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STABILITY OF HOMOGENEOUS REACTORSSUMMARY

A comparison between one- and two-region reactors is given with regard to their stability. There appears to be no distinct advantage of one type over the other, since the stability criteria changes with operating conditions and nuclear parameters in a manner which leads to about the same stability for each reactor type.

It has been shown that the effect of fluid circulation can be neglected with relatively small effect upon the stability criteria.

The most pessimistic HRT operating conditions have been shown to lie in the stable region of operation. Also, the TBR and similar high power-density reactors have stable-type behavior.

INTRODUCTION

The conventional equations of motion for a one-region reactor can be written as

$$\dot{x} = [m + \xi t + z] x + \lambda c \quad (1a)$$

$$\dot{y} = x - 1 - \epsilon y \quad (1b)$$

$$\dot{z} = -v \quad (1c)$$

$$\dot{v} = \omega_h^2 \left\{ (1+C_2)z + \omega_n^2 y \right\} - \frac{\alpha_1}{2\gamma_3} \left[(v+\gamma_3) |v+\gamma_3| - \gamma_3^2 \right] \quad (1d)$$

$$\dot{c} + \lambda c = \gamma x \quad (e)$$

In this discussion it is assumed that the stability relations in a two-region reactor are determined by the core system characteristics. (However, the mean lifetime of prompt neutrons is that for the reactor as a whole.) Under most circumstances the above will be true if a large fraction of the power is

generated in the core region. Equations 1 will therefore be applicable to both one- and two-region machines, and so the indicated comparison can only concern differences in operating conditions associated with the two reactor types. The fundamental differences in the two types concern their operating power densities and nuclear parameter values.

STABILITY CRITERIA

The stability analysis is based upon the linearized equations of motion from which the criteria of stability is obtained. The stability criteria concern operating conditions which assure that a system disturbance does not build up in amplitude with time. Previous work pertaining to stability analysis is given elsewhere.*

Linearization of Equation 1 gives

$$\dot{x} = z + \lambda c - \gamma x \quad (2a)$$

$$\dot{z} = -v \quad (2b)$$

$$\dot{c} + \lambda c = \gamma x \quad (2c)$$

$$\dot{v} = -\alpha_1 v + \omega_h^2 (1+C_2) \left[z + \frac{\omega_h^2}{1+C_2} y \right] \quad (2d)$$

$$\dot{y} + \epsilon y = x - 1 \quad (2e)$$

The above system of equations is stable against disturbances so long as its characteristic algebraic equation has no roots of real, positive part. The characteristic equation for Equation 2 is

$$s^5 + as^4 + bs^3 + cs^2 + ds + e = 0 \quad (3)$$

* G. T. Trammell, HRP Quarterly Progress Report, ORNL-1121, pp. 88-93, December 27, 1951.

P. R. Kasten, Linearized Stability Criteria for HRT Type Reactors, April 21, 1954, CF-54-4-183.

where $s = \text{roots of Equation 3}$ (4a)

$$a = \lambda + \epsilon + \alpha_1 + \gamma \quad (4b)$$

$$b = \omega_h^2(1+c_2) + \alpha_1\epsilon + \lambda(\epsilon + \alpha_1) + \gamma(\epsilon + \alpha_1) \quad (4c)$$

$$c = \omega_h^2(1+c_2)\epsilon + \lambda \left[\omega_h^2(1+c_2) + \alpha_1\epsilon \right] + \gamma \left[\omega_h^2(1+c_2) + \alpha_1\epsilon \right] \quad (4d)$$

$$d = \omega_h^2(1+c_2)\epsilon\lambda + \omega_h^2\omega_n^2 + \gamma\omega_h^2\epsilon(1+c_2) \quad (4e)$$

$$e = \omega_h^2\omega_n^2\lambda \quad (4f)$$

The conditions which have to be satisfied for s to have no roots of real positive part are

$$(ab - c) c > a(ad - e) > \frac{ae(ab - c)^2}{(ab - c)c - a(ad - e)} > 0 \quad (5)$$

Condition 5 is quite burdensome if all terms are kept in Equation 4; however, little is gained by adding the delayed neutron equation so long as λ is small in comparison with associated parameters, and ω_h^2 is large compared with α_1 , ϵ and γ . The stability condition then reduces to

$$\begin{aligned} & \epsilon^3 \left\{ \omega_h^2(1+c_2) + \gamma(\alpha_1+\gamma) \right\} \alpha_1 + \epsilon^2 \left\{ (\alpha_1+\gamma)\alpha_1 \left[\omega_h^2(1+c_2) + \gamma(\alpha_1+\gamma) \right] \right\} \\ & + \epsilon \left[\omega_h^2(1+c_2) + \alpha_1\gamma \right] \left[\omega_h^2(1+c_2) + \gamma(\alpha_1+\gamma) \right] \alpha_1 \\ & + \alpha_1\gamma \omega_h^2(1+c_2) \left[\omega_h^2(1+c_2) + \gamma(\alpha_1+\gamma) \right] \\ & \epsilon^2\omega_h^2\omega_n^2 + \epsilon \left\{ 2\omega_h^2\omega_n^2(\alpha_1+\gamma) \right\} + (\alpha_1+\gamma)^2\omega_h^2\omega_n^2 > 0 \end{aligned} \quad (6)$$

If fluid flow effects within the reactor can be neglected in Equation 2e, $\epsilon = 0$, and the stability criteria simplifies to

$$\alpha_1 \gamma \left[\omega_h^2 (1+C_2) + \gamma (\alpha_1 + \gamma) \right] > (\alpha_1 + \gamma)^2 \frac{\omega_h^2}{1+C_2} > 0 \quad (7)$$

Since all quantities are positive, the inequality on the right in Equation 6 is always satisfied. Replacing the other inequality by an equal sign and fixing all parameters but one, the value of the remaining parameter which barely does not satisfy the inequality can be obtained. Equation 6 is plotted in Figures 1-7 ($\epsilon = 0$) (using an equality sign in the expression) as a function of parameter values. Stable operating conditions are then operating conditions lying above the appropriate curve in Figures 1-7.

Reasonable values for ϵ lie between .2 and 2 sec^{-1} , corresponding to core residence times between 10 and 1 seconds. As illustrated in Figures 2-6, increasing ϵ increases the stringency of the stability condition at low values of α_1 . However, since the influence of ϵ is small, condition 7 can be used, which assumes $\epsilon = 0$, greatly simplifying any additional calculations.

Fluid Circulation Effects

Equations 2 tacitly assume the heat exchanger always returns core fluid at a constant temperature. A more reasonable assumption would be to consider the power removed from the shell side of the heat exchanger to be constant. The equations of motion then become

$$\dot{x} = z - \gamma(x - 1) \quad (8a)$$

$$\dot{z} = -v \quad (8b)$$

$$\dot{v} = -\alpha_1 v + \omega_h^2 (1+C_2) \left[z + \frac{\omega_h^2}{1+C_2} y \right] \quad (8c)$$

$$y = \frac{(T_c - T_{co})S_c}{P_o} \quad (8d)$$

$$T_c = \frac{T_{in} + T_{out}}{2} = T_h \quad (8e)$$

$$S_c \frac{dT_c}{dt} = wC_p [T_{in} - T_{out}] + P_o x \quad (8f)$$

$$S_h \frac{dT_h}{dt} = wC_p [T_{out} - T_{in}] - hA_h [T_h - T_s] \quad (8g)$$

$$S_s \frac{dT_s}{dt} = hA_h [T_h - T_s] - P_o \quad (8h)$$

The characteristic equation for the above mathematical system is

$$\begin{aligned} s^5 + s^4 [\gamma + \alpha_1 + \alpha_2] + s^3 \left\{ \alpha_1 \alpha_2 + \omega_h^2 (1 + C_2) + \gamma (\alpha_1 + \alpha_2) \right\} \\ + s^2 \left\{ \alpha_2 \omega_h^2 (1 + C_2) + \gamma [\alpha_1 \alpha_2 + \omega_h^2 (1 + C_2)] \right\} + s \left\{ \gamma \alpha_2 \omega_h^2 (1 + C_2) + \omega_h^2 \omega_n^2 \alpha_3 \right\} \\ + \omega_h^2 \omega_n^2 \alpha_3 \alpha_4 = 0 \end{aligned} \quad (9)$$

where

$$\alpha_2 = \frac{hA_h}{S_s} \left(1 + \frac{S_s}{S_c + S_h} \right)$$

$$\alpha_3 = \frac{S_c}{S_c + S_h}$$

$$\alpha_4 = \frac{hA_h}{S_s}$$

Equation 9 is of the form of Equation 3 and so the stability criteria is given by conditions 5. As S_s and $S_h \rightarrow 0$, α_4 and α_2 approach ∞ and α_3 approaches 1, and the characteristic Equation 9 becomes

$$s^4 + s^3(\alpha_1 + \gamma) + s^2 \left[\omega_h^2(1+C_2) + \gamma\alpha_1 \right] + s\gamma\omega_h^2(1+C_2) + \omega_h^2\omega_n^2 = 0$$

corresponding to replacing Equation 8f, 8g and 8h by $y = x - 1$. The same result is obtained from Equation 3 with $\epsilon = 0$ and $\lambda c = \gamma$.

EFFECT OF CIRCULATION UPON HRT STABILITY

The values appropriate for the HRT are approximately given by

$$S_s = 3 S_c$$

$$S_h = .2 S_c$$

$$\alpha_3 = 5/6$$

$$\frac{S_s}{S_h + S_c} = 2.5$$

$$\alpha_4 = \frac{hA_h}{S_s} = 1.1$$

$$\alpha_2 = 3.8$$

Insofar as stability is concerned, the worst case would correspond to a heavy slurry in the blanket region. For that case the appropriate values are approximately

$$\alpha_1 = 1.0 \text{ sec}^{-1}$$

$$\gamma = 50 \text{ sec}^{-1}$$

(10)

For numerical work, the following parameter values were chosen:

Table I
Parameter Values Chosen for HRT
Stability Criteria (heavy slurry blanket)

$\frac{S_s}{S_c}$	α_4	α_3
1	3.3	6.33
3	1.1	3.80
10	.33	3.10

Inserting the values given in Equation 10 and Table I into Equation 9 (in conjunction with Equation 3 and condition 5) gives the stability criteria in terms of allowable values of $\omega_h^2(1+C_2)$ for a chosen value of $\frac{\omega_n^2}{1+C_2}$.

Following the above procedure, the curves given in Figure 8 were obtained. The actual HRT operating condition is plotted in Figure 8, which shows that the net effect of circulation in the HRT results in a less stringent stability criterion than if circulation is considered zero ($C = 0$ curve in Figure 8).

Previous analytical studies* have shown that the effects of circulation will always tend to damp any power oscillations so long as the Fourier sine transform of $K(t)$ is > 0 , where $K(t)$ is a weighting kernel relating the actual power removal from the reactor to the past reactor power. In such studies, the pressurizer system was neglected. Based on the results given in Figure 8, inclusion of the pressurizer system does not appear to alter the effect of fluid circulation, namely, circulation tends to damp out any pressure (or power) oscillations. The magnitude of the effect is illustrated in Figure 8, which indicates that for large values of ω_n^2 little damping action is afforded by circulation. Under such circumstances, neglect of fluid circu-

* W. K. Ergen and A. M. Weinberg, Some Aspects of Non-Linear Reactor Dynamics, Physica XX, pp. 413-426, 1954.

lation is justified since such neglect introduces only a small change in the stability criteria. At lower values of ω_n^2 the effect of circulation upon the stability criteria can be appreciable insofar as degree of stability is concerned. However, the $\mathcal{C} = 0$ curve appears to be reasonably accurate (and also conservative) and is much more convenient for design bases.

Figure 8 shows that some advantage is gained in stability by increasing the value of $\frac{S_s}{S_c} \approx$ volume of fluid in steam chest/vol. of fluid in core. The HRT value of $\frac{S_s}{S_c} \approx 3$ is about as good as the $\frac{S_s}{S_c} = 10$ value with regard to stability. The HRT operating condition at 5 Mw for a heavy slurry blanket is given in Figure 8, and is well within the stable region.

ONE- AND TWO-REGION REACTOR CONDITIONS

The differences in the above reactor types will be restricted to power densities and nuclear parameter values. Generally a one-region machine will operate at a lower power density than the corresponding two-region reactor, and the physical size of the one-region reactor will be larger. On the basis of economic studies* the optimum two-region reactor will operate at a core power density of 100-200 kw/l, and a one-region machine at power densities of 15-30 kw/l. For the same output power, then, the volume of the one-region reactor is roughly 8 times the core volume of the two-region system. Assuming the exit velocities, U_o , of the core fluid are about the same and that the effective length of piping between the core and pressurizer are the same, α_1 will not vary for the two reactor types.

With the same operating conditions with respect to fluid temperature and pressure, the ratio of ω_n^2 for the two reactors will be given approximately by

* H. C. Claiborne and M. Tobias, Thorium Breeder Reactor Studies, ORNL-1810, to be issued.

$$\frac{(\omega_h^2)_1}{(\omega_h^2)_2} = \frac{\left(\frac{A}{V_c}\right)_1}{\left(\frac{A}{V_c}\right)_2} \approx \frac{1}{8}$$

where the subscripts 1 and 2 refer to one- and two-region reactors, respectively.

The above assumes that the core fluid temperature rise in passing through the core and fluid exit velocity are the same for each reactor.

The ω_n^2 ratio is given by

$$\frac{(\omega_n^2)_1}{(\omega_n^2)_2} = \frac{(\gamma)_2}{(\gamma)_1} \frac{\left(\frac{P_o}{V_c}\right)_1 \left(\frac{\partial k}{\partial T}\right)_1}{\left(\frac{P_o}{V_c}\right)_2 \left(\frac{\partial k}{\partial T}\right)_2}$$

For comparable conditions, reasonable values are

$$\frac{\tau_2}{\tau_1} \approx 2, \quad \left(\frac{\partial k}{\partial T}\right)_1 \bigg/ \left(\frac{\partial k}{\partial T}\right)_2 \approx \frac{1}{2}$$

so

$$(\omega_n^2)_1 \approx \frac{1}{8} (\omega_n^2)_2$$

The value of $1 + C_2$ will be about the same for the two reactor types, so the net effect in going from a two-region to a one-region machine is to decrease ω_h^2 and ω_n^2 by the same ratio, namely, 8 in the above case. Examination of the stability criteria given in Figures 1-8 shows that such a decrease results in about the same degree of stability, since the stability criteria becomes more stringent at the higher values of ω_n^2 (requiring higher values of ω_h^2).

Assuming that the value of $\frac{A_1}{A_2} \geq 2$, then $(\alpha_1)_2 \geq 2(\alpha_1)_1$ and $(\omega_h^2)_1 \geq \frac{1}{4} (\omega_h^2)_2$. An advantage may be gained by such a change if the desirable increase in ω_h^2 is not offset by the undesirable decrease in α_1 .

Results given in Figures 1 and 2 indicate that some but no significant advantage is gained by such a change.

To tie down the above results and also to see where a high power density machine may fall on the stability curves, consider the TBR which has a core power density of ~ 190 kw/l. Reasonable values for the parameters are

$$\alpha_1 \approx 1 - 3 \text{ sec}^{-1}$$

$$\omega_n^2 \approx 70 \text{ sec}^{-2}$$

$$\omega_h^2 \approx 13,000 \text{ sec}^{-2}$$

$$\gamma \approx 5$$

By plotting such an operating condition in Figures 1-3, it is possible to illustrate that operation is well within the stable region.

In general, stability problems are most likely to occur in small size, high power density reactors (for example, in the HRT with a heavy slurry blanket, instabilities are indicated at power densities above ~ 60 kw/l).

/alg

TABLE OF NOMENCLATURE

a_f	= friction coefficient, $\text{psi}/(\text{ft}/\text{sec})^2$
A	= cross sectional area of relief pipe, ft^2
b	= rate of reactivity addition, sec^{-1}
C_i	= relative latent power associated with the delayed neutrons, dimensionless
C_2	= conversion factor = $\frac{144}{v_s^2} \frac{g_c}{\rho_o} \frac{P_o}{V_p}$
hA_h	= heat transfer coefficient of heat exchanger times heat exchange area, $\text{kw}/^\circ\text{C}$
g_c	= dimensional constant, $\frac{32.2 \text{ lb mass-ft}}{\text{lb force-sec}^2}$
k_e	= effective multiplication constant, dimensionless
$k_e - 1$	= reactivity
$\frac{\partial k_e}{\partial \rho}$	= density coefficient of reactivity, $\text{ft}^3/\text{lb mass}$
$\left \frac{\partial k_e}{\partial T} \right $	= absolute value of temperature coefficient of reactivity, $^\circ\text{C}^{-1}$
L	= length of piping between core and pressurizer surface, ft
m	= $\frac{\delta - \beta}{\gamma}$, sec^{-1}
n_f	= number of velocity heads of fluid lost due to friction, dimensionless
P	= reactor power, kw
P_o	= P evaluated under initial conditions, kw
S_c	= heat capacity of core fluid volume, $^\circ\text{C}/\text{kw-sec}$
S_h	= heat capacity of heat exchanger fluid volume, $^\circ\text{C}/\text{kw-sec}$

- S_s = heat capacity of shell side fluid volume of heat exchanger, $^{\circ}\text{C}/\text{kw-sec}$
 t = time, sec
 T_c = average core fluid temperature, $^{\circ}\text{C}$
 T_{co} = T evaluated under initial steady-state conditions
 T_{in} = temperature of core inlet fluid, $^{\circ}\text{C}$
 T_{out} = temperature of core outlet fluid, $^{\circ}\text{C}$
 T_h = average temperature of fuel fluid within heat exchanger, $^{\circ}\text{C}$
 T_s = average temperature of shell side fluid of heat exchanger, $^{\circ}\text{C}$
 U = average velocity of fluid leaving reactor core through exit piping, ft/sec
 U_o = U evaluated under initial conditions, ft/sec
 v = $\frac{1}{\gamma} \frac{\partial k_e}{\partial \rho} A \frac{\rho_o}{V_c} (U - U_o)$, normalized deviation of velocity in core exit pipe, sec^{-2}
 v_s = velocity of sound in core fluid, ft/sec

$$= \sqrt{\frac{dp}{d\rho}} 144 g_c$$

 V_c = volume of fluid in reactor core, ft^3
 V_p = volume of pressurizing fluid in pressurizer, ft^3
 wC_p = fluid mass flow rate (lb/sec) times heat capacity of fluid, $\left(\frac{\text{kw-sec}}{\text{lb-}^{\circ}\text{C}}\right); \text{kw}/^{\circ}\text{C}$
 x = relative reactor power = $\frac{P}{P_o}$, dimensionless
 y = $\frac{S_c(T_c - T_{co})}{P_o}$ = normalized temperature rise of core fluid, sec
 z = $\frac{1}{\gamma} \frac{\partial k_e}{\partial \rho} (\rho - \rho_o)$ = normalized change in core fluid density, sec^{-1}

$$\alpha_1 = \frac{288 g_c a_f U_o}{\rho_o L} = \text{normalized friction coefficient, sec}^{-1}$$

$$= \frac{n_f U_o}{L}$$

$$\alpha_2 = \frac{hA}{S_s} \left(1 + \frac{S_s}{S_c + S_h}\right), \text{ sec}^{-1}$$

$$\alpha_3 = \frac{S_c}{S_c + S_h}, \text{ dimensionless}$$

$$\alpha_4 = \frac{hA_h}{S_s}, \text{ sec}^{-1}$$

$$\beta = \text{effective fraction of fission neutrons which are delayed, dimensionless, } \beta = \sum_1 \beta_1$$

$$\gamma = \frac{\beta}{\gamma}, \text{ sec}^{-1}$$

$$\gamma_3 = \text{conversion factor} = U_o A \frac{\rho_o}{V_c} \frac{\partial k_e}{\partial \rho}, \text{ ft}^{-1} \text{ sec}^{-1}$$

$$\delta = \text{instantaneous reactivity addition, dimensionless}$$

$$\epsilon = 2/t_r, \text{ where } t_r \text{ is the average residence time of a fluid particle in the core; sec}^{-1}$$

$$\lambda = \text{decay constant associated with average one group of delayed neutron emitters, sec}^{-1}$$

$$\xi = \frac{b}{\gamma}, \text{ sec}^{-1}$$

$$\rho = \text{average core fluid density, lb mass/ft}^3$$

$$\rho_o = \rho \text{ evaluated under initial steady-state conditions, lb mass/ft}^3$$

$$\gamma = \text{mean lifetime of prompt neutrons, sec}^{-1}$$

$$\omega_h^2 = \frac{A v_s^2}{V_c L} = \text{square of frequency of hydraulic system, sec}^{-2}$$

$$\omega_n^2 = \frac{1}{\gamma} \left| \frac{\partial k_e}{\partial T} \right| \frac{\rho_o}{S_c} = \text{square of frequency of nuclear system, sec}^{-2}$$

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