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LEAD WIRE INDUCED MEASUREMENT
UNCERTAINTY IN COPPER-CONSTANTAN
THERMOCOUPLE CIRCUITS WHEN MEASURING
LIQUID HYDROGEN TEMPERATURES

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I. INTRODUCTION

The content of this report is limited to a discussion of cable induced measurement uncertainties within thermocouple data acquisition systems. Measurement uncertainties contributed by the thermocouple transducer, signal conditioners, and recorders are not discussed. Special emphasis has been placed on the measurement uncertainties encountered when employing extension wire in copper-constantan thermocouple channels that are used to measure liquid hydrogen temperatures.

II. SUMMARY OF SIGNIFICANT CONCLUSIONS

Only those measurement uncertainties contributed by the thermocouple circuit lead wires were considered. To minimize measurement uncertainties, recommendations are as follow:

1. Always use fully annealed special grade wire in thermocouple cables, thermocouple extension cables, and two conductor copper cables.
2. Minimize the relative amount of constantan wire by placing the thermocouple reference as close to the measurement junction as environmental conditions permit.
3. Reduce the number of transitions to an absolute minimum by using continuous wire lengths whenever possible.
4. Minimize the number of bulkhead type connectors by using continuous lengths of wire through partition walls whenever practical. If it is absolutely necessary to use large bulkhead plugs, move the thermocouple reference to the measurement junction side. If this cannot be done, use matching thermocouple pin materials.
5. Use the largest practical wire diameter and route the wires in a manner that will yield the shortest practical overall wire length.

II. Summary of Significant Conclusions (cont.)

6. Take great care not to cold work thermocouple circuit wiring by excessive handling during wire installation.
7. Use double sheath cable.
8. Use solid metal foil shielding instead of metal braid.
9. If practical, equalize the resistance per unit length of constantan and copper wire by increasing the diameter of the constantan wire relative to the copper wire diameter.
10. Short the shield to ground at only one point.

III. BASIC PRINCIPLES

A closed thermocouple circuit (Figure 1) is composed of two homogeneous conductors of dissimilar materials A and B, the ends of which are joined at points called junctions. If the junctions are at different temperatures, a thermoelectric emf (electromotive force) is generated and current will flow around the circuit.

FIGURE 1

BASIC THERMOCOUPLE CIRCUIT

III, Basic Principles (cont.)

Since the discovery of the thermoelectric phenomenon by Thomas Seebeck in 1821, it was found that the net thermoelectric emf is generated by two different sources; the Peltier emf and the Thomson emf, named after their discoverers.

The Thomson effect is explained with the aid of Figure 2 which pictures an insulated metal conductor that is heated at one end to produce a temperature gradient.

FIGURE 2

THE THOMSON EMF

It is an experimental fact that a temperature gradient along a wire is accompanied by the appearance of an electric field E_T . Electron motion is initiated by the appearance of the thermally induced Thomson field (E_T) and continues until the accumulation of electrons produces a retarding field which is strong enough to prevent further electron drift. The Thomson field (E_T) will continue to exist as long as the temperature gradient exists. If the heat source were removed, both the temperature gradient and the Thomson field would vanish and electron redistribution would occur. In some conductors the Thomson field (E_T) is directed toward cooler regions, as in the above example, while in others it is directed toward warmer regions. The magnitude of E_T at any point along a wire depends upon the temperature gradient (dT/dL) existing at the point and is described analytically by:

$$E_T = - \sigma \left(\frac{dT}{dL} \right)$$

III, Basic Principles (cont.)

where σ is the Thomson coefficient. The voltage (emf) developed within a wire as a result of the Thomson effect is:

$$\begin{aligned} V_T &= - \int E_T \, dL \\ &= \int \sigma \left(\frac{dT}{dL} \right) \, dL \\ &= \int_{T_1}^{T_2} \sigma \, dT \end{aligned}$$

where T_1 and T_2 are the terminal temperatures of an incremental wire segment and V_T is the Thomson electromotive force generated along the segment. The sign and magnitude of the Thomson coefficient (σ) depends upon the chemical composition and metallurgical structure of the wire as well as its absolute temperature.

The Thomson coefficient (σ) is considered to be positive if both the Thomson potential gradient (dV_T/dL) and temperature gradient (dT/dL) have the same sign (i.e., both gradients are either positive or negative). Thus, the Thomson coefficient for the metal conductor shown in Figure 2 is positive as would be the case for a copper conductor. Constantan possesses a negative Thomson coefficient.

The second thermoelectric source, the Peltier effect, is described with the aid of Figure 3 which shows two conductors at the same temperature and physically connected.

FIGURE 3

THE PELTIER EMF

III, Basic Principles (cont.)

The valance electrons in any one metal have a different maximum energy for a given temperature than do the valance electrons within other metals. Thus when two dissimilar metals are joined, the physical barrier between them is destroyed and electron migration is experienced. In the above figure, metal "A" is assumed to contain higher energy electrons than metal "B". Thus, there will be a flow of electrons from A to B until the Peltier potential (π_{AB}) generated by the resulting charge unbalance prevents further electron migration. Note that when double-subscript voltage symbols are used, the positive reference direction is from higher to lower potentials. Thus,

$$\pi_{AB} = \pi_A - \pi_B$$

where π_A is at a higher potential than π_B . Also, a potential generator tends to drive current in the direction of the reference arrow which is directed out of the positive generator terminal.

The magnitude and direction of the electromotive force developed across the junction of two dissimilar metals depends upon the chemical composition and metallurgical structure of each metal as well as the absolute temperature of their common junction. The emf generated at the junction of two thermocouple elements A and B is known as the Peltier emf and is designated as π_{AB} .

The Peltier emf (π_{AB}) is considered to be positive when a "positive charge carrier" experiences an increase in its potential energy when moving across the junction from metal B to metal A. Thus the junction shown in Figure 3 possesses a positive Peltier emf. If the junction of Figure 3 represented a copper-constantan terminal, metal B would be constantan and metal A would be copper.

We shall now return to a basic thermocouple circuit consisting of two homogeneous conductors A and B whose junctions are maintained at temperatures T_1 and T_2 where T_2 is greater than T_1 (i.e., $T_2 = T_1 + \Delta T$).

III, Basic Principles (cont.)

FIGURE 4

ADDITIVE EFFECTS OF THE PELTIER AND THOMSON EMF'S IN A CLOSED THERMOCOUPLE CIRCUIT

Due to the existence of a temperature gradient along each thermocouple element, a Thomson emf will appear in each. Unequal Peltier emf's will also be generated since the junction temperatures differ. If we assume positive Thomson coefficients for each element and if element A is assumed to be thermoelectrically positive relative to element B, the circuit emf's will have the directions shown in Figure 4. The net circuit emf (V_{AB}) can now be determined by summing the potentials encountered as we move around the circuit in a clockwise direction as shown below:

$$V_{AB} = (V_T)_B + (\pi_{AB})_{T_2} - (V_T)_A - (\pi_{AB})_{T_1}$$

$$V_{AB} = (\pi_{AB})_{T_2} - (\pi_{AB})_{T_1} + \int_{T_1}^{T_2} (\sigma_B - \sigma_A) dT$$

where (V_{AB}) states, symbolically, that the circuit potential of element A exceeds that of element B at the cool junction. Thus the net emf generated by a thermocouple circuit is the algebraic sum of the Thomson and Peltier emf's. Stating the above generalization analytically yields:

III, Basic Principles (cont.)

$$V_{NET} = \sum (\pi_{ji} + \int \sigma dT) \quad (1)$$

where (V_{NET}) equals (V_{AB}) for the circuit of Figure 4.

As a general rule, conductor A is said to be positive with respect to B if the thermoelectric induced current flows from A to B at the cooler junction¹.

Theoretical reasoning based upon the second law of thermodynamics shows that, for the basic thermocouple circuit of Figure 4, the net thermoelectric (V_{AB}) , Peltier (π_{AB}) , and Thomson (V_T) emf's are related by:²

$$(\pi_{AB}) = T \left(\frac{dV_{AB}}{dT} \right) \quad (2)$$

$$\sigma_B - \sigma_A = -T \left(\frac{d^2 V_{AB}}{dT^2} \right) \quad (3)$$

where T is absolute temperature.

Thus, access to a calibration table relating the net thermoelectric emf to absolute junction temperatures will provide values for (dV_{AB}/dT) and $(d^2 V_{AB} / dt^2)$. These values, when combined with equations (2) and (3), will yield analytical values for the parameters of equation (1).

1. D. I. Finch (General Principles of Thermocouple Thermometry), Temperature, Its Measurement and Control in Science and Industry, Vol. 3, Part 2, Reinhold Publishing Corp., New York, 1962, p. 3.
2. Van Der Ziel, Solid State Physical Electronics, Prentice Hall Inc., Englewood Cliffs, N.J., 1957, pp 460-464.

IV. TYPICAL EMF'S ENCOUNTERED IN COPPER-CONSTANTAN THERMOCOUPLE CIRCUITS OPERATING AT TEMPERATURES NEAR THE ICE POINT

By employing equation 3 and experimental results, it has been determined that the Thomson coefficient (σ) at temperatures near 0°C in microvolts per degree C is +2 and -23, respectively, for copper and constantan.³ Using these values for the Thomson coefficient and calculating π_{AB} with the aid of equation 2 and thermocouple calibration tables, the thermoelectric emf of a copper-constantan circuit is calculated to demonstrate the magnitude of typical emf's encountered by circuits utilizing these materials near ice point temperatures.

FIGURE 5

THERMAL EMF'S WITHIN A COPPER-CONSTANTAN CIRCUIT NEAR ICE POINT TEMPERATURES

By convention, the emf's are added as we move around the circuit in a clockwise direction to yield

$$V_{NET} = 10689 \mu\text{V} - 4 \mu\text{V} - 10561 \mu\text{V} - 46 \mu\text{V} = 78 \mu\text{V}$$

3. W. F. Roeser (Thermoelectric Thermometry), Temperature, Its Measurement and Control in Science and Industry, Reinhold Publishing Corp., New York, 1941, p. 186.

IV, Typical emf's Encountered in Copper-Constantan Thermocouple Circuits
Operating at Temperatures Near the Ice Point (cont.)

Since the net positive emf is in a clockwise direction, current will flow from copper to constantan at the cooler junction so that, by definition, copper is considered to be thermoelectrically positive to constantan and the net emf is designated as $V_{Cu, Cn}$. By referring to the same thermocouple tables used to compute ϵ_{AB} , we find that the actual measured emf generated by the circuit of Figure 5 is 77.6 μv which is in good agreement with the computed value.⁴

As seen from the above example for copper-constantan elements at temperatures near 0°C, the emf's produced by the junctions are much larger than those produced by temperature gradients along individual wires. Also, constantan is eleven times more sensitive to temperature gradients than copper.

V. THERMOCOUPLE CIRCUIT LAWS

Work by many investigators has shown that the operation of thermocouple circuits comprised of homogeneous metals may be summarized by fundamental laws which have been established experimentally beyond any doubt.⁵ However, it must be emphasized that these laws only apply to circuits containing homogeneous wires; that is, wires that are uniform in chemical composition and physical structure throughout their length (i.e., wires containing no dissimilar thermoelectric sections).

A. Law of Homogeneous Metal Conductors - An electric current cannot be sustained in a single homogeneous metal wire by the application of heat alone. That is, the output of a thermocouple is independent of local temperature variations along any homogeneous wire, but is only dependent upon the net temperature difference between the ends of each wire (i.e., the overall temperature gradient).

4. Powel, Lindsay, Caywood, and Bunch (Low Temperature Thermocouples), Temperature, Its Measurement and Control in Science and Industry, Vol. 3, Part, 2 Reinhold Publishing Corp., New York, 1962, p. 69.

5. D. I. Finch, op. cit., p. 6.

V, A, Law of Homogeneous Metal Conductors (cont.)

FIGURE 6

DEMONSTRATION OF THE FIRST LAW OF THERMOCOUPLES

As a consequence of the first law, we find that if the junctions of two dissimilar homogeneous metals (A and B) were maintained at temperatures T_1 and T_2 as shown in Figure 6, the thermoelectric emf developed would be independent of "local" temperature variations along the wires. That is, the emf is unaffected by the temperature zones T_3 and T_4 .

B. Law of Intermediate Metals - The algebraic sum of thermoelectric emf's in a circuit composed of any number of dissimilar metals is zero if all junctions are at the same temperature.

FIGURE 7

DEMONSTRATION OF THE SECOND LAW OF THERMOCOUPLES

V. B. Law of Intermediate Metals (cont.)

As a consequence of the second law, the net thermoelectric emf of the circuit shown in Figure 7A will be identical to that generated by the circuit of Figure 7B if all of the metals are homogeneous.

VI. APPLICATION OF THE THERMOCOUPLE LAWS

By using the two thermocouple laws, it is possible to construct the thermocouple circuit of Figure 8 which is capable of making precise temperature measurements.

FIGURE 8

STANDARD THERMOCOUPLE MEASUREMENT CIRCUIT

By employing the first law, temperature gradients along the homogeneous copper lead wires will not contribute to the total emf so that the meter used to measure the thermoelectric emf may assume the temperature of its surrounding environment (T_3). Using the second law we find that the thermoelectric emf generated by the circuit is identical to that which would have been generated if the third metal (copper wire) had not been inserted. That is, the circuit emf is only a function of the temperature zones T_1 and T_2 since we are assuming that all wires are homogeneous.

VI, Application of the Thermocouple Laws (cont.)

Normally, the junction located in the T_2 temperature zone is referred to as the measurement junction and T_2 is the environmental temperature to be measured. The junctions located in the T_1 temperature zone are referred to as the reference junctions and T_1 is the reference temperature. The reference junctions are kept at a controlled temperature, usually by immersion in a solid or liquid that is normally maintained at the ice point of distilled water (0°C).

By holding the reference junctions at a constant temperature (T_1) and varying the temperature of the measuring junction, an empirical relationship between the measurement junction temperature and the corresponding net thermoelectric emf may be expressed in tables, graphs, or empirical equations. This calibration process⁶ has been carried out by many experimenters and a representative table has been obtained⁷ and published by the National Bureau of Standards. A plot of the emf-temperature relationship for copper-constantan thermocouples is presented in Figure 9.

The temperature of the reference junctions shown in Figure 8 are most easily controlled by placing them in a wide-mouth Dewar flask filled with shaved (distilled water) ice and distilled water. Although the ice bath reference is most satisfactory for laboratory temperature measurements, its use is somewhat awkward under rocket test conditions. As a result, other techniques are often employed to provide a substitute "ice bath" reference. A substitute reference which has been used for nuclear rocket engine testing employs two heated metal blocks to which thermocouple wires are thermally coupled as shown in Figure 10.

6. Roeser and Lonberger, Methods of Testing Thermocouples and Thermocouple Materials, National Bureau of Standards Circular 590, 1953.

7. Shenker, Lauritzen, Corruccini, and Lonberger, Reference Tables for Thermocouples, National Bureau of Standards Circular 561, 1955.

TEMPERATURE-MILLIVOLT CALIBRATION
CURVE FOR COPPER-CONSTANTAN THERMOCOUPLES
(REFERENCE JUNCTION AT 32°F)

VI, Application of the Thermocouple Laws (cont.)

FIGURE 10

EMPLOYMENT OF AN ELECTRICAL THERMOCOUPLE REFERENCE

The three thermocouple junctions contained within the unit generate a net absolute millivolt signal whose polarity and magnitude equals that which would be produced by a single copper-constantan junction at 0°C . That is, the total thermoelectric emf generated by the circuit of Figure 10 will equal that generated by the circuit of Figure 8 if an equal environmental temperature (T_2) were encountered by both circuits. Functionally, both circuits are identical.

VII. TYPICAL THERMOCOUPLE DATA CHANNEL

So far we have considered only those thermocouple circuits which employ continuous lengths of homogeneous thermocouple wire. However, rocket test facilities frequently use numerous terminal strips, patch boards, plugs, etc., to facilitate the routing and installation of instrumentation cabling. Thus a typical thermocouple channel would more closely resemble the circuit shown in Figure 11.

VII, Typical Thermocouple Data Channel (cont.)

FIGURE 11

TYPICAL THERMOCOUPLE DATA CHANNEL

The thermocouple circuit encountered in most data acquisition systems contains b_n cable transitions between the measurement junction and the thermocouple reference and a_n cable transitions between the thermocouple reference and the meter. Notice that both the input and output terminals of the thermocouple reference are counted as transition points as is the meter input terminal.

VIII. SPLICE INDUCED SPURIOUS EMF'S

It should be noted that all wire materials having the same nominal composition do not possess identical emf-temperature characteristics. This should be obvious when one considers the impossibility of controlling, exactly, the chemical and physical properties of wire. Thus, a typical thermocouple circuit employs several discrete wire sections, each of which may differ thermoelectrically from all other sections. It should also be noted that each transition region is characterized by a temperature which differs from all other transition regions. Thus a Thomson emf will be generated along both wires of each section and thermoelectric mismatch induced Peltier emf's will be generated at each transition. The magnitude

VIII, Splice Induced Spurious emf's (cont.)

of the spurious Thomson and Peltier emf's are, to a considerable extent, controlled by purchasing or selecting wire materials that approximate a particular temperature-emf relation which is representative for the alloy combination employed.

The sections connecting the measurement junction to the thermocouple reference may be thermocouple wire or a combination of thermocouple wire and thermocouple extension wire. Thermocouple extension wire is generally less expensive and yields practically the same temperature-emf relation as thermocouple wire over limited temperature ranges (i.e., -75°F to $+200^{\circ}\text{F}$ for copper-constantan extension wire). When copper-constantan thermocouples are used, both thermocouple wire and thermocouple extension wire are obtainable in two grades, standard and special, whose limits of calibration error are $\pm 1.5^{\circ}\text{F}$ and $\pm 0.75^{\circ}\text{F}$, respectively, when compared to the National Bureau of Standards thermocouple calibration tables over the temperature range -75°F to $+200^{\circ}\text{F}$.⁸

Caldwell of NBS has stated that the reproducibility of copper-constantan thermocouples is primarily limited by the constantan leg since copper has a high degree of reproducibility.⁹ Finch of Leeds and Northrup Co. has found that copper wires conforming to ASTM Specification B3-45 are thermoelectrically alike to about one microvolt and that the error limit of copper-constantan thermocouples is governed by the melt-to-melt variability of constantan.¹⁰

8. I.S.A. American Standard for Temperature Thermocouples, Instrument Society of America, 530 William Penn Place, Pittsburgh, Pennsylvania, 1964, p. 9.

9. Caldwell, F. R., Thermocouple Materials, National Bureau of Standards Monograph 40, March 1, 1962, p. 20.

10. D. I. Finch, op. cit., p. 6.

VIII, Splice Induced Spurious emf's (cont.)

Because of the melt-to-melt variability of both copper and constantan, copper and constantan wires are normally selected and matched to obtain a thermocouple which approximates the NBS calibration tables to a given limit of error. For wires that have been matched to form "special" grade thermocouple and thermocouple extension cables, the thermoelectric variability of constantan is normally five times that of copper. In general, the thermoelectric variability of thermocouple grade copper is the same for both special and standard grade cables, while the allowable variability of constantan is greater when it is to be used in standard grade thermocouple cables. In other words, the variability of the constantan leg generally limits the calibration uncertainty of the copper-constantan combination.¹¹

If we assume that all wire transitions are within the temperature range -75 to +200°F and that special or premium grade wire is used, the calibration uncertainty of each thermocouple cable or thermocouple extension cable will be $\pm 0.75^\circ\text{F}$.

By determining the average (emf/°F) sensitivity of copper-constantan thermocouples over the temperature range -75 to 200°F,

$$\begin{array}{l} \text{Average Copper-Constantan} \\ \text{Thermocouple Sensitivity} \\ \text{Over the Range -75 to 200}^\circ\text{F} \end{array} = \frac{3967 - (-2117)}{200 - (-75)} = 22.12 \mu\text{V}/^\circ\text{F}$$

the equivalent microvolt calibration uncertainty within the same range may be found as follows:

$$\begin{aligned} \text{Microvolt Calibration Uncertainty} &= \left(\begin{array}{c} \text{Calibration} \\ \text{Uncertainty} \\ \text{in } ^\circ\text{F} \end{array} \right) \left(\begin{array}{c} \text{Average} \\ \text{Thermocouple} \\ \text{Sensitivity} \end{array} \right) \\ \text{for Copper-Constantan Thermocouples} &= (\pm 0.75^\circ\text{F})(22.12 \frac{\mu\text{V}}{^\circ\text{F}}) \\ \text{Over the Range -75 to 200}^\circ\text{F} &= \pm 16.59 \mu\text{V} \end{aligned}$$

11. J. D. Sine (personal communication), Industrial Division, Honeywell, Fort Washington, Pa., March 24, 1966.

VIII, Splice Induced Spurious emf's (cont.)

However, reference to Figure 8 demonstrates that the calibration induced uncertainty results from the terminal uncertainty of two junctions, one at temperature T_1 and the other at temperature T_2 . By using statistical square-root-of-the-sum-of-the-squares (RSS) techniques for determining the most probable uncertainty of any function composed of independent quantities,¹² the calibration uncertainty may be considered to be the (RSS) summation of the two terminal uncertainties. That is:

$$\text{Microvolt Calibration Uncertainty} = \sqrt{\left[\{T_1\} \text{ Terminal Uncertainty} \right]^2 + \left[\{T_2\} \text{ Terminal Uncertainty} \right]^2}$$

Since both terminals are formed from the same wire pair, the magnitude of both terminal uncertainties are equal and may be determined as shown:

$$\begin{array}{l} \text{Terminal Uncertainty for Special Grade} \\ \text{Copper-Constantan (Cu/Cn) Thermocouple} \\ \text{Cable and Thermocouple Extension Cable} \\ \text{Over the Range } -75 \text{ to } 200^\circ\text{F} \end{array} = \pm \frac{16.59}{\sqrt{2}} = 11.73 \text{ } \mu\text{v}$$

Repeating the above calculations for standard grade wire yields:

$$\begin{array}{l} \text{Terminal Uncertainty for Standard Grade} \\ \text{Copper-Constantan (Cu/Cn) Thermocouple} \\ \text{Cable and Thermocouple Extension Cable} \\ \text{Over the Range } -75 \text{ to } 200^\circ\text{F} \end{array} = \pm 23.47 \text{ } \mu\text{v}$$

By recalling that the thermoelectric variability of constantan is normally five times that of copper in special grade thermocouple cable and thermocouple extension cable, U_C may be assigned to represent the terminal uncertainty of copper wire and

12. Scarborough, J. B., Numerical Mathematical Analysis, The Johns Hopkins Press, 1958, pp. 443-446.

VIII, Splice Induced Spurious emf's (cont.)

$5U_C$ to represent the terminal uncertainty of constantan wire. Then the use of (RSS) summation techniques yields:

$$\left(\begin{array}{c} \text{Terminal} \\ \text{Uncertainty} \\ \text{of Copper} \end{array} \right)^2 + \left(\begin{array}{c} \text{Terminal} \\ \text{Uncertainty} \\ \text{of Constantan} \end{array} \right)^2 = \left(\begin{array}{c} \text{Terminal Uncertainty of Special Grade} \\ \text{Copper-Constantan (Cu/Cn) Thermocouple} \\ \text{Cable and Thermocouple Extension Cable} \end{array} \right)^2$$

$$(U_C)^2 + (5U_C)^2 = (11.73)^2$$

Terminal Uncertainty
of Copper Wire (U_C)
Over the Range -75
to 200°F = $\pm 2.30 \mu v$

Since the same type of copper wire is normally used in the fabrication of both special and standard grade thermocouple cables, the above terminal uncertainty value for copper wire applies to both cable grades. With this assumption, the terminal uncertainty of constantan wire is found to be:

Terminal Uncertainty of Special
Grade Constantan Wire Over the = $\pm 11.51 \mu v$
Range -75 to 200°F

Terminal Uncertainty of Standard
Grade Constantan Wire Over the = $\pm 23.35 \mu v$
Range -75 to 200°F

By knowing the terminal uncertainty of copper and constantan wire, the thermoelectric transition uncertainty produced by joining one wire to another may be found by using RSS techniques. As an example, consider the transition uncertainty

VIII, Splice Induced Spurious emf's (cont.)

of a copper-to-copper junction. Using a RSS summation of individual wire terminal uncertainties, we have:

$$\begin{aligned} \text{Copper-to-Copper} \\ \text{Transition Uncertainty} &= \pm \sqrt{\left(\text{Terminal Uncertainty of Copper} \right)^2 + \left(\text{Terminal Uncertainty of Copper} \right)^2} \\ &= \pm 3.25 \mu\text{v} \end{aligned}$$

Wire transition uncertainty values for all possible combinations of copper and constantan wire are presented in Table I.

TABLE I
WIRE TRANSITION UNCERTAINTY VALUES
OVER THE RANGE -75 to 200°F

Wire Material	Copper Wire	Special Grade Constantan Wire
Copper Wire	$\pm 3.25 \mu\text{v}$	$\pm 11.73 \mu\text{v}$
Special Grade Constantan Wire	$\pm 11.73 \mu\text{v}$	$\pm 16.28 \mu\text{v}$
Standard Grade Constantan Wire	$\pm 23.47 \mu\text{v}$	$\pm 33.02 \mu\text{v}$

The (RSS) summation technique may also be used to obtain the thermoelectric transition uncertainty resulting from the joining of two thermocouple cables. As an example, the transition uncertainty resulting from joining two special grade copper-constantan cables is found to be:

$$\begin{aligned} \text{Transition Uncertainty} \\ \text{of a Special Grade (Cu/Cn)} \\ \text{Cable Splice Over the} \\ \text{Range -75 to 200°F} &= \sqrt{(2.30)^2 + (2.30)^2 + (11.51)^2 + (11.51)^2} \\ &= \pm 16.60 \mu\text{v} \end{aligned}$$

VIII, Splice Induced Spurious emf's (cont.)

To determine the equivalent cable transition induced measurement uncertainty in degrees Rankine, it is necessary to determine the thermoelectric sensitivity ($^{\circ}\text{R}/\mu\text{v}$) and multiply it by the splice microvolt uncertainty. For instance, the uncertainty in degrees Rankine induced in a thermocouple circuit as a result of a splice between two special grade copper-constantan cables when measuring liquid hydrogen temperatures is:

$$\begin{aligned} \text{Transition Induced Uncertainty} \\ \text{in } ^{\circ}\text{R of a Special Grade} \\ \text{(Cu/Cn) Cable Splice When} \\ \text{Measuring LH}_2 \text{ Temperatures} &= \left(\text{Microvolt Uncertainty} \right) \left(\text{Thermoelectric Sensitivity} \right) \\ & \quad \left(\text{at LH}_2 \text{ Temperatures} \right) \\ &= (\pm 16.60 \mu\text{v})(0.3636 ^{\circ}\text{R}/\mu\text{v}) \\ &= \pm 6.04^{\circ}\text{R} \end{aligned}$$

Cable transition induced measurement uncertainties for other splice configurations are presented in Table II.

TABLE II

THE MEASUREMENT UNCERTAINTY INDUCED BY
THE PRESENCE OF VARIOUS INDIVIDUAL
TRANSITIONS WHEN MEASURING LH₂
TEMPERATURES WITH COPPER-
CONSTANTAN (Cu/Cn) THERMOCOUPLES

TYPE OF CABLE TRANSITION	GRADE OF CABLE WIRE	INDUCED MEASUREMENT UNCERTAINTY	
		MICROVOLTS	DEGREES RANKINE
Cu/Cn to Cu/Cn	Special to Special	± 16.60	± 6.04
Cu/Cn to Cu/Cn	Special to Standard	± 26.24	± 9.54
Cu/Cn to Cu/Cn	Standard to Standard	± 33.18	± 12.07
Cu/Cu to Cu/Cu	Thermocouple to Thermocouple	± 4.60	± 1.67

By referring to Figure 11, it becomes obvious that the number of possible cable transition combinations can be quite large. However, the measurement uncertainty

VIII, Splice Induced Spurious emf's (cont.)

induced by any transition combination can be found by using (RSS) summation techniques. Table III has been prepared to demonstrate the magnitude of measurement uncertainties generated by using various transition (splice) combinations in a copper-constantan thermocouple circuit when measuring liquid hydrogen temperatures.

TABLE III

TRANSITION INDUCED MEASUREMENT UNCERTAINTY
IN DEGREES RANKINE WHEN SPECIAL GRADE
COPPER-CONSTANTAN (CU/CN) CABLE IS USED
BEFORE THE THERMOCOUPLE REFERENCE AND
COPPER-COPPER (CU/CU) CABLE IS USED AFTER
THE THERMOCOUPLE REFERENCE

NUMBER OF COPPER- COPPER CABLE TRANSITIONS AFTER THE THERMOCOUPLE REFERENCE	NUMBER OF SPECIAL GRADE COPPER-CONSTANTAN THERMOCOUPLE CABLE TRANSITIONS BEFORE THE THERMOCOUPLE REFERENCE					
	0	1	2	3	4	5
0	0°R	± 6.0°R	± 8.4°R	± 10.5°R	± 12.1°R	± 13.5°R
1	± 1.7°R	± 6.3°R	± 8.7°R	± 10.6°R	± 12.2°R	± 13.6°R
2	± 2.4°R	± 6.5°R	± 8.9°R	± 10.7°R	± 12.3°R	± 13.7°R
3	± 2.9°R	± 6.7°R	± 9.0°R	± 10.9°R	± 12.4°R	± 13.8°R
4	± 3.3°R	± 6.9°R	± 9.2°R	± 11.0°R	± 12.5°R	± 13.9°R
5	± 3.7°R	± 7.1°R	± 9.3°R	± 11.1°R	± 12.6°R	± 14.0°R

From the above discussions, several general conclusions can be made concerning transitions in thermocouple circuits.

1. Use special grade wire whenever possible.
2. Eliminate all unnecessary transitions.

VIII, Splice Induced Spurious emf's (cont.)

3. If the test facility requires the use of a finite number of transitions, place the thermocouple reference as close to the measurement function as the test environment permits to increase the percentage of Cu/Cu transitions occurring in the circuit.

Up to this point we have limited our discussion to simple cable transitions which are located within uniform temperature regions. However, when large bulkhead plugs are employed to route cables in and out of inclosures, cabinets, or buildings; it is not uncommon to find that temperature gradients exist along the plug pins. Thus a plug really represents a double cable transition, one at each end of the plug pins. As an example of the kinds of errors which can exist because of temperature differentials across plugs, we shall assume that a connector is used as a transition mechanism in a copper-constantan thermocouple circuit and that gold plated brass pins are used. By referring to tabulated listings of the thermoelectric emf of copper, constantan, and brass against platinum¹³, we can calculate the spurious emf's generated by various wire-pin-wire combinations as a function of temperature differentials across the pins. Calculations begin by reproducing part of the tabulation of Reference 13 and then calculating the emf-temperature characteristic of brass-constantan and copper-brass thermocouples with a 0°C reference assumed. The calculated results are included in Table IV.

13. American Institute of Physics Handbook, McGraw-Hill Book Co., Inc., New York, 1963, pp. 4-9.

VIII, Splice Induced Spurious emf's (cont.)

TABLE IV

THERMOELECTRIC EMF'S AS A FUNCTION OF TEMPERATURE FOR
VARIOUS MATERIAL COMBINATIONS WITH A 0°C REFERENCE JUNCTION

MEASUREMENT JUNCTION TEMPERATURE °C	THERMAL EMF'S FROM REFERENCE 13			CALCULATED THERMAL EMF'S	
	Brass-Pt (mv)	Cu-Pt (mv)	Cn-Pt (mv)	Cu-Brass (mv)	Brass-Cn (mv)
0	0	0	0	0	0
+100	0.60	0.76	-3.51	0.16	4.11
+200	1.49	1.83	-7.45	0.34	8.94
+300	2.58	3.15	-11.71	0.57	14.29

NOTE: A positive sign means that in a single thermocouple circuit, current flows from the first listed element to the second listed element at the 0°C reference junction.

Since we are concerned with the transition arrangement of Figure 12, it will be necessary to calculate the Peltier emf (π_{AB}) of the copper-brass and brass-constantan junctions. The computation is performed with the aid of a previously presented thermoelectric equation which is repeated here for convenience.

$$\pi_{AB} = T \left(\frac{dV_{AB}}{dT} \right)$$

VIII, Splice Induced Spurious emf's (cont.)

FIGURE 12

PLUG TRANSITION FOR A
COPPER-CONSTANTAN THERMOCOUPLE CIRCUIT

Values for (dV_{AB}/dT) are calculated from the listing in Table IV and are assumed to be nearly constant over the temperature range $(-75^{\circ}\text{F to } +200^{\circ}\text{F})$.

$$\left(\frac{dV_{\text{Cu-Br}}}{dT} \right) = \frac{0.16}{100} \text{ mv}/^{\circ}\text{C} = 0.9 \text{ } \mu\text{V}/^{\circ}\text{F}$$

$$\left(\frac{dV_{\text{Br-Cn}}}{dT} \right) = \frac{4.11}{100} \text{ mv}/^{\circ}\text{C} = 22.8 \text{ } \mu\text{V}/^{\circ}\text{F}$$

VIII, Splice Induced Spurious emf's (cont.)

The spurious potentials created by the two wire-pin-wire combinations (going from left to right) are:

$$\begin{aligned} (\text{emf})_{\text{Cn-Br-Cn}} &= \left[(T_o + \Delta T) \left(\frac{dV_{\text{Br-Cn}}}{dT} \right) - (T_o) \left(\frac{dV_{\text{Br-Cn}}}{dT} \right) \right] = (\Delta T) \left(\frac{dV_{\text{Br-Cn}}}{dT} \right) \\ &= 22.8 (\Delta T) \mu\text{V} \end{aligned}$$

$$\begin{aligned} (\text{emf})_{\text{Cu-Br-Cu}} &= - (T_o + \Delta T) \left(\frac{dV_{\text{Cu-Br}}}{dT} \right) + (T_o) \left(\frac{dV_{\text{Cu-Br}}}{dT} \right) = - \Delta T \left(\frac{dV_{\text{Cu-Br}}}{dT} \right) \\ &= - 0.9 (\Delta T) \mu\text{V} \end{aligned}$$

Thus, when brass connector pins are used, the spurious Peltier emf's in the constantan leg are 25.3 times larger than those generated in the copper leg. The total spurious Peltier emf generated by passing a copper-constantan cable through a plug with brass pins is found by going around the circuit in a clockwise direction to obtain

$$\begin{aligned} &\text{Idealized} \\ &\text{plug induced} \\ &\text{uncertainty} \quad = - [0.9 (\Delta T) + 22.8 (\Delta T)] \mu\text{V} = - 23.7 (\Delta T) \mu\text{V} \\ &\text{when copper-} \\ &\text{constantan} \\ &\text{wire is used} \end{aligned}$$

which is equivalent to a $(- 8.6 \Delta T)$ degree Rankine bias when measuring LH_2 temperatures. If two conductor copper cables were routed through the same plug, the total spurious Peltier emf generated would be

VIII, Splice Induced Spurious emf's (cont.)

Idealized
plug induced
uncertainty = $-0.9 (\Delta T) + 0.9 (\Delta T) = 0 \mu V$
when copper-
copper wire
is used

which is equivalent to a 0 degree Rankine bias.

Although the above results yield the most probable bias that will be inserted into a copper-constantan thermocouple circuit when a plug with gold plated brass pins is used as a cable transition mechanism, we must include the uncertainties resulting from the thermoelectric variability of each wire-pin material. Since terminal variability values for brass are not available, we shall assume the variability of brass to be zero. Thus the additional plug induced uncertainty will equal the normal uncertainty for Cu/Cn and Cu/Cu transitions. When these are added to the bias uncertainty, we have the results presented in Table V.

TABLE V
MEASUREMENT UNCERTAINTY INDUCED BY EMPLOYING
A PLUG WITH GOLD PLATED BRASS PINS AS A
TRANSITION MECHANISM IN A COPPER-CONSTANTAN
THERMOCOUPLE CIRCUIT WHEN MEASURING LH_2 TEMPERATURES

TYPE OF CABLE BEING ROUTED THROUGH THE PLUG	WIRE GRADE	
	SPECIAL	STANDARD
Copper-Constantan	$- 8.6 (\Delta T) \pm 6.0^\circ R$	$- 8.6 (\Delta T) \pm 12.1^\circ R$
Copper-Copper	$\pm 1.7^\circ R$	$\pm 1.7^\circ R$

In deriving the uncertainty values of Table V, the effect of the gold plating was not included because of its insignificant contribution to the total uncertainty.

VIII, Splice Induced Spurious emf's (cont.)

To justify the above statement, note that the thickness (d) of the wire-gold-pin transition region is typically a few thousandths of an inch. Since the thermal conductivity of gold (k) is large, the temperature gradient ($\Delta T = Qd/k$) appearing across the wire-gold-pin transition as a result of heat flow (Q) across the transition region is very small. Then, since the wire-gold-pin transition occurs within a near isotherm region, the second thermocouple law may be applied and the effects of the gold plating ignored. The Thomson emf's generated within the pins were also ignored as the emf's would tend to cancel when summed algebraically around the thermocouple circuit in the normal clockwise direction.

IX. WIRE INHOMOGENEITY

Up to this point, we have assumed that all cable sections were composed of homogeneous wire; that is, wire that is uniform in both chemical composition and physical condition throughout its length. However, it is impossible to acquire wire that is perfectly homogeneous and the effects of inhomogeneity must be considered when designing thermocouple circuits. Inhomogeneity effects are most simply explained by imagining a length of wire (L) that is composed of many small discrete (ΔL) sections. The two end sections possess identical thermoelectric properties while all other sections differ from the end sections and from each other as shown in Figure 13.

FIGURE 13

INHOMOGENEOUS WIRE SECTION

IX, Wire Inhomogeneity (cont.)

If the temperature of all sections were equal, the second thermocouple law would apply and the net thermoelectric emf appearing across the wire would be zero. However, if a uniform temperature gradient existed along the wire; a Thomson emf would be generated along each section, the summation of Peltier emf's would no longer be zero, and a net thermoelectric emf would appear across the wire. If the length of each incremental section were to approach zero, and the number of sections were to approach infinity, the wire would then appear as a single section of length (L) with continuously variable physical and chemical properties. However, the discussion of emf's generated in the wire of Figure 13 still applies since we have only increased the number of sections from a finite number to an infinite number. The above discussion may be summarized by stating that spurious emf's will be generated along the length of any metal conductor possessing inhomogeneities in its chemical composition or physical condition if the inhomogeneous region occurs in a temperature gradient zone. The emf generated by wire inhomogeneity may be thought of as arising from distortions of the normal Thomson emf, while the appearance of the spurious emf is known as the Becquerel effect.¹⁴

In general, thermocouple wire produced by manufacturers in this country is sufficiently homogeneous in chemical composition for most purposes. More often, physical inhomogeneity is introduced by inadequate post fabrication annealing and improper installation handling.¹⁵ The thermoelectric properties of copper are exceptionally uniform and cold working, resulting from mishandling, will not produce the large thermoelectric changes that occur in constantan. In general, work by various experimenters has shown that the spurious emf's generated in constantan wire

14. Lion, K. S., Instrumentation in Scientific Research, McGraw-Hill Book Co. Inc., New York, 1959, p. 170.

15. Roeser, W. F., and H. T. Wensel, pp. cit., p. 287.

IX, Wire Inhomogeneity (cont.)

are approximately ten times as large as those generated in copper wire when both materials have been subjected to the same amount of cold working.¹⁶

The effects of inhomogeneity in both wires may be either additive or subtractive and, as the emf developed along an inhomogeneous wire depends upon local temperature distributions and wire characteristics, attempts to predict inhomogeneity effects are impractical if not impossible. About all that can be done is to compare the relative sensitivity of different thermocouple circuits to inhomogeneity effects. As an example, consider the two circuits of Figure 14.

FIGURE 14

RELATIVE SENSITIVITY OF THERMOCOUPLE CIRCUITS
TO INHOMOGENEITY EFFECTS

16. Pollack, D. D., and Finch, D. I. (The Effects of Cold Working Upon Thermoelements), Temperature, Its Measurement and Control In Science and Industry, Vol. 3, Part 2, Reinhold Publishing Corp., New York, 1962, p. 238.

IX, Wire Inhomogeneity (cont.)

If we assume that the probability of encountering inhomogeneity induced emf's is proportional to the length of the cable used, we may employ a proportional constant (C_x) to formulate an expression for the probable sensitivity of a thermocouple cable to inhomogeneity effects:

$$\text{Probable sensitivity of a wire to inhomogeneity effects} = C_x (L)$$

Noting that constantan is ten times more sensitive than copper, the proportional constant for constantan (C_{cn}) will be ten times as large as the proportional constant of copper (C_{cu}). Thus the probable magnitude of inhomogeneity induced emf's in the top circuit of Figure 14 is

$$(EMF)_T = C_{cn} (L) + C_{cu} (5L) = 15 C_{cu} (L)$$

and that of the bottom circuit is

$$(EMF)_B = C_{cn} (2L) + C_{cu} (4L) = 24 C_{cu} (L)$$

Therefore, the bottom circuit is 1.6 times more sensitive to inhomogeneity effects than the top circuit.

X. INADEQUATE SHIELDING INDUCED MEASUREMENT UNCERTAINTIES

The next error mode to be discussed results from the currents induced in both thermocouple wires due to the presence of electrical wire to ground coupling. Potential gradients along the earth's surface interact with thermocouple cables and generate erroneous emf signals. The generation mechanism is explained with the aid of Figure 15 where Z represents the wire to ground coupling impedance and R_{cu} and R_{cn} are the resistances of copper and constantan wires, respectively.

X, Inadequate Shielding Induced Measurement Uncertainties (cont.)

FIGURE 15

COMMON MODE ERROR

The existence of a ground potential will cause two currents, the conductive path of one being (4), (1), (2), (4) and the other (4), (1), (3), (4). The potential generated in the upper leg will be:

$$V_1 = \frac{R_{cu1} + R_{cu2}}{R_{cu1} + R_{cu2} + Z} V_g$$

and the potential generated in the lower leg will be:

$$V_2 = \frac{R_{cu1} + R_{cu2}}{R_{cu1} + R_{cu2} + Z} V_g$$

If we were to use an amplifier in the meter circuit that was sensitive only to voltage differences between the two conductors (i.e., a differential amplifier), and if the line resistance of both lead wires were equal, then V_1 would equal V_2 and no spurious emf's would be detected. Lines of equal resistance are referred to as balanced lines and the two equal voltages V_1 and V_2 are referred to as common mode voltages (i.e., they are common to both lines).

X, Inadequate Shielding Induced Measurement Uncertainties (cont.)

However, when copper-constantan cables are used, the lines are normally unbalanced since the resistivity of constantan and copper are 49×10^{-6} and 1.7×10^{-6} ohm-cm, respectively. Thus, the resistance of the constantan leg will exceed the resistance of the copper leg and V_1 will exceed V_2 . Then, since the ideal differential amplifier is only sensitive to voltage differences between the two lines, the indicated thermocouple output would be:

$$\text{Indicated Thermocouple Signal} = \left[\text{Actual Thermocouple Signal} \right] + (V_1 - V_2)$$

where $(V_1 - V_2)$ is the spurious emf generated by the combined effects of the ground potential gradient, line unbalance, and electrical wire to ground coupling. From equations describing V_1 and V_2 , we have:

$$(V_1 - V_2) = \frac{(R_{cn} - R_{cul}) Z V_g}{(R_{cul} + R_{cu2} + Z) (R_{cn} + R_{cu2} + Z)}$$

which demonstrates that electrical ground coupling errors may be reduced by:

1. Minimizing the length of constantan wire used in the circuit (i.e., moving the thermocouple reference toward the measurement junction).
2. Reducing the resistance per unit length of the constantan wire by increasing its diameter. That is, the resistance per unit length of both the copper and constantan leads should be equal (i.e., $R_{cn} = R_{cul}$).
3. Increasing the wire to ground coupling impedance (Z) by using insulating jackets about each wire, wrapping an electrical shield over the wire insulation, and insulating the shield from ground everywhere except at one point in the system.

X. Inadequate Shielding Induced Measurement Uncertainties (cont.)

Since a typical copper braid shield has a shield coverage of about 90 to 95%, electrical ground coupling errors can still occur with this type of shielding¹⁷. Even if perfect shielding is used (i.e., a solid metal covering), the use of a single sheathed cable can result in multipoint shield grounding and electrical ground coupling errors will occur in the metal sheathed section. Ideally, a double sheathed cable is recommended in addition to the three general recommendations listed previously.

As in the case of inhomogeneity induced errors, we can only compare the relative sensitivity of various thermocouple circuits to electrical ground coupling errors.

XI. THE EFFECT OF FINITE METER INPUT IMPEDANCES

The final error mode to be discussed results from current flow around the circuit because of a finite meter input impedance (R_M). The circuit current is shown in Figure 16 where R_{cn} and R_{cu} are resistances of constantan and copper wires.

FIGURE 16

CURRENT INDUCED VOLTAGE DROPS

By referring to both Figures 5 and 16, the measurable emf appearing at the meter input terminals (V_M) is:

17. Grounding and Noise Reduction Practices for Instrumentation Systems, SDS Data Systems, 600 East Bonia Ave., Pomona, Calif., 1965, p. 2-73.

XI, The Effect of Finite Meter Input Impedances (cont.)

$$V_M = iR_M = \frac{(\epsilon_{cu-cn})_{T_2} - (\epsilon_{cu-cn})_{T_1} + \int_{T_1}^{T_2} \sigma_{cn} dT - \int_{T_1}^{T_2} \sigma_{cu} dT}{1 + [(R_{cn} + R_{cu1} + 2R_{cu2})/R_M]}$$

Thus, for a given (R_M) , the measured voltage will increase and approach the open circuit thermoelectric voltage as the total wire resistance of the circuit $(R_{cn} + R_{cu1} + 2R_{cu2})$ decreases. Then, since the resistance per unit length of constantan is about 29 times that of copper, a reduction in the relative amount of constantan will tend to improve the measurement accuracy. In general, there are three techniques that may be employed to reduce current induced measurement errors

1. Reduce the relative amount of constantan used in the circuit by moving the reference junction toward the measurement junction.
2. Reduce the total circuit wire resistance by increasing the wire diameters or by reducing overall wire lengths.
3. Use meters possessing high input impedances.