the stresses and deflections of all flexible elements in the subassembly cross section in terms of pressure loading and temperature distribution.

The input and output cards for code S0131 are described under the heading "Input and Output Data."

SIMPLE PLATE THEORY

Cantilever Plate Notation

Each parallel plate can be separated into free bodies consisting of uniform plates with uniform pressure and temperature distributions. The deflections of each uniform plate are found relative to the tangent line at one end, which is the usual cantilever orientation shown in Fig. 4, along with the positive directions of all variables.

Fig. 4 Uniform Cantilever

Let

- x = distance along centerline of cantilever beam from left end, in.
- y = distance in cross section from centerline, in.
- z = axial distance to cross section from bottom of subassembly, in.
- h = thickness of plate in y direction, in.
- L = length of plate in x direction, in.
- M = bending moment per unit depth (unit depth is in z direction), lb

$$M_1 = M \text{ at } x = L$$

$$Q = \text{shear force per unit depth, lb/in.}$$

$$Q_1 = Q \text{ at } x = L$$

$$N = \text{tensile force per unit depth (negative if compressive), lb/in.}$$

$$N_1 = N \text{ at } x = L$$

$$p = \text{pressure on surface } y = 0.5h \text{ minus pressure on surface } y = -0.5h, \text{ psi}$$

$$\theta_{M_1} \begin{cases} = \text{working deflection of } M_1 \\ = \text{angular deflection at right end, radians} \end{cases}$$

$$w_{Q_1} \begin{cases} = \text{working deflection of } Q_1 \\ = \text{deflection in the } -y \text{ direction at right end, in.} \end{cases}$$

$$w_{N_1} \begin{cases} = \text{working deflection of } N_1 \\ = \text{elongation of the plate in the } x \text{ direction, in.} \end{cases}$$

$$T = \text{temperature in plate at distance } y \text{ from the middle surface of the plate, measured above the side plate temperature, } ^\circ\text{F (each side plate is assumed to be at zero reference temperature)}$$

$$T_N \begin{cases} = \text{effective tensile temperature, } ^\circ\text{F} \\ = \frac{1}{h} \int_{-0.5h}^{+0.5h} T \, dy \end{cases}$$

and

$$T_M \begin{cases} = \text{effective bending temperature, } ^\circ\text{F} \\ = \left(\frac{12}{h^2} \right) \int_{-0.5h}^{+0.5h} T \, y \, dy \end{cases}$$

Plate Theory

Bending Theory Stress

The basic elasticity equations (Ref 1) for the cantilever plate are

$$E \epsilon_x = S_x - \nu S_z + E\alpha T \quad (1A)$$

$$E \epsilon_z = S_z - \nu S_x + E\alpha T \quad (1B)$$

and the basic assumptions are

$$\epsilon_x = A + By, \quad (2A)$$

$$S_y = 0, \quad (2B)$$

$$\epsilon_z = \alpha T_z, \quad (2C)$$

where

E = tensile modulus of elasticity, psi,

ϵ_i = strain in direction i , in./in.,

S_i = stress in direction i , psi,

ν = Poisson's ratio,

α = thermal expansion coefficient, in./in.-°F,

A = average strain coefficient, in./in.,

B = bending strain coefficient, in./in.², and

T_z = average temperature of subassembly cross section, measured above side plate temperature, °F.

The assumptions of Eq (2) state, in effect, that

- 1) the plate cross section remains a plane during bending,
- 2) the stress S_y in the y direction is negligible in comparison with S_x and S_z , and
- 3) subassembly cross sections remain parallel.

Eliminating ϵ_z between Eq (1B) and Eq (2C) gives

$$S_z = \nu S_x + E\alpha(T_z - T), \quad (3)$$

which is used to determine axial stresses in code S0131. The cross section stress is determined from equilibrium relations between M and N obtained from Eqs (1A), (2A), (3). First, elimination of ϵ_x and S_z in Eqs (1A), (2A), and (3) gives

$$S_x = \left(\frac{E}{1 - \nu^2} \right) \left[A + By + \nu\alpha T_z - (1 + \nu)\alpha T \right]. \quad (4)$$

Next, consideration of equilibrium on the plate cross section at a general position x gives

$$N = \int_{-0.5h}^{+0.5h} S_x dy \quad (5A)$$

$$M = \int_{-0.5h}^{+0.5h} S_x y dy. \quad (5B)$$

Substituting Eq (4) into Eq (5) and solving for the strain coefficients A and B give

$$A = \left(\frac{1 - \nu^2}{Eh} \right) N + (1 + \nu) \alpha T_N - \nu \alpha T_z \quad (6A)$$

$$B = \left(\frac{12}{h^3} \cdot \frac{1 - \nu^2}{E} \right) M + \left(\frac{1 + \nu}{h} \right) \alpha T_M \quad (6B)$$

Finally, elimination of A and B from Eqs (4) and (6) gives

$$S_x = \left(\frac{1}{h} \right) N + \left(\frac{12y}{h^3} \right) M + \left(\frac{E\alpha}{1 - \nu} \right) \left[T_N + \frac{y}{h} T_M - T \right] \quad (7)$$

which is a useful equation for cross section stresses. It is to be noted that if the temperature distribution is linear, then

$$T = T_N + (y/h) T_M$$

and

$$S_x = (1/h) N + (12y/h^3) M$$

Bending Theory Deflections

Physically, the coefficient A in Eq (2) is the elongation of the plate per unit length x, and the coefficient B is rate of change of slope per unit length x; that is,

$$\frac{dw_N}{dx} = A$$

and

$$\frac{d\theta_M}{dx} = B$$

Substitution of A and B from Eq (6) gives

$$\frac{dw_N}{dx} = \left(\frac{1 - \nu^2}{Eh} \right) N + (1 + \nu) \alpha T_N - \nu \alpha T_z \quad (8A)$$

$$\frac{d\theta_M}{dx} = \left(\frac{12}{h^3} \cdot \frac{1 - \nu^2}{E} \right) M + \left(\frac{1 + \nu}{h} \right) \alpha T_M \quad (8B)$$

Before these equations can be integrated with respect to x, the loads must be given as functions of x. By inspection of Fig. 4a, it is seen that

$$N = N_1 \quad (9A)$$

$$Q = Q_1 + p (L - x) \quad (9B)$$

$$M = M_1 + Q_1 (L - x) + 0.5 p (L - x)^2 \quad (9C)$$

Substituting these values into Eq (8) and integrating over the general range 0 to x gives the elongation and slope as

$$w_N = \left[\left(\frac{1 - \nu^2}{Eh} \right) N_1 + (1 + \nu) \alpha T_N - \nu \alpha T_z \right] x \quad (10A)$$

$$\theta_M = \left(\frac{12}{h^3} \cdot \frac{1 - \nu^2}{E} \right) \left[M_1 x + Q_1 \left(Lx - \frac{x^2}{2} \right) + p \left(\frac{L^2 x}{2} - \frac{Lx^2}{2} + \frac{x^3}{6} \right) \right] + \left[\left(\frac{1 + \nu}{h} \right) \alpha T_M \right] x \quad (10B)$$

Thus, at $x = L$

$$w_{N_1} = \left(\frac{1 - \nu^2}{E} \cdot \frac{L}{h} \right) N_1 + \alpha L \left[(1 + \nu) T_N - \nu T_z \right] \quad (11A)$$

$$\theta_{M_1} = \left(\frac{12}{h^3} \cdot \frac{1 - \nu^2}{E} \right) \left[(L) M_1 + \left(\frac{L^2}{2} \right) Q_1 + \left(\frac{L^3}{6} \right) p \right] + \alpha \left(\frac{L}{h} \right) (1 + \nu) T_M \quad (11B)$$

Since slope is the rate of change of deflection,

$$dw_Q/dx = \theta_M$$

Substitution of θ_M from Eq (10) and integration from 0 to L gives the end deflection as

$$w_{Q_1} = \left(\frac{12}{h^3} \cdot \frac{1 - \nu^2}{E} \right) \left[\left(\frac{L^2}{2} \right) M_1 + \left(\frac{L^3}{3} \right) Q_1 + \left(\frac{L^4}{8} \right) p \right] + \alpha \left(\frac{L^2}{2h} \right) (1 + \nu) T_M \quad (12)$$

Shear Theory Stress

Consider a unit depth of the cantilever plate in Fig. 5 bounded by the surfaces x , $x + dx$, y , and $y = 0.5h$. Let S_s be the shear stress on the surface at y , positive as shown. Summing forces on the element in the x direction gives

$$0 = - \int_y^{0.5h} S_x dy + S_s dx + \int_y^{0.5h} \left(S_x + \frac{\partial S_x}{\partial x} dx \right) dy$$

$$S_s = - \int_y^{0.5h} \left(\frac{\partial S_x}{\partial x} \right) dy \quad (13)$$

In Eq (7) for S_x , the only quantity varying with x is M . From Eq (9C), $\partial M / \partial x = -Q_1 - p(L - x)$ which, according to Eq (9B), is

$$\frac{\partial M}{\partial x} = -Q \quad (14)$$

Substituting S_x from Eq (7) and $\partial M / \partial x$ from Eq (14) into Eq (13) and integrating give

$$S_s = \left(\frac{1.5}{h} \right) \left(1 - \frac{4y^2}{h^2} \right) Q \quad (15)$$

The maximum shear stress at $y = 0$ is seen to be 50% more than the average stress Q/h .

Shear Theory Deflection

The shear strain energy in the plate per unit cross section area is

$$dU_s = \frac{S_s^2}{2} \left(\frac{2(1 + \nu)}{E} \right) dx dy$$

so that the total shear strain energy is, by use of Eq (9) and Eq (15),

$$U_s = \frac{1 + \nu}{E} \int_0^L dx \int_{-0.5h}^{+0.5h} dy \left[\left(\frac{1.5}{h} \right) \left(1 - \frac{4y^2}{h^2} \right) (Q_1 + pL_1 - px) \right]^2$$

By Castigliano's theorem, the working deflection of load Q_1 produced by the shear strain energy U_s is

$$\left(w_{Q_1}\right)_s = \frac{\partial U_s}{\partial Q_1}.$$

Substituting U_s into this equation and simplifying gives

$$\left(w_{Q_1}\right)_s = \left[\frac{2.4(1+\nu)}{Eh} \right] \left[(L) Q_1 + \left(\frac{L^2}{2} \right) p \right]$$

Thus, the total working deflection of Q_1 is

$$w_{Q_1} = \left[w_{Q_1} \text{ from Eq (12)} \right] + \left(w_{Q_1}\right)_s$$

or

$$\begin{aligned} w_{Q_1} = & \left(\frac{6L^2}{h^3} \cdot \frac{1-\nu^2}{E} \right) M_1 + \left(\frac{4L^3}{h^3} \cdot \frac{1-\nu^2}{E} + \frac{2.4L}{h} \cdot \frac{1+\nu}{E} \right) Q_1 \\ & + \left(\frac{1.5L^4}{h^3} \cdot \frac{1-\nu^2}{E} + \frac{1.2L^2}{h} \cdot \frac{1+\nu}{E} \right) p + \alpha \left(\frac{L^2}{2h} \right) (1+\nu) T_M. \end{aligned} \quad (16)$$

PARALLEL PLATE

Notation

Geometry

Each of the parallel plates can be considered to be composed of uniform plates with uniform pressure and temperature loadings. The loads on the uniform plates, determined from the loads on the parallel plate, give the deflections of the uniform plates. These deflections can be combined to give the total weld deflections of the parallel plate. A systematic notation, however, is needed first. As shown in Fig. 2, let

- a = subscript for the middle surface of the side plate at the left weld
- b = subscript for the middle surface of the side plate at the right weld
- c = subscript for the middle surface of the parallel plate itself at the center
- h_a = thickness of left side plate, in.
- h_b = thickness of right side plate, in.
- h_f = thickness of parallel plate, in.
- L_T = half of the heated length of the parallel plate, in.
- L_{fa} = half of the length of the channel at weld a, in.
- L_{fb} = half of the length of the channel at weld b, in.
- L_f = the smaller of L_{fa} and L_{fb} , in. (In Fig. 2, $L_f = L_{fa}$)
- L_{pa} = "overhang" length of side plate at weld a, in.
- L_{pb} = "overhang" length of side plate at weld b, in.
- $L_{sa} = L_{pa} + 0.5h_f$ = distance of weld a from the middle surface of the parallel plate, in.
- $L_{sb} = L_{pb} + 0.5h_f$ = distance of weld b from the middle surface of the parallel plate, in.

and

- f = subscript for the middle surface of the side plate that is a distance L_f from the centerline

Forces

Free bodies of the flexible elements of the parallel plate are shown in Fig. 3, for which the notation is

$i = a, b, \text{ or } f$

$N_i =$ tensile force per unit depth at i , lb/in.

$Q_i =$ shear force per unit depth at i , lb/in.

$M_i =$ bending moment per unit depth at i , lb

$L_a =$ effective flexible length of the side plate near weld a, in.

$L_b =$ effective flexible length of the side plate near weld b, in.

$P_a =$ pressure in the channel at weld a, psi

and

$P_b =$ pressure in the channel at weld b, psi

By symmetry there is no shear at c, so that summing forces horizontally and solving for N_b gives

$$N_b = N_a - P_a L_{fa} + P_b L_{fb} \quad (17)$$

By inspection of Fig. 3, the reactions at f are seen to be

$$N_f = -Q_a + Q_b + P_a L_{pa} + P_b L_{pb} \quad (18A)$$

$$Q_f = (P_a - P_b)L_f \quad (18B)$$

$$\begin{aligned} M_f = & + (L_{fa} + 0.5h_a - L_f)N_a + (L_{sa})Q_a - M_a \\ & - (L_{fb} + 0.5h_b - L_f)N_b + (L_{sb})Q_b + M_b \\ & + (L_{sa} - 0.5L_{pa})(P_a - P_{pa}) - 0.5(L_f - L_{fa})^2 P_a \\ & + (L_{sb} - 0.5L_{pb})(P_b - P_{pb}) + 0.5(L_f - L_{fb})^2 P_b \end{aligned} \quad (18C)$$

It is to be noted that the channel pressures are measured relative to the pressure on the outside surfaces of the two side plates. The outside pressure is taken to be at zero reference pressure.

The flexible lengths L_a and L_b can be estimated by an empirical formula.* If h_a is less than h_f , L_a should be near L_{pa} , and if h_a is greater than h_f , L_a should be near L_{sa} ; therefore, let

$$L_a = (h_a L_{sa} + h_f L_{pa}) / (h_a + h_f)$$

and

$$L_b = (h_b L_{sb} + h_f L_{pb}) / (h_b + h_f)$$

Deflections

Positive values of the following deflections are shown in Fig. 6:

$\theta_a =$ working deflection of $M_a =$ slope at weld a, radians

$w_a =$ working deflection of $Q_a =$ inward deflection at weld a, in.

* A study of photoelastic results would be better.

Fig. 6 Deflections of a Parallel Plate

θ_b = working deflection of M_b = slope at weld b, radians

w_b = working deflection of Q_b = outward deflection at weld b, in.

θ_f = slope of parallel plate at f, radians

w_f = outward deflection of side plate at f, in.

and

w_c = lateral bow of parallel plate towards the right at c, in.

Deflection Equations

The only part of the parallel plate that has a nonzero temperature distribution is the heated length $2L_T$. If the effective tensile temperature T_N and effective bending temperature T_M are assumed constant over the length $2L_T$, then the deflections at f and c are,

from Eq (11) and Eq (16),

$$w_f = \left(\frac{1 - \nu_f^2}{E_f} \cdot \frac{L_f}{h_f} \right) N_f + \alpha_f L_T (1 + \nu_f) T_N, \quad (19A)$$

where

T_z is not used, since $\alpha_f (1 + \nu_f) T_z$ is the same for all plates and so has no effect on weld reactions,

$$\theta_f = \left[\left(\frac{1 - \nu_f^2}{E_f} \cdot \frac{12 L_f}{h_f^3} \right) M_f + \left(\frac{1 - \nu_f^2}{E_f} \cdot \frac{6 L_f^2}{h_f^3} \right) Q_f + \left(\frac{1 - \nu_f^2}{E_f} \cdot \frac{2 L_f^3}{h_f^3} \right) (P_b - P_a) + \alpha_f \left(\frac{L_T}{h_f} \right) (1 + \nu_f) T_M \right], \quad (19B)$$

and

$$w_c = \left[\left(\frac{1 - \nu_f^2}{E_f} \cdot \frac{6 L_f^2}{h_f^3} \right) M_f + \left(\frac{1 - \nu_f^2}{E_f} \cdot \frac{4 L_f^3}{h_f^3} + \frac{1 + \nu_f}{E_f} \cdot \frac{2 \cdot 4 L_f}{h_f} \right) Q_f + \left(\frac{1 - \nu_f^2}{E_f} \cdot \frac{1.5 L_f^4}{h_f^3} + \frac{1 + \nu_f}{E_f} \cdot \frac{1.2 L_f^2}{h_f^3} \right) (P_b - P_a) + \alpha_f \left(\frac{L_T^2}{2 h_f} \right) (1 + \nu_f) T_M \right] + \left[\alpha_f \left(\frac{L_T}{h_f} \right) (1 + \nu_f) T_M \right] (L_f - L_T). \quad (19C)$$

The last term in the w_c equation is the lateral deflection at f caused by the thermal slope $\alpha(L_T/h_f)(1 + \nu)T_M$ acting over the length $(L_f - L_T)$.

The deflections at a are produced by the movement of the side plate caused by the deflections at f and by the deformation of the side plate over the flexible length L_a ; that is, if P_a is assumed to act over the flexible length L_a ,

$$\theta_a = -\theta_f + \left[\left(\frac{1 - \nu_a^2}{E_a} \cdot \frac{12 L_a}{h_a^3} \right) M_a - \left(\frac{1 - \nu_a^2}{E_a} \cdot \frac{6 L_a^2}{h_a^3} \right) Q_a + \left(\frac{1 - \nu_a^2}{E_a} \cdot \frac{2 L_a^3}{h_a^3} \right) P_a \right] \quad (20A)$$

$$\begin{aligned}
w_a = & -w_f + (L_{sa})\theta_f + \left[-\left(\frac{1-\nu_a^2}{E_a} \cdot \frac{6L_a^2}{h_a^3}\right)M_a \right. \\
& + \left(\frac{1-\nu_a^2}{E_a} \cdot \frac{4L_a^3}{h_a^3} + \frac{1+\nu_a}{E_a} \cdot \frac{2 \cdot 4L_a}{h_a^3}\right)Q_a \\
& \left. - \left(\frac{1-\nu_a^2}{E_a} \cdot \frac{1.5L_a^4}{h_a^3} + \frac{1+\nu_a}{E_a} \cdot \frac{1.2L_a^2}{h_a^3}\right)P_a \right] \quad (20B)
\end{aligned}$$

Similarly at b,

$$\theta_b = \theta_f + \left[\left(\frac{1-\nu_b^2}{E_b} \cdot \frac{12L_b}{h_b^3}\right)M_b + \left(\frac{1-\nu_b^2}{E_b} \cdot \frac{6L_b^2}{h_b^3}\right)Q_b + \left(\frac{1-\nu_b^2}{E_b} \cdot \frac{2L_b^3}{h_b^3}\right)P_b \right] \quad (21A)$$

$$\begin{aligned}
w_b = & w_f + (L_{sb})\theta_f + \left[\left(\frac{1-\nu_b^2}{E_b} \cdot \frac{6L_b^2}{h_b^3}\right)M_b \right. \\
& + \left(\frac{1-\nu_b^2}{E_b} \cdot \frac{4L_b^3}{h_b^3} + \frac{1+\nu_b}{E_b} \cdot \frac{2 \cdot 4L_b}{h_b^3}\right)Q_b \\
& \left. + \left(\frac{1-\nu_b^2}{E_b} \cdot \frac{1.5L_b^4}{h_b^3} + \frac{1+\nu_b}{E_b} \cdot \frac{1.2L_b^2}{h_b^3}\right)P_b \right] \quad (21B)
\end{aligned}$$

MATRIX EQUATIONS

Matrix Rules

To keep track of the many simultaneous equations, the equations will be rewritten in matrix form in blocks of 2 x 2. The following rules will be used:

Matrix

A matrix is a block of 4 numbers arranged in two rows and two columns. For example, let

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

where A_{ij} is the number in row i and column j .

Factor

A number k times a matrix A is given by

$$kA = k \times \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} kA_{11} & kA_{12} \\ kA_{21} & kA_{22} \end{bmatrix}$$

If $k = -1$, then $-A =$

$$-A = \begin{bmatrix} -A_{11} & -A_{12} \\ -A_{21} & -A_{22} \end{bmatrix}$$

Addition

A matrix A plus a matrix B is given by

$$A + B = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} A_{11} + B_{11} & A_{12} + B_{12} \\ A_{21} + B_{21} & A_{22} + B_{22} \end{bmatrix}$$

Subtraction

$$A - B = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} - \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} A_{11} - B_{11} & A_{12} - B_{12} \\ A_{21} - B_{21} & A_{22} - B_{22} \end{bmatrix}$$

Multiplication

$$A \times B = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \times \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} A_{11}B_{11} + A_{12}B_{21} & A_{11}B_{12} + A_{12}B_{22} \\ A_{21}B_{11} + A_{22}B_{21} & A_{21}B_{12} + A_{22}B_{22} \end{bmatrix}$$

In general, $A \times B \neq B \times A$. Note $A \times B \times C = (A \times B) \times C = A \times (B \times C)$.

Zero

A zero matrix has each element equal to 0.

$$0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Equality

$A = B$ if and only if $A_{11} = B_{11}$, $A_{12} = B_{12}$, $A_{21} = B_{21}$, and $A_{22} = B_{22}$.

Unity

The unit matrix U is defined by

$$U = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Note that $U \times A = A \times U = A$ for any A.

Transpose

The transpose A^T of matrix A is

$$A^T = \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix}; \text{ that is, } A_{ij}^T = A_{ji}.$$

Inverse

The inverse A^I of matrix A is

$$A^I = \begin{bmatrix} \left(\frac{A_{22}}{A_{11}A_{22} - A_{12}A_{21}} \right) & \left(\frac{-A_{12}}{A_{11}A_{22} - A_{12}A_{21}} \right) \\ \left(\frac{-A_{21}}{A_{11}A_{22} - A_{12}A_{21}} \right) & \left(\frac{A_{11}}{A_{11}A_{22} - A_{12}A_{21}} \right) \end{bmatrix}$$

and it exists only if $A_{11}A_{22} \neq A_{12}A_{21}$. Note that $A^I \times A = A \times A^I = U$.

Basic Formulae

Side Plate Deflections at f

Each force and deflection is separated into a term produced by pressure loading (p) and a term produced by thermal loading (t). From Eq (19), *

$$\begin{bmatrix} \theta_{fp} & \theta_{ft} \\ w_{fp} & w_{ft} \end{bmatrix} = \begin{bmatrix} \frac{1 - \nu_f^2}{E_f} \cdot \frac{12L_f}{h_f^3} & 0 \\ 0 & \frac{1 - \nu_f^2}{E_f} \cdot \frac{L_f}{h_f} \end{bmatrix} \times \begin{bmatrix} M_{fp} & M_{ft} \\ N_{fp} & N_{ft} \end{bmatrix} + \begin{bmatrix} \left(\frac{1 - \nu_f^2}{E_f} \cdot \frac{4L_f^3}{h_f^3} \right) (P_a - P_b) & \alpha_f \left(\frac{I_{xI}}{h_f} \right) (1 + \nu_f) T_M \\ 0 & \alpha_f L_T \left[(1 + \nu_f) T_N \right] \end{bmatrix}$$

Let

$$R_f = \begin{bmatrix} M_{fp} & M_{ft} \\ N_{fp} & N_{ft} \end{bmatrix}$$

represent the unknown reactions at f, and

$$D_f = \begin{bmatrix} \theta_{fp} & \theta_{ft} \\ w_{fp} & w_{ft} \end{bmatrix}$$

represent the corresponding deflections. Then the matrix equation can be written as

$$D_f = A_{ff} R_f + D_{fe} \quad (22)$$

where

$$A_{ff} = \begin{bmatrix} \frac{1 - \nu_f^2}{E_f} \cdot \frac{12L_f}{h_f^3} & 0 \\ 0 & \frac{1 - \nu_f^2}{E_f} \cdot \frac{L_f}{h_f} \end{bmatrix}$$

represents the flexibility matrix giving the deflections D_f produced by the reactions R_f and

* Note that $Q_f = (P_a - P_b)L_f$. Also, the expansion by T_z is the same at each weld of neighboring parallel plates and so produced no effect on weld reactions.

$$D_{fe} = \begin{array}{|c|c|} \hline \left(\frac{1 - \nu_f^2}{E_f} \cdot \frac{4L_f^3}{h_f^3} \right) (P_a - P_b) & \alpha_f \left(\frac{L_T}{h_f} \right) (1 + \nu_f) T_M \\ \hline 0 & \alpha_f L_t (1 + \nu_f) T_N \\ \hline \end{array}$$

represents the deflections D_f produced by the plate environment conditions of pressure and temperature. The reactions R_f are given with respect to the weld reactions and the environment pressure in Eq (18). Here the equations are put in matrix form with the assumption that N_a and N_b are affected only by pressure, not temperature.

$$\begin{array}{|c|c|} \hline M_{fp} & M_{ft} \\ \hline N_{fp} & N_{ft} \\ \hline \end{array} = \begin{array}{|c|c|} \hline -1 & L_{sa} \\ \hline 0 & -1 \\ \hline \end{array} \times \begin{array}{|c|c|} \hline M_{ap} & M_{at} \\ \hline Q_{ap} & Q_{at} \\ \hline \end{array} + \begin{array}{|c|c|} \hline 1 & L_{sb} \\ \hline 0 & 1 \\ \hline \end{array} \times \begin{array}{|c|c|} \hline M_{bp} & M_{bt} \\ \hline Q_{bp} & Q_{bt} \\ \hline \end{array}$$

$$+ \begin{array}{|c|c|} \hline + (L_{fa} + 0.5h_a - L_f)N_a - (L_{fb} + 0.5h_b - L_f)N_b \\ - (L_{sa} - 0.5h)(P_a L_{pa}) - 0.5(L_f - L_{fa})^2 P_a \\ + (L_{sb} - 0.5L_{pb})(P_b L_{pb}) + 0.5(L_f - L_{fb})^2 P_b \\ \hline P_a L_{pa} + P_b L_{pb} \\ \hline \end{array} \begin{array}{|c|} \hline 0 \\ \hline 0 \\ \hline \end{array}$$

Let

$$R_a = \begin{array}{|c|c|} \hline M_{ap} & M_{at} \\ \hline Q_{ap} & Q_{at} \\ \hline \end{array} \quad \text{and} \quad R_b = \begin{array}{|c|c|} \hline M_{bp} & M_{bt} \\ \hline Q_{bp} & Q_{bt} \\ \hline \end{array}$$

represent the reactions at welds a and b. Then,

$$R_f = C_{fa} R_a + C_{fb} R_b + R_{fe} \quad (23)$$

where

$$C_{fa} = \begin{array}{|c|c|} \hline -1 & L_{sa} \\ \hline 0 & -1 \\ \hline \end{array}, \quad C_{fb} = \begin{array}{|c|c|} \hline 1 & L_{sb} \\ \hline 0 & 1 \\ \hline \end{array}$$

and

$$R_{fe} = \begin{array}{|c|c|} \hline (L_{fa} + 0.5h_a - L_f)N_a - (L_{fb} + 0.5h_b - L_f)N_b \\ - (L_{sa} - 0.5L_{pa})(P_a L_{pa}) - 0.5(L_f - L_{fa})^2 P_a \\ + (L_{sb} - 0.5L_{pb})(P_b L_{pb}) + 0.5(L_f - L_{fb})^2 P_b \\ \hline P_a L_{pa} + P_b L_{pb} \\ \hline \end{array} \begin{array}{|c|} \hline 0 \\ \hline 0 \\ \hline \end{array}$$

Deflections at a

Putting Eq (20) in matrix form gives

θ_{ap}	θ_{at}
w_{ap}	w_{at}

$$=$$

$\left(\frac{1 - \nu_a^2}{E_a} \cdot \frac{12L_a^2}{h_a^3} \right)$	$-\left(\frac{1 - \nu_a^2}{E_a} \cdot \frac{6L_a^2}{h_a^3} \right)$
$-\left(\frac{1 - \nu_a^2}{E_a} \cdot \frac{6L_a^2}{h_a^3} \right)$	$\frac{1 - \nu_a^2}{E_a} \cdot \frac{4L_a^3}{h_a^3} + \frac{1 + \nu_a}{E_a} \cdot \frac{2.4L_a}{h_a}$

$$\times$$

M_{ap}	M_{at}
Q_{ap}	Q_{at}

$$+$$

-1	0
L_{sa}	-1

$$\times$$

θ_{fp}	θ_{ft}
w_{fp}	w_{ft}

$$+$$

$\left(\frac{1 - \nu_a^2}{E_a} \cdot \frac{2L_a^3}{h_a^3} \right) P_a$	0
$-\left(\frac{1 - \nu_a^2}{E_a} \cdot \frac{1.5L_a^4}{h_a^3} + \frac{1 + \nu_a}{E_a} \cdot \frac{1.2L_a^2}{h_a} \right) P_a$	0

Let

$$D_a = \begin{bmatrix} \theta_{ap} & \theta_{at} \\ w_{ap} & w_{at} \end{bmatrix}$$

represent the pressure and temperature components of the deflections at weld a.

Then,

$$D_a = A_{aa} R_a + C_{fa}^T D_f + D_{ae} \quad (24)$$

where

$$A_{aa} = \begin{bmatrix} \left(\frac{1 - \nu_a^2}{E_a} \cdot \frac{12L_a^2}{h_a^3} \right) & -\left(\frac{1 - \nu_a^2}{E_a} \right) \left(\frac{6L_a^2}{h_a^3} \right) \\ -\left(\frac{1 - \nu_a^2}{E_a} \cdot \frac{6L_a^2}{h_a^3} \right) & \frac{1 - \nu_a^2}{E_a} \cdot \frac{4L_a^3}{h_a^3} + \frac{1 + \nu_a}{E_a} \cdot \frac{12L_a}{h_a} \end{bmatrix}$$

represents the flexibility matrix giving the weld deflections D_a in terms of the weld reactions R_a .
 C_{fa}^T is the transpose of C_{fa} and

$$D_{aa} = \begin{bmatrix} \left(\frac{1 - \nu_a^2}{E_a} \cdot \frac{2L_a^3}{h_a^3} \right) P_a & 0 \\ -\left(\frac{1 - \nu_a^2}{E_a} \cdot \frac{1.5L_a^4}{h_a^3} + \frac{1 + \nu_a}{E_a} \cdot \frac{1.2L_a^2}{h_a} \right) P_a & 0 \end{bmatrix}$$

represents the deflections at weld a in terms of the environment loading on the side plate near weld a.

Deflections at b

In the same way as for weld a, Eq (21) is put in the matrix form

$$D_b = A_{bb} R_a + C_{fb}^T R_f + D_{be} \quad (25)$$

where

$$D_b = \begin{bmatrix} \theta_{bp} & \theta_{bt} \\ w_{bp} & w_{bt} \end{bmatrix}$$

$$A_{bb} = \begin{bmatrix} \frac{1 - \nu_b^2}{E_b} \cdot \frac{12L_b^2}{h_b^3} & \frac{1 - \nu_b^2}{E_b} \cdot \frac{6L_b^2}{h_b^3} \\ \frac{1 - \nu_b^2}{E_b} \cdot \frac{6L_b^2}{h_b^3} & \frac{1 - \nu_b^2}{E_b} \cdot \frac{4L_b^3}{h_b^3} + \frac{1 + \nu_b}{E_b} \cdot \frac{2 \cdot 4L_b}{h_b} \end{bmatrix}$$

and

$$D_{be} = \begin{bmatrix} \left(\frac{1 - \nu_b^2}{E_b} \cdot \frac{2L_b^3}{h_b^3} \right) P_b & 0 \\ \left(\frac{1 - \nu_b^2}{E_b} \cdot \frac{1.5L_b^4}{h_b^3} + \frac{1 + \nu_b}{E_b} \cdot \frac{1.2L_b^2}{h_b} \right) P_b & 0 \end{bmatrix}$$

Stresses at a, b, c, and f

Cross Section Stress—Let "in" represent the bottom surface of the side plate or the right surface of the parallel plate in Fig. 3, and let "out" represent the other surface. Then for any location on either plate, from Eq (7),

$$\begin{bmatrix} S_{xp(in)} & S_{xt(in)} \\ S_{xp(out)} & S_{xt(out)} \end{bmatrix} = \begin{bmatrix} \frac{6}{h^2} & \frac{1}{h} \\ -\frac{6}{h^2} & \frac{1}{h} \end{bmatrix} \times \begin{bmatrix} M_p & M_T \\ N_p & N_T \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & \left(\frac{E\alpha}{1 - \nu} \right) \left[T_N + 0.5T_M - T(in) \right] \\ 0 & \left(\frac{E\alpha}{1 - \nu} \right) \left[T_N - 0.5T_M - T(out) \right] \end{bmatrix}$$

Since $T_{(in)} = T_{(out)} = T_N = T_M = 0$ at a, b, and f, and since $T_{(in)}$ and $T_{(out)}$ are not specified at c in the S0131 code, the equation used in the code is simply

$$\begin{bmatrix} S_{xp(in)} & S_{xt(in)} \\ S_{xp(out)} & S_{xt(out)} \end{bmatrix} = \begin{bmatrix} \frac{6}{h^2} & \frac{1}{h} \\ -\frac{6}{h^2} & \frac{1}{h} \end{bmatrix} \times \begin{bmatrix} M_p & M_T \\ N_p & N_t \end{bmatrix} \quad (26)$$

code

Thus, in examining the output of the code, the following correction should be made to S_x at c:

$$S_x = (S_x)_{code} + \left(\frac{E\alpha}{1 - \nu} \right) \left[T_N + \left(\frac{y}{h} \right) T_M - T \right] \quad (27)$$

Axial Stress—The equation used for S_z in the S0131 code is

$$(S_z)_{code} = \nu(S_x)_{code} + E\alpha(T_z - T_N) \quad (28)$$

Comparison of Eqs (3), (27), and (28) shows that the following correction should be made to obtain the true value of S_z at c:

$$S_z = (S_z)_{\text{code}} + \left(\frac{E\alpha}{1-\nu} \right) \left[T_N + \left(\frac{\nu Y}{h} \right) T_M - T \right]. \quad (29)$$

Shear Stress—According to Eq (15), the shear stress at either surface is zero and the stress at the middle surface of a plate is maximum. This maximum shear stress in the S0131 code is correctly given by the following formula:

$$(S_s)_{\text{code}} = 1.5 Q/h \quad (30)$$

for locations a, b, c, and f. Note that $S_s = 0$ at c.

Parallel Plate Formulae

Weld Deflections

The weld deflections can be obtained in terms of weld reactions and environmental pressure and temperature loads by substituting the reaction R_f from Eq (23) and the deflection D_f from Eq (22) into the weld deflection equations. Thus, in Eq (24)

$$D_a = A_{aa} R_a + C_{fa}^T \left[A_{ff} (C_{fa} R_a + C_{fb} R_b + R_{fe}) + D_{fe} \right] + D_{ae}$$

or

$$D_a = \left(A_{aa} + C_{fa}^T A_{ff} C_{fa} \right) R_a + \left(C_{fa}^T A_{ff} C_{fb} \right) R_b + \left(C_{fa}^T A_{ff} R_{fe} + C_{fa}^T D_{fe} + D_{ae} \right) \quad (31)$$

Similarly, substitution in Eq (25) gives

$$D_b = \left(C_{fb}^T A_{ff} C_{fa} \right) R_a + \left(A_{bb} + C_{fb}^T A_{ff} C_{fb} \right) R_b + \left(C_{fb}^T A_{ff} R_{fe} + C_{fb}^T D_{fe} + D_{be} \right) \quad (32)$$

Weld Deflection Notation

To keep track of individual plates and welds, the following notation is used.

Let

n = total number of parallel plates ($n \leq 8$ in S0131)

j = any plate, 1 to 8, numbered from left to right, or the weld at the right end of the j^{th} plate

R_j = weld reactions at b on plate j or at a on plate $j+1$ (Note $R_0 = R_a$ on plate $j=1$.)

D_j = working deflections of R_j at weld j (Note $D_0 = D_a$ on plate $j=1$.)

Suppose that the plates between welds i and j have been welded together so that the weld deflections D_i and D_j are determined by the weld reactions R_i , R_j and the environment pressure and thermal loading between weld i and j ; that is, let

$$(D_i)_{ij} = (A_{ii})_{ij} R_i + (A_{ij})_{ij} R_j + (D_{ie})_{ij} \quad (33A)$$

$$(D_j)_{ij} = (A_{ji})_{ij} R_i + (A_{jj})_{ij} R_j + (D_{je})_{ij}, \quad (33B)$$

where

$(D_i)_{ij}$ = D_i on the plates $i+1, i+2, \dots, j$ welded together between welds i and j ,

$(A_{ij})_{ij}$ = flexibility matrix giving $(D_i)_{ij}$ in terms of R_j ,

and

$(D_{ie})_{ij}$ = D_i produced by environment between i and j .

In particular, if $i = j - 1$, then for plate j , a comparison of Eqs (31), (32), and (33) shows that

$$(A_{j-1,j-1})_{j-1,j} = A_{aa} + C_{fa}^T A_{ff} C_{fa} \quad (34A)$$

$$(A_{j-1,j})_{j-1,j} = C_{fa}^T A_{ff} C_{fb} \quad (34B)$$

$$(D_{j-1,e})_{j-1,1} = C_{fa}^T A_{ff} R_{fe} + C_{fa}^T D_{fe} + D_{ae} \quad (34C)$$

$$(A_{j,j-1})_{j-1,j} = C_{fb}^T A_{ff} C_{fa} \quad (34D)$$

$$(A_{jj})_{j-1,j} = A_{bb} + C_{fb}^T A_{ff} C_{fb} \quad (34E)$$

$$(D_{je})_{j-1,j} = C_{fb}^T A_{ff} R_{fe} + C_{fb}^T D_{fe} + D_{be} \quad (34F)$$

Because of the reciprocal theorem*, the following relationships exist for the flexibility matrices in Eq (33):

- 1) $(A_{ii})_{ij}$ and $(A_{jj})_{ij}$ are symmetrical; that is, $(A_{ii})_{ij} = (A_{ii}^T)_{ij}$ and $(A_{jj})_{ij} = (A_{jj}^T)_{ij}$.
- 2) $(A_{ij})_{ij}$ and $(A_{ji})_{ij}$ are transposes of each other; that is $(A_{ij})_{ij} = (A_{ji}^T)_{ij}$ and $(A_{ji})_{ij} = (A_{ij}^T)_{ij}$.

Combination of Parallel Plates

The basic equations are now available for combining groups of parallel plates. Suppose the equations for the group of plates between welds i and j and for the group of plates between welds j and k are known. At the joint j , the working deflections of the weld reactions R_j on the group of plates between i and j are $(D_j)_{ij}$ and the working deflections of R_j on the group of plates between j and k are $(D_j)_{jk}$. When the groups ij and jk are joined to form a single group ik , the reactions R_j become internal and can do no work; that is, the net working deflection must be zero. Thus,

$$0 = (D_j)_{ij} + (D_j)_{jk}$$

Substituting deflections from Eq (33) into the preceding equation gives

$$0 = \left[(A_{ji})_{ij} R_i + (A_{jj})_{ij} R_j + (D_{je})_{ij} \right] + \left[(A_{jj})_{jk} R_j + (A_{jk})_{jk} R_k + (D_{je})_{jk} \right]$$

or

$$0 = (A_{ji})_{ij} R_i + \left[(A_{jj})_{ij} + (A_{jj})_{jk} \right] R_j + (A_{jk})_{jk} R_k + \left[(D_{je})_{ij} + (D_{je})_{jk} \right]$$

This can be solved for R_j by multiplying each term by the inverse matrix of $\left[(A_{jj})_{ij} + (A_{jj})_{jk} \right]$. The result is

$$R_j = (C_{ji})_{ik} R_i + (C_{jk})_{ik} R_k + (R_{je})_{ik} \quad (35)$$

where

$$(C_{ji})_{ik} = - \left[(A_{jj})_{ij} + (A_{jj})_{jk} \right]^{-1} \times (A_{ji})_{ij},$$

$$(C_{jk})_{ik} = - \left[(A_{jj})_{ij} + (A_{jj})_{jk} \right]^{-1} \times (A_{jk})_{jk},$$

and

$$(R_{je})_{ik} = - \left[(A_{jj})_{ij} + (A_{jj})_{jk} \right]^{-1} \times \left[(D_{je})_{ij} + (D_{je})_{jk} \right].$$

* The working deflection of one load produced by a unit value of a second load equals the working deflection of the second load produced by a unit value of the first load.

Substitution of the reactions R_j from Eq (35) into equations for $(D_i)_{ij}$ and $(D_k)_{kj}$ of the form of Eq (33) gives

$$(D_i)_{ij} = (A_{ii})_{ij} R_i + (A_{ij})_{ij} \left[(C_{ji})_{ik} R_i + (C_{jk})_{ik} R_k + (R_{je})_{ik} \right] + (D_{ie})_{ij}$$

or

$$(D_i)_{ij} = \left[(A_{ii})_{ij} + (A_{ij})_{ij} (C_{ji})_{ik} \right] R_i + \left[(A_{ij})_{ij} (C_{jk})_{ik} \right] R_k + \left[(A_{ij})_{ij} (R_{je})_{ik} + (D_{ie})_{ij} \right]$$

and similarly,

$$(D_k)_{jk} = \left[(A_{kj})_{jk} (C_{ji})_{ik} \right] R_i + \left[(A_{kj})_{jk} (C_{jk})_{ik} + (A_{kk})_{jk} \right] R_k + \left[(A_{kj})_{jk} (R_{je})_{ik} + (D_{ke})_{jk} \right]$$

These equations are seen to be in the standard form

$$\left. \begin{aligned} (D_i)_{ik} &= (A_{ii})_{ik} R_i + (A_{ik})_{ik} R_k + (D_{ie})_{ik} \\ (D_k)_{ik} &= (A_{ki})_{ik} R_i + (A_{kk})_{ik} R_k + (D_{ke})_{ik} \end{aligned} \right\} \quad (36)$$

where

$$(A_{ii})_{ik} = (A_{ii})_{ij} + (A_{ij})_{ij} (C_{ji})_{ik},$$

$$(A_{ik})_{ik} = (A_{ij})_{ij} (C_{jk})_{ik},$$

$$(D_{ie})_{ik} = (A_{ij})_{ij} (R_{je})_{ik} + (D_{ie})_{ij},$$

$$(A_{ki})_{ik} = (A_{kj})_{jk} (C_{ji})_{ik},$$

$$(A_{kk})_{ik} = (A_{kj})_{jk} (C_{jk})_{ik} + (A_{kk})_{jk},$$

and

$$(D_{ke})_{ik} = (A_{kj})_{jk} (R_{je})_{ik} + (D_{ke})_{jk}.$$

By repeated use of Eqs (35) and (36), all the parallel plates of the subassembly are joined together and the weld reactions are determined. This procedure is followed in code S0131, as shown in the flow diagram in Fig. 7.

INPUT AND OUTPUT DATA

Input Cards

The regular Wolontis format is used on the input cards.

Columns	Contents
7-9	Location of first word
10	Number of words on card, 1 to 6
11-21	First word (11 columns)
22-32	Second word (11 columns)
	Etc. (in general, word k starts in column kk)

The sign of a word is given first and 10 digits follow. If the word is fixed (fix), the significant digits are at the right side, for example, - 23 (fix) is - 0000000023. If the word is not fixed, it is in Wolontis floating form; that is, if a number N is put on a card in the form $\pm a b c d e f g h i j$, the actual number is $N = \pm (a.b c d e f g h) \times 10^{1j-50}$. Thus, - 23 is - 2300000051 and + .0173 is + 1730000048. Table I gives the format of input cards.

There are three cards of preliminary data and $2n$ sets of plate data, each set consisting of 4 cards or 1 card. Four cards are used for the first plate and each plate different from the preceding

Fig. 7 Flow Diagram for S0131

TABLE I
INPUT DATA FORMAT

7-9	10	11-21	22-32	33-43	44-54	55-65	66-76	Columns
001	5	#	α	T_z	$n(\text{fix})$	N_{op}		Three cards of preliminary data
006	4	M_{op}	Q_{op}	M_{ot}	Q_{ot}			
010	4	M_{np}	Q_{np}	M_{nt}	Q_{nt}			
014	6	P_b	T_M	T_N	L_T	+0	$J(\text{fix})$	4-card set of data for plate j (j=1 to N)
019	6	E_s	v_s	E_f	v_f	h_f	$J(\text{fix})$	
024	5	h_a	h_b	L_{sa}	L_{sb}	$J(\text{fix})$		
028	5	L_a	L_b	L_{fa}	L_{fb}	$J(\text{fix})$		1-card set of data for plate j
014	6	P_b	T_M	T_N	L_T	$J(\text{fix})$	E_s	

plate in materials or geometry; only one modified card need be used for a plate which is the same in materials and geometry as the preceding plate. The input data cards are arranged in the following order:

- (x) 3 cards of preliminary data
- (y) {
 - 4 cards of data for first plate
 - 4 cards (or 1 modified card) of data for second plate
 -
 - 4 cards (or 1 modified card) of data for last plate
- (z) {
 - 4 cards of data for first plate
 - 4 cards (or 1 modified card) of data for second plate
 -
 - 4 cards (or 1 modified card) of data for last plate

Note that the plate data cards are repeated; that is, group (z) is the same as group (y). The running time on the IBM-650 is about two minutes per plate per problem. The following notations are used:

- # = problem number (arbitrary digits)
- α = linear thermal expansion coefficient, assumed to be the same for all plates, in. / in. $\cdot$ $^{\circ}$ F
- T_z = average temperature of whole subassembly cross section measured above side plate temperature, $^{\circ}$ F
- n = number of plates ($1 \leq n \leq 8$)
- p = subscript for pressure loading
- t = subscript for thermal loading
- N_o, Q_o, M_o = direct force, shear force, and moment per unit depth on weld o, lb/in., lb/in., and lb

Note symmetry:

- Q_n, N_n = shear force and moment per unit depth on weld n, lb/in. and lb/in.
- p_b = channel pressure on right side of parallel plate j measured above external pressure, psi

T_M	= effective bending temperature over length $2L_T$ in plate j, tending to bow the center of the parallel plate towards the right if positive, °F
T_N	= effective tensile temperature over length $2L_T$ in plate j measured above side plate temperature, °F
L_T	= half of heated length of plate j, in.
+0	= + 0000000000
J	= plate number, 1 to n
f	= subscript for parallel plate
E	= modulus of elasticity, psi
ν	= Poisson's ratio
h	= plate thickness, in. (see Fig. 2)
L	= plate length, in. (see Fig. 2)

Output Cards

The following additional notations are used for the output cards shown in Table II:

θ	= slope, radians (see Fig. 6)
w	= deflection in. (see Fig. 6)
S_x	= cross section stress, psi [see Eqs (26) and (27)]
S_z	= axial stress, psi [see Eqs (28) and (29)]
S_s	= shear stress in middle surface, psi [see Eq (30)]
o	= subscript for left weld on plate 1
n	= subscript for right weld on plate n
in	= notation for bottom surface of side plate or right surface of parallel plate (see Fig. 3)
out	= notation for top surface of side plate or right surface of parallel plate
a, b, c, f	= subscripts for locations shown in Fig. 3
M	= bending moment per unit depth, lb (see Fig. 3); note $M_c = M_f + Q_f L_f + \left(L_f^2 / 2 \right) (P_b - P_a)$
Q	= shear force per unit depth, lb/in. (see Fig. 3)
N	= tensile force per unit depth, lb/in. (see Fig. 3); note $N_c = N_f$

Figure 8 shows a sample problem solved with code S0131. The input cards and data cards are listed in Table III, along with the output cards for the first three plates. Since the stresses and deflections decrease in magnitude away from the end plate, only the results in the first two or three plates are important.

REFERENCE

1. S. Timoshenko, Theory of Plates and Shells, (McGraw-Hill Book Co., New York, 1940).

TABLE II
OUTPUT DATA FORMAT

7-9	10	21-31	22-32	33-43	44-54	55-65	66-76	Columns
001	1	#						Preliminary output data
112	4	θ_{op}	w_{op}	θ_{ot}	w_{ot}			
116	4	θ_{np}	w_{np}	θ_{nt}	w_{nt}			
187	6	M_{op}	Q_{op}	M_{ot}	Q_{ot}	N_{op}	N_{ot}	Location 0
193	6	$S_{xop}(in)$	$S_{xop}(out)$	$S_{xot}(in)$	$S_{xot}(out)$	S_{sop}	S_{sot}	
199	4	$S_{zop}(in)$	$S_{zop}(out)$	$S_{zot}(in)$	$S_{zot}(out)$			
038	1	J(fix)						Output data for plate J (J = 1 to n)
205	6	θ_{ap}	w_{ap}	θ_{at}	w_{at}	θ_{fp}	w_{fp}	
211	6	θ_{ft}	w_{ft}	θ_{bp}	w_{bp}	θ_{bt}	w_{bt}	
217	2	w_{cp}	w_{ct}					Location f
187	6	M_{fp}	Q_{fp}	M_{ft}	Q_{ft}	N_{fp}	N_{ft}	
193	6	$S_{xfp}(in)$	$S_{xfp}(out)$	$S_{xft}(in)$	$S_{xft}(out)$	S_{sfp}	S_{sft}	
199	4	$S_{zfp}(in)$	$S_{zfp}(out)$	$S_{zfp}(in)$	$S_{zfp}(out)$			Location c
187	6	M_{cp}	Q_{cp}	M_{ct}	Q_{ct}	N_{cp}	N_{ct}	
193	6	$S_{xcp}(in)$	$S_{xcp}(out)$	$S_{xct}(in)$	$S_{xct}(out)$	S_{scp}	S_{sct}	
199	4	$S_{zcp}(in)$	$S_{zcp}(out)$	$S_{zct}(in)$	$S_{zct}(out)$			Location b
187	6	M_{bp}	Q_{bp}	M_{bt}	Q_{bt}	N_{bp}	N_{bt}	
193	6	$S_{xbp}(in)$	$S_{xbp}(out)$	$S_{xbt}(in)$	$S_{xbt}(out)$	S_{sbp}	S_{sbt}	
199	4	$S_{zbp}(in)$	$S_{zbp}(out)$	$S_{zbt}(in)$	$S_{zbt}(out)$			

TABLE III INPUT AND DATA CARDS

				S0131	INPUT				
002	1	001	5	+3510010002	+3000000044	+6440000051	+0000000008	+0000000000	
002	2	006	4	+0000000000	+0000000000	+0000000000	+0000000000		
002	3	010	4	+0000000000	+0000000000	+0000000000	+0000000000		
002	4	014	6	+1000000051	+0000000000	+0000000000	+1250000050	+0000000000	+0000000001
002	5	019	6	+1200000057	+3000000049	+1200000057	+3000000049	+1000000049	+0000000001
002	6	024	5	+1000000049	+1000000049	+1000000049	+1000000049	+0000000001	
002	7	028	5	+7500000048	+7500000048	+1500000050	+1500000050	+0000000001	
002	8	014	6	+1000000051	+0000000000	+1000000052	+1250000050	+0000000002	+1200000057
002	9	014	6	+1000000051	+0000000000	+1000000052	+1250000050	+0000000003	+1200000057
002	10	014	6	+1000000051	+0000000000	+1000000052	+1250000050	+0000000004	+1200000057
002	11	014	6	+1000000051	+0000000000	+1000000052	+1250000050	+0000000005	+1200000057
002	12	014	6	+1000000051	+0000000000	+1000000052	+1250000050	+0000000006	+1200000057
002	13	014	6	+1000000051	+0000000000	+1000000052	+1250000050	+0000000007	+1200000057
002	14	014	6	+1000000051	+0000000000	+1000000052	+1250000050	+0000000008	+1200000057
002	15	014	6	+1000000051	+0000000000	+0000000000	+1250000050	+0000000000	+0000000001
002	16	019	6	+1200000057	+3000000049	+1200000057	+3000000049	+1000000049	+0000000001
002	17	024	5	+1000000049	+1000000049	+1000000049	+1000000049	+0000000001	
002	18	028	5	+7500000048	+7500000048	+1500000050	+1500000050	+0000000001	
002	19	014	6	+1000000051	+0000000000	+1000000052	+1250000050	+0000000002	+1200000057
002	20	014	6	+1000000051	+0000000000	+1000000052	+1250000050	+0000000003	+1200000057
002	21	014	6	+1000000051	+0000000000	+1000000052	+1250000050	+0000000004	+1200000057
002	22	014	6	+1000000051	+0000000000	+1000000052	+1250000050	+0000000005	+1200000057
002	23	014	6	+1000000051	+0000000000	+1000000052	+1250000050	+0000000006	+1200000057
002	24	014	6	+1000000051	+0000000000	+1000000052	+1250000050	+0000000007	+1200000057
002	25	014	6	+1000000051	+0000000000	+1000000052	+1250000050	+0000000008	+1200000057

TABLE III (Cont)

			S0131	DATA				
111	1	001 1	+3510010002					
111	2	112 4	+6914410046	-1030340046	-1732864547	+1070276446		
111	3	116 4	+9105778043	+1255134444	+2936383344	+4876832146		
111	4	187 6	+0000000000	+0000000000	+0000000000	+0000000000	+0000000000	+0000000000
111	5	193 6	+0000000000	+0000000000	+0000000000	+0000000000	+0000000000	+0000000000
111	6	199 4	+0000000000	+0000000000	+2318400053	+2318400053		
111	7	038 1	+0000000001					
111	8	205 6	+6914410046	-1030340046	-1732864547	+1070276446	-6914410046	+3388989045
111	9	211 6	+1732864547	+6625880445	-2898240046	-1345910045	+1571037547	+2466989046
111	10	217 2	-2467362047	+1299648447				
111	11	187 6	+6993450050	-1500000051	+1269497850	+0000000000	+2979331051	+5824949851
111	12	193 6	+4494003153	-3898136953	+1344193753	-1792037052	-2250000052	+0000000000
111	13	199 4	+1348200953	-1169441153	+2721658153	+2264638953		
111	14	187 6	-4256550050	+0000000000	+1269497850	+0000000000	+2979331051	+5824949851
111	15	193 6	-2255996953	+2851863153	+1344193753	-1792037052	+0000000000	+0000000000
111	16	199 4	-6767990752	+8555589352	+2721658153	+2264638953		
111	17	187 6	+4776619050	+2929331051	-4555452050	+5824949851	+1500000051	+0000000000
111	18	193 6	+3015971453	-2715971453	-2733271253	+2733271253	+4393996552	+8737424752
111	19	199 4	+9047914252	-8147914252	+1498418653	+3138381452		

TABLE III (Cont)

111	20	038	1	+0000000002							
111	21	205	6	+2898236046	+1345898045	-1571038047	-2466988046	-3815229645	-2014767945		
111	22	211	6	+1111045847	+3882752946	+4152250045	-1848815045	+5817570046	+4729530046		
111	23	217	2	-2861422245	+8332843846						
111	24	187	6	-2795040048	+0000000000	+8139530049	+0000000000	-1771224551	-8723051151		
111	25	193	6	-1938926952	-1603522152	-3839333152	-1360676953	+0000000000	+0000000000		
111	26	199	4	-5816780751	-4810566351	+2203220053	+1910196953				
111	27	187	6	-2795040048	+0000000000	+8139530049	+0000000000	-1771224551	-8723051151		
111	28	193	6	-1938926952	-1603522152	-3839333152	-1360676953	+0000000000	+0000000000		
111	29	199	4	-5816780751	-4810566351	-1396780053	-1689803153				
111	30	187	6	-7612311449	+1058106551	-6668348050	-2898101351	+1500000051	+0000000000		
111	31	193	6	-6067386852	-3067386852	-4001008853	+4001008853	+1587159852	-4347152052		
111	32	199	4	-1820216052	-5202160451	+1118097453	+3518702653				
111	33	038	1	-0000000003							
111	34	205	6	-4152190045	+1848821745	-5817587046	-4729531646	+6703485045	-1042478145		
111	35	211	6	-2008173046	+5007417646	+4898837045	-4218393044	+2750430045	+5103882146		
111	36	217	2	-5027613745	+1506129746						
111	37	187	6	-4910978048	+0000000000	+1471189049	+0000000000	-0164642250	+1164110651		
111	38	193	6	-6218055451	-1211122952	+2046824052	+2813972051	+0000000000	+0000000000		
111	39	199	4	-1865416651	-3633368751	+2379804753	+2326841953				
111	40	187	6	-4910978048	+0000000000	+1471189049	+0000000000	-0164642250	+1164110651		
111	41	193	6	-6218055451	-1211122952	+2046824052	+2813972051	+0000000000	+0000000000		
111	42	199	4	-1865416651	-3633368751	-1220195353	-1273158153				
111	43	187	6	-2894079049	+4164228449	-1889137150	-1733990751	+1500000051	+0000000000		
111	44	193	6	-2364474051	+3236447452	-1133482353	+1133482353	+6246342650	-2600986152		
111	45	199	4	-7093422050	+9709342251	+1978355353	+2658444753				