FOUNDATIONS OF S-MATRIX THEORY

APPENDIX A

THE PRAGMATIC INTERPRETATION OF QUANTUM THEORY[†]

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ABSTRACT

The pragmatic interpretation of quantum theory is defined. This refinement of the orthodox Copenhagen interpretation covers both the relativistic S-matrix form of quantum theory and also the ordinary nonrelativistic form. The connection of the pragmatic interpretation to the Copenhagen interpretation is described. Contrasts with several other interpretations are drawn.

NOTICE

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I. THE PRAGMATIC INTERPRETATION

Quantum theory is a theory by which scientists calculate the probabilities that observations of specified kinds occur in situations of specified kinds. These situations are such that each can be characterized as a preparation and subsequent observation of a physical system. The preparation is characterized by a set of specifications A , and the subsequent observation is characterized by a set of specifications B . These specifications A and B are formulated in a language that allows properly trained technicians to prepare systems that meet the specifications A , and to decide whether the subsequent observations meet the specifications B . These specifications A and B are mapped, by a procedure to be discussed later, onto density matrices ρ_A and ρ_B respectively. (These matrices are discussed in appendix B.) A dynamical assumption, in the form of a Schrodinger equation for nonrelativistic theory, or the S-matrix for relativistic theory, allows the difference between the time of preparation and the time of observation to be taken into account. The probability that an observation meeting specifications B occurs in situations meeting specifications A is then

$$P(A; B) = \text{Tr } \rho_A \rho_B, \quad (1)$$

where $\text{Tr } \rho_A \rho_B$ is the trace of the product $\rho_A \rho_B$. The probability $P(A; B)$ is interpreted as the predicted relative frequency that observations meeting specifications B occur in systems that are prepared in accordance with specifications A and examined

ed in accordance with specifications B .

The expression (1) for $P(A; B)$ can be converted, by a simple mathematical transformation described in appendix B, to an equivalent expression

$$P(A; B) = \int dp dq \rho_A(p, q) \rho_B(p, q) , \quad (2)$$

where p and q are the canonical variables of the classical analog of the observed system. Equation (2) is identical to the formula for $P(A; B)$ that arises from classical statistical mechanics. There $\rho_B(p, q)$ is the efficiency function associated with observation B. For example, if observation B corresponds to detecting a particle by a counter that detects with efficiency ϵ particles in the momentum interval Δp and in the coordinate interval Δq then expression (2) becomes

$$P(A; B) = \int_{\Delta p} dp \int_{\Delta q} dq \rho_A(p, q) \epsilon . \quad (2')$$

The logical set-up just described is identical to that by which classical mechanics would be used to make predictions about observations obtained under specified conditions. In classical physics there are experimental uncertainties associated with both the preparation and subsequent observation, and hence both of these must in principle be represented by probability density functions. The important difference between classical statistical mechanics and quantum theory is the uncertainty principle limitation. In

classical statistical mechanics the widths Δp and Δq of the probability functions can in principle be shrunk to zero. In this limit one obtains a deterministic result: for each pair (A,B) either $P(A; B) = 0$ or $P(A; B) = 1$. However, the mathematics of quantum theory entails the uncertainty principle limitation

$$\Delta p \Delta q > \hbar, \quad (3)$$

as is shown in appendix B.

To determine the probability $P(A; B)$ in either classical statistical mechanics or quantum theory it is necessary to determine the mappings $A \rightarrow \rho_A$ and $B \rightarrow \rho_B$. These mappings translate the language of the experimentalist into the language of the theorist. The construction of these mappings is an awkward task, because the two languages operate in different realms. Nevertheless classical physicists have learned how to perform these translations quite well, in the sense that predictions of the theory agree well with what is observed.

In quantum theory the translations $A \rightarrow \rho_A$ and $B \rightarrow \rho_B$ are more difficult to construct, because complex measuring devices involving unstable equilibrium are needed to effect the relevant observations. But quantum physicists have nevertheless also succeeded in calibrating their devices; they can often determine very accurately the probability functions of beams emerging from accelerators, and the efficiency functions of their counters. (See appendix B for more details.) It is by means of these empirical calibrations

that physicists transform the language of the theory into the language of actions and observations, and hence into a useful tool for man.

The variables p and q occur in quantum theory simply as the arguments of the functions $\rho_A(p, q)$ and $\rho_B(p, q)$: they are simply the parameters or free variables upon which the theory is constructed. All interpretation is via the probability formula (1) or (2).

In classical statistical mechanics the arguments p and q of the functions $\rho(p, q)$ are regarded also as properties of possible individual physical systems. In quantum theory, on the other hand, no description of individual physical systems is provided beyond that given by the statistical theory itself.

From the pragmatic viewpoint the statistical theory is the basic scientific theory, both classically and quantum mechanically. For it is the statistical theory that provides the experimentally testable predictions. Any proposed "deeper" theory that adds more assumptions or logical elements, but yields no further testable predictions is regarded as metaphysical, rather than scientific.

2. THE COPENHAGEN INTERPRETATION

The pragmatic interpretation of quantum theory is a refinement of the standard or orthodox Copenhagen interpretation propounded by Bohr and Heisenberg. However, the exact meaning of the Copenhagen interpretation is a subject of continuing controversy. This arises, at least in part, from the fact that Bohr and Heisenberg express themselves very differently and have developed their ideas differently over the years. In particular, some of the

later ideas of Heisenberg find no echo in the words of Bohr, and in fact seem to run counter to the trend of Bohr's thinking.

For these reasons the sum of the words of Bohr and Heisenberg may define no single coherent position. However, the logical essence of the common ground in their positions is summarized by the following two assertions:

(1) The quantum theoretical formalism is to be interpreted pragmatically.

(2) Quantum theory is the complete scientific theory of atomic phenomena.

The problem to be resolved by the Copenhagen interpretation is this: assertions (1) and (2) appear to be contradictory. The first says that quantum theory is fundamentally a set of rules by which scientists ~~make~~ predictions about what they will observe under specified conditions. Yet any such atomic theory seems manifestly incomplete. For the traditional concept of the complete scientific theory of atomic phenomena requires it to describe the atoms themselves, rather than merely supply rules by which scientists calculate predictions about their observations.

The fundamental question at issue is this: Can human ideas, which are presumably geared to the problem of human survival, ever become accurate copies or representations of the essences of external reality? Human thoughts are associated with human brains, and hence the structures that can occur in our ideas would naturally be expected to be reflections of the structure of these gross neurological organs. Yet the structure of the essences of external reality might be quite different.

$$\sum_{n=0}^{\nu\max} \left[\frac{e^{i\phi} + (\epsilon-1)}{\epsilon} \right]^n f_n = \sum_{\nu=0}^{\nu\max} p_{\nu} e^{i\nu\phi}.$$

Performing a binomial expansion of the expression in brackets, gives

$$\sum_{n=0}^{\nu\max} \frac{f_n}{\epsilon^n} \sum_{j=0}^n e^{ij\phi} (\epsilon-1)^{n-j} \binom{n}{j} = \sum_{\nu=0}^{\nu\max} p_{\nu} e^{i\nu\phi}.$$

Multiplying both sides by $e^{i\nu'\phi}$, where ν' is an integer, and integrating over ϕ from 0 to 2π gives

$$\sum_{n=0}^{\nu\max} \frac{f_n}{\epsilon^n} \sum_{j=0}^n (\epsilon-1)^{n-j} \binom{n}{j} 2\pi \delta_{\nu',j} = \sum_{\nu=0}^{\nu\max} p_{\nu} 2\pi \delta_{\nu\nu'}.$$

$$\sum_{n=\nu'}^{\nu\max} \frac{f_n}{\epsilon^n} (\epsilon-1)^{n-\nu'} \binom{n}{\nu'} = p_{\nu'},$$

and finally

$$p_{\nu'} = \sum_{n=\nu'}^{\nu\max} f_n (1/\epsilon)^{\nu'} (1-1/\epsilon)^{n-\nu'} \binom{n}{\nu'}.$$

ism amounts only to predictions, of determinate or statistical character, pertaining to individual phenomena appearing under conditions defined by classical physical concepts" (II.64). "... the formalism does not allow pictorial representation on accustomed lines, but aims directly at establishing relations between observations obtained under well-defined conditions" (II.71). "The sole aim of (the quantum mechanical formalism) is the comprehension of observations obtained under experimental conditions described by simple physical concepts" (II.90). "Strictly speaking, the mathematical formalism of quantum mechanics and electrodynamics merely offers rules of calculation for the deduction of expectations about observations obtained under well-defined experimental conditions specified by classical physical concepts" (III.60).

Bohr's insistence on the use of classical concepts in the description of the experimental conditions has raised the question of whether it is consistent to introduce into quantum theory, in this fundamental way, concepts that are basically incompatible with it. What must be recognized here is that the descriptions of the experimental conditions are basically operational directives to technicians: they instruct technicians how to prepare systems, and how to recognize whether subsequent observations fall into specified classes. Thus these descriptions are not really exact descriptions individual physical systems; rather they are operational directives that define classes of systems.

The essential logical requirement on the language used to formulate these directives is that it should provide for unambiguous communication: the instructions should mean the same thing to all

properly trained technicians, at least to the accuracy needed to make significant tests and useful applications. It is by virtue of this property of the language that the theory becomes "objective", in the sense that it makes no explicit reference to any individual observer, even though it is "subjective" in the sense that it deals primarily with human experiences. This pragmatic way of reconciling objective and subjective qualities resolves an old philosophic problem: it yields an objective scientific theory that is securely rooted in human experience.

The pragmatism used here is different from positivism. Positivism seeks to narrow the scope of philosophy by imposing rigid criteria of meaning, whereas the pragmatism used here seeks instead to broaden the scope of science by releasing it from the stricture that basic theory must represent nonideas by equivalent ideas.

The second essential part of the Copenhagen interpretation is the claim that quantum theory is complete. This claim must be interpreted pragmatically. Thus the claim is that no theory can make predictions about observations (on atomic systems) that go beyond those that can be made by quantum theory.

The argument is this. Because of experimental uncertainties, if nothing else, the problem is inherently statistical. The task, therefore, is not to predict exactly what will be observed, but rather to predict the probability that the observation will lie in a certain

range, i.e., meet certain specifications. The general statistical framework appropriate to this task is the same for quantum theory and classical statistical mechanics: the basic formula (2) is identical in the two theories. Thus if the quantum theoretical predictions are valid, then the only way to obtain a valid new prediction is to devise a preparation or observation that corresponds to a $\rho(p, q)$ that does not fulfill the quantum theoretical constraints (e.g., which violates the uncertainty principle limitations).

Einstein tried hard to devise an experimental set-up that, with due regard to the experimentally known diffraction effects, would correspond to a probability density function $\rho(p, q)$ not satisfying the uncertainty principle limitation. He failed, as have all others who tried. Thus unless someone can succeed where Einstein failed no valid predictions about observations that go beyond those of quantum theory can ever be obtained within the general framework that includes both quantum theory and classical statistical mechanics.

This argument is, of course, not a rigorous proof. Thus the Copenhagen claim of completeness must be regarded as an opinion based on massive studies, rather than a rigorously established fact.

One final remark: according to the Copenhagen interpretation the system that is prepared and later observed is not the whole universe. As Heisenberg says² "in natural science... we direct our attention to some part of the universe and make that the object of our studies. ... a large part of the universe, including ourselves, does not belong to the object." "...we always start with a division of the world into an object, which we are going to study, and the rest of the world." The quantum probabilities refer to

our observations, and hence to the "rest of the world".

3. SEVERAL OTHER INTERPRETATIONS

An alternative to the Copenhagen interpretation has its roots in von Neumann's theory of the process of measurement. According to this view the physical world itself is characterized by a wave function: i.e., the whole universe is represented by some "absolute" wave function.

This absolute- ψ interpretation leads to difficulties. The first is raised by the question "To what do the quantum probabilities refer?" In the Copenhagen interpretation the wave functions (or density functions) are functions of the degrees of freedom of some limited physical system, which is first prepared and then examined according to certain specifications, and the quantum probabilities refer to something outside this limited system, namely the observations of the scientists who are studying the system. But if the entire universe is represented by a wave function then the notions of preparations and observations according to operational directives lose their meanings, and it becomes unclear what the quantum probabilities refer to.

Put another way, the question is this: if, in the probability formula, ρ_A corresponds to the whole universe, then to what does ρ_B correspond?

To cope with this problem von Neumann³ suggests that the wave function must develop in time in two radically different ways. Ordinary it develops lawfully in accordance with the Schrodinger equation. However, whenever a measurement is made the wave func-

tion makes a sudden fitful jump to a new form. This idea has never been formulated in a precise way. One problem is to define exactly when a "measurement" takes place. Evidently "measurements" must be rigorously distinguished from other interactions, and the instants at which they occur defined exactly. This would presumably require a detailed theory that goes far beyond quantum theory. A second problem is that it is not possible to rigorously eliminate the interference terms between the parts of the wave function, that correspond to the different possible results of a measurement. Thus there is no natural way to define the precise form of the wave function after the quantum jump.

von Neumann links measurements to the subjective perception of the observer, and Wigner⁴ follows this line by suggesting that perhaps the sudden quantum jumps are effects of the action of mind on matter; i.e., that the whole universe itself actually undergoes a giant quantum jump whenever a human observer perceives a result of a measurement.

An alternative proposal is that of Everett⁵ and Wheeler⁶, who suggest that the universe is represented by a wave function that evolves always according to the Schrodinger equation; i.e., that undergoes no quantum jumps. In this case the universe would be continually splitting into parts corresponding to all of the various quantum possibilities; e.g., the atom would decay at every instant, and the observer would split into a corresponding continuum of observers, one for each instant of decay.

These varied proposals provide different mental images of the external world. However, none has provided a testable predic-

tion that goes beyond what is provided by the pragmatic Copenhagen interpretation. Nor are these alternative proposals recommended by logical or mathematical considerations. Indeed, the proposals are all based on the notion that one should interpret the wave function of quantum theory, which has the mathematical structure of a probability function, and hence naturally ought to be interpreted as such, as a representation of reality itself. This strange unnatural notion evidently arises from trying to interpret nonpragmatically the Copenhagen claim of the completeness of quantum theory.

FOOTNOTES AND REFERENCES

- * This work was supported by the U. S. Atomic Energy Commission.
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