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Rock Heat-Loss Shape Factors for Subterrene Penetrators

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Rock Heat-Loss Shape Factors for Subterranean Penetrators

by

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ROCK HEAT-LOSS SHAPE FACTORS FOR SUBTERRENE PENETRATORS

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ABSTRACT

Results of calculations to predict heat losses to surrounding rock for Subterrene rock-melting penetrators of various shapes are presented in nondimensional form that can be applied to a range of axisymmetric penetrator geometries. Theoretical upper and lower limits are placed on a dimensionless time parameter, which includes penetrator dimensions, rock properties, and penetration rate. At these limits, heat losses either approach zero or are unaffected by penetrator geometry and velocity. The estimated heat losses from the nondimensional parameters agree well with experimental data and numerically calculated results.

I. INTRODUCTION

Rock-melting penetrator systems (Subterrenes) are under development at the Los Alamos Scientific Laboratory to pierce rock and soil by progressive melting rather than by chipping, abrading, or spalling.¹ When perfected, this technique will offer the possibility for desirable advancements in excavation and tunneling technology, and thus will permit enhanced and more efficient utilization of the earth's resources.

The small-diameter prototype penetrator devices developed successfully thus far are of two basic types: (1) The melting-consolidating penetrator² (MCP) is a solid body of revolution, which melts loose or low-density rock or soil to form a dense glass lining on the hole; no debris remains to be removed. (2) The universal extruding penetrator³ (UEP) has a central extrusion hole through which the rock-melt flows to be chilled and hardened behind the penetrator into discrete, small particles (glass rods, pellets, or rock wool); the debris is then removed by the stem coolant flowing in an annular passage in the stem. The UEP can also be

designed to produce a glass lining, if desired. The "universal" aspect of this type penetrator results from its ability to penetrate dense, hard rocks, such as basalt and granite, as well as loose and low-density materials.

The shape factors presented herein are applicable to any penetrator design, provided the location of the melt-to-rock interface can be estimated from rock porosity and melt-flow predictions. Heat losses for right circular cylindrical penetrator geometries have been calculated⁴ to be nearly equal to the energy required to melt some rock types. The conical penetrators currently being tested are much more efficient because most of the heat conducted into unmelted rock ahead of its tapered sides is regained as sensible heat in the melted material as the penetrator advances. The specific advantage is strongly dependent on the details of penetrator shape, on rock properties, and on the rate of penetration. Although the cost of energy lost^{*} to the

^{*}Power losses for penetrators of very large diameter can also be estimated by the techniques derived in this study.

rock is inconsequential in comparison to total rock penetration cost,⁵ minimizing these losses can improve the penetration rate, heater effectiveness, and penetrator life.

II. SYSTEM HEAT LOSSES

$$\dot{E}_H = \dot{E}_E + \dot{E}_C + \dot{E}_R \quad (1)$$

 $\dot{E}_H$ = power supplied to heater,

The heater power, $\dot{E}_H$, is the measured and controlled variable and the melting power, $\dot{E}_M$, has been shown⁹ to be predictable from the properties of the rock or soil and the penetration rate. The internal heat loss, $\dot{E}_C$, can be either obtained from experimental data or calculated numerically. The heat loss to the rock, $\dot{E}_R$, can be found by difference from experimental data.

For a melting-consolidating penetrator, the ratio of outer radius to inner radius of the glass lining is simply related to the rock porosity by the conservation of mass.⁹

Low-porosity rock requires large volumes of rock to be melted to form a hole by consolidation. Melt removal by extrusion (or in combination with consolidation) would likely be more economical. The porosity of local Los Alamos Tuff is ~ 0.44 , which results in a radius ratio, R , of ~ 1.5 .

The constant temperature, T_m , along the interface between molten and solid rock is a convenient thermal-boundary condition. Figure 1 shows the

Fig. 1. Geometry of penetrators and melt layers.

simplified representation of the three penetrator shapes* considered and the corresponding interface between melt and solid rock. The radius of the melting interface was defined as 1.5 times the penetrator radius for both sets of rock properties (dense and porous). Although this definition is not strictly true for fully consolidating penetration in basalt, a combination of melt extrusion and consolidation could result in such a radius ratio, and this assumption allows us to separate the effects of geometry and thermophysical properties.

Another simplification consists in eliminating the effects of heat conduction in both the stem and the solid glass liner, as was done in Ref. 4. With these simplifications, the only part of the system that must be analyzed is the infinite region of solid rock external to the isothermal melting interface.

B. Mesh Grid and Boundary Conditions

Figure 2 shows a typical finite-element mesh with thermal boundary conditions for the rock surrounding the melting interface. For purposes of analysis, we assume that the penetrator and the layer of molten rock are stationary, with the solid rock moving steadily by.

A variable mesh whose element thickness increases with distance from the melting interface was used to represent a large region (approximately an infinite region) and to obtain a more accurate solution in the vicinity of the penetrator. The region was considered large enough when the sum of the conduction heat losses across Boundaries ③ and ④ was at least four orders of magnitude less than the total heat conducted into the model across Boundary ① (the melting interface).

The left-hand vertical axis of Fig. 2 is the center line of the penetrator and, by symmetry, the heat flux across this boundary is zero. It was assumed that the conductive heat flow across the bottom horizontal plane ($z = 0$, or Boundary ② in Fig. 2) was also zero. This assumption is not strictly true for finite-sized penetrators, but the error is small, because of poor heat-conduction properties of solid rock and small thermal gradients in the z -direction in comparison with those in the r -direction (shown

Fig. 2. Typical finite-element mesh and thermal boundary conditions.

later in isotherm plots). The melting interface (Boundary ①) is taken to be constant at the melting temperature. The remaining two region boundaries (③ and ④) are assumed to be at ambient temperature because this condition would exist at an infinite distance from the penetrator.

The net heat loss to unmelted rock, therefore, consists of the conduction losses across Boundaries ③ and ④ (which are negligibly small because of the choice of model dimensions) coupled with the net increase in sensible heat flowing out of Boundary ② with the moving rock.

C. Computer Program

The thermal analysis was carried out with the AYER⁶ finite-element computer program. The code solves the general two-dimensional heat-conduction equation including terms involving material velocity.

* A cylindrical penetrator was included, even though thoroughly analyzed in Ref. 4, to provide a more convenient basis for comparison with the other geometries having identical mesh spacing and boundary conditions. However, the present results agree with those of Ref. 4.

TABLE I
REPRESENTATIVE PHYSICAL PROPERTIES OF ROCKS USED IN THIS STUDY^{*}

General Rock Type	Density, ρ , (Mg/m ³)	Specific Heat, C, (J/g·K)	Room Temperature Conductivity, λ , (W/m·K)	Room Temperature Diffusivity, D, (m ² /s)	Melting Point, T_m , (K)
Basalt	2.7	1.0	1.5	5.56×10^{-7}	1420
Local tuff	1.4	1.0	0.25	1.79×10^{-7}	1420

*All properties were assumed constant with temperature, except the thermal conductivity of local tuff which was:

$\lambda = 0.25$ for $T \leq 1000$ K, and

$\lambda = 0.0005T - 0.25$, for $T > 1000$ K.

D. Rock Properties

Published data list widely varying rock properties because of great variations in composition and structure for a given type throughout the world. To a lesser degree, the variations are the result of differences in methods of measuring rock properties. The two sets of properties chosen for this study come close to spanning the range expected and are listed in Table I. The generalized results will permit an estimation of heat loss for any type of rock.

IV. HEAT LOSS RESULTS

Heat losses were calculated for the three geometries shown in Fig. 1, each with the two sets of rock properties given in Table I, and each at three penetration rates, giving a total of eighteen different calculations. The penetration rates were 0.15, 0.30, and 0.45 mm/s.

Figures 3 and 4 are typical isotherm plots showing the effect of rock properties and penetration velocity, respectively. Note that the heat loss from the numerical calculations is much less for the former case than for the latter.

The heat-loss results are summarized in Table II, which includes the corresponding inefficiency factors, $\eta_R = \dot{E}_R / \dot{E}_m$.

Table II shows the expected dependence of heat loss on rock properties and penetrator shape: Losses are higher for basalt and cylindrical geometry, and are lowest for tuff and the most sharply pointed conical design (Shape 3). Note also that the dependence of heat loss on penetration velocity is a strong

function of both penetrator shape and rock properties. The variables are thus not separable.

V. GENERALIZATION TO OTHER SHAPES

A. Cylindrical Geometry

The heat loss from a cylindrical penetrator can be estimated from available analytical solutions.⁷ Reference 4 showed that the dimensionless integrated heat loss, $\hat{E}$, could be found analytically as a function of the Fourier number, τ , as shown in Fig. 5, where,

TABLE II

SUMMARY OF CALCULATED HEAT LOSSES, $\dot{E}_R$,
 AND EFFICIENCY FACTORS, η_R ,
 AT DIFFERENT PENETRATION VELOCITIES, v ,
 FOR VARIOUSLY SHAPED PENETRATORS

	$v = 0.15 \text{ mm}\cdot\text{s}^{-1}$		$v = 0.30 \text{ mm}\cdot\text{s}^{-1}$		$v = 0.45 \text{ mm}\cdot\text{s}^{-1}$	
	$\dot{E}_R$ (W)	η_R	$\dot{E}_R$ (W)	η_R	$\dot{E}_R$ (W)	η_R
Shape 1 (50 mm consolidating)						
Tuff	367	0.24	421	0.14	433	0.09
Basalt	1637	0.95	1887	0.55	2019	0.39
Shape 2 (cylindrical)						
Tuff	850	0.53	1158	0.37	1346	0.29
Basalt	2839	1.65	3786	1.10	4508	0.87
Shape 3 (conical)						
Tuff	145	0.09	53	0.02	~ 0	0.00
Basalt	955	0.55	850	0.24	677	0.13

^a η_R based on $\dot{E}_R = (1.067 \times 10^4) v$, for tuff and $\dot{E}_R = (1.15 \times 10^4) v$, for Basalt.

Fig. 3. Isotherm plot for Shape-1 penetrator in tuff at 0.45 mm/s.

Fig. 4. Isotherm plot for Shape-1 penetrator in basalt at 0.15 mm/s.

$$E = \frac{DE_R}{\lambda H r^2 \Delta T} \quad (3)$$

$$\tau = \frac{Dt^*}{r^2}$$

D = rock thermal diffusivity

E_R = heat loss to rock, integrated over time t^*

λ = rock thermal conductivity

H = penetrator length

r = radius of melt-to-rock interface (an effective value for a conical penetrator)

$$\Delta T = T_m - T_o$$

t^* = time during which heat loss occurs, i.e., the time it takes the penetrator to travel its own length, H/V , when V is penetration rate.

The instantaneous rate of heat loss, $\dot{E}_R$, is E_R/t^* .

Using a mean radius r for a typical 50-mm-diam consolidating penetrator, the value of $\dot{E}_R$ was found to agree reasonably well with numerical solutions (and with experimental values). This apparent agreement may be accidental because the analytical solution is strictly applicable only to the cylindrical geometry. Figure 6 compares numerical

heat-loss calculations (data from Table II) with estimates using the nondimensional parameters from

Fig. 5. Dimensionless heat-loss parameters derived from analytical solution.

Fig. 6. Heat loss versus penetration velocity for 50-mm-diam penetrators.

Fig. 5, with a mean radius for the conical penetrators. The numerical results again indicate that the velocity dependence of heat loss is a function of penetrator shape, but the estimates based on nondimensional parameters with a mean penetrator radius indicate no such effect.

B. Derivation of Heat Loss for Arbitrary Shape

Axisymmetric penetrators of arbitrary shape can be approximately represented by the following equation,

$$\frac{y}{H} = \left(\frac{r}{r_0}\right)^B \quad (4)$$

where

y = axial distance from the leading edge

r = radius at y

r_0 = maximum radius

H = total length

B = a shape factor, > 0 .

When B equals 1, the penetrator is a cone; for $B < 1$, the penetrator has concave sides; and when

Fig. 7. Penetrator shapes.

$B > 1$, it has convex sides. $B = 2$ represents a paraboloid and $B \approx 100$ is very close to a cylinder. Figure 7 shows penetrator shapes for various values of B , in comparison with actual shapes used in experiments and in numerical analysis of preceding sections.

Although a precise definition cannot be made, the shape factor for the 50-mm consolidating penetrator² is ~ 4 (certainly between 2 and 5), whereas the value for the more sharply pointed conical penetrator (Shape 3) is ~ 1.5 .

Note that the shape factor is not affected, whether the radius is defined at the penetrator surface or at the melting interface, as long as the radii are in constant ratio per Eq. (2).

If we assume that all heat leaving the melting interface is radial and none is recovered (neglecting the effect of the tapered sides encountering preheated rock as the penetrator moves forward), the nondimensional relations in Fig. 5 can be modified to apply to penetrators with shape defined by Eq. (4).

First, a curve "fit" to the nondimensional relationship in Fig. 5 is found by standard techniques to be

$$\hat{E} = 9.836 \tau^{0.605} \quad (5)$$

Fig. 8. Relative heat loss versus shape factor.

Substituting the nondimensional parameters defined previously, and rearranging,

$$\dot{E}_R' = 9.836 (\lambda H \Delta T) \left(\frac{H}{V} \right)^{0.605} \left(\frac{r^2}{D} \right)^{0.395} \quad (6)$$

where $\dot{E}_R'$ applies only to cylindrical geometry.

For a conical penetrator, the value of r may be taken as the value over a small axial increment, Δy , given by

$$r = r_o \left(\frac{y}{H} \right)^{\frac{1}{B}} \quad \text{from Eq. (4).}$$

Substituting in Eq. (6) and integrating,

$$\dot{E}_R = C \frac{r_o^{0.79}}{H^{0.79/B}} \int_0^H y^{0.79/B} dy \quad (7)$$

where

$$C = \frac{9.836 \lambda \Delta T}{D^{0.395}} \left(\frac{H}{V} \right)^{0.605}$$

Finally,

$$\dot{E}_R = C H r_o^{0.79} \left(\frac{1}{1 + \frac{0.79}{B}} \right) \quad (8)$$

Because t^* is the same for both shapes, the ratio of instantaneous losses is equal to the ratio of integrated losses.

Comparing Eq. (8) with Eq. (6),

$$\frac{\dot{E}_R'}{\dot{E}_R} = \frac{1}{1 + \frac{0.79}{B}} \quad (9)$$

Equation (9) behaves as expected because $\dot{E}_R'$ approaches $\dot{E}_R$ when B becomes large, and $\dot{E}_R'$ approaches zero when B becomes small (needlepoint penetrator with concave sides). Equation (9) gives an upper limit to the heat loss. It includes all the heat conducted from the melting interface, neglecting the velocity term, $V_z \frac{\partial T}{\partial z}$, which tends to recover some energy. The upper limit is applicable when the thermal diffusivity is very high and the penetration velocity very low, or, more precisely, when the Fourier number, τ , is ≥ 1.8 as will be shown.

C. Comparison With Numerical Results.

The numerical results in Table II were converted into relative heat losses, $\dot{E}_R'/\dot{E}_R$, and are shown versus Shape Factor B in Fig. 8. For comparison, the relative heat loss from Eq. (9) is also shown. This figure and Fig. 9 permit the direct comparison of heat losses for cylindrical versus conical penetrators under identical conditions.

The Fourier number, τ , is obviously a significant parameter affecting the relative heat loss, in addition to the shape factor. To account numerically for the effect of τ in conjunction with B is difficult, because, unfortunately, a simple relationship such as that of Eq. (9) does not exist. However, the analytical solutions mentioned in Section V,A, indicate that surrounding rock temperatures are proportional to the logarithm of the Fourier number over much of the range of interest. Because the heat loss to unmelted rock is proportional to its temperature, it is reasonable to assume that the heat loss is also proportional to $\ln \tau$ for a given shape factor.

An empirical relationship showing $\dot{E}_R'/\dot{E}_R$ versus $\ln \tau$ is given in Fig. 9. A very good linear dependence is indicated. Extrapolating these numerical

Fig. 9. Relative heat loss versus natural logarithm of Fourier number.

results to the value of $\dot{E}_R/\dot{E}_R'$ given by Eq. (9) yields a nearly constant value of 1.8 for the Fourier number. This can be considered an empirical Fourier-number limit for penetrators with $B < 4$, above which there is no heat recovery as the penetrator advances, regardless of shape. Alternatively, at Fourier numbers above 1.8, the net heat loss from the melting interface is equal to the total heat conduction given by the closed-form solutions in the literature⁷ and by Section V,A. Extrapolation of the data in Fig. 9 in the opposite direction leads to a low Fourier-number limit, at which the value of $\dot{E}_R/\dot{E}_R'$ is zero. This Fourier number, which is uniquely determined for each penetrator shape, represents the combination of penetrator size and shape, penetration rate, and rock thermal diffusivity at which losses to unmelted rock are zero. The usefulness and applicability of the numerical solutions are thus greatly extended.

The low Fourier-number limit permits numerical evaluation of the limiting condition discussed in Appendix C of Ref. 4, including the influence of penetrator shape. Briefly, the high-speed approximation states that whenever the quantity $\frac{V\lambda}{D} \gg 1$, the convective term, $\frac{V}{D} \frac{\partial T}{\partial z}$, dominates the conduction terms in the quasi-stationary heat-conduction equation. The value of the characteristic length, λ , is then taken as r_o^2/H , and by inverting the inequality above we obtain,

$$\tau = \frac{D H}{V r_o^2} \ll 1$$

for the high-speed approximation to be valid. Now, with the aid of the numerical data in Fig. 9, a definite value can be placed on τ and the inequality above becomes an equality whose value depends on the shape factor, B .

The velocity limit is obtained easily by substitution in the above expression.

For example, Fig. 9 indicates that $\dot{E}_R/\dot{E}_R'$ is zero for $B = 1.5$ when the Fourier number is less than 0.08. For the penetrator size used in the analysis (Fig. 1, Shape 3) this value of τ occurs in tuff at a penetration velocity of 0.45 mm/s.

Extrapolating the data in Fig. 9 for the 50-mm consolidating penetrator² ($B = 4$) to $\dot{E}_R/\dot{E}_R' = 0$ yields a Fourier number of 1.5×10^{-3} . In tuff, this value of τ occurs at a velocity of 8.00 mm/s, well above present or future contemplated penetration velocities. These examples illustrate the extreme sensitivity of the heat loss to the shape of the penetrator. Of course, the loss is also sensitive to penetrator size for a fixed shape. For example, if penetrators are scaled up in size by a factor of ten, the velocity is reduced by a factor of ten if $\dot{E}_R/\dot{E}_R' = 0$.

Another interpretation that may be given to the concept of the low Fourier-number limit is that the velocity obtained therefrom represents the maximum

TABLE III
VELOCITY LIMITS FOR CONICAL PENETRATORS ($B \leq 4$)

Penetrator Size, mm	Velocity Limits, mm/s					
	Tuff ⁽¹⁾			Basalt ⁽¹⁾		
	$v_L^{(2)}$	$v_H^{(3)}$		$v_L^{(2)}$	$v_H^{(3)}$	
		$B = 1.5$	$B = 4$		$B = 1.5$	$B = 4$
Diameter = 50, H = 95	0.007	0.15	8.02	0.021	0.47	25.0
Diameter = 75, H = 180	0.006	7×10^{-3}	0.38	0.017	0.022	1.2
		$\left(\begin{matrix} B = 2.5 \\ v_H = 0.18 \end{matrix} \right)$				
Diameter = 114, H = 55	0.002	0.017	0.89	0.007	0.052	2.8
Diameter = 300, H = 300	0.001	0.013	0.70	0.004	0.014	2.2
Diameter = 3000, H = 600	3.0×10^{-5}	6.0×10^{-4}	0.032	8.0×10^{-5}	2.0×10^{-3}	0.099
Diameter = 6000, H = 1000	5.0×10^{-6}	1.0×10^{-4}	6.0×10^{-3}	2.0×10^{-5}	3.0×10^{-4}	0.018

(1) Rock properties given on Table I; porosity assumed 0.44.

(2) Low speed-velocity limit based on $\tau = 1.8$, insensitive to penetrator shape.

(3) High speed-velocity limits for shape factors indicated based on $\tau = 0.08$ for $B = 1.5$ and on $\tau = 1.5 \times 10^{-3}$ for $B = 4$.

theoretically obtainable. At this velocity, the heat conducted across the melting interface is exactly balanced by the sensible energy convected across the boundary by unmelted rock. Therefore, there is no excess energy leaving the melting interface which could be used to raise the temperature of additional unmelted rock if the penetration velocity were to increase. Of course, the actual limit on penetration rate is likely to be controlled by other factors such as melt removal or maximum penetrator or heater material temperatures.

Although of lesser interest, the high Fourier number is also of significance because it defines the opposite extreme. For $\tau > 1.8$, heat losses to unmelted rock are not affected by penetrator advance and are insensitive to penetrator shape. This extreme occurs when the thermal pulse is essentially able to "outrun" the penetrator's advance in such a way that the energy conducted in the radial direction from the melting interface is never recovered.

Table III gives both low and high velocity limits for some typical penetrators in both tuff and basalt. Experimentally determined penetration rates⁹ are well above the low velocity limits. The high velocity limit increases with the heated length H, and decreases as the square of the radius of the

melting interface. Consequently, high velocities are more difficult to obtain with large-diameter penetrators operating in the fully consolidating mode. Note that the high-velocity limit for the 75-mm-diam consolidating penetrator in tuff is 0.18 mm/s with $B = 2.5$. This value is very close to actual maximum penetration rates that have been attained in the laboratory with this design in tuff. However, experimental penetration rates could very well exceed the theoretical limit for several reasons. There is, of course, the general problem of inaccuracies in rock properties and in the value of shape factor for the penetrator. Also, the porous rock can be locally crushed by high thrust loads in the vicinity of the melting interface, which tends to locally increase the thermal conductivity. The same mechanism could force molten rock into the pores with the same effect. The velocity limits nevertheless provide a useful guide to the influence of the various parameters.

D. Examples of Heat-Loss Calculations

The procedure to estimate the rate of heat loss to unmelted rock for a specific penetrator design, rock type, and penetration velocity is as follows. First, calculate the loss, $\dot{E}_R$, for a cylindrical penetrator having the same overall heated length

Fig. 10. 75-mm consolidating penetrator.

and diameter. Second, multiply the result by the ratio $\hat{E}_R/\hat{E}'_R$ (from Fig. 9) to account for penetrator shapes other than cylindrical. The shape factor B is estimated with the aid of Fig. 7 (judgment may be necessary). Linear interpolation in Fig. 9 for values of B between 1.5 and 4.0 may be made with the aid of lines provided. Extrapolation beyond these limits should be based on the slope of Eq. 9 as illustrated in Fig. 8.

The example below is based on a recent penetrator design shown in Fig. 10, in tuff:

- Step 1. Instantaneous rate of heat loss for a cylinder.[‡]

[‡]This step may utilize the method of Ref. 4 with equal validity.

1.1 Fourier Number, τ :

$$\tau = \frac{D H}{V r_o^2}.$$

Given: D = rock thermal diffusivity =

$$5.34 \times 10^{-7} \frac{\text{m}^2}{\text{s}},$$

H = penetrator length = 180 mm

V = penetration rate = 0.15 mm/s

r_o = maximum melt outer radius.

Because the porosity of tuff is ~44%; r_o is 1.5 times the maximum penetrator radius, using Eq. (3). Therefore, $r_o = (1.5)(37.5) = 56$ mm, and

$$\tau = 0.204.$$

1.2 Nondimensional Integrated Heat Loss, $\hat{E}$:

$$\hat{E} = 9.836 (\tau)^{0.605}$$

Figure 5 could also be used:

$$\hat{E} = 3.76$$

1.3 Integrated Heat Loss, E'_R , from definition in Section V:

$$E'_R = \frac{\hat{E} \lambda H r_o^2 \Delta T}{D}.$$

Given: λ = rock thermal conductivity = 0.75 W/m·K

ΔT = temperature difference between melting interface and ambient = 1420 - 300 = 1120 K

$$E'_R = 3.35 \text{ MJ.}$$

1.4 Instantaneous rate of heat loss, $\dot{E}'_R$:

$$\dot{E}'_R = E'_R/t^*.$$

Given: t^* = time interval over which integrated heat loss occurs, as used in calculation of Fourier number:

$$t^* = \frac{H}{V} = \frac{180}{0.15} = 1200 \text{ s}$$

$$\dot{E}'_R = 2800 \text{ W.}$$

The above calculation may be compared with values obtained from Ref. 4 and will be seen to agree within $\pm 10\%$. Extrapolation of Fig. 9 in Ref. 4 will be necessary to make this comparison.

TABLE IV
RATE OF HEAT LOSS TO UNMELTED ROCK, $\dot{E}_R$

Penetrator Design				Rock (1)	Velocity (mm/s)	Fourier No. τ	$\dot{E}_R$ (kW)	
Diameter, mm	Height, mm	Shape Factor B	Ref.				Calculated	Experimental
75	120	2.5	Fig. 10	Tuff (2)	0.15	0.204	1.3	1.4 (3)
50	95	4	Ref. 2	Tuff	0.15	0.06	0.32	0.3 to 0.5 Ref. 8
"	"	"	"	"	0.3	0.04	0.34	
114	55	11	Dug. 597-32723	Tuff	0.2	0.01	0.21	< 0.5 (4)
66 (5)	70	~ 100 (cylindrical)	Ref. 3	Basalt	0.06	0.59	1.5	1.67 (6)
3000	5800	~ 100 (cylindrical)	none	Tuff	0.15	1.4×10^{-4}	538.0	362 (6)
"	"	"	"	"	0.30	6.8×10^{-5}	707.0	510 (6)

(1) Rock properties given by Table I except where noted.

(2) Thermal conductivity increased to 0.75 W/m·K.

(3) Interpolated from data.

(4) Limited information indicates loss is < 0.5 kW.

(5) All penetrators are melting-consolidating (MCP) except the 66-mm unit, which is universal-extruding (UEP).

(6) By numerical analysis.

Because the method used to estimate $\dot{E}_R'$ in this report does not include the small amount of heat that diffuses radially from the front surface out of the path of the cylindrical penetrator, a correction may be applied. One example for obsidian in Ref. 4 estimates the loss by lateral diffusion from the front surface of a cylindrical penetrator as ~10% of the loss from the side surface. Rock properties, penetration rate, and penetrator length-to-diameter (L/D) ratio will affect this percentage significantly, but based on the example, the percentage should not exceed 10% for L/D ratios greater than 0.5, penetration rates greater than 0.15 mm/s, and any rock type of interest. A 5% correction is assumed for this example:

$$\dot{E}_R' = 1.05 \times 2800 = 3080 \text{ W.}$$

• Step 2. Ratio $\dot{E}_R/\dot{E}_R'$ for conical penetrator versus cylinder.

2.1 With the aid of Figs. 7 and 10, $B \approx 2.5$.

The bounds on the effect of B on the final result can easily be placed by using the figures.

2.2 Using the result of Step 1.1, $\theta_m \tau = -1.59$.

2.3 From Fig. 9, $\dot{E}_R/\dot{E}_R' = 0.41$

$$\dot{E}_R = 3080 \times 0.41 = 1260 \text{ W.}$$

For a penetrator of this shape moving through dry tuff, measured heat losses have been recorded as 1291 W at 0.134 mm/s and as 1462 W at 0.166 mm/s. The most uncertain quantity in estimating the heat loss to unmelted rock is the thermal conductivity. The value of 0.75 W/m·K used in the example is based on literature data and is the same as that used (for the most part) to represent tuff in Ref. 4. The heat-loss estimate is almost directly proportional to the rock's thermal conductivity and, obviously, can be no more accurate than this value.

Table IV presents estimates of heat loss to unmelted rock for a variety of penetrator geometries, rock types, and penetration rates. Where available, results of experimental measurements and numerical computations are given for comparison.

VI. CONCLUSIONS

- The numerical computer methods of Ref. 4 have been extended to include conical axisymmetric penetrators as well as cylindrical shapes.

- A nondimensional analytical method for predicting heat loss to the rock has been shown to agree well with both numerical and experimental results for penetrators with a wide range of axisymmetric geometries.
- Upper and lower bounds have been determined for the Fourier number, based on penetrator shape. These bounds permit determination of theoretical penetration-velocity limits based on rock properties and penetrator dimensions. The maximum velocity limit increases linearly with penetrator heated length and decreases as the square of the radius of the melting interface. For large-diameter penetrators operating in the melt-consolidating mode, the velocity limits are too low to be practical.

REFERENCES

1. M. C. Smith, Editor, "A Preliminary Study of the Nuclear Subterrene," Los Alamos Scientific Laboratory report LA-4547 (April 1971).
2. J. W. Neudecker, "Design Description of Melting-Consolidating Prototype Subterrene Penetrator," Los Alamos Scientific Laboratory report LA-5212-MS (February 1973).
3. J. W. Neudecker, A. J. Giger, and P. E. Armstrong, "Design and Development of Prototype Universal Extruding Subterrene Penetrators," Los Alamos Scientific Laboratory report LA-5205-MS (March 1973).
4. D. J. Murphy and R. G. Gido, "Heat Loss Calculations for Small Diameter Subterrene Penetrators," Los Alamos Scientific Laboratory report LA-5207-MS (April 1973).
5. J. A. Altseimer, "Systems and Cost Analysis for a Nuclear Subterrene Tunneling Machine - A Preliminary Study," Los Alamos Scientific Laboratory report LA-5354-MS (July 1973).
6. R. G. Lawton, "Description of the AYER Heat Conduction Computer Program," Los Alamos Scientific Laboratory report, in preparation.
7. H. S. Carslaw and J. C. Jaeger, Conduction of Heat in Solids, 2nd Ed. (Oxford Univ. Press, 1959).
8. R. G. Gido, "Internal Temperature Distribution of a Rock-Melting Penetrator," Los Alamos Scientific Laboratory report LA-5135-MS (January 1973).
9. R. G. Gido, "Subterrene Penetration Rate: Melting Power Relationship," Los Alamos Scientific Laboratory report LA-5204-MS (March 1973).