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Argonne National Laboratory

STUDIES OF LOW-ENERGY NUCLEAR REACTIONS
AND LEVEL DENSITIES
FOR MEDIUM-MASS NUCLEI

by

Anastasios A. Katsanos

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Chemistry Division

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the Faculty of the Division of the Physical Sciences
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Department of Chemistry

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CHAPTER I INTRODUCTION

The atomic nucleus can exist in a series of quantum levels, each characterized by a specific energy E , parity, and angular momentum J . Each level is composed of $(2J + 1)$ degenerate states. A state is characterized by energy E , parity, angular momentum J , and the projection M of the angular momentum J on some arbitrary axis. Every level has an energy width Γ , related to its mean lifetime τ by $\tau \Gamma = \hbar$. The spacing D between the levels decreases exponentially with increasing excitation energy U , and the density of levels $\rho(U)$ increases. The behavior is typical of a system with many degrees of freedom, and statistical thermodynamics can be applied.

Four statistical models have been proposed for the nuclear level density; (1) the independent-particle Fermi-gas model, (2) the Fermi-gas model with pairing energy between nucleons in twofold degenerate levels, (3) the superconductor, and (4) the constant-temperature model. In the simple Fermi-gas case¹⁻⁷ the nucleus is described as a gas of independent fermions which have a quasiadditive spectrum. The Fermi gas with pairing energy⁷⁻¹³ is an extension of the first method with a correction for the interaction among the nucleons. It has been fairly successful in explaining the odd-even difference in the level densities of odd and even nuclei. The superconductor model,^{12,14-17} by analogy to the superconducting metal state, has been proposed to explain the energy gap observed in low-lying nuclear levels. Finally, the constant-temperature model^{7,18,19} is a simplified thermodynamic representation, which is a good approximation for the extension of the density of energy levels in the neighborhood of an energy where the absolute value of the density is known.

Experimental information about level densities is obtained primarily from "compound nucleus" reactions. According to the independence hypothesis,* the probability of decay of the compound nucleus is independent of the mode of formation. The decay probability depends on the level densities of

*The independence hypothesis is being studied by a number of investigators.^{20,21}

the residual and compound nuclei, and the appropriate transmission coefficients. Compound-nucleus processes occur at low bombarding energies, for which a relatively stable intermediate system is formed, with a long lifetime in comparison with the time required for the incoming particle to pass through the nucleus. As the incoming energy increases, direct interaction also takes place.

The main source of information about nuclear level densities was at first the experiments on slow-neutron resonance capture. During the last few years extended information has been obtained by use of magnetic spectrographs.²²⁻²⁵ For a given incoming energy, the energy distribution of the emitted particles consists of a series of peaks corresponding to the energy levels of the residual nucleus. One can count the individual levels up to an excitation energy where the spacing D begins to approach the experimental resolution. The only errors in this method are those due to unresolved levels. By using a Tandem Van de Graaff accelerator in conjunction with a magnetic spectrograph for the analysis of the outgoing particles, total resolution of 5 to 10 keV can be obtained. The separation of more than 100 levels up to excitation energies 5 to 8 MeV for medium-mass nuclei has been achieved in this way, and numerous publications using this technique have appeared. A third method is based on the analysis of the yields of particles of the evaporation spectra.^{8,23,26-33} Although this is an indirect method and larger errors are involved, it has the advantage that it can be used up to higher excitation energies than in the previous method. Most of the reported measurements are at energies in the neighborhood of the neutron binding energy.

The experimental testing of the level-density models has been limited so far up to excitation energies of 12 to 13 MeV. Researchers analyse their data in terms of either the Fermi-gas or the constant-temperature model and report, respectively, the Fermi-gas constant or the nuclear temperature. There is not sufficient evidence to favor one model over the other. The differences in the level densities derived by the two models are within the limits of the experimental errors. Moreover, by either model, one can fit curves to the experimental data in this small energy region with various combinations of the level-density constants. These facts call for measurements in the region of high excitation energy.

The theory of statistical fluctuations, which has been widely tested experimentally in the recent years, enables one to calculate the level density of the compound nucleus in the continuum. At low bombarding energies, where $\Gamma/D < 1$, one observes resonances in the reaction cross section corresponding to individual levels of the compound nucleus. As the excitation energy increases, the level widths broaden because more exit channels become available and the transparency of the Coulomb barrier increases. Since D decreases exponentially, the ratio Γ/D grows rapidly with excitation energy. In the region where $\Gamma/D \gg 1$ there are many overlapping

levels, and one can no longer observe the individual resonances. This condition is fulfilled for medium-mass nuclei at an excitation energy of about 20 MeV. In experiments with a beam spread smaller than Γ , the excitation functions show strong fluctuations with energy. From the analysis of the excitation functions according to the statistical theory, developed by T. Ericson^{7,34-38} and by D. Brink and R. Stephen,³⁹⁻⁴² the average width Γ can be obtained. From the width Γ , the experimental differential cross section, and the optical-model transmission coefficients, the level density of the compound nucleus can be calculated. Only preliminary results have been published by this method.^{19,43,44}

The level densities of S^{32} , Cr^{52} and Fe^{56} in an excitation energy range up to about 20 MeV were studied in the present work, with a combination of the above-described methods. The following considerations were taken into account in the choice of these nuclei. The mass number A should be low, so that the width Γ , which decreases with increasing A , be larger than the available experimental resolution. On the other hand, A should be high enough so that Γ/D is large such that statistical theory can be applied at a relatively low excitation energy, where the probability of direct interaction is fairly small. These conditions are fulfilled with A between 30 and 60. Moreover, if odd-mass targets with odd Z are used to study the statistical fluctuations, the products are spinless in the (p,α) reactions, for the ground state of the residual nuclei. Only one reaction channel is open at 0° and at 180° in this case. The mean square fluctuation of the excitation functions is then predicted from the statistical theory to be equal to unity for pure compound-nucleus reactions,* and the amount of direct interaction can be checked this way. Finally, the resolution of the energy levels at low excitation energy of these nuclei should be experimentally possible with the magnetic spectrograph.

The energy levels of Cr^{52} and Fe^{56} up to excitation energies of 6.3 and 6.7 MeV, respectively, were resolved from the reactions $Cr^{52}(p,p')Cr^{52}$, $Mn^{55}(p,\alpha)Cr^{52}$, $Fe^{56}(p,p')Fe^{56}$, and $Co^{59}(p,\alpha)Fe^{56}$ with thin targets. The proton beam from the ANL Tandem Van de Graaff (TVDG) accelerator was used, and the outgoing particles were analysed with a single-gap broad-range magnetic spectrograph. By analyses of the spectra in the continuum according to the evaporation theory, the level densities of the residual nuclei, up to an excitation energy of 8 MeV, were calculated. In addition, the excitation functions for the reactions $P^{31}(p,\alpha)Si^{28}$, $V^{51}(p,\alpha)Ti^{48}$, and $Mn^{55}(p,\alpha)Cr^{52}$ were obtained from experiments with surface-barrier solid-state detectors, and the average widths Γ of the compound nuclei S^{32} , Cr^{52} and Fe^{56} were calculated at excitation energies around 20 MeV. The absolute value of the cross sections for the last two reactions are also reported. The experimental widths are compared with theoretical values computed with optical-model transmission coefficients. Finally, the

*Recent theoretical investigation indicates that the prediction may not be true.²¹ This point will be further discussed in Ch. IV, Sect. B.

level density up to about 20 MeV of excitation energy was calculated for each of the three nuclides. The results obtained are compared with curves derived from the Fermi-gas and constant-temperature models. The data on the level density of S^{32} at low excitation energy and the absolute values of the cross section for the reaction $P^{31}(p,p')P^{31}$ were taken from the literature.

CHAPTER II

EXPERIMENTAL PROCEDURE

A. Targets

Targets of V^{51} , Cr^{52} , Fe^{56} , and Mn^{55} were prepared by vacuum evaporation of metallic elements onto carbon backing foils of 20 to 100 $\mu g/cm^2$. The phosphorus target was prepared with the ANL mass separator. Thin targets of about 10 $\mu g/cm^2$ were prepared for the (p, α) experiments with the magnetic spectrograph, in order to obtain the best possible resolution. Thicker targets, of about 20 $\mu g/cm^2$, were used in the (p,p') experiments, since the stopping power for protons is smaller. Targets of about 50 $\mu g/cm^2$ were used in the experiments studying the statistical fluctuations of the cross section. Finally, thick targets of about 0.5 to 1.0 mg/cm² were desired for the measurements of the angular and energy distributions of the absolute values of the cross sections, in order to damp the statistical fluctuations. The thickness and the stopping power^{45,46} for 10-MeV protons and 10-MeV alpha particles for each of the targets used are listed in Table 1. The thickness of targets 3, 8, and 9, which were used for measurements of the absolute values of the cross section, were determined by weighing and by comparison of the elastic-scattering cross section for protons, with

TABLE 1. List of Targets

Target	Purity, Thickness		Stopping Power (keV) for	
	%	($\mu g \cdot cm^{-2}$)	10-MeV Protons	10-MeV α Particles
1. $^{15}P^{31}$	100	25	1	10
2. $^{23}V^{51}$	99.9	40 ± 4	1.5	30
3. $^{23}V^{51}$	99.9	607 ± 30	17	180
4. $^{23}V^{51}$	99.9	4280 ± 200	120	1300
5. $^{24}Cr^{52}$	99.9	20	<1	6
6. $^{25}Mn^{55}$	99.0	10	<1	3
7. $^{25}Mn^{55}$	99.0	50	1.5	15
8. $^{25}Mn^{55}$	99.0	320 ± 25	9	95
9. $^{25}Mn^{55}$	99.0	770 ± 40	22	230
10. $^{25}Mn^{55}$	99.0	2050 ± 100	58	600
11. $^{26}Fe^{56}$	99.9	20	<1	6
12. $^{27}Co^{59}$	99.99	10	<1	3

that of rolled vanadium and manganese foils (targets 4 and 10 of Table 1). The effective thickness of the targets, depending on the angle with respect to the beam, was 20 to 30 percent higher than the absolute value.

B. Beam Characteristics

The entrance and exit slits for the beam-analysing magnet of the TVDG accelerator, which has a radius of 86 cm, were set at 0.9 mm in all experiments in which good resolution was desired. The energy resolution of the proton beam has been estimated from experiments with the magnetic spectrograph. The width of the observed peaks depends on the energy width of the beam, the thickness of the target, and the aberration of the spectrograph,* which is 0.075 percent (5 keV at $E_p = 7$ MeV). Peak widths of 6 keV (FWHM) were observed in 6- to 8-MeV proton groups from (p,p') reactions with targets of $10 \mu\text{g}/\text{cm}^2$ (<1 keV) and incoming energies of 9 to 11 MeV. Therefore, the resolution of the beam is assumed to be better than 5 keV at about 10 MeV. The maximum beam current on the target is 0.5 to 1.0 μA , and the maximum energy about 13 MeV.

C. Detection and Recording of Particles

1. The Magnetic Spectrograph

The energies of the excited levels of Cr^{52} and Fe^{56} were measured by high-resolution magnetic analysis of the particles emitted from (p,p') and (p, α) reactions, with the TVDG and the broad-range single-gap magnetic spectrograph, which is described in Appendix I and references 47 and 48. Nuclear emulsion plates Kodak NTA, 50 microns thick, were used for recording the protons, and Ilford KO plates of 50-micron thickness were used for the alpha-particle spectra. By proper development, proton and deuteron tracks were suppressed in the plates used for the determination of α particles. The particles were recorded in a focal plane of dimensions $1 \times 127 \text{ cm}^2$. According to the calibration used, the projection on the base line of a point $1/3$ the maximum height on the high-energy side of a peak was taken as the position of the corresponding particle group. The calibration of J. R. Erskine,⁴⁹ based on a value of 5.3045 MeV for the alpha particles of Po^{210} , was used. The accuracy and reproducibility of this calibration were checked by observing the position of the second and third excited levels of C^{13} in the (p,p') runs. The energy of the second excited level of C^{13} has been measured very accurately by Carter and Motz.⁵⁰ The experimental energies of the two levels in C^{13} for several runs along with the energy of the second excited level of Motz and Carter are listed in Table 2 and indicate a possible error of ± 0.004 MeV in the calibration. In addition, levels 1 to 8 in Cr^{52} were determined spectrographically by M. S. Freedman et al.,⁵¹

*The aberration of the spectrograph is determined by the size of the beam spot on the target. It is 0.075 percent with a beam spot of $0.5 \times 4.0 \text{ mm}^2$.

and are compared with the present data in Ch. III. The cross section vs. excitation energy of the residual nucleus was computed from the intensity of particles vs. distance along the plates, and plotted with an IBM-1620 computer. Relativistic kinematics were used in the transformation program (see Appendix I).

TABLE 2. Comparison of the Energies (MeV) of the Second and Third Excited Levels of C^{13} from Different Measurements

Reaction	Angle, deg	2nd Excited Level	3rd Excited Level
$Cr^{52}(p,p')$	90	3.684	
$Cr^{52}(p,p')$	120	3.683	3.853
$Cr^{52}(p,p')$	140		
$Cr^{52}(p,p')$	140	3.683	
$Fe^{56}(p,p')$	60	3.681	3.851
$Fe^{56}(p,p')$	140	3.684	3.853
Carter and Motz ⁵⁰		3.6843	

The resolving power of the spectrograph is determined by the following five factors:⁴⁸

- a) the resolution of the proton beam from the TVDG;
- b) the vertical dimension of the beam spot on the target ($0.5 \times 4.0 \text{ mm}^2$);
- c) the width of the strips scanned with the microscope across the nuclear track plate (0.5 mm along the plate corresponds to about 2 to 4 keV for magnetic inductions of 5 to 6 kG, respectively);
- d) the effective target thickness (negligible for protons for the targets used in the present experiments, and 3 to 4 keV for the alpha particles);
- e) the aberration of the spectrograph.

Half-widths of 6 to 8 keV were observed for proton groups and of about 10 to 15 keV for the alpha-particle groups. The poorer resolution for the alpha particles is partly due to their higher energy loss and straggling in the target. Most of the increase in the half-width for the alpha-particle groups, however, is probably due to the fact that these particles were observed at higher energies than the protons because of their larger Coulomb

barrier. Since the relative energy resolution $\Delta E/E$ of the spectrograph is approximately independent of the energy, the line width ΔE for the alpha particles is larger.

Measurements were made at different angles for each reaction in order to check the proper variation of the energy of the particle groups with energy. The incident proton energies were chosen so that they would bring the highest resolvable group just above the Coulomb barrier (the use of still higher bombarding energies would decrease the intensity of the resolvable particle groups due to the opening of additional exit channels).

2. Solid-state Detectors

The energy and angular distributions of the cross sections from (p, α) reactions of P^{31} , V^{51} , and Mn^{55} were studied in the 18-in.-diameter scattering chamber of the

TVDG accelerator. A schematic diagram of the chamber is shown in Fig. 1. The proton beam was collimated by two 1/16-in. tantalum apertures and a 3/32-in. skimmer. The beam was collected for current integration in a copper Faraday cup. The ring between the chamber and the

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Fig. 1. Schematic Drawing of the 18-in. Scattering Chamber

Faraday block, insulated from both by Teflon rings, was negatively biased at 900 V, to prevent the escape of electrons scattered backwards from the cup, so that an accurate current reading could be achieved.

The alpha particles from the reactions were detected simultaneously at 4 to 6 angles, by surface-barrier solid-state detectors. In order to discriminate against protons, the detectors were biased to just stop the ground-state alpha particles. Deuterons and tritons were energetically inhibited due to their large negative Q -values for all reactions studied. Silicon n-type detectors were used with specific resistivity of 600 to 2,000 ohm-cm and active area 50 mm². Collimators of 0.63-cm diameter were placed in front of the detectors, covered with thin nickel foils of 100 $\mu\text{g}/\text{cm}^2$, in order to exclude low-energy electrons and light. One detector at 90° was used as a monitor, the others being on a movable plate which was controlled from the counting room. They were all placed in the same reaction plane and subtended an angle of 7°. The geometry was checked by observing the counting rate of the alpha particles from a natural Cm^{244} source, the absolute counting rate of which was determined with a proportional counter. The target was placed at such an angle with respect to the beam so that it would bisect the angle between the detector at 90° and the detector at the maximum angle (170-175°). This eliminates any differences in the solid angles of the two detectors, due to the uncertainty of the position of the beam spot on the target.

A block diagram of the electronics associated with each detector is shown in Fig. 2. The pulses were recorded by two RIDL-400 channel and two Nuclear Data-512 channel analysers. A beam current of about 50 to 100 nano-amperes was used in most of the experiments, in order to have low dead-time. Corrections for the dead-time were made from the counting rate of the pulse generators. The large energy widths of the alpha-particle groups were mainly due to the effective thickness of the targets and the angular energy distribution of the particles. Half-widths (FWHM) of 100 to 150 keV were observed with targets of about 50 to 100 $\mu\text{g}/\text{cm}^2$, and of 500 keV with targets of about 1 mg/cm^2 . This was adequate to resolve the first two levels in Si^{28} , Ti^{48} , and Fe^{56} , which are about one MeV apart. Typical spectra of the alpha particles from the reactions $\text{P}^{31}(\text{p},\alpha)\text{Si}^{28}$, $\text{V}^{51}(\text{p},\alpha)\text{Ti}^{48}$, and $\text{Mn}^{55}(\text{p},\alpha)\text{Cr}^{52}$ are displayed in Figs. 3-5.

Fig. 2
Block Diagram of Electronics

121-1694

Fig. 3
Spectrum of Alpha Particles from
the Reaction $\text{P}^{31}(\text{p},\alpha)\text{Si}^{28}$

121-1695 Rev. 1

121-1696

Fig. 4
Spectrum of Alpha Particles from
the Reaction $\text{V}^{51}(\text{p},\alpha)\text{Ti}^{48}$

Fig. 5

Spectra of Alpha Particles from
 the Reaction $\text{Mn}^{55}(\text{p},\alpha)\text{Cr}^{52}$

121-670

The output from the analysers was punched on computer cards, and the results were analysed with the CDC-3600 computer. Corrections made for the center-of-mass system are described in Appendix I.

CHAPTER III

ENERGY LEVELS OF Cr^{52} AND Fe^{56}

A. Energy Levels and Particle-group Intensities

1. Spectra Analysis

The nuclear energy levels of Cr^{52} and Fe^{56} were determined to excitation energies 6.3 and 6.7 MeV, respectively, from (p,p') and (p, α) reactions with the magnetic spectrograph. In order to obtain good resolution, thin targets of about $10 \mu\text{g}/\text{cm}^2$ were used in the (p, α) reactions (Nos. 6 and 12 of Table 1) and targets of about $20 \mu\text{g}/\text{cm}^2$ for the (p,p') reactions (Nos. 5 and 11 of Table 1). For better resolution in the higher excitation region, the energy scale in this range was expanded on the spectrograph in some (p,p') runs by proper combination of the incident energy and the magnetic field. As a consequence, the well-known low-excitation-energy groups fell outside the range of the spectrograph. Detailed information about each of the experiments is given in Table 3. Typical spectra of protons and alpha particles from the reactions are displayed in Figs. 6-9.

TABLE 3. List of Reactions Studied with the Magnetic Spectrograph

Reaction	Projectile Energy, MeV	Angle (L), deg	Magnetic Field, kG	Excitation Energy up to Which Levels Were Resolved, MeV
1. $\text{Cr}^{52}(\text{p},\text{p}')\text{Cr}^{52}$	8.972	90	4.40	5.5
2. $\text{Cr}^{52}(\text{p},\text{p}')\text{Cr}^{52}$	9.969	140	5.10	5.0
3. $\text{Cr}^{52}(\text{p},\text{p}')\text{Cr}^{52}$	10.966	140	4.50	6.3
4. $\text{Cr}^{52}(\text{p},\text{p}')\text{Cr}^{52}$	11.963	120	4.90	6.3
5. $\text{Mn}^{55}(\text{p},\alpha)\text{Cr}^{52}$	11.963	90	5.93	6.3
6. $\text{Mn}^{55}(\text{p},\alpha)\text{Cr}^{52}$	11.963	120	5.82	6.2
7. $\text{Fe}^{56}(\text{p},\text{p}')\text{Fe}^{56}$	10.966	60	4.40	6.7
8. $\text{Fe}^{56}(\text{p},\text{p}')\text{Fe}^{56}$	10.966	140	4.40	6.7
9. $\text{Co}^{59}(\text{p},\alpha)\text{Fe}^{56}$	10.966	90	5.30	5.3
10. $\text{Co}^{59}(\text{p},\alpha)\text{Fe}^{56}$	10.966	120	6.00	6.5
11. $\text{Co}^{59}(\text{p},\alpha)\text{Fe}^{56}$	8.972	120	4.75	5.0

121-970

Fig. 6. Spectrum of Protons Inelastically Scattered from Cr⁵²

121-969

Fig. 7. Spectrum of Alpha Particles from the Reaction $\text{Mn}^{55}(p, \alpha)\text{Cr}^{52}$

120-9224

Fig. 8. Spectrum of Protons Inelastically Scattered from Fe^{56}

120-9226 Rev. 1

Fig. 9. Spectrum of Alpha Particles from the Reaction $\text{Co}^{59}(\text{p}, \alpha) \text{Fe}^{56}$

The fact that the same levels were excited in both the (p,p') and the (p, α) experiments indicates a compound-nucleus process. The relative intensities of the particle groups are different in the two types of experiments, depending on the compound-nucleus spin distributions of the two reactions. For a given bombarding energy, the relative cross section for each final level of the residual nucleus depends also on the spin and excitation energy of the level. The cross section decreases with increasing excitation energy of the residual nucleus, due to the change of the transparency of the outgoing particles through the Coulomb barrier. The spin distribution of the outgoing particles depends on the energy of the projectile, and the spin and mass of the target, and of the incoming and outgoing particles. Heavier incoming and outgoing particles can carry in and out higher orbital angular momentum, and hence populate final levels with higher spin. The energy, angular, and spin distributions of the cross section for a compound-nucleus reaction can be computed with equations (1a, 1b, 1c) or (9) of Appendix IV. By comparing the distributions from different nuclear reactions for the same residual nucleus, one can obtain information about the spin of each level.

The relative cross section for each particle group was calculated by summing the number of tracks in the group, divided by the respective solid angle. Since the experiments were carried out with very thin targets and good energy resolution, strong statistical fluctuations are expected. On the other hand, the angular anisotropy between 60 and 140° can be neglected, being much smaller than the statistical fluctuations. For these reasons, the average intensities of the particle groups from all runs are given for each type of reaction, instead of the intensities for each individual run. More detailed discussion of this has been published elsewhere.⁵²

Systematic and random errors arise in the determination of level energies. Systematic errors are due to the limited accuracy and reproducibility of the calibration, and in this experiment are probably less than 4 keV. Random errors are introduced due to the statistical uncertainty involved in the determination of the position of the particle groups. For well-separated groups of average intensity the random error is expected to be 2 keV, whereas for partially overlapping groups the random error might be as large as 8 keV.

2. Levels of Cr⁵²

The energies of the excited levels of Cr⁵² and the particle-group intensities are listed in Table 4. The energies were averaged from four (p,p') and two (p, α) experiments (see Table 3). The energies and spins of levels 1 to 8 and level 10 have been determined by M. S. Freedman et al.⁵¹ and are listed for comparison. The intensities of the particle groups

for each of the two reactions were calculated relative only to levels 4 to 12 in order to avoid errors due to background in the higher excitation energy region; levels 1 to 3 were outside the spectrograph range in some (p,p') runs.

TABLE 4. Energy Levels of Cr^{52} and Particle-group Intensities, Averaged from Four $\text{Cr}^{52}(\text{p},\text{p}')\text{Cr}^{52}$ and Two $\text{Mn}^{55}(\text{p},\alpha)\text{Cr}^{52}$ Experiments with the Magnetic Spectrograph

Level	Excitation Energy, MeV		Intensity(a)	
	This Work	Freedman et al. ⁵¹	$\text{Cr}^{52}(\text{p},\text{p}')$	$\text{Mn}^{55}(\text{p},\alpha)$
0	0.000	0.000		0.1
1	1.434 ± 0.005	1.435	3.0	1.0
2	2.371 ± 0.005	2.370	1.1	1.5
3	2.650 ± 0.005	2.648	0.5	0.1
4	2.767 ± 0.005	2.769	1.1	1.4
5	2.965 ± 0.005	2.965	1.3	1.0
6	3.114 ± 0.005	3.115	0.4	0.9
7	3.163 ± 0.005	3.161	1.6	0.8
8	3.416 ± 0.005	3.415	0.9	1.0
9	3.472 ± 0.005		1.4	1.0
10	3.619 ± 0.005	3.618	0.5	1.2
11	3.772 ± 0.005		1.5	0.7
12	3.947 ± 0.005		0.6	0.9
13	4.016 ± 0.005		0.3	0.9
14	4.040 ± 0.005		0.7	1.1
15	4.563 ± 0.005		1.2	0.8
16	4.630 ± 0.005		0.9	1.4
17	4.706 ± 0.005		1.4	0.6
18	4.743 ± 0.005		1.3	1.2
19	4.808 ± 0.005		0.4	0.7
20	4.837 ± 0.005		0.9	1.0
21	4.950 ± 0.005		0.8	0.5
22	5.097 ± 0.005		1.2	1.4
23	5.141 ± 0.005		0.2	0.8
24	5.211 ± 0.005		0.5	0.3
25	5.281 ± 0.005		0.3	0.7
26	5.346 ± 0.005		0.5	0.7
27	5.410 ± 0.005		1.4	1.3
28	5.422 ± 0.008		0.4	0.4
29	5.432 ± 0.008		0.8	1.3
30	5.450 ± 0.006		1.0	0.6
31	5.494 ± 0.005		0.8	0.7
32	5.538 ± 0.005		1.1	0.4

TABLE 4 (Contd.)

Level	Excitation Energy, MeV		Intensity(a)	
	This Work	Freedman <u>et al.</u> ⁵¹	Cr ⁵² (p,p')	Mn ⁵⁵ (p, α)
33	5.546 $\pm$ 0.008		0.8	0.4
34	5.571 $\pm$ 0.005		1.3	1.1
35	5.585 $\pm$ 0.007		0.5	0.1
36	5.664 $\pm$ 0.005		0.9	0.5
37	5.725 $\pm$ 0.005		0.7	1.1
38 ^(b)	5.798 $\pm$ 0.005		0.4	0.5
39	5.812 $\pm$ 0.008			
40	5.816 $\pm$ 0.008		0.5	
41	5.853 $\pm$ 0.005		1.0	
42	5.866 $\pm$ 0.007		1.0	
43	5.879 $\pm$ 0.006		1.1	
44	5.916 $\pm$ 0.006		0.6	
45	5.922 $\pm$ 0.010		0.6	
46	5.953 $\pm$ 0.006		0.7	
47	5.960 $\pm$ 0.007		1.0	
48	5.996 $\pm$ 0.006		0.4	0.7
49	6.026 $\pm$ 0.007			
50	6.035 $\pm$ 0.010			
51	6.057 $\pm$ 0.007			
52	6.065 $\pm$ 0.010			
53	6.107 $\pm$ 0.007			
54	6.145 $\pm$ 0.007			
55	6.155 $\pm$ 0.012			
56	6.164 $\pm$ 0.012			
57	6.175 $\pm$ 0.010		0.8	
58	6.192 $\pm$ 0.008		0.8	
59	6.205 $\pm$ 0.006			
60	6.210 $\pm$ 0.010			
61	6.220 $\pm$ 0.008			
62	6.233 $\pm$ 0.010		0.5	
63	6.253 $\pm$ 0.008		0.3	
64	6.272 $\pm$ 0.008		0.3	
65	6.282 $\pm$ 0.010		0.3	
66	6.294 $\pm$ 0.010		0.3	

(a) Intensity relative to the average of levels 4 to 12.

(b) Possible doublet.

The ground-state Q-value for the reaction $\text{Mn}^{55}(\text{p},\alpha)\text{Cr}^{52}$ was found to be 2.60 ± 0.01 MeV.

3. Levels of Fe^{56}

The energies of 100 excited levels in Fe^{56} resolved in the present work have already been reported elsewhere⁵² and are listed in Table 5 for the sake of completeness of the present report. G. Brown *et al.*⁵³ have also reported the energies of 135 levels in Fe^{56} , and the results are listed for comparison. The numbers in column 1 of Table 5 correspond to the numbers of Figs. 21 and 22 (see below). Column 2 contains the average energies from two (p,p') and two (p, α) experiments in the present work (see Table 3). Column 3 of Table 5 shows the results obtained from the (p,p') experiments by G. Brown *et al.* Columns 4 and 5 give the average intensities of the particle groups in the (p,p') and (p, α) experiments, respectively, relative to the average intensity of levels 11 to 20. Seven of the levels found in the present work, Nos. 51, 77, 83, 89, 93, 98, and 99, have not been resolved by G. Brown *et al.* On the other hand, their levels with Nos. 10A, 18A, 23A, 27A, 32A, 37A, 45A, 47A, 49A, 54A, 59A, and 60A were not resolved in our experiments, with the exception of level 10A which was indicated as possible doublet. Levels 6, 9, and 10 have been investigated by M. Shapiro *et al.*⁵⁴ and found to be doublets.

TABLE 5. Energy Levels of Fe^{56} and Particle-group Intensities Averaged from Two $\text{Fe}^{56}(\text{p},\text{p}')\text{Fe}^{56}$ and Two $\text{Co}^{59}(\text{p},\alpha)\text{Fe}^{56}$ Experiments with the Magnetic Spectrograph

Level	Excitation Energy, MeV		Intensity ^(a)	
	This Work	Brown <i>et al.</i> ⁵³		
			$\text{Fe}^{56}(\text{p},\text{p}')$	$\text{Co}^{59}(\text{p},\alpha)$
0	0.000	0.000		0.3
1	0.845 ± 0.005	0.845		2.0
2	2.078 ± 0.005	2.081		1.6
3	2.652 ± 0.005	2.651		1.8
4	2.938 ± 0.008	2.935		0.3
5	2.954 ± 0.007	2.953		0.6
6(b)	3.114 ± 0.005	3.112	2.6	1.9
7	3.363 ± 0.005	3.359	1.6	1.0
8	3.381 ± 0.007	3.379	0.2	1.8
9(b)	3.440 ± 0.005	3.438	2.4	1.8
10(b)	3.597 ± 0.005	3.595	2.1	1.1
10A	-	3.606	-	-
11	3.748 ± 0.005	3.747	0.2	1.0
12	3.823 ± 0.005	3.821	1.2	0.7

TABLE 5 (Contd.)

Level	Excitation Energy, MeV		Intensity ^(a)	
	This Work	Brown et al. ⁵³		
			Fe ⁵⁶ (p,p')	Co ⁵⁹ (p, α)
13	3.851 $\pm$ 0.005	3.847	0.8	1.0
14	4.042 $\pm$ 0.005	4.046	0.6	1.1
15	4.093 $\pm$ 0.005	4.094	0.8	1.1
16	4.113 $\pm$ 0.005	4.111	0.8	1.1
17	4.290 $\pm$ 0.005	4.288	0.5	1.1
18	4.388 $\pm$ 0.005	4.384	2.6	1.2
18A	-	4.396	-	-
19	4.449 $\pm$ 0.005	4.450	0.5	0.9
20	4.501 $\pm$ 0.005	4.501	1.8	0.9
21	4.531 $\pm$ 0.005	4.532	0.7	0.4
22	4.545 $\pm$ 0.007	4.547	0.4	1.2
23	4.602 $\pm$ 0.005	4.602	1.1	1.6
23A	-	4.612	-	-
24	4.651 $\pm$ 0.005	4.652	0.5	0.7
25	4.675 $\pm$ 0.005	4.680	0.5	1.2
26	4.688 $\pm$ 0.008	4.698	0.3	1.5
27	4.727 $\pm$ 0.005	4.721	0.8	0.7
27A	-	4.736	-	-
28	4.802 $\pm$ 0.005	4.808	0.3	0.8
29	4.858 $\pm$ 0.005	4.860	0.6	0.4
30	4.873 $\pm$ 0.005	4.869	1.1	0.6
31	5.021 $\pm$ 0.005	5.015	1.0	1.1
32	5.032 $\pm$ 0.007	5.030	0.9	0.5
32A	-	5.046	-	-
33	5.123 $\pm$ 0.005	5.124	0.7	0.4
34	5.140 $\pm$ 0.006	5.142	0.4	0.5
35	5.181 $\pm$ 0.008	5.179	0.3	0.5
36	5.188 $\pm$ 0.005	5.190	0.6	0.4
37	5.224 $\pm$ 0.005	5.219	1.0	1.1
37A	-	5.229	-	-
38	5.249 $\pm$ 0.005	5.250	0.7	1.4
39	5.274 $\pm$ 0.005	5.276	0.4	0.5
40	5.296 $\pm$ 0.005	5.295	0.6	0.5
41	5.386 $\pm$ 0.007	5.389	0.2	0.1
42	5.444 $\pm$ 0.008	5.448	0.3	0.3
43	5.476 $\pm$ 0.005	5.477	1.1	0.7
44	5.496 $\pm$ 0.005	5.490	0.7	0.5
45	5.503 $\pm$ 0.008	5.505	0.6	0.5
45A	-	5.515	-	-

TABLE 5 (Contd.)

Level	Excitation Energy, MeV		Intensity ^(a)	
	This Work	Brown et al. ⁵³	Fe ⁵⁶ (p,p')	Co ⁵⁹ (p, α)
46	5.528 $\pm$ 0.005	5.531	0.6	0.1
47	5.557 $\pm$ 0.005	5.553	0.9	0.5
47A	-	5.568	-	-
48	5.583 $\pm$ 0.005	5.582	0.9	0.3
49	5.612 $\pm$ 0.005	5.611	0.8	1.1
49A	-	5.621	-	-
50	5.663 $\pm$ 0.006	5.660	0.3	0.3
51	5.673 $\pm$ 0.008	-	0.6	0.1
52	5.684 $\pm$ 0.005	5.678	1.6	0.4
53	5.697 $\pm$ 0.008	5.692	0.4	0.5
54	5.725 $\pm$ 0.005	5.722	0.6	0.4
54A	-	5.737	-	-
55	5.768 $\pm$ 0.005	5.765	0.5	0.3
56	5.797 $\pm$ 0.005	5.792	0.5	0.3
57	5.813 $\pm$ 0.007	5.807	0.4	0.1
58	5.863 $\pm$ 0.005	5.864	1.2	1.6
59	5.908 $\pm$ 0.005	5.904	1.6	0.9
59A	-	D	-	-
60	5.932 $\pm$ 0.005	5.927	1.6	0.3
60A	-	5.941	-	-
61	5.962 $\pm$ 0.007	5.958	0.6	0.3
62	5.980 $\pm$ 0.008	5.977	0.6	0.3
63	6.002 $\pm$ 0.007	5.997	0.3	0.3
64	6.013 $\pm$ 0.010	6.009	0.2	0.4
65	6.024 $\pm$ 0.010	6.020	0.2	0.5
66	6.041 $\pm$ 0.008	6.035	0.5	0.3
67	6.055 $\pm$ 0.008	6.053	0.6	0.3
68	6.074 $\pm$ 0.008	6.075	0.7	0.3
69	6.089 $\pm$ 0.010	6.089	0.4	0.3
70	6.118 $\pm$ 0.010	6.120	0.4	0.4
71	6.138 $\pm$ 0.007	6.135	0.6	0.5
72	6.174 $\pm$ 0.007	6.172	0.6	0.3
73	6.201 $\pm$ 0.010	6.198	0.1	0.4
74	6.226 $\pm$ 0.008	6.221	0.4	0.1
75	6.243 $\pm$ 0.010	6.239	0.5	0.1
76	6.265 $\pm$ 0.008	6.262	0.7	0.3
77	6.289 $\pm$ 0.010	-	0.1	0.1
78	6.306 $\pm$ 0.010	6.303	0.8	0.4
79	6.316 $\pm$ 0.008	6.316	0.6	0.4

TABLE 5 (Contd.)

Level	Excitation Energy, MeV		Intensity ^(a)	
	This Work	Brown et al. ⁵³	Fe ⁵⁶ (p,p')	Co ⁵⁹ (p, α)
80	6.351 $\pm$ 0.008	6.349	0.7	0.1
81	6.364 $\pm$ 0.007	6.362	1.0	0.3
82	6.382 $\pm$ 0.008	6.382	0.4	0.3
83	6.397 $\pm$ 0.008	-	0.1	0.3
84	6.432 $\pm$ 0.008	6.424		0.7
85	6.448 $\pm$ 0.008	6.439		0.7
86	6.463 $\pm$ 0.008	6.459	0.6	
87	6.489 $\pm$ 0.010	6.490	0.5	
88	6.509 $\pm$ 0.008	6.507	0.7	
89	6.527 $\pm$ 0.010	-	0.2	
90	6.543 $\pm$ 0.010	6.540	0.2	
91	6.555 $\pm$ 0.010	6.553	0.8	
92	6.563 $\pm$ 0.010	6.568	0.6	
93	6.593 $\pm$ 0.012	-	0.1	
94	6.613 $\pm$ 0.010	6.609	0.8	
95	6.630 $\pm$ 0.012	6.620	0.1	
96	6.652 $\pm$ 0.010	6.649	0.7	
97	6.662 $\pm$ 0.010	6.664	0.4	
98	6.670 $\pm$ 0.012	-	0.2	
99	6.695 $\pm$ 0.012	-	0.5	
100	6.700 $\pm$ 0.012	6.699	0.5	

(a) Intensity relative to the average of levels 11 to 20.

(b) The levels 6, 9, and 10 have been reported to be doublets.⁵⁴

The ground-state Q-value for the reaction Co⁵⁹(p, α)Fe⁵⁶ was found to be 3.22 $\pm$ 0.01 MeV.

B. The Evaporation Model*

The evaporation model describes the energy and spin distributions of particles emitted from a compound nucleus at moderate excitation energy. The compound system is assumed to lose its memory of formation, and the

* The following notations are used throughout the present report: Subscripts A, a, b, B, and C denote quantities of the target, projectile, outgoing particle, and the recoil and the compound nucleus, respectively, for the nuclear reaction A(a,b)B. Subscripts 1 and 2 are referring to the entrance and exit channels, respectively. The symbol U is used for the excitation energy, E for the kinetic energy, and ϵ for the channel energy (the sum of the kinetic energies in the center-of-mass system of either a and A or b and B).

decay depends on phase-space considerations and the transmission coefficients. The differential cross section $d^2\sigma/d\Omega d\epsilon_2$ for the emission of particles b with channel energy ϵ_2 , going to a residual nucleus B at excitation energy U_B with level density $\rho(U_B)$, is given by^{55a}

$$d^2\sigma/d\Omega d\epsilon_2 \propto \rho(U_B) \epsilon_2 \sigma_1(\epsilon_2). \quad (1)$$

The inverse cross section $\sigma_1(\epsilon_2)$ is calculated with the optical-model transmission coefficients. Relation (1) is an approximation derived from the exact expression assuming an infinite moment of inertia $\mathcal{J}$.²⁶ In this case Eq. (3) of Appendix II for the level density at excitation energy U and spin J

$$\rho(U, J) = (2J+1) \rho(U, J=0) \exp[-J(J+1) \hbar^2 / \mathcal{J} t] \quad (2)$$

is reduced to

$$\rho(U, J) = (2J+1) \rho(U, J=0). \quad (3)$$

Sources of errors in the calculation of the level density from relation (1) are the following:

(a) uncertainties in the inverse cross section calculated from the optical-model transmission coefficients;

(b) the approximation of the infinite moment of inertia. The angular momentum dependence of the level density was investigated by Thomas,²⁶ who compared the results from relation (1) with an exact relation derived with a spin distribution given by (2); a possible error of slightly larger than 20% was indicated. Also, Vonach and Huizenga³² concluded that, at low energies, the deviations between Eq. (1) and an exact form depend only weakly on the bombarding energy and the moment of inertia. However, the change in level density with excitation energy derived from Eq. (1) is not strictly valid, and the nuclear temperatures derived this way are low;

(c) direct interaction that occurs at higher incoming energies;

(d) experimental errors, primarily due to background.

C. Level Densities

1. Level Density of Cr⁵²

The level density of Cr⁵² up to an excitation energy of 7.8 MeV was calculated from the experiments of Table 3, using relation (1), in which $d^2\sigma/d\Omega d\epsilon_2$ was the integrated cross section in 0.5-MeV intervals. Since the relation is only a proportionality, absolute values were obtained by

normalizing the number of levels between 4 and 5 MeV of excitation energy, to 9 levels. The results from two (p,p') and two (p, α) experiments are listed in Table 6. The average value of the density is given in column 7. The errors are standard deviations calculated from the deviations of the individual values from the average. Column 8 shows the level density, in levels per MeV, calculated by summing the number of resolved levels per 1/2-MeV interval from Table 4 and multiplying by two. Possible errors in the level densities of column 8 are only due to unresolved levels. Therefore, these values represent a minimum limit of the level density. The good agreement between the values in columns 7 and 8 at excitation energies above 5 MeV indicates that the levels were well resolved in this region.

TABLE 6. Level Density of Cr^{52} from the Experiments with the Magnetic Spectrograph; All Numbers Are Given in Levels per MeV

Excitation Energy, MeV	Energy Interval, MeV	Evaporation Spectra				Average Density, MeV ⁻¹	No. of Levels from Table 4, MeV ⁻¹
		Cr ⁵² (p,p')		Mn ⁵⁵ (p,α)			
		120° 12 MeV	140° 11 MeV	90° 12 MeV	120° 12 MeV		
2.75	2.5-3.0	4		3	3	3 ± 1	6
3.25	3.0-3.5	5	6	6	4	5 ± 1	8
3.75	3.5-4.0	5	4	6	4	5 ± 1	6
4.25	4.0-4.5	3	2	4	3	3 ± 1	4
4.75	4.5-5.0	16	16	14	15	15 ± 1	14
5.25	5.0-5.5	20	25	22	21	22 ± 2	20
5.75	5.5-6.0	37	36	35	36	36 ± 1	34
6.25	6.0-6.5	58	54	63	86	65 ± 12	
6.75	6.5-7.0	96	77	86	123	96 ± 17	
7.25	7.0-7.5		92	138	211	147 ± 49	
7.65	7.5-7.8			179	358	269 ± 90	

2. Level Density of Fe^{56}

A similar analysis was made of the spectra for Fe^{56} , with the results listed in Table 7. The number of levels between 2.5 and 4.5 MeV

TABLE 7. Level Density of Fe^{56} from the Experiments with the Magnetic Spectrograph; All Numbers Are Given in Levels per MeV

Excitation Energy, MeV	Energy Interval, MeV	Evaporation Spectra				Average Density, MeV ⁻¹	No. of Levels from Table 5, MeV ⁻¹
		Fe ⁵⁶ (p,p')		Co ⁵⁹ (p,α)			
		60° 11 MeV	140° 11 MeV	120° 9 MeV	120° 11 MeV		
2.75	2.5-3.0			3	5	4 ± 1	6
3.25	3.0-4.0		13	10	11	11 ± 1	12
3.75	3.5-4.0		10	8	8	9 ± 1	10
4.25	4.0-4.5	14	13	21	19	17 ± 3	14
4.75	4.5-5.0	25	24	38	38	31 ± 7	26
5.25	5.0-5.5	35	30	74	44	46 ± 17	32
5.75	5.5-6.0	53	74	171	68	92 ± 48	48
6.25	6.0-6.5	72	107		122	100 ± 22	50
6.75	6.5-7.0	99	140		148	129 ± 22	(58) ^(a)
7.25	7.0-7.5	220	299		275	265 ± 30	

(a) From reference 53.

of excitation energy was normalized to 21 levels. Significant differences exist above 5 MeV between the values of column 7, from the evaporation model, and the values of column 8, derived by counting the number of resolved levels. Although most of the differences are within the limits of the noted errors, it is possible that a number of levels have not been resolved, especially above an excitation energy of 6 MeV.

The results for both nuclides, Cr^{52} and Fe^{56} , are analysed with the Fermi-gas and constant-temperature models in Ch. VI.

CHAPTER IV

ENERGY-LEVEL WIDTHS

A. Predictions of the Statistical Theory

1. Compound-nucleus Reactions

At low bombarding energies one observes resonances in the reaction cross section at those energies, at which the sum of the entrance-channel energy and the binding energy is equal to the excitation energy of a level in the compound nucleus. This occurs if the widths Γ of the levels are smaller than their spacing D . The cross section for the reaction $A(a,b)B$ is then given by the Breit-Wigner one-level formula:^{55b}

$$\sigma_{ab} = (2\ell_1 + 1) \pi \lambda_a^2 \frac{\gamma_\lambda^1 \gamma_\lambda^2}{(E - E_\lambda)^2 + (\Gamma_\lambda/2)^2}, \quad (1)$$

where λ_a is the wavelength of the incident particle, ℓ_1 the orbital angular momentum, Γ_λ the total energy-level width, and γ_λ^1 and γ_λ^2 the partial energy widths for the formation and decay, respectively. The angular distribution of the emitted particles is described by even-order Legendre polynomials and hence there is symmetry about 90° . Furthermore, the excitation functions for different final levels of the residual nucleus are correlated.

In the energy region where the width of the resonances is comparable to the spacing D , symmetry about 90° no longer exists, due to interference between the resonances.

Finally, in the region where $\Gamma/D \gg 1$ there are many overlapping levels and a statistical theory is employed.³⁴⁻⁴² The cross section for one-channel compound-nucleus reactions is given by

$$\sigma = 2\pi \lambda^2 |S|^2, \quad (2)$$

where the matrix element S is defined by

$$S = \sum \frac{A_\lambda}{(E - E_\lambda) + i \Gamma/2}, \quad (3)$$

with

$$A_\lambda = \gamma_\lambda^1 \gamma_\lambda^2.$$

The amplitudes γ_λ^1 and γ_λ^2 for the formation and decay of the compound nucleus are assumed to have random phases, and hence A_λ must have a random distribution. For this reason, the excitation functions are expected to show strong fluctuations with energy. This is because at energies less than Γ apart the sums in (3) are not very different and the cross sections are strongly correlated; at energies that differ much more than Γ the sums in (3) are completely different and the cross sections are uncorrelated. The fluctuations are uncorrelated for angles more than $(kR)^{-1}$ apart,⁴¹ k being the wave number of the incident particle and R the radius of the target nucleus. Finally, the excitation functions of different final levels are also uncorrelated. One can expect to see the fluctuations experimentally only if the resolution is less than or of the order of the width Γ . The angular distributions are not symmetrical about 90° in this case. The fluctuations are damped if more than one exit channel is open. One can re-establish symmetry about 90° in the angular distribution by using a thick target such that the experimental resolution is much larger than Γ , i.e., taking an average in an energy interval much larger than Γ .

2. The Correlation Function

The autocorrelation function $R(\Gamma, \delta)$ for energy points of the excitation function varying by the energy interval δ is defined by

$$R(\Gamma, \delta) = \frac{\langle [\sigma(\epsilon_i) - \langle \sigma \rangle][\sigma(\epsilon_i + \delta) - \langle \sigma \rangle] \rangle}{\langle \sigma \rangle^2}, \quad (4)$$

where $\sigma(\epsilon_i)$ is the cross section at energy ϵ_i and the brackets denote an average over i . The function $R(\Gamma, \delta)$ has a Lorentzian form and is equal to

$$R(\Gamma, \delta) = \frac{1}{N} \frac{\Gamma^2}{\Gamma^2 + \delta^2}, \quad (5)$$

where N is the number of uncorrelated channels. If the reaction is not a pure compound-nucleus reaction, then

$$R(\Gamma, \delta) = \frac{1}{N} \left(1 - \frac{\sigma_D^2}{\langle \sigma \rangle^2} \right) \frac{\Gamma^2}{\Gamma^2 + \delta^2}, \quad (6)$$

where $\sigma_D/\langle \sigma \rangle$ is the relative contribution to the cross section from direct interaction. The correlation function $R(\Gamma, \delta)$ has its maximum value at $\delta = 0$, where it is equal to the mean-square fluctuation. For a pure-compound nucleus reaction in which both of the reaction products have zero spin, there is only one reaction channel open at 0° and 180° . In this case

$$R(\Gamma, 0) = 1.$$

For the general case,

$$R(\Gamma, 0) = \frac{1}{N} \left(1 - \frac{\sigma_D^2}{\langle \sigma \rangle^2} \right) \quad (7)$$

and

$$R(\Gamma, \delta = \Gamma) = \frac{1}{2N} \left(1 - \frac{\sigma_D^2}{\langle \sigma \rangle^2} \right) = \frac{1}{2} R(\Gamma, 0). \quad (8)$$

Thus, one can extract Γ as the energy δ for which the experimental correlation function becomes half its maximum value.

The cross-correlation function between two excitation functions 1 and 2 is defined by

$$R_{1,2}(\Gamma, \delta) = \frac{[\sigma_1(\epsilon_i) - \langle \sigma_1 \rangle][\sigma_2(\epsilon_i + \delta) - \langle \sigma_2 \rangle]}{\langle \sigma_1 \rangle \langle \sigma_2 \rangle}. \quad (9)$$

3. Effect of Finite Energy Range

The curve which is obtained from Eq. (6) proceeds smoothly to zero for large values of δ . The experimental correlation function from Eq. (4) oscillates, due to the fact that it is calculated from the data over a finite energy interval. The errors that are introduced by the limited experimental data have been studied by I. Hall⁵⁶ and W. Gibbs.⁵⁷ The effective number of independent cross sections in an energy range ΔE is given by

$$n = (\Delta E / \pi \Gamma) + 1, \quad (10)$$

and the value of $R(\Gamma, 0)$ was found for small N to be equal to

$$R(\Gamma, 0) = \frac{(n-1)(4n-4+N)}{4n^2N}. \quad (11)$$

The width Γ , calculated by comparing $R(\Gamma, \delta)$ with a Lorentzian form near $\delta = 0$, is related to the actual width Γ_0 by

$$\Gamma = \Gamma_0 \left[\frac{(n-1)(4n-4+N)}{4n^2} \right]^{1/2}. \quad (12)$$

The mean-square fluctuation of the oscillations in the correlation function due to the effect of a finite energy range has been calculated by Gibbs and Monahan.⁵⁸ For

$$\delta > 2\Gamma, \quad (13)$$

we have

$$\sqrt{\frac{\Delta E - \delta}{\Delta E}} R_{1,2}^2(\Gamma, \delta) = \frac{\Gamma}{\Delta E} R_1(\Gamma, 0) R_2(\Gamma, 0) \tan^{-1}\left(\frac{\Delta E - \delta}{\Gamma}\right). \quad (14)$$

For

$$\delta \ll \Delta E - \Gamma, \quad (15)$$

we have

$$\tan^{-1}\left(\frac{\Delta E - \delta}{\Gamma}\right) \simeq \frac{\pi}{2}, \quad (16)$$

and Eq. (14) reduces to

$$\left\langle \sqrt{\frac{\Delta E - \delta}{\Delta E}} R_{1,2}^2(\Gamma, \delta) \right\rangle \simeq \frac{\Gamma}{\Delta E} R_1(\Gamma, 0) R_2(\Gamma, 0) \frac{\pi}{2}, \quad (17)$$

the average taken in the range $2\Gamma < \delta \ll \Delta E - \Gamma$. The width Γ can be calculated from Eq. (17).

4. The Number of Fluctuation Maxima

The number of maxima in the excitation function per energy unit, $\nu_{\max}$, is given by the equation⁵⁹

$$\nu_{\max} = b_N \frac{0.55}{\Gamma}, \quad (18)$$

in which b_N is equal to unity for one channel. One can also calculate Γ from Eq. (18).

5. The Effect of the Experimental Resolution

The statistical fluctuations in the cross section are damped through the averaging process if the experimental resolution, ρ , is of the order of Γ or larger. The experimental value of Γ is then larger than the actual average width of the compound-nucleus levels. The autocorrelation function (6) is reduced by the factor⁶⁰ $1/[(\rho/\pi\Gamma) + 1]$. Curves for the correction of Γ for different forms of experimental resolution have been given by D. Lang.⁶¹

B. Results

1. Excitation Functions

The differential cross sections for the reactions $P^{31}(p, \alpha)Si^{28}$, $V^{51}(p, \alpha)Ti^{48}$, and $Mn^{55}(p, \alpha)Cr^{52}$ were measured with thin targets, in order to determine the average level widths of the compound nuclei S^{32} , Cr^{52} , and Fe^{56} , respectively. The outgoing particles were detected with solid-state detectors in the 18-in. scattering chamber.

The statistical-theory calculation of the average width Γ in the continuum is discussed in Sect. C of this Chapter, and the calculation of the ratio Γ/D in Ch. V. Some of the results for S^{32} , Cr^{52} , and Fe^{56} , with rather large variations of the necessary parameters, are listed in Table 8. The calculation shows that the requirement of the statistical theory that Γ/D should be larger than unity is fulfilled in this energy region. However, the low limits of the calculated widths Γ for Cr^{52} and Fe^{56} are of the same order of magnitude as the experimental resolution, indicating that corrections might be necessary.

TABLE 8. Approximate Values for the Width Γ and the Ratio Γ/D from Statistical-theory Calculations

Reaction	Incoming Energy, MeV	Compound Nucleus	Excitation Energy, MeV	Γ , keV	Γ/D
$P^{31}(p, \alpha)Si^{28}$	8	S^{32}	16.9	20-50	4-10
	12		20.9	50-90	60-120
$V^{51}(p, \alpha)Ti^{48}$	8	Cr^{52}	18.5	3-7	30-70
	12		22.5	6-10	$>10^3$
$Mn^{55}(p, \alpha)Cr^{52}$	8	Fe^{56}	18.2	4-10	60-150
	12		22.2	8-15	$>10^3$

A typical spectrum of the alpha particles from the reaction $P^{31}(p, \alpha)Si^{28}$ is displayed in Fig. 3. Two levels were resolved in the residual nucleus Si^{28} , the ground state (0+) and the first excited level (2+) at 1.78 MeV. Excitation functions were measured in two proton-energy regions, with 10-keV steps. Measurements were made in the region from 8.37 to 9.00 MeV at laboratory angles 30, 60, 90, 120, 150, 170, and 175°, and in the region from 10.00 to 11.77 MeV at 39, 69, 81, 111, 141, 161, and 175°. In a third experiment the excitation functions were reproduced in the energy region from 10.00 to 10.50 MeV. The results from the last two experiments at 140° are displayed in Fig. 10 and show that the functions are well reproduced. The differences in the two curves in Fig. 10 are within the limits of the errors due to counting statistics, the latter being about 5% for the ground state and about 2% for the first excited level.

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Fig. 10. Excitation Functions of $P^{31}(p, \alpha)Si^{28}$ from Two Different Experiments

The first two levels in Ti^{48} , the ground state (0^+) and the first excited level (2^+) at 0.99 MeV, were also resolved from the reaction $\text{V}^{51}(\text{p}, \alpha)\text{Ti}^{48}$. A spectrum of alpha particles from the reaction is displayed in Fig. 4. Excitation functions with 5-keV steps were measured for proton energies from 9.0 to 9.7 MeV at the laboratory angles 60, 75, 140, 160, and 171°, and from 10.0 to 10.7 MeV at 115, 140, 155, and 170°.

Finally, the excitation functions for the ground and first two excited levels of Cr^{52} were obtained from the reaction $\text{Mn}^{55}(\text{p}, \alpha)\text{Cr}^{52}$, in the proton-energy region from 9.5 to 9.9 MeV, with 5-keV steps. The ground state of Cr^{52} is a 0^+ state; the first and second excited levels at 1.434 and 2.371 MeV have spins 2^+ and 4^+ , respectively. Measurements were made at four laboratory angles: 90, 130, 150, and 170°. A spectrum of alpha particles is displayed in Fig. 5.

The excitation functions from the above three reactions are displayed in Figs. 11 to 20. The cross sections are not normalized for the different angles.

121-1698

Fig. 11. Excitation Functions of $\text{P}^{31}(\text{p}, \alpha)\text{Si}^{28}$ for the Ground and First Excited Levels in Si^{28}

121-1699

Fig. 12. Excitation Functions from the Reaction $\text{P}^{31}(\text{p}, \alpha_0)\text{Si}^{28}$

121-1700

Fig. 13. Excitation Functions from the Reaction $p^{31}(p, \alpha_1)Si^{28}$

121-1701

Fig. 14. Excitation Functions from the Reaction $v^{51}(p, \alpha_0)Ti^{48}$

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Fig. 15. Excitation Functions from the Reaction $v^{51}(p, \alpha_1)Ti^{48}$

121-1703

Fig. 16. Excitation Functions from the Reaction $v^{51}(p, \alpha_0)Ti^{48}$

121-1704

Fig. 17. Excitation Functions from the Reaction $V^{51}(p, \alpha_1)Ti^{48}$

121-1705

Fig. 18. Excitation Functions from the Reaction $Mn^{55}(p, \alpha_0)Cr^{52}$

121-1706

Fig. 19. Excitation Functions from the Reaction $Mn^{55}(p, \alpha_1)Cr^{52}$

121-1707

Fig. 20. Excitation Functions from the Reaction $Mn^{55}(p, \alpha_2)Cr^{52}$

2. The Autocorrelation Functions; Coherence Widths

Calculations of the ratio Γ/D for the compound nuclei S^{32} , Cr^{52} , and Fe^{56} at the excitation energies of Table 8 show this quantity to be much larger than unity. Hence, the excitation functions were analysed in terms of

the statistical fluctuation theory. The autocorrelation functions were calculated for each of the 58 excitation functions of Figs. 11 to 20.

The correlation functions of Eqs. (4) and (9) are defined for small intervals ΔE of the incoming energy, assuming that the average cross section does not change in the interval ΔE . It was found that the assumption is not well-justified, and for this reason the excitation functions were corrected for the change of the average. A straight line was fitted to each individual excitation function by the least-squares method. The average slope b of the lines was calculated from all levels and all angles for the same reaction and energy region. The cross section σ_i was then corrected at each energy point by the factor

$$\{1 + b[i - 0.5(N+1)] (\delta E)\}, \quad (19)$$

where δE is the energy step between σ_i and σ_{i+1} , and N the number of energy points ($i_{\max} = N$). For the same reason, the excitation functions for the reaction $P^{31}(p, \alpha)Si^{28}$ in the energy region from 10.00 to 11.77 MeV were divided into two groups: 10.00 to 10.90 MeV, and 10.90 to 11.77 MeV.

The correlation functions were computed and plotted by the CDC-3600 computer with the aid of a computer program which provided corrections for the change of the average.

A typical autocorrelation function is displayed in Fig. 21(a), along with the theoretical curve calculated from

$$R(\Gamma, \delta) = R(\Gamma, 0) \frac{\Gamma^2}{\Gamma^2 + \delta^2}. \quad (20)$$

The most striking difference between the experimental and the theoretical correlation functions is the sizable fluctuation about zero in the experimental curve for large δ . This results from the fact that the experimental data represent a finite energy interval. The statistical nature of the fluctuations becomes apparent by comparing the correlation functions displayed in Figs. 21(b) and 21(c). The data in Fig. 21(c) represent an average of 12 autocorrelation functions (three levels and four angles). The data were normalized to 1 at $\delta = 0$ before averaging. The fluctuations at large δ are considerably damped in the average correlation function, as expected.

The average widths Γ , calculated as the FWHM of the autocorrelation functions, from all angles and all final levels of the residual nucleus for each reaction, are listed in column 4 of Table 9. The errors are standard deviations, calculated with the assumption that the values of Γ from all levels and angles are independent. This was not true in all cases, since several of the excitation functions are correlated. Column 5 of Table 9 contains the widths Γ_2 , computed from the mean-square fluctuation in the correlation functions, using Eq. (17). The average was taken in the interval

$$5\Gamma < \delta < \Delta E/2,$$

so that both conditions (13) and (15) would be fulfilled. Column 6 shows the widths, denoted as Γ_3 , calculated from the number of maxima in the excitation functions [see Eq. (18)]. The values of Γ_1 and Γ_2 agree remarkably well on the average for Cr^{52} and Fe^{56} . The widths Γ_3 , from the number of maxima, are higher by about a factor of two than the values calculated with the other two methods. However, the third method is expected to overestimate Γ if either the energy resolution or the magnitude of the energy steps approaches Γ . In the case of S^{32} the values of Γ_3 are expected to be more realistic than Γ_1 and Γ_2 . The resolution and the energy steps are much smaller than Γ in this case. On the other hand, there is a considerable gross structure in the excitation functions of $\text{P}^{31}(\text{p}, \alpha)$, which is expected to effect the values of Γ_1 and Γ_2 .

Fig. 21

Excitation and Autocorrelation Functions for the Reaction $\text{Mn}^{55}(\text{p}, \alpha)\text{Cr}^{52}$. (a) Excitation function at 170° . (b) Autocorrelation function for the data displayed in (a). The dashed line is a theoretical Lorentzian line with $\Gamma = 6$ keV. (c) Normalized autocorrelation function where this function is the average of 12 normalized [$R(0) = 1$] autocorrelation functions derived from the excitation functions for three levels (each at four angles) in the $\text{Mn}^{55}(\text{p}, \alpha)\text{Cr}^{52}$ reaction.

TABLE 9. Observed Values of Level Widths, Averaged from All Final Levels of the Residual Nuclei and All Angles. From the autocorrelation function Γ_1 is the FWHM and Γ_2 the width calculated from the mean-square fluctuation; the width Γ_3 was calculated from the number maxima in the excitation function. The quantity $(\text{kR})^{-1}$ is the coherence angle and n is the number of independent cross sections from Eq. (10).

Compound Nucleus	Average Excitation Energy, MeV	$(\text{kR})^{-1}$, degrees	Γ , keV			Reaction	Incoming Energy, MeV	n	No. of Auto-correlation Functions
			Γ_1	Γ_2	Γ_3				
S ³²	17.3	23	32 ± 15	20	33	P ³¹ (p,α)Si ²⁸	8.37-9.00	7	14
	19.0	21	41 ± 11	29	46		10.00-10.90	8	14
	19.9	20	38 ± 12	32	50		10.90-11.77	8	14
Cr ⁵²	19.7	19	8.3 ± 3.0	9.0	16.6	V ⁵¹ (p,α)Ti ⁴⁸	9.00-9.66	26	10
	20.7	18	9.3 ± 1.5	8.5	14.8		10.00-10.68	24	8
Fe ⁵⁶	19.7	18	6.8 ± 1.4	5.6	13.3	Mn ⁵⁵ (p,α)Cr ⁵²	9.50-9.90	20	12

Errors in the calculation of the widths and the mean-square fluctuations $R(\Gamma, 0)$ are introduced from the experimental resolution, the effect of finite energy range, and the direct interaction.

The effect of the experimental resolution Δ was studied in the case of Fe⁵⁶, where the ratio Δ/Γ has the largest value. Resolutions of 10, 15, 20, and 30 keV were simulated by averaging 2, 3, 4, and 6 of the original cross-section values of the excitation functions of Figs. 18, 19, and 20, and the quantities $R(\Gamma, 0)$, Γ_1 , and Γ_2 were recalculated from the new excitation functions. The excitation and correlation functions that were produced this way are plotted in Figs. 22 and 23. The dependence of the quantities $R(\Gamma, 0)$, Γ_1 , and Γ_2 on the resolution (averaged over all 12 excitation functions) are shown in Figs. 24(a) and 24(b). From qualitative extrapolations of the curves to $\Delta = 0$, it was estimated that the values of $R(\Gamma, 0)$ for Fe⁵⁶ in Fig. 25 should be increased by $15 \pm 10\%$, and the values of Γ_1 and Γ_2 in Table 9 decreased by $20 \pm 10\%$, in order to correct for the resolution effect. If one makes corrections for the resolution according to the curves of D. Lang, Fig. 3 of Ref. 61, for $\Gamma = \Delta$ one finds that $\Gamma_{\text{exp}}/\Gamma_{\text{theor}}$ varies from 1.2 to 1.4, depending on the type of the resolution function. This is in good agreement with the 10 to 30% correction of the present work, assuming energy resolution equal to the level width.

A large energy interval is required in order to minimize the finite-range errors. On the other hand, other types of errors may be introduced if too large an energy range is taken. These errors could be due to the change of the average cross section, and probably also due to the change in the reaction mechanism over a large energy range. It is argued by W. Gibbs that the effective number of independent cross sections is given by Eq. (10) as long as the energy range ΔE is not too large. The quantity n was calculated for the reactions that were studied, using $\Gamma = \Gamma_1$, and is listed in Table 9. The number of independent cross sections, n , is sufficiently large for Cr⁵² and Fe⁵⁶, but not large enough for the experiments with P³¹, due to the large value of Γ for S³². If one wants to make

corrections from Eqs. (11) and (12) using $N = 1$, the values of Γ for S^{32} in Table 9 should be increased by 10% and the values of $R(\Gamma, 0)$ in Fig. 25 increased by 20%. The corrections for Cr^{52} and Fe^{56} are negligible.

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Fig. 22. Excitation Function from the Reaction $Mn^{55}(p, \alpha_0)Cr^{52}$ at 170° , and Functions Calculated from This By Summing 2, 3, 4, and 6 Points

121-1897

Fig. 23. Autocorrelation Functions for the Excitation Functions of Fig. 22

120-8528

Fig. 24

Dependence of Observed Fluctuation Parameters on Energy Resolution. Each parameter is derived from the average of 12 parameters, each of which is deduced from a separate excitation function. The results at $\Delta = 5$ are calculated from all the experiment data which were taken at 5-keV energy intervals. The results at $\Delta = 10, 15, 20$, and 30 are calculated by averaging 2, 3, 4, and 6 of the original cross-section values. (a) Increase of apparent width Γ as a function of energy resolution. (b) Decrease of apparent fluctuation amplitude with energy resolution.

Fig. 25

Mean-square Cross-section Fluctuation as a Function of Observation Angle. The theoretical curve in (d) was taken from Ref. 63 and calculated from Eq. (20) for the reaction $\text{Sc}^{45}(p, \alpha)\text{Ca}^{42}$.

121-1710

3. Evidence for Direct Interaction

Each of the three (p, α) reactions that were studied has an even-even residual nucleus, which has spin I_B equal to zero in the ground state. Since the spin I_b of the alpha particle is zero, so is the exit-channel spin,

$$S_2 = I_B + I_b. \quad (21)$$

The projection of the orbital angular momentum l_2 on the z-axis (which is defined by the direction of the beam) is also zero, and therefore only one reaction channel is open for the ground state of the residual nucleus at 0 and 180° with respect to the beam. The mean-square fluctuation of the excitation functions, $R(\Gamma, 0)$ from Eq. (5), is then expected to be equal to 1 at these two angles for pure compound-nucleus reactions.

The experimental values of the mean-square fluctuation $R(\Gamma, 0)$ for each of the excitation functions are shown in Fig. 25. Data are available at a maximum angle of 175° for P^{31} , and at 170° for the other two reactions. However, the values of $R(\Gamma, 0)$ for the ground state seem to be much smaller than unity at these angles, and a very large nuclear moment of inertia is required to drop the value of $R(\Gamma, 0)$ that fast between 180 and 170°.

The theoretical value of $R(\Gamma, 0)$ decreases away from 180 and 0°, and it is symmetrical about its minimum value at 90°. The angular distribution is given by the expression^{62,63}

$$R_{th}^{\theta}(\Gamma, 0) = \sum_m \langle \sigma_m^{\theta} \rangle^2 / \left(\sum_m \langle \sigma_m \rangle \right)^2; \quad (22)$$

$$m = \{m_A, m_a, m_B, m_b\},$$

where the summation is over all sets of magnetic quantum numbers of the target, the residual nucleus, and the incoming and outgoing particles. The decrease away from 180° is faster for the reaction $V^{51}(p, \alpha)Ti^{48}$ due to the high spin of $V^{51}(I_A = 7/2)$, and smaller for the reaction $P^{31}(p, \alpha)Si^{28}$ with $I_A = 1/2$ for P^{31} . The relative cross section for direct interaction, $\sigma_D / \langle \sigma \rangle$, can be calculated from Eq. (8):

$$R_{exp}^{\theta}(\Gamma, 0) = R_{th}^{\theta}(\Gamma, 0)(1 - \sigma_D^2 / \langle \sigma \rangle^2), \quad (23)$$

where R_{exp}^{θ} and R_{th}^{θ} are the experimental and theoretical values of the mean-square fluctuation, respectively, at angle θ . The quantity $R_{th}^{\theta}(\Gamma, 0)$ was calculated as a function of angle by A. Richter *et al.*⁶⁴ for the reaction $Sc^{45}(p, \alpha)Ca^{42}$, and it is displayed in Fig. 25(d), along with the present experimental data for $V^{51}(p, \alpha)Ti^{48}$ in the same energy region. The cross section

121-1711

Fig. 26. Cross Section of Formation and Spin Distribution of the Compound Nuclei Formed by Bombardment of P^{31} , Sc^{45} , V^{51} , and Mn^{55} with Protons. P_J is the probability for formation of the compound nucleus with spin J .

of formation of the compound nucleus and the angular distributions are the same for the last two reactions, all the spins and parities are the same, and the rest of the parameters are very much alike. The cross sections of formation and the spin distributions are shown in Fig. 26, the details being described in Appendix III. Therefore, the variation of $R_{th}(\Gamma, 0)$ as a function of angle is expected to be the same for the two reactions. The relative cross section for direct interaction from Eq. (23) was found to be approximately 50% for V^{51} at 171° in the energy region 9.0-9.7 MeV [see Fig. 25(d)], this being the lowest value calculated from the present data. As high as 90% direct interaction was calculated for the same reaction in the energy region 10.0-10.7 MeV [see Fig. 25(c)]. Similar results were obtained by other investigators for different reactions.⁶⁴

From the angular distributions of the experimental cross section in this energy region and other evidence (see Ch. V), it seems very unlikely that such a large percentage of a noncompound-nucleus process takes place. In fact, pure compound-nucleus reaction is usually assumed at backward angles.⁶⁵ It has been suggested by P. Moldauer²¹ that the small values of mean-square fluctuations could be due to correlations in the resonance amplitudes in Eq. (3), imposed by the unitarity of the S-scattering matrix for resonance reactions. The independence hypothesis that goes into the derivation of Eq. (22) has also been questioned by Gibbs.⁶⁶

For the above reasons, one may also expect correlations between the cross sections to different final levels, contrary to what was previously believed. The cross-correlation function between two excitation functions 1 and 2, at two different angles, or two different final levels of the residual nucleus, is calculated from Eq. (9). Typical normalized cross-correlation functions

$$R_{1,2}^N(\Gamma, \delta) = R_{1,2}(\Gamma, \delta) / [R_1(\Gamma, 0) R_2(\Gamma, 0)]^{1/2} \quad (23a)$$

are displayed in Fig. 27. The correlation amplitude

$$R_{1,2}^N(\Gamma, 0) = R_{1,2}(\Gamma, 0) / [R_1(\Gamma, 0) R_2(\Gamma, 0)]^{1/2} \quad (24)$$

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Fig. 27. Normalized Cross-correlation Functions for the Reaction $\text{Mn}^{55}(\text{p}, \alpha)\text{Cr}^{52}$, Averaged over the Three Excited Levels

is equal to 1 for completely correlated functions 1 and 2, and equal to zero if no correlation exists. The experimental correlation amplitudes between the different final levels of all residual nuclei are listed in Tables 10(a), 10(b), and 10(c). No correlation was found between the ground and the first excited levels of the residual nucleus in the $\text{P}^{31}(\text{p}, \alpha)\text{Si}^{28}$ reaction for the energy regions 8.37-9.00 MeV and 10.00-10.90 MeV. However, there is some indication of correlation between the two levels in the energy region 10.90-11.77 MeV [see Table 10(a)]. There are also indications of correlation between the ground and first excited levels in Ti^{48} from the reaction $\text{V}^{51}(\text{p}, \alpha)\text{Ti}^{48}$ [see Table 10(b)]. There is a strong correlation at 60° , probably due to direct interaction in the forward angles. Finally, there was no correlation found between the three final levels in Cr^{52} from the reaction $\text{Mn}^{55}(\text{p}, \alpha)\text{Cr}^{52}$ [see Table 10(c)], the correlation amplitudes showing only statistical fluctuations.

TABLE 10. Cross-correlation Functions between Different Final Levels

a. Normalized cross-correlation amplitude $R_{1,2}^N(\Gamma, 0)$ from Eq. (24) between the ground and first excited levels in Si^{28} , from the reaction $\text{P}^{31}(\text{p}, \alpha)\text{Si}^{28}$

$E_p = 8.37-9.00 \text{ MeV}$		$E_p = 10.00-10.90 \text{ MeV}$		$E_p = 10.90-11.77 \text{ MeV}$	
θ_L, deg	$R_{1,2}^N(\Gamma, 0)$	θ_L, deg	$R_{1,2}^N(\Gamma, 0)$	θ_L, deg	$R_{1,2}^N(\Gamma, 0)$
175	0.17	175	-0.33	175	0.41
170	0.22	161	0.26	161	0.52
150	0.00	141	0.14	141	0.33
120	-0.16	111	-0.10	111	0.39
90	0.06	81	-0.52	81	0.26
60	0.01	69	-0.36	69	-0.50
30	-0.16	39	0.49	39	0.50

b. Normalized cross-correlation amplitude $R_{1,2}^N(\Gamma, 0)$ from Eq. (24) between the ground and first excited levels in Ti^{48} , from the reaction $\text{V}^{51}(\text{p}, \alpha)\text{Ti}^{48}$

$E_p = 9.00-9.66 \text{ MeV}$		$E_p = 10.00-10.68 \text{ MeV}$	
θ_L, deg	$R_{1,2}^N(\Gamma, 0)$	θ_L, deg	$R_{1,2}^N(\Gamma, 0)$
171	0.28	170	0.57
160	0.56	150	0.22
140	0.32	130	0.31
75	0.57	90	0.36
60	0.82		

c. Normalized cross-correlation amplitude $R_{1,2}^N(\Gamma, 0)$ between different levels in Cr^{52} , from the reaction $\text{Mn}^{55}(\text{p}, \alpha)$

Angle, deg	Levels		
	G-E ₁	G-E ₂	E ₁ -E ₂
170	-0.28	0.09	0.13
150	-0.07	0.32	-0.15
130	0.37	0.02	0.18
90	0.59	0.11	0.26

The cross-correlation functions were also calculated for each reaction between all excitation functions at different angles, and the correlation amplitudes

$$R_{1,2}^N(\Delta\theta, 0) = R(\Delta\theta, \delta = 0) / [R_1(\theta_1, 0) R_2(\theta_2, 0)] \quad (25)$$

are listed in Tables 11(a, b, c, d, e). The coherence angles $(kR)^{-1}$ listed in Table 9 were calculated by the equation⁴¹

$$(kR)^{-1} = \frac{209.67(M_a + M_A)}{E_a^{1/2} M_a^{1/2} M_A^{4/3}} \quad (\text{in degrees}), \quad (26)$$

where k is the wave number of the incoming particle, R the radius of the target nucleus, M_a and M_A the masses (in amu) of the projectile and the target, respectively, and E_a the energy in MeV of the projectile in the laboratory system. Excitation functions at two different angles that have a difference smaller than $(kR)^{-1}$ are expected to be strongly correlated, and the correlation amplitude of Eq. (24) to be close to 1. In Tables 11a-11e, however, one observes correlations between excitation functions at angles differing much more than $(kR)^{-1}$ as well. This is against the predictions of the statistical theory, and could be due to direct interaction.

TABLE 11. Cross-correlation Functions between Different Angles

a. Normalized cross-correlation amplitude $R_{1,2}^N(\Delta\theta, 0)$ from Eq. (25) between excitation functions at different angles from the reaction $P^{31}(p, \alpha)Si^{28}$

Proton Energy, MeV	Level	Angle, deg	Amplitude					
			30°	60°	90°	120°	150°	170°
8.37-9.00	Ground	60	0.12					
		90	0.32	-0.07				
		120	0.39	0.56	0.01			
		150	0.31	0.19	0.03	0.05		
		170	0.52	0.32	-0.06	0.56	0.42	
		175	0.51	0.28	-0.04	0.59	0.29	0.98
8.37-9.00	E ₁	60	-0.20					
		90	-0.28	0.07				
		120	0.33	0.54	-0.37			
		150	0.37	0.23	-0.25	0.40		
		170	0.50	-0.13	-0.25	0.32	0.70	
		175	0.48	-0.18	-0.24	0.32	0.58	0.99

TABLE 11 (Contd.)

b. Normalized cross-correlation amplitude $R_{1,2}^N(\Delta\theta, 0)$ from Eq. (25) between excitation functions at different angles from the reaction $P^{31}(p, \alpha)Si^{28}$

Proton Energy, MeV	Level	Angle, deg	Amplitude					
			39°	69°	81°	111°	141°	161°
10.00-10.90	Ground	69	-0.27					
		81	-0.18	0.42				
		111	-0.19	0.46	-0.21			
		141	0.39	0.06	0.08	-0.34		
		161	0.56	0.24	0.05	-0.05	0.60	
		175	0.40	0.32	-0.02	0.07	0.41	0.93
10.00-10.90	E ₁	69	0.19					
		81	0.11	0.87				
		111	0.62	0.33	0.19			
		141	0.20	0.57	0.35	0.45		
		161	-0.03	-0.17	-0.29	0.00	0.39	
		175	-0.08	-0.51	-0.45	-0.04	-0.31	0.70
10.90-11.77	Ground	69	-0.40					
		81	-0.34	0.91				
		111	-0.03	0.26	0.34			
		141	0.33	-0.03	0.09	0.09		
		161	0.04	0.40	0.49	0.68	0.42	
		175	0.17	0.19	0.12	0.59	-0.18	0.70
10.90-11.77	E ₁	69	0.16					
		81	-0.15	1.00				
		111	0.06	0.32	0.50			
		141	0.95	0.14	-0.01	0.33		
		161	0.54	0.33	0.23	-0.01	0.31	
		175	0.21	0.41	0.36	0.03	-0.08	1.00

c. Normalized cross-correlation amplitude $R_{1,2}^N(\Delta\theta, 0)$ from Eq. (25) between excitation functions at different angles from the reaction $V^{51}(p, \alpha)Ti^{48}$

Proton Energy, MeV	Level	Angle, deg	Amplitude			
			60°	75°	140°	160°
9.00-9.66	Ground	75	0.58			
		140	0.58	0.51		
		160	0.35	0.36	0.55	
		171	0.27	0.31	0.47	0.81
9.00-9.66	E ₁	75	0.82			
		140	0.71	0.62		
		160	0.54	0.39	0.48	
		171	0.35	0.13	0.34	0.70

TABLE 11 (Contd.)

- d. Normalized cross-correlation amplitude $R_{1,2}^N(\Delta\theta, 0)$ from Eq. (25) between excitation functions at different angles from the reaction $V^{51}(p, \alpha)Ti^{48}$

Proton Energy, MeV	Level	Angle, deg	Amplitude		
			115°	140°	155°
10.00-10.68	Ground	140	0.43		
		155	0.37	0.66	
		170	0.11	0.26	0.47
10.00-10.68	E_1	140	0.63		
		155	0.49	0.63	
		170	-0.03	-0.01	0.20

- e. Normalized cross-correlation amplitude $R_{1,2}^N(\Delta\theta, 0)$ from Eq. (25) between excitation functions at different angles from the reaction $Mn^{55}(p, \alpha)Cr^{52}$

Proton Energy, MeV	Level	Angle, deg	Amplitude		
			90°	130°	150°
9.5-9.9	Ground	130	0.00		
		150	-0.13	0.52	
		170	-0.05	0.03	0.49
9.5-9.9	E_1	130	0.37		
		150	0.01	0.49	
		170	0.06	0.14	0.35
9.5-9.9	E_2	130	0.35		
		150	0.21	0.18	
		170	0.06	-0.04	0.70

C. Theoretical Level Widths

1. Calculation of Widths

The partial energy widths $\Gamma_J^b(U_C)$ of the compound nucleus with spin J at excitation energy U_C , for the emission of particles b , can be calculated by summing the transmission coefficients over all exit channels that are available to particles b :

$$\Gamma_J^b(U_C) = \frac{1}{2\pi\rho_C(U_C, J)} \int_{U_B} \sum_{\ell_2, S_2, I_B} T_{b\ell_2} \rho_B(U_B, I_B), \quad (27)$$

where $\rho_C(U_C, J)$ and $\rho_B(U_B, I_B)$ are the level densities for the particular spin and both parities of the compound and residual nuclei, respectively, S_2 the exit-channel spin, and I_B the spin of the residual nucleus. The optical-model transmission coefficients were computed with the program ABACUS-2 by E. H. Auerbach,⁶⁷ revised for the ANL CDC-3600 computer. The optical-model parameters used were from F. Bjorklund and S. Fernbach⁶⁸ for neutrons, F. Perey⁶⁹ for protons, and J. Huizenga and G. Igo⁷⁰ for alpha particles. Other kinds of particles were inhibited at low energies because of the negative Q -value for those reactions.

The absolute value of the theoretical widths, the energy and spin dependence, and the relative values of Γ^b for the emission of different kinds of particles depend primarily on the level density form for ρ_C and ρ_B , and the level-density parameters which are used in the calculation.

Three methods of calculations, using different forms of level density, are described in Appendix III. In the first method, called the "Fermi-gas" calculation, the Fermi-gas form of level density with pairing-energy correction is used for ρ_C and ρ_B . In the second, the "constant-temperature" calculation, the constant-temperature form of the level density is used for ρ_C and ρ_B . Finally, in the "modified constant-temperature" calculation, the constant-temperature form of density is used only for the residual nuclei, and the Fermi-gas form for the relative density of the compound nucleus with respect to the density of the residual nuclei. The "constant-temperature" calculation gives average level widths which are independent of the excitation energy and is discussed in Appendix III. It is not a realistic calculation because one does not expect the nuclear temperature to be constant in a large energy region. The "modified constant-temperature" calculation has been studied by Vonach and Huizenga.¹⁹ Here the spin dependence of Γ for various nuclear temperatures, spin cut-off factors, and pairing energies have been described.

2. Results and Comparisons

Some of the results from the calculation of the level widths are displayed in Figs. 28-33. The constants are given in the figures, and U_C is the excitation energy of the compound nucleus; σ_{rigid}^2 and $\sigma_{1/2 \text{ rigid}}^2$ are the spin cut-off factors calculated assuming rigid and half-rigid body moments of inertia for the nucleus, respectively; t and T are the thermodynamic and the nuclear temperatures, respectively; a is the Fermi-gas constant and A the mass number; 2Δ is the pairing energy for an even-even nucleus, Δ for an odd-mass nucleus, and zero for an odd-odd nucleus. The level-density models and the constants are discussed in Appendix II.

Fig. 29

Spin Dependence of the Total Energy-level Widths of S^{32} , Cr^{52} , and Fe^{56} . The solid lines were calculated using Fermi-gas level densities with $a = A/10$ and pairing energies 2Δ for even-even nuclei and Δ for odd-even nuclei. The dotted lines were calculated using constant-temperature level densities for the residual nuclei with $T_{Res}(S^{32}) = 2.0$ MeV, $T_{Res}(Cr^{52}) = 1.5$ MeV, $T_{Res}(Fe^{56}) = 1.4$ MeV, and pairing energies as above.

Fig. 28
Spin Dependence of the Partial Energy-level Widths of Fe^{56} . (a) The solid lines were calculated using Fermi-gas level densities with $a = A/10$, $\sigma^2 = (\mathcal{J} \text{ rigid}/\hbar^2) t$ and pairing energies $\Delta(Fe^{56}) = \Delta(Cr^{52}) = 3.0$ MeV; $\Delta(Fe^{55}) = \Delta(Mn^{55}) = 1.5$ MeV. (b) The dotted lines were calculated using constant-temperature level densities for the residual nuclei, with $T_{Res} = 1.4$ MeV, $\sigma^2 = (\mathcal{J} \text{ rigid}/\hbar^2) t$ and the same pairing energies as in (a).

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Fig. 30. Energy Dependence of the Average Level Width of S^{32} with Spin Distribution Given in Fig. 26

Fig. 31. Energy Dependence of the Average Level Width of Cr^{52} with Spin Distribution Given in Fig. 26

Fig. 32. Energy Dependence of the Average Level Width of Fe^{56} with Spin Distribution Given in Fig. 26

Fig. 33. Energy Dependence of the Average Level Widths of S^{32} , Ti^{46} , Cr^{52} , and Fe^{56} with Spin Distributions Given in Fig. 26. The level density constants are those given in Figs. 30-32 for $\sigma_{1/2}^{rigid}$. The constants for the widths of Ti^{46} are the same as for Cr^{52} .

The spin dependences of the partial widths of Fe^{56} for the emission of neutrons, protons, and alpha particles are shown in Fig. 28, for two types of calculation. The width Γ_{Tot}^J or Γ_J is given by

$$\Gamma_J = \Gamma_{\text{Tot}} = \sum_b \Gamma_b^J, \{b = n, p, \alpha\}.$$

The Fermi-gas constants for the compound nucleus in the "modified constant-temperature" calculation are the same as in the "Fermi-gas" calculations. The spin dependences of the total widths for S^{32} , Cr^{52} , and Fe^{56} are shown in Fig. 29 for two types of calculations at each excitation energy. The total width Γ for the decay of the compound nucleus is calculated from the equation

$$\frac{1}{\Gamma} = \frac{\sum_J (P_J / \Gamma_J)}{\sum_J P_J}, \quad (28)$$

where P_J is the probability of formation of the compound nucleus with spin J . The spin distributions of the compound nuclei S^{32} , Cr^{52} , and Fe^{56} , formed by the bombardment of P^{31} , V^{51} , and Mn^{55} , respectively, with protons, were calculated by Eq. (1) of Appendix III and are displayed in Fig. 26. The calculated total width Γ is plotted as a function of excitation energy in Figs. 30-33 for S^{32} , Cr^{52} , and Fe^{56} . The experimental points of S^{32} in Fig. 30 are the values of Γ_3 from Table 9, calculated from the number of maxima in the excitation functions. The experimental points for Cr^{52} and Fe^{56} in Figs. 31 and 32 are the values of Γ_1 of Table 9. As was pointed out in Sect. B.2 of this chapter, the above values of Γ are believed to represent the best values in each case.

In the present work, for the calculation of the level density at high excitation energy, one is primarily interested in the energy and spin dependence of the level width Γ . Level widths were calculated with the "Fermi-gas" and "modified constant-temperature" formulae for about 10 nuclides with mass numbers from 27 to 100, with large variations in the level-density parameters. The values of the parameters a , T , and 2Δ for the calculations that are displayed in Figs. 28-33 are those given in the literature.⁷¹⁻⁷³ The spin cut-off factors were calculated for rigid and half-rigid moments of inertia, as indicated from the study of the angular distributions of the cross section (see Ch. V). From the above calculations the following conclusions are made: The deviation of the theoretical level width as a function of energy depends to a small extent on the level-density form used in the calculations. It is rather insensitive to the level-density parameters and is in good agreement with the experiment. The spin dependence of the widths, on the other hand, is completely different for the

"Fermi-gas" and the "modified constant-temperature" calculations. It also depends to some extent on the spin cut-off factor and the Fermi-gas constant or the nuclear temperature. The modified constant-temperature calculation gives widths which are almost independent of the spin, sometimes increasing with increasing spin, especially in the low-mass region. On the other hand, the "Fermi-gas" calculation gives, almost invariably, widths which decrease with increasing spin. The only experimental evidence on the spin dependence of the level width comes from the experiments by M. Halbert *et al.*⁷⁴ and of L. Put *et al.*⁷⁵ They measured the excitation functions for the reactions $\text{Al}^{27}(\text{p}, \alpha)\text{Mg}^{24}$, $\text{Mg}^{24}(\alpha, \alpha')\text{Mg}^{24}$, and $\text{C}^{12}(\text{O}^{16}, \alpha)\text{Mg}^{24}$, all having the same compound nucleus, Si^{28} . The energies were chosen in such a way that the excitation energy of Si^{28} was the same in each of the three reactions. The higher the mass of the projectile, the higher the average spin of the compound nucleus. The widths Γ of Si^{28} at 26 MeV of excitation energy, calculated from the excitation functions, were found to be 58, 87, and 120 keV, respectively, for the three reactions, indicating that Γ_J increases with increasing spin J . The results are consistent with the "modified constant-temperature" calculations for this reaction.

The absolute values of the theoretical widths depend primarily on the Fermi-gas constant and the nuclear temperature T for the two types of calculations, respectively. The width increases with increasing T or decreasing a . Finally, variation of the pairing energy effects the absolute values of Γ , as well as the relative widths, for the emission of different kinds of particles.

CHAPTER V

REACTION CROSS SECTIONS

A. Experimental Results

1. Experimental Requirements

The differential cross section to a single final level of the residual nucleus for a "compound-nucleus" reaction is predicted by the statistical theory to be symmetrical about 90° only if the Ericson fluctuations are damped, by averaging the cross section over a large energy interval in comparison with the average width Γ of the levels. The averaging can be achieved experimentally either by summing over a relatively large region of incoming energies, or by using broad energy resolution (e.g., using a thick target). An energy interval of about 50 to 100 keV is estimated from Figs. 14-20 to be necessary for the reactions $V^{51}(p,\alpha)Ti^{48}$ and $Mn^{55}(p,\alpha)Cr^{52}$. This corresponds to target thickness of about 1.5 to 3.0 mg/cm². On the other hand, it would be impossible to resolve the particle groups experimentally, corresponding to different final levels of the residual nucleus, using such a thick target which would have a stopping power of about 0.5 to 1.0 MeV for 10-MeV alpha particles. Therefore, the thickest targets from which good separation of the first two particle groups could be achieved were used. A typical spectrum of alpha particles from the reaction $Mn^{55}(p,\alpha)Cr^{52}$ with a 0.77-mg/cm² target is displayed in Fig. 5. Some angular distributions from the same reaction with a thin target of 0.32 mg/cm² are shown in Fig. 34. The energy resolution in the compound nucleus is about 15 keV, the target being at 50° with respect to the beam, and the average width Γ of the compound nucleus about 6 keV. One observes statistical fluctuations in the angular distributions, primarily in the ground state of the residual nucleus, which has spin zero. This is in agreement with the predictions of the statistical theory. The fluctuations are damped in Figs. 34(f) and 34(l), by averaging over an energy interval of the incoming particles of 80 keV, corresponding to about 13 times the width Γ .

2. Cross Sections from the Reaction $V^{51}(p,\alpha)Ti^{48}$

The differential cross sections for the ground state and the first excited level in Ti^{48} were measured, with the proton beam from the TVDG accelerator, in the 18-in. scattering chamber. A natural vanadium target of 0.607 ± 0.030 mg/cm² thickness was used at an angle of 50° with respect to the beam having an effective thickness of 0.792 mg/cm². The outgoing alpha particles were recorded simultaneously at 3 angles with solid-state detectors. Measurements were made at the laboratory angles 50, 70, 90, 110, 145, 155, and 175° , and at 3 proton energies: 8.5, 9.0, and 9.5 MeV. Corrections made for the center-of-mass system and the dead-time are described in Sect. B of Appendix I. The energy resolution was

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Fig. 34. Differential Cross Sections from the Reaction $\text{Mn}^{55}(\text{p}, \alpha)\text{Cr}^{52}$ with a 0.32-mg cm^{-2} Target. The energy spread in the compound nucleus is about 2Γ . The statistical fluctuations are damped in Figs. f and l by averaging over an energy region of 80 keV ($\sim 12\Gamma$).

about 22 keV, and the width Γ of the compound nucleus Cr^{52} about 9 keV (see Table 9). The differential cross section is plotted as a function of angle in Fig. 35, for a proton energy of 9 MeV. Errors of 15% for the ground state and 10% for the first excited level are mainly due to statistics of counting and uncertainty in the target thickness. The curves in Fig. 35 represent theoretical calculations of the "compound-nucleus" cross section and will be discussed in Sect. B of this chapter. The total cross section for the reaction is plotted as a function of energy in Fig. 36.

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Fig. 35. Differential Cross Sections from the Reaction $\text{V}^{51}(\text{p}, \alpha)\text{Ti}^{48}$. The curves were calculated from Eqs. [9(a,b)] of Appendix IV for rigid and half-rigid moments of inertia, assuming Γ independent of spin.

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Fig. 36. Total Cross Sections for the Ground State and the First Excited Level from the Reaction $\text{V}^{51}(\text{p}, \alpha)\text{Ti}^{48}$, Calculated from the Data Obtained at 8 Angles

3. Cross Sections from the Reaction $\text{Mn}^{55}(\text{p}, \alpha)\text{Cr}^{52}$

The differential cross sections for the ground state and for the first and second excited levels in Cr^{52} were measured with a 0.320-mg/cm² natural manganese target in the 18-in. scattering chamber at the TVDG accelerator. The target was at an angle of 50° with respect to the beam and had an effective thickness of 0.418 mg/cm² (corresponding to a proton-stopping power of about 14 keV). The outgoing particles were recorded with four solid-state detectors. Measurements were made at the laboratory angles 50, 70, 110, 145, 160, and 170°, and for proton energies from 9.50 to 9.90 MeV with 20-keV steps. Some of the results have already been discussed (see Fig. 34). The differential cross section averaged over the energy interval from 9.5 to 9.9 MeV (21 energies) is plotted as a function of angle in Figs. 37(a) and 37(b).

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Fig. 37(a). Differential Cross Sections from the Reaction of $\text{Mn}^{55}(\text{p}, \alpha)\text{Cr}^{52}$. The curves were calculated from Eqs. [9(a,b)] of Appendix IV for rigid and half-rigid moments of inertia, assuming Γ independent of spin.

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Fig. 37(b). Differential Cross Sections for the First and Second Excited Levels from the Reaction $\text{Mn}^{55}(\text{p}, \alpha)\text{Cr}^{52}$. The curves were calculated from Eqs. [9(a,b)] of Appendix IV for rigid and half-rigid moments of inertia, assuming Γ independent of spin.

A target with effective thickness of 1 mg/cm^2 (target 6 of Table 1) was used in a second experiment in which the ground and first excited levels were resolved (see Fig. 6). An energy resolution of about six times the level width of the compound nucleus Fe^{56} was thus obtained. Measurements were taken at the laboratory angles 90° , 130° , 160° , and 170° , and with proton energies from 7.0 to 12.5 MeV with 0.25-MeV steps. The total cross section is plotted as a function of the incoming energy in Fig. 38, along with the results by Ogata *et al.*^{76a}

B. Theoretical Differential Cross Sections

1. Calculation of the Differential Cross Section

When the compound nucleus is in the energy region where $\Gamma \gg D$, many overlapping levels are excited simultaneously and the random-sign approximation implies that the angular distribution of the cross section is symmetric about 90° . The differential cross section from the reaction $A(a,b)B$, to a single final level in the residual nucleus, per unit solid angle, is given by Eqs. (9a) and (9b) of Appendix IV:

$$\frac{d\sigma}{d\Omega}(\theta) = \frac{D_0}{\Gamma} \frac{\lambda^2}{8\pi(2I_A + 1)(2I_a + 1)}$$

$$\sum_{L, J, \ell_1, S_1, \ell_2, S_2} \frac{\delta_n^m (-1)^{S_2 - S_1} T_{\ell_1} T_{\ell_2} Z_1 Z_2 P_L(\cos\theta)}{(\Gamma_J/\Gamma)(2J+1) \exp[-J(J+1)/2\sigma_C^2]}, \quad (1)$$

where D_0 is the spacing of levels with spin zero in the compound nucleus and Γ_J the energy width for the decay of levels with spin J ; λ is the wavelength of the incoming particle;

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Fig. 38. Total Cross Section for the Ground State and the First Excited Level from the Reaction $Mn^{55}(p, \alpha)Cr^{52}$

I_x the spin of particle x , $S_1 = I_A + I_a$, $S_2 = I_B + I_b$, and T_{ℓ_1} and T_{ℓ_2} are the transmission coefficients for the incoming and outgoing particles; the coefficients Z_1 and Z_2 are defined in terms of the Clebsch-Gordon and Racah coefficients; δ_n^m is a delta function, where $m = (-1)^{\ell_1 + \ell_2}$, $n = \pi_1 \pi_2$, with ℓ_1 and ℓ_2 being the orbital angular moments of the entrance and exit channels, and π_1 and π_2 are the parities of the two channels, respectively. The spin cut-off factor of the compound nucleus is given by

$$\sigma_C^2 = \frac{\mathcal{I}_C}{\hbar^2} t, \quad (2)$$

t being the thermodynamic temperature and $\mathcal{I}_C$ the moment of inertia of the compound nucleus. The symmetry about 90° is imposed by selection rules allowing only even values of L for the Legendre polynomials $P_L(\cos\theta)$.

If the Γ_J is assumed to be independent of J and equal to an average

value Γ , the ratio D_0/Γ for a given incoming energy can be taken as a constant which characterizes the compound nucleus. One can calculate then from Eq. (1) the relative differential cross sections for the emission of different kinds of particles, for particles emitted to different final levels of the residual nuclei, and the angular distributions. The relative values of the cross sections to different final levels of the residual nuclei depend on the penetrabilities of the outgoing particles, expressed in terms of the transmission coefficients, and on the spins of the final levels, and their coupling

is expressed with the Z-coefficients. The Z-coefficients are kinematical, in the sense that they do not depend on the mechanism of the reaction, since they are expressed in terms of the vector-addition coefficients. The anisotropy $\sigma(180^\circ)/\sigma(90^\circ)$ in the angular distributions from Eq. (1) depends very sensitively on the spin cut-off factor σ_c^2 of the compound nucleus. However, for a given incoming energy, the same value of σ_c^2 should fit the experimental angular distributions of all final levels and all residual nuclei.

The assumption that Γ_J is independent of J is well-justified for the calculation of $1/D_0$ and level densities in the present work, for nuclear reactions with low-energy light ions and medium-mass target nuclei. The spin distribution in the compound nucleus and the spin dependence of the level width Γ_J have already been discussed, and curves are displayed in Figs. 26 and 29. For the reactions and the incoming energies of the present work, over 95% of the total cross section of formation of the compound nucleus is weighted with spins 1 to 6 (see Fig. 26). From the spin distributions of Γ_J in Fig. 29, one expects the approximation that Γ_J is equal to an average Γ for all J in the above spin limits, to be good to within 30% or better, depending on the level-density form and the level-density parameters used in the calculation. Furthermore, in order to test the possible errors which are introduced with the above assumption, the product $(\Gamma/D_0)(d\sigma/d\Omega)$ was evaluated from Eq. (1) for the first two levels in Cr^{52} from the reaction $\text{Mn}^{55}(p,\alpha)\text{Cr}^{52}$ in two ways. In the first, the ratio Γ_J/Γ was set equal to unity for all J. In the second, the ratio Γ_J/Γ was calculated from the "Fermi-gas" spin distribution of Γ_J (solid line at $U_c = 19.7$ MeV of Fig. 29), all the other parameters in (1) being the same. The angular anisotropy $\sigma(180^\circ)/\sigma(90^\circ)$ was about 15% higher in the second calculation, but there was negligible difference in the value of $(1/D_0)(d\sigma/d\Omega)$ that was calculated. For the above reasons, and also due to the fact that the correct form of the spin distribution of Γ_J is not known, the parameter Γ_J/Γ of Eq. (1) was set equal to unity for all J for the rest of the calculations.

2. Results

Typical angular distributions of the differential cross section, calculated from Eq. (1), are displayed in Figs. 35, 37(a), and 37(b), along with the experimental distributions at the same energies. Figure 35 shows the angular distributions for the ground and first excited levels from the reaction $\text{V}^{51}(p,\alpha)\text{Ti}^{48}$ at 9.0 MeV, and Figs. 37(a) and 37(b) the distributions for the ground and the first two excited levels from the reaction $\text{Mn}^{55}(p,\alpha)\text{Cr}^{52}$ at 9.7 MeV. The ratio Γ/D_0 was adjusted from the experimental value of the cross section for the first excited level around 90° for each of the reactions. Curves were derived with σ_c^2 calculated with rigid and half-rigid nuclear moments of inertia for the compound nucleus.

By comparing the distributions from (1) with the experimental data* from 5 (p, α) reactions with targets which have mass numbers from

*The experimental data for 3 of the reactions were taken from the literature.^{64,76b,77}

37 to 60, and in the energy region between 8 and 12 MeV, the following points are deduced. The angular distributions obtained using spin cut-off factors calculated with half the rigid nuclear moment of inertia fit the experimental data better than the ones with the rigid moment of inertia. However, this can be partly due to the approximation that Γ_J/Γ is equal to unity for all J . The relative cross sections for the different final levels, calculated by using the same ratio Γ/D_0 , agree very well with the experimental results. This indicates a compound-nucleus process for the mass and energy regions of the present reactions.

The products $(\Gamma/D_0)(d\sigma/d\Omega)$ from Eq. (1) for two final levels of the residual nuclei for each of the reactions $P^{31}(p,p')P^{31}$, $V^{51}(p,\alpha)Ti^{48}$, and $Mn^{55}(p,\alpha)Cr^{52}$ are displayed in Fig. 39 for incoming energies from about 7 to 12.5 MeV. Calculations were made for rigid and half-rigid moments of inertia for the compound nuclei. The solid lines represent the average cross section per steradian calculated from the equation

$$\sigma_{Avg} = \sigma_{Tot}/4\pi.$$

The details of the calculation are presented in Appendix IV.

3. Calculation of (Γ/D_0)

The ratio Γ/D_0 for each compound nucleus was calculated as a function of energy from the product $(\Gamma/D_0)(d\sigma/d\Omega)$ and the experimental cross sections $d\sigma/d\Omega$, in two ways. First, from the average differential cross section at each energy, values were calculated from all angles at which experimental data were taken, and the average values of $(\Gamma/D_0)(d\sigma/d\Omega)$ (solid lines in Fig. 39). The results for S^{32} , Cr^{52} , and Fe^{56} from the reactions $P^{31}(p,p')P^{31}$, $V^{51}(p,\alpha)Ti^{48}$, and $Mn^{55}(p,\alpha)Cr^{52}$, respectively, are displayed in Figs. 40 and 41. The straight lines in Fig. 41 represent least-square fits of the points from the two levels in Cr^{52} . Second, a separate value of Γ/D_0 was calculated from the data at each angle, and a straight line was fitted again to the points from all angles and all levels of the residual nuclei. There was good agreement in the results from the two calculations. The experimental cross sections used for the first and second excited levels from the reaction $P^{31}(p,p')$ were taken from J. Kokame et al.⁷⁸

Figure 42 shows the ratio Γ/D_0 as a function of excitation energy for 6 nuclides. The values for Ca^{40} , Ar^{38} , and Ni^{60} were calculated from the reactions $Ar^{39}(p,\alpha)Cl^{36}$, $Cl^{37}(p,\alpha)S^{34}$, and $Co^{59}(p,\alpha)Fe^{56}$. The experimental data for the three reactions were taken from D. von Ehrenstein et al.,⁷⁷ A. Richter et al.,⁶⁴ and H. Vonach and J. Huizenga,³² respectively.

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Fig. 39. The Differential Cross Sections Times the Ratios Γ/D_0 of the Compound Nuclei Computed from Eqs. [9(a,b)] for Spin Cut-off Factors Calculated with Rigid and Half-rigid Moments of Inertia

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Fig. 40. The Ratio Γ/D_0 (a) for S^{32} , Calculated from the Curves of Fig. 39 and the Experimental Cross Sections Obtained by Jum Kokame *et al.*,⁷⁸ (b) for Cr^{52} , Calculated from the Data Displayed in Figs. 37 and 39

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Fig. 41. The Ratio Γ/D_0 for Fe^{56} Calculated from the Data Displayed in Figs. 38 and 39. The straight lines represent least-square fits of the points.

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Fig. 42

The Ratio Γ/D_0 as a Function of Excitation Energy for Six Nuclides. The experimental data for Ca^{40} are from D. von Ehrenstein *et al.*, Ni^{60} from H. Vonach and J. Huizenga, and for Ar^{38} from A. Richter *et al.*; the ratio Γ/D_0 for Ar^{38} was calculated only at one energy.

CHAPTER VI

LEVEL DENSITIES

A. Experimental Results

1. Low-energy Calculations

The level densities of S^{32} , Cr^{52} , and Fe^{56} are shown in Figs. 43-46. The densities of Cr^{52} and Fe^{56} up to 6.0 and 6.5 MeV of excitation energies, respectively, were calculated by counting the number of resolved levels from (p, p') and (p, α) experiments with the magnetic spectrograph.

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Fig. 43

Level Density of S^{32} as a Function of Excitation Energy U . The data at $U = 9-11$ MeV were calculated from the number of levels with spin 0, 1, and 2 from the Nuclear Data Sheets. The data at $U = 16.6-18.1$ MeV were calculated from the average values of Γ/D_0 of Fig. 40, and the Γ -values from the "Fermi-gas" curve of Fig. 33, normalized to the experimental values of Γ_1 of Table 9. The curves were calculated with the Fermi-gas formula [Eq. (13) of Appendix II].

Fig. 44

Level Density of Cr^{52} as a Function of Excitation Energy U . The data at $U = 1-7.65$ MeV are from the experiments with the magnetic spectrograph (see Table 6). The data at $U = 18.8-19.8$ MeV were calculated from the average values of Γ/D_0 (see Fig. 40) and the Γ -values from the "Fermi-gas" curve of Fig. 33, normalized to the experimental values of Γ_1 of Table 9. The curves were calculated with the Fermi-gas formula [Eq. (13) of Appendix II].

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Fig. 45
Level Density of Fe^{56} as a Function of Excitation Energy U . The data at $U = 1-7.25$ MeV are from the experiments with the magnetic spectrograph (see Table 7). The data at $U = 17-23$ MeV were calculated from the average values of Γ/D_0 (see Fig. 41 and the Γ -values from the "Fermi-gas" curve of Fig. 33, normalized to the experimental value of 5.4 keV at $U = 19.7$ MeV. The curves were calculated with the Fermi-gas formula [Eq. (13) of Appendix II].

The results have already been discussed, and are listed in column 8 of Tables 6 and 7. Column 7 of the same tables shows the level densities up to excitation energies of 7.8 and 7.5 MeV for the two nuclides, respectively, calculated by analysing the same experimental data according to the evaporation model. The level density of Fe^{56} has also been calculated by H. K. Vonach and J. R. Huizenga³² up to 9 MeV of excitation energy, from the evaporation spectra of alpha particles from the reactions $\text{Co}^{59}(\text{p}, \alpha)\text{Fe}^{56}$ and $\text{Fe}^{56}(\text{p}, \text{p}')\text{Fe}^{56}$, which were studied with silicon surface-barrier detectors, with proton energies from 9 to 13.5 MeV. The results are shown in Fig. 45 and are in good agreement with the present data. It should be noted that both were normalized to the number of resolved levels from the same experiments with the magnetic spectrograph.

The level densities of S^{32} at 9.5 and 10.5 MeV were calculated from the number of resonances observed in the $\text{P}^{31}(\text{p}, \gamma)\text{S}^{32}$ and $\text{P}^{31}(\text{p}, \alpha)\text{Si}^{28}$ reactions with proton energies from 0.355 to 2.110 MeV, corresponding to excitation energies in S^{32} from 9.207 to 10.91 MeV. The experimental data were taken from Ref. 79. Since the spins of the proton and the nucleus P^{31} are both $1/2^+$, the entrance channel spin is 1^+ . From the transmission coefficients in Table 12, it is deduced that only protons with orbital angular momenta $\ell = 0, 1$, and 2 have significant values of the cross section. Therefore, only the resonances of the

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Fig. 46. Comparison of the Level Densities of S^{32} , Cr^{52} , and Fe^{56}

compound nucleus with spins $J = 0^\pm, 1^\pm, 2^\pm$, and $3^\pm$ would be observed. The ratio of the total level density $\rho(U)$ to the density $\rho(U, J = 0^\pm, 1^\pm, 2^\pm, 3^\pm)$ was calculated from Eq. (8) of Appendix II:

$$\rho(U) = \frac{2\sigma^2 \rho(U, J = 0^\pm, 1^\pm, 2^\pm, 3^\pm)}{1 + 3e^{(-1/\sigma^2)} + 5e^{(-3/\sigma^2)} + 3.5e^{(-6/\sigma^2)}} \quad (1)$$

If we assume a rigid-body nuclear moment of inertia for S^{32} and a nuclear temperature⁷¹ of 2.0 MeV, the spin cut-off factor σ^2 was calculated to be 8.8 (see Eqs. (4) and (6) of Appendix II). Therefore,

$$\rho(U) = 1.95 \rho(U, J = 0^\pm, 1^\pm, 2^\pm, 3^\pm). \quad (2)$$

Twelve resonances were observed between 9 and 10 MeV of excitation energy in S^{32} , and 19 resonances between 10 and 11 MeV. From Eq. (2),

$$\rho(U = 9.5) = 23 \pm 7 \text{ MeV}^{-1}$$

and

$$\rho(U = 10.5) = 37 \pm 12 \text{ MeV}^{-1},$$

the errors being attributed primarily to uncertainties in the experimental data and the approximation that ℓ has values between 0 and 2.

TABLE 12. Transmission Coefficients of Protons for P^{31} , Calculated with the Optical-model Parameters from Perey,⁶⁹ J is the Spin of the Compound Nucleus

E, MeV	J			
	$0^+, 1^+$	$0^-, 1^-, 2^-$	$0^+, 1^+, 2^+, 3^+$	$0^-, 1^-, 2^-, 3^-, 4^-$
	ℓ			
	0	1	2	3
1.0	4.5×10^{-4}	2.4×10^{-4}	1.7×10^{-5}	4.0×10^{-6}
2.0	4.6×10^{-2}	5.7×10^{-2}	2.5×10^{-3}	4.2×10^{-4}
3.0	1.8×10^{-1}	3.6×10^{-1}	2.1×10^{-2}	4.4×10^{-3}

2. High-energy Calculations

It was argued in Sect. C-2 of Ch. IV that the theoretical energy dependence of the total level width for the decay of the compound nucleus is

rather insensitive to the level-density form and the level-density parameters used in the calculation, and in good agreement with the experiments. The Fermi-gas model for the nuclear level density is in general believed to be more realistic than the constant-temperature one, especially at high excitation energies. Therefore, the values of Γ for S^{32} , Cr^{52} , and Fe^{56} in Table 13, which were used in the calculation of the level densities, were taken from the "Fermi-gas" curves in Fig. 33, normalized to the experimental values. The widths for S^{32} were normalized to the values of Γ_3 of Table 9, for Cr^{52} to the values of Γ_1 , and for Fe^{56} to the value Γ_1 after a correction of -20% for the experimental resolution. The reasons for the above choices are discussed in Sect. B-2 of Ch. IV. The parameter D_0 was then calculated as a function of excitation energy from the values of Γ/D_0 (see Figs. 40-42), for rigid and half-rigid moments of inertia.

TABLE 13. Nuclear Level Densities for S^{32} , Cr^{52} , and Fe^{56} at High Excitation Energies, Calculated from the Values of $(\Gamma/D_0)(d\sigma/d\Omega)$, the Experimental Cross Sections, and Values of Γ from the "Fermi-gas" Curves of Fig. 33 Normalized to the Experimental Values

Nuclide	UC (MeV)	Γ (keV)	σ^2 Rigid	$\rho(U)$ (MeV ⁻¹)	σ^2 Half-rigid	$\rho(U)$ (MeV ⁻¹)
S^{32}	16.6	21	9.5	$(5.0 \pm 1.7) \times 10^2$	4.8	4.4×10^2
	17.1	25	9.7	$(5.5 \pm 1.8) \times 10^2$	4.9	5.1×10^2
	18.1	35	10.0	$(6.4 \pm 2.1) \times 10^2$	5.0	6.0×10^2
Cr^{52}	18.84	5.5	18.8	$(1.1 \pm 0.4) \times 10^5$	9.4	9.5×10^4
	19.33	6.7	19.0	$(1.2 \pm 0.4) \times 10^5$	9.5	1.1×10^5
	19.82	7.9	19.2	$(1.7 \pm 0.6) \times 10^5$	9.6	1.6×10^5
Fe^{56}	17.0	2.5	18.8	$(6.6 \pm 2.2) \times 10^4$	9.4	4.7×10^4
	18.0	3.4	19.4	$(9.6 \pm 3.2) \times 10^4$	9.7	7.2×10^4
	19.0	4.5	20.0	$(1.4 \pm 0.5) \times 10^5$	10.0	1.1×10^5
	20.0	6.0	20.6	$(2.1 \pm 0.7) \times 10^5$	10.3	1.6×10^5
	21.0	8.2	21.2	$(2.9 \pm 1.0) \times 10^5$	10.6	2.5×10^5
	22.0	11.2	21.6	$(4.2 \pm 1.4) \times 10^5$	10.8	3.7×10^5
	23.0	15.0	22.2	$(6.1 \pm 2.0) \times 10^5$	11.1	5.7×10^5

The total level density $\rho(U)$ is related to the spacing D_0 of zero-spin levels by Eq. (7) of Appendix II:

$$\rho(U) = 2\sigma^2/D_0. \quad (3)$$

The spin cut-off factor was calculated from the equation

$$\sigma^2 = \frac{\mathcal{J}}{\hbar^2} t, \quad (4)$$

and the thermodynamic temperature t of the nucleus from

$$U_C - \Delta = a t^2 - t, \quad (5)$$

using $a = A/10$, where A is the mass number, and pairing energies Δ of 4 MeV for S^{32} and of 3 MeV for Cr^{52} and Fe^{56} . The level density as a function of excitation energy, calculated using rigid and half-rigid moments of inertia, and the respective spin cut-off factors are listed in Table 13. The densities are also shown in Figs. 43-46.

Errors in the calculation of the level density can be introduced due to uncertainties in the transmission coefficients T_ℓ , the spin cut-off factor σ^2 , the decay width Γ , and the experimental cross section $d\sigma/d\Omega$. The transmission coefficients can be considered as introducing negligible errors, for they are believed to be known with uncertainties below the denoted errors of the level densities in Table 13. The values of the level density are also not very different for values of the spin cut-off factor, differing by a factor of two. The value of σ^2 is believed, as indicated from the angular distributions, to be between the ones calculated with rigid and half-rigid nuclear moments of inertia. Finally, the experimental values of $d\sigma/d\Omega$ are known to within 10-15%. All the above sources introduce a standard deviation of about $\pm 30\%$.

On the other hand, possible sources of much larger errors are considered to be the decay width and the contribution from direct interaction in the measured cross section. Poor experimental energy resolution and gross structure in the excitation functions are the primary sources of errors in Γ , all of them increasing the observed experimental values. Smaller values of $\rho(U)$ would be calculated in this case. Errors in the same direction are introduced from the direct-interaction contributions to the experimental values of the cross section. However, the probability of direct interaction in the (p, α) reactions at low bombarding energies (8-12 MeV) is expected to be very small, for the following reasons:

(a) The angular distributions of the cross sections, when the conditions of the experiments are such that the statistical fluctuations are

negligible, are symmetrical about 90° , except for very small (forward) angles at the higher energies.

(b) The statistical-model calculations of the relative cross sections for different levels of the residual nuclei are in good agreement with experiments.

(c) The cross section for direct interaction, measured at higher energies, is much smaller than the cross section measured at low energies. For example, the cross section for the reaction $\text{Co}^{59}(p,\alpha)\text{Fe}^{56}$ at $E_p = 22$ MeV was found to be^{76a} smaller by about a factor of 15 than at $E_p = 12$ MeV, and smaller by about a factor of 100 than at $E_p = 8.7$ MeV. Since the cross section for direct interaction is increasing with increasing energy, it is indicated that it is less than 5% at bombarding energies below 12 MeV for this reaction.

B. Statistical Models

1. Constant Temperature

In the excitation energy region from 2 to 4 MeV the level densities of Cr^{52} and Fe^{56} show a definite structure, which is much more pronounced in the nucleus of Cr^{52} than Fe^{56} . At energies from 4 to 7.5 MeV the results are consistent with the constant-temperature model of level density increasing exponentially with excitation energy U as $\exp(U/T)$. Nuclear temperatures T of 1.1 ± 0.1 and 1.3 ± 0.1 MeV were derived for Cr^{52} and Fe^{56} , respectively. The nuclear temperature for Fe^{56} is smaller than the value of 1.37 MeV determined by Vonach and Huizenga³² for the excitation energy range from 5.0 to 9.5 MeV, and 1.54 MeV determined by Sherr and Brady⁸⁰ in the energy range from 7.0 to 13.0 MeV. The results indicate that the temperature is increasing with excitation energy. A much higher temperature of 2.5 ± 0.2 MeV is deduced in the high-energy region from 17 to 23 MeV for the same nucleus from data obtained with the statistical model. If one attempts to analyse according to the constant-temperature model the whole energy range of Figs. 43-45, average nuclear temperatures of 2.7, 1.7, and 1.7 MeV are derived for the nuclei S^{32} , Cr^{52} , and Fe^{56} , in the energy regions 9-18 MeV, 4-20 MeV, and 4-23 MeV, respectively. The above data indicate that a simple constant-temperature model is not sufficient to describe the level density over large energy ranges.

2. Fermi-gas

The experimental results in Figs. 43-45 are compared with curves derived with the Fermi-gas form of level density with pairing-energy correction, as given by Eq. (13) of Appendix II:

$$\rho(U) = \frac{ta^{1/2}}{12\sqrt{2} C^{1/2}(U+t-\Delta)^2} \exp[2a^{1/2}(U-\Delta)^{1/2}], \quad (6)$$

where the thermodynamic temperature t is related to the excitation energy by

$$U - \Delta = at^2 - t; \quad (7)$$

the parameter a is the Fermi-gas constant, Δ the pairing energy, and

$$C = \mathcal{I}/\hbar^2, \quad (8)$$

$\mathcal{I}$ being the nuclear moment of inertia.

The slope of the natural logarithm of the level density, calculated as a function of excitation energy, depends primarily on the parameter a , and is rather insensitive to C and Δ , the variation of which causes more or less parallel shifting of the level-density curve. The variation of the level density by varying each of the three parameters separately is shown in Figs. 43-45. The curves representing the best fits of the experimental level densities for S^{32} , Cr^{52} , and Fe^{56} , and the respective Fermi-gas parameters a , C , and Δ , are shown in Fig. 46. The pairing energies derived in the present work are independent of the parameter C , and strikingly smaller than the values given by Newton⁷² and by Nemirovski and Adamchuk,⁷³ the latter being about 3 MeV for Cr^{52} and Fe^{56} , and well above 3 MeV for S^{32} .

C. Discussion

The experimental level densities in small energy regions (5 to 10 MeV) are consistent with the constant-temperature and the Fermi-gas models. Furthermore, theoretical curves consistent with the experimental data can be derived with various combinations of the level density parameters, i.e., the parameters are not defined uniquely from the data in a small energy region. In order to explain the differences in the level densities at low excitation energies between nuclei with even and odd numbers of protons and neutrons, people redefine the ground state for the Fermi-gas model to be used in each case. Thus, one defines the excitation energy for the calculation of the level density from a fictitious energy level, which is Δ_n MeV above the ground state for a nucleus with even number of neutrons, and Δ_p MeV above the ground state for a nucleus with even number of protons; for an even-even nucleus this fictitious level is $\Delta_n + \Delta_p$ above the ground state. The quantities Δ_n and Δ_p are the pairing energies of two neutrons and two protons, respectively, and can be calculated from the differences in the masses of odd and adjacent even nuclei. One can fit the low-energy data with the standard technique, using the above pairing energies and large

values of the Fermi-gas parameter a . This, however, does not fit the high-energy data. Far too large level densities are calculated at 20 MeV, because the slope of density as a function of excitation energy is too large. Level densities, consistent with the experimental data in both energy regions, can be calculated using smaller values of a and reduced Δ . This indicates that the influence of the pairing energy gap disappears at a moderate excitation energy.

APPENDIX I

Experimental DetailsA. The Magnetic Spectrograph

A complete description of the magnetic spectrograph is given in references 47 and 48. A schematic drawing is displayed in Fig. 47. The focal surface has a length of 127 cm and a variable width of 1 or 3 cm. The particles are recorded on a set of five nuclear emulsion plates each of dimensions 5.1 x 25.4 cm². Reference lines, approximately 50 microns wide, are marked at accurately known positions 7 cm apart along the plates, by exposure to collimated light. After being exposed and developed, the plates are scanned under a microscope with a dark-field condenser, and magnification of 10 x 20. The number of tracks within each 0.5-mm zone along the plate is recorded as a function of distance relative to the reference marks.

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Fig. 47. Schematic Drawing of the Magnetic Spectrograph

The radius of the trajectory is related to the distance D along the plates by⁴⁷

$$dD/dr = 4R^2(9R^4 + 10R^2r^2 + r^4)^{1/2} / (3R^2 - r^2), \quad (1)$$

R being the radius of the uniform magnetic field (see Fig. 47). In practice it has been found that the relation between r and D can be expressed to a high accuracy through a power series of the form⁴⁸

$$r^2 = a_0 + a_1D + a_2D^2 + a_3D^3 + a_4D^4, \quad (2)$$

the constants a_0 through a_4 being determined experimentally. For normal operations magnetic inductions in the range from 2 to 10 kG can be set. The relativistic kinetic energy E_b of the particles is given by⁸¹

$$1 + E_b \frac{10^3 F}{M_b C^2} = \left[\left(B_r \frac{zF}{10 M_b C} \right)^2 + 1 \right]^{1/2}, \quad (3)$$

where

- C = velocity of light in vacuum, cm/sec⁻¹;
- E_b = kinetic energy of the particle, MeV;
- M_b = rest mass of the particle, amu;
- F = 96,520 Coulombs (g/mole)⁻¹;
- B = magnetic induction, gauss;
- z = atomic number of the particle,

and the Q-value for the reaction $A(a,b)B$ is given by the relativistic relation⁸²

$$Q = E_b - E_a + M_B C^2 \left\{ \left[1 + \left(\frac{M_b}{M_B} \right)^2 \frac{E_b}{M_b C^2} \left(2 + \frac{E_b}{M_b C^2} \right) + \left(\frac{M_a}{M_B} \right)^2 \frac{E_a}{M_a C^2} \left(\frac{E_a}{M_a C^2} + 2 \right) - \frac{2 M_a M_b}{M_B^2} \right. \right. \\ \left. \left. \cdot \left[\frac{E_a E_b}{M_a C^2 M_b C^2} \left(2 + \frac{E_b}{M_a C^2} \right) \left(2 + \frac{E_a}{M_b C^2} \right) \right]^{1/2} \cos \theta_b \right]^{1/2} - 1 \right\}, \quad (4)$$

where

M_a, M_b, M_A, M_B are the masses of the particles a, b, A and B , respectively, amu;

$Q = m_a + m_A - m_b - m_B - U_B$; m_a, m_A, m_b, m_B are the masses of a, A, b and B , units of MeV;

U_B = excitation energy, MeV;

E_a = projectile energy, MeV;

$C^2 = 931.441 \text{ MeV}(\text{amu})^{-1}$;

θ_b = laboratory angle of observation.

Finally, the solid angle of the spectrograph as a function of distance on the plates is given in Fig. 48.

Fig. 48
Solid Angle of the Magnetic Spectrograph vs Distance
along the Emulsion Plates

A computer program was written to calculate the Q-value of the reaction corresponding to each point on the plates. The number of tracks per 0.5-mm zone or the cross section of the reaction can be plotted by the computer as a function of distance on the plates or the excitation energy of the residual nucleus.

B. Center-of-mass Corrections

The angle of observation, θ_C , in the center-of-mass system for a nuclear reaction $A(a, b)B$, is related to the angle θ_L in the laboratory system by⁸³

$$\frac{\sin(\theta_C - \theta_L)}{\sin \theta_L} = \frac{V}{v_b}, \quad (5)$$

where V is the velocity of the center of mass and v_b the velocity of particle b in the center-of-mass system. The ratio of the velocities is given by

$$\frac{V}{v_b} = \left[\frac{M_a M_b}{M_B (M_b + M_B - M_a)} \right]^{1/2} \left(1 + \frac{M_b + M_B}{M_b + M_B - M_a} \frac{Q}{E_a} \right)^{-1/2}, \quad (6)$$

where M_a , M_A , M_b , and M_B are the masses of a , A , b , and B , E_a the energy of the incident particle in the laboratory system, $Q = m_a + m_A - m_b - m_B - U_B$, m_a , m_A , m_b , m_B being the masses of the particles in energy units, and U_B the excitation energy of the residual nucleus. The ratio of the intensities of particles $I(\theta_C)$ and $J(\theta_L)$ in the center-of-mass and the laboratory systems, respectively, is given by

$$G(\theta_C, \theta_L) \equiv \frac{I(\theta_C)}{J(\theta_L)} = \frac{\sin \theta_L d\theta_L}{\sin \theta_C d\theta_C} = \cos(\theta_C - \theta_L) \frac{\sin^2 \theta_L}{\sin^2 \theta_C}. \quad (7)$$

The cross section of a proton-induced reaction in an experiment with solid-state detectors can be calculated from

$$\sigma = \frac{N}{\ln \Delta x} (\text{cm}^2), \quad (8)$$

or from

$$\sigma = 0.264 \frac{RA}{\Omega_{Lqm}} \frac{R_p}{R_t} G(\theta_C, \theta_L) (\text{microbarns/sr}), \quad (9)$$

where

N = number of events, $\text{cm}^{-2}\text{sec}^{-1}$;

I = number of incident particles, $\text{cm}^{-2}\text{sec}^{-1}$;

n = number of nuclei in the target, cm^{-3} ;

Δx = range of incident particle in the target, cm;

R = total number of counts on the detector;

Ω_L = solid angle of the detector, steradians;

A = atomic mass of the target, amu;

m = target thickness, $\mu\text{g}/\text{cm}^2$;

q = total current through the target, μC ;

R_t = pulse rate of the pulse generator, min^{-1} ;

R_p = observed number of counts from the pulse generator, min^{-1} .

APPENDIX II

Nuclear Level Densities

A. Thermodynamic and Nuclear Temperature

When the excitation energy of the nucleus is considerably larger than the spacing of the levels, statistical thermodynamics is used to derive the density of levels and states. The thermodynamic temperature t of the nucleus is defined by

$$\frac{1}{t} = \frac{dS}{dE}, \quad (1)$$

where S is the entropy and E the energy of the system. The natural logarithm of the average density $\rho(E)$ of states or levels is referred to as the "nuclear entropy" and its derivative with respect to the entropy is defined as the "nuclear temperature" T :

$$\frac{1}{T} = \frac{d \ln \rho(E)}{dE}. \quad (2)$$

Several nuclear temperatures can be defined depending on $\rho(E)$, which can be defined as the density of states, levels, or states characterized by a specific quantum number. The following symbols are used in the present work: $\rho(U)$ for the total density of levels at excitation energy U and T for the nuclear temperature; $\rho(U, J)$ for the density of levels with spin J and T_J for the respective nuclear temperature. Some level density expressions from references 4, 7, 8, 12, 19, 26, 84, and 85 are given below in the exact form they were used in the present calculation.

B. Spin Dependence of the Level Density

The number of levels with a given angular momentum J is assumed to be given by

$$\rho(U, J) = \rho(U, J=0)(2J+1) \exp[-J(J+1)/2\sigma^2], \quad (3)$$

where σ^2 is called the spin cut-off factor:

$$\sigma^2 = \frac{\mathcal{J}}{\hbar^2} t, \quad (4)$$

and $\mathcal{J}$ has the form of a moment of inertia. The rigid-body moment of inertia is given by

$$\mathcal{J} = \frac{2}{5} M A R^2. \quad (5)$$

If R is assumed to be given by $R = 1.25 \times 10^{-13} A^{1/3}$ cm, then

$$c = \mathcal{J}/\hbar^2 = 0.0137 A^{5/3} \text{ MeV}^{-1}. \quad (6)$$

Integration of (3) over all J leads to the total density

$$\rho(U) = \rho(U, J=0) 2\sigma^2. \quad (7)$$

From Eqs. (3) and (7) one can also write

$$\rho(U, J) = \frac{\rho(U)}{2\sigma^2} (2J+1) \exp[-J(J+1)/2\sigma^2]. \quad (8)$$

For values of σ^2 calculated for medium-mass nuclei with moment of inertia not much different than the rigid moment, Eq. (8) is an approximation correct to within 5% of the expression⁸⁶

$$\rho(U, J) = \rho(U) \left\{ \left[\exp \frac{-J^2}{2\sigma^2} \right] - \left[\exp \frac{-(J+1)^2}{2\sigma^2} \right] \right\}. \quad (9)$$

C. Fermi-gas Model

The density of levels, $\rho(U)$, for a nucleus with an excitation energy U and pairing-energy correction Δ can be written as⁸

$$\rho(U) \cong \rho'(U - \Delta), \quad (10)$$

where Δ is the pairing energy and $\rho'(U - \Delta)$ is the independent-particle-model level density without pairing correction. The thermodynamic temperature is then related to the excitation energy by

$$U - \Delta = at^2 - t, \quad (11)$$

where the parameter a is the Fermi-gas constant, and the density of levels with angular momentum J of both parities, at excitation energy U , is given by

$$\rho(U, J) = \frac{a^{1/2}(2J+1)}{24\sqrt{2} c^{3/2}(U+t-\Delta)^2} \exp \left[2a^{1/2}(U-\Delta)^{1/2} - J(J+1)/2ct \right]. \quad (12)$$

The total level density can be calculated from Eqs. (8) and (12):

$$\rho(U) = \frac{ta^{1/2}}{12\sqrt{2} c^{1/2}(U+t-\Delta)^2} \exp \left[2a^{1/2}(U-\Delta)^{1/2} \right]. \quad (13)$$

D. The Constant-temperature Model

If the nuclear temperature is assumed to be constant in the energy region between U and U' , one can write from Eq. (2)

$$\rho(U) = \rho(U') \exp\left(\frac{U - U'}{T}\right), \quad (14)$$

where $\rho(U')$ is a constant term which can be calculated by any other model. The density of levels with spin J is given by

$$\rho(U, J) = \frac{\rho(U')}{2\sigma^2} (2J + 1) \exp\left[\frac{U - U'}{T} - \frac{J(J + 1)}{2\sigma^2}\right]. \quad (15)$$

APPENDIX III

Energy-level Widths

The cross section of formation and the spin distribution of the compound nucleus from a nuclear reaction $A + a \rightarrow C^* \rightarrow B + b$, and the average widths Γ for the decay by emission of neutrons, protons, deuterons, and alpha particles, were calculated with the CDC-3600 computer by a program based on the following formulae.

A. Cross Section of Formation and Spin Distribution of the Compound Nucleus

The cross section for the formation of the compound nucleus with spin J at an excitation energy U_C , from the reaction $A + a \rightarrow C^*$, is given by⁸⁷

$$\sigma(U_C, J) = \frac{\pi(2J+1) \lambda_1^2}{(2I_a+1)(2I_A+1)} \sum_{S_1=|I_A-I_a|}^{I_A+I_a} \sum_{\ell_1=|J-S_1|}^{J+S_1} T_{a\ell_1}(\epsilon_1), \quad (1)$$

where

λ_1 = de Broglie wavelength of the entrance channel;

I_A = spin of target nucleus;

I_a = spin of projectile;

S_1 = spin of entrance channel;

$S_1 = I_A + I_a$;

$T_{a\ell_1}(\epsilon_1)$ = optical-model transmission coefficient of particle a with orbital angular momentum ℓ_1 and channel energy ϵ_1 ;

ϵ_1 = center-of-mass energy of the entrance channel, sometimes referred to as the entrance channel energy;^{55c}

$\epsilon_1 = E_a M_A / (M_a + M_A)$;

M_x = mass of particle x ;

E_a = energy of projectile in the laboratory system.

The channel wavelength λ_1 is related to the channel energy by^{55d}

$$\frac{1}{\lambda_1} = \frac{(2M_0\epsilon_1)^{1/2}}{\hbar}, \quad (2)$$

where M_0 is the reduced mass of the system.

$$\chi^2 \text{ (in barns)} = 4.784 E_a \text{ (in MeV)} \frac{M_a M_A^2}{(M_a + M_A)^2}. \quad (3)$$

The total cross section is given by

$$\sigma(U_C) = \sum_J \sigma(U_C, J), \quad (4)$$

and the probability P_J for the formation of the compound nucleus with spin J by

$$P_J = \frac{\sigma(U_C, J)}{\sigma(U_C)}. \quad (5)$$

B. Energy-level Widths

1. The "Fermi-gas" Calculation

The energy width $\Gamma_J^b(U_C)$ of the compound nucleus with spin J at excitation energy U_C for the decay by emission of particles b leading to a residual nucleus B is given by³⁷

$$\Gamma_J^b(U_C) = \frac{D(U_C, J)}{2\pi} \int_0^{U_{B\max}} dU_B \sum_{\ell_2} T_{b\ell_2}(\epsilon_2) \sum_{S_2=|J-\ell_2|}^{J+\ell_2} \sum_{I_B=|S_2-I_b|}^{S_2+I_b} \rho_B(U_B, I_B), \quad (6)$$

where I_b and I_B are the spins of the particles b and B , S_2 the exit-channel spin, ℓ_2 the orbital angular momentum of b and $T_{b\ell_2}$ its transmission coefficient. The spacing $D_J(U_C, J)$ of the levels with spin J of the compound nucleus at an excitation energy U_C and the level densities $\rho_B(U_B, I_B)$ of the residual nuclei are calculated from Eq. (12) of Appendix II. The exit-channel energy ϵ_2 and the maximum excitation energy $U_{B\max}$ of the residual nuclei are given by

$$\epsilon_2 = U_{B\max} - U_B \quad (7)$$

and

$$U_{B\max} = \epsilon_1 + Q_{ab}, \quad (8)$$

where Q_{ab} is the ground state Q -value for the reaction. The summations are performed up to $\ell_{2\max} = 17$ and $S_{2\max} = 14$, and the integral is approximated by sums according to Simpson's rule, in 0.5-MeV steps. The partial widths Γ_J^b are calculated for neutrons, protons, deuterons, and alpha particles for $J = 0-17$.

2. The "Constant-temperature" Calculation

The partial width $\Gamma_J^b(U_C)$ of Eq. (6) is calculated using constant-temperature form of level density. Equation (15) of Appendix II can be written as

$$\rho_B(U_B, I_B) = \rho_n(U_C - B_n) \exp\left[\frac{U_B - (U_C - B_n) + (\Delta_n - \Delta_b)}{T}\right] \cdot \frac{2I_B + 1}{2\sigma_B^2} \exp\left[-\frac{I_B(I_B + 1)}{2\sigma_B^2}\right], \quad (9)$$

in which the level densities $\rho_B(U_B, I_B)$ of each residual nucleus B is calculated relative to the level density $\rho_n(U_C - B_n)$ of the residual nucleus reached by emission of neutrons with zero energy. The quantity B_n is the neutron binding energy, T the nuclear temperature, σ_B^2 the spin cut-off factor, and $\Delta_n - \Delta_b$ a correction term for differences in shell effects and pairing energies in the residual nuclei reached by emission of a neutron and a particle b . Equation (3) of Appendix II can be written as

$$1/D(U, J) = \rho(U, J=0)(2J+1) \exp[-J(J+1)/2\sigma^2]. \quad (10)$$

Now from Eqs. (6), (9), and (10) one can write for the partial width Γ_J^b of the compound nucleus for the emission of particles b

$$\Gamma_J^b(U) = \frac{\rho_n(U_C - B_n, I_B=0)}{2\pi\rho_C(U_C, J=0)} \frac{\exp[J(J+1)/2\sigma_C^2]}{2J+1} \int_0^{U_{B\max}} dU_B \sum_{\ell_2} T_b \ell_2(\epsilon_2) \sum_{S_2=|J-\ell_2|}^{J+\ell_2} \sum_{I_B=|S_2-I_b|}^{S_2+I_b} (2I_B+1) \exp\left[\frac{U_B - (U_C - B_n) + (\Delta_n - \Delta_b)}{T_B} - \frac{I_B(I_B+1)}{2\sigma_B^2}\right]. \quad (11)$$

When the ratio $\rho_n(U_C - B_n, J=0)/\rho_C(U_C, J=0)$ is calculated using the constant-temperature form of level density [Eq. (9)], it is independent of the excitation energy U_C of the compound nucleus:

$$\frac{\rho_n(U_C - B_n, J=0)}{\rho_C(U_C, J=0)} = \frac{\mathcal{I}_n}{\mathcal{I}_C \exp \left[\frac{B_n + (\Delta_n - \Delta_C)}{T} \right]}, \quad (12)$$

where $\mathcal{I}_C$ and $\mathcal{I}_n$ are the moments of inertia of the compound nucleus and the nucleus reached by neutron emission, respectively. Since the integral from 0 to U_{Bmax} in Eq. (10) varies very little with U_C , if one assumes a constant-temperature level density to hold for all excitation energies, covering the excitation-energy regions of the residual nuclei and the compound nucleus, Γ_J is essentially independent of the excitation energy.

3. The "Modified Constant-temperature" Calculation

One expects the temperature to be approximately constant over small energy intervals but probably not to be constant for the excitation energies of the residual nuclei all the way to the excitation energy of the compound nucleus. For this reason in the "modified constant-temperature" calculation the ratio ρ_n/ρ_C in Eq. (10) is calculated with the Fermi-gas form of level density [Eq. (12) of Appendix II], the constant-temperature form being used only for the residual nuclei. In this case

$$\begin{aligned} \frac{\rho_n(U_C - B_n, J=0)}{\rho_C(U_C, J=0)} &= \left(\frac{a_n \mathcal{I}_C^3}{a_C \mathcal{I}_n^3} \right)^{1/2} \left(\frac{U_C + t - \Delta_C}{U_C - B_n + t_n - \Delta_n} \right)^2 \\ &\cdot \exp[2a_n^{1/2}(U_C - B_n - \Delta_n)^{1/2} - 2a_C^{1/2}(U_C - \Delta_C)^{1/2}]. \end{aligned} \quad (13)$$

C. Total Energy Widths

The total decay width Γ_J of the compound nucleus with spin J , is calculated by summing the partial widths for all outgoing particles:

$$\Gamma_J = \sum_b \Gamma_J^b. \quad (14)$$

The average life τ of the compound nucleus is found from the sum of the lifetimes of all nuclei divided by the total number of nuclei. And since $\tau = \hbar/\Gamma$,

$$\frac{1}{\Gamma} = \frac{\sum_J \frac{\sigma_J}{\Gamma_J}}{\sum_J \sigma_J}, \quad (15)$$

where σ_J is the cross section of formation of the compound nucleus with spin J .

APPENDIX IV

Calculation of Differential Cross SectionsA. Energy and Angular Distributions of the Differential Cross Section

The differential cross section for particles emitted from a nuclear reaction $A(a,b)B$ to a single final level in the residual nucleus B with spin I_B , parity Π_B , and excitation energy U_B can be calculated by modifying Eq. (3.4) of reference 86. A compound nucleus process is assumed.

$$\frac{d\sigma_{ab}}{d\Omega}(I_B, \Pi_B, U_B, \theta) = \sum_{L \text{ even}} A_L P_L(\cos \theta); \quad (1a)$$

$$A_L = \frac{1}{4k_a^2(2I_A+1)(2I_a+1)} \sum_{S_1, \ell_1, S_2, \ell_2, J} \frac{\delta_n^{m(-)S_2-S_1}}{G(J)} \cdot T_{a\ell_1}(\epsilon_1) T_{b\ell_2}(\epsilon_2) Z(\ell_1 J \ell_1 J; S_1 L) Z(\ell_2 J \ell_2 J; S_2 L); \quad (1b)$$

$$G(J) = \sum_{b'} \int_0^{U_{B'}^{\max}} dU_{B'} \sum_{\ell_2'} T_{b'\ell_2'}(\epsilon_2) \sum_{S_2=|J-\ell_2|}^{J+\ell_2} \sum_{I_{B'}=|S_2-I_{b'}|}^{S_2+I_{b'}} \rho_{B'}(U_{B'}, I_{B'}), \quad (1c)$$

where

I_a = spin of projectile;

I_b = spin of emitted particle;

I_A = spin of target;

I_B = spin of residual nucleus;

S_1 = spin of entrance channel;

$S_1 = I_A + I_a$;

S_2 = spin of exit channel;

ℓ_1 = orbital angular momentum of projectile;

ℓ_2 = orbital angular momentum of emitted particle;

J = spin of compound nucleus;

k_a = wave number of bombarding particle;

ϵ_1 = entrance channel energy; $\epsilon_1 = E_a M_a / (M_a + M_A)$;

ϵ_2 = exit channel energy; $\epsilon_2 = \epsilon_1 + Q_{ab'} - U_B$;

E_a = energy of projectile in the laboratory system;

M_x = mass of particle x ;

$Q_{ab} = m_a + m_A - m_b - m_B$, in energy units;

$T_{a\ell_1}(\epsilon_1)$ = optical-model transmission coefficient of bombarding particle;

$T_{b\ell_2}(\epsilon_2)$ = optical-model transmission coefficient of outgoing particle;

$P_L(\cos \theta)$ = Legendre polynomial of order L ;

the delta function δ_n^m is the parity conservation factor:

$$\begin{aligned} \delta_n^m &= 1; & m &= n, \\ &= 0; & m &\neq n. \end{aligned}$$

$$m = (-1)^{\ell_1 + \ell_2}.$$

$$n = \Pi_1 \Pi_2,$$

$$\Pi_1 = \Pi_a \Pi_A; \quad \Pi_2 = \Pi_b \Pi_B, \quad (1d)$$

where Π_a , Π_A , Π_b , and Π_B are the parities of particles a , A , b , and B , respectively, Π_1 and Π_2 the entrance- and exit-channel parities, respectively; the "Z-coefficients" are defined in terms of the Racah coefficient W and the Clebsch-Gordan coefficient $(\ell\ell 00; L0)$:

$$Z(\ell J \ell J; SL) = (2\ell + 1)(2J + 1)(\ell\ell 00|L0) W(\ell J \ell J; SL); \quad (2)$$

the random sign assumption implies that L is even and that the angular distribution is symmetric about 90° ; the denominator $G(J)$ is the summation of the transmission coefficients over all exit channels, b' referring to all emitted particles (neutrons, protons, deuterons, and alpha particles); the integrals were approximated by sums according to Simpson's rule, with 0.5-MeV steps up to

$$U_{B' \max} = \epsilon_1 + Q_{ab'}.$$

Two forms of level densities $\rho_{B'}(U_{B'}, I_{B'})$ for the residual nuclei formed by emission of particles b' were used in the calculations; the Fermi-gas form described by Eq. (12) of Appendix II, and the constant-temperature form defined by Eq. (9) of Appendix III.

In Eqs. (1a, 1b, 1c) the differential cross section is expressed as a function of parameters characterizing the initial and final states. The differential cross section can also be expressed in terms of parameters characterizing the compound nucleus. From such expressions one can learn something about the properties of the compound nucleus, such as the spin distribution and the spin cut-off factor, the energy and spin dependence of the energy-level widths, and the level density.

The energy width Γ_J for the decay levels with spin J of the compound nucleus is related to $G(J)$ of Eq. (1c) by [Eq. (6), Appendix III].

$$\Gamma_J = \frac{D_J}{2\pi} G(J). \quad (3)$$

Also from Eqs. (3) and (7) of Appendix II,

$$D_J = \frac{D_0}{(2J+1) \exp\left[\frac{-J(J+1)}{2\sigma_C^2}\right]} = \frac{1}{(2J+1) \rho_C(U_C, J=0) \exp\left[\frac{-J(J+1)}{2\sigma_C^2}\right]} \quad (4)$$

and

$$\frac{1}{D_0} = \rho_C(U_C, J=0) = \frac{\rho_C(U_C)}{2\sigma_C^2}, \quad (5)$$

where U_C is the excitation energy of the compound nucleus, D_0 and D_J are the spacings of levels with spins 0 and J , respectively, σ_C^2 the spin cut-off factor, $\rho_C(U_C)$ the total density of levels of the compound nucleus, and $\rho_C(U_C, J=0)$ the density of zero-spin levels. One then can write the following expressions for $G(J)$:

$$G'(J) = \frac{\Gamma_J}{D_0} 2\pi(2J+1) \exp\left[\frac{-J(J+1)}{2\sigma_C^2}\right] \quad (6)$$

and

$$G'(J) = \pi \Gamma_J \frac{\rho_C(U_C)}{\sigma_C^2} (2J+1) \exp\left[\frac{-J(J+1)}{2\sigma_C^2}\right]. \quad (7)$$

Equations (1a) and (1b) can be rewritten in the following form:

$$\frac{d\sigma_{ab}}{d\Omega}(I_B, \Pi_B, U_B, \theta) = \frac{\sigma_C^2}{4\pi k^2 (2I_A + 1)(2I_a + 1)} \sum_{L_{\text{even}}} P_L(\cos \theta) \sum_{S_1, \ell_1, S_2, \ell_2, J} \frac{\delta_n^m (-1)^{S_2 - S_1} T_{a\ell_1} T_{b\ell_2} Z_1 Z_2}{\Gamma_J \rho_C(U_C) (2J + 1) \exp[-J(J + 1)/2\sigma_C^2]} \quad (8)$$

The angular anisotropy $\sigma(180^\circ)/\sigma(90^\circ)$ of the differential cross section calculated from Eqs. (1a, 1b, 1c) depends primarily on the spin cut-off factors of the residual nuclei. In general, the absolute value of the cross section, as well as the energy and angular distributions, depend on the accuracy of evaluation of $G(J)$, that is, on the level-density form and the density parameters of the residual nuclei.

B. Calculation of $(\Gamma/D_0)(d\sigma/d\Omega)$

In Eq. (8) the angular anisotropy depends very sensitively on the spin cut-off factor σ_C^2 of the compound nucleus, and also on the spin distribution of Γ_J . Assuming a certain spin distribution of Γ_J , one can adjust the parameter σ_C^2 to fit the experimental angular anisotropy $\sigma(180^\circ)/\sigma(90^\circ)$ of the differential cross section. Then the absolute value of the calculated cross section depends on the value of $\Gamma \rho_C(U_B)$, for which one can obtain information from such a calculation. Assuming Γ_J to be independent of J and equal to the average width Γ , we calculate the product $(\Gamma/D_0)(d\sigma/d\Omega)$ from Eqs. (1a), (1b), and (6) as

$$\frac{\Gamma}{D_0} \frac{d\sigma_{ab}}{d\Omega}(I_B, \Pi_B, U_B, \theta) = \sum_{L_{\text{even}}} A'_L P_L(\cos \theta), \quad (9a)$$

where

$$A'_L = \frac{1}{8\pi k_a^2 (2I_A + 1)(2I_a + 1)} \sum_{S_1, \ell_1, S_2, \ell_2, J} \frac{\delta_n^m (-1)^{S_2 - S_1} T_{a\ell_1} T_{b\ell_2} Z_1 Z_2}{(2J + 1) \exp[-J(J + 1)/2\sigma_C^2]} \quad (9b)$$

A Fortran program was developed for the numerical evaluation of Eqs. (1a, 1b, 1c), (9), and (9a, 9b), based on a similar program by Dr. H. K. Vonach. The transmission coefficients were calculated with the program ABACUS-2⁶⁷ and the optical-model parameters of Bjorkund and Fernbach⁶⁸ for neutrons, Perey⁶⁹ for protons, and Huizenga and Igo⁷⁰ for alpha particles. The deuteron emission was negligible in all of our cases. The Clebsch-Gordan and Racah coefficients were computed from Eqs. (17') and (36'), respectively, of Ref. 88.

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