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SUBJECT: The Calculation of Gamma Heating in Reactors of
Rectanguloid Geometry

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SUMMARY

A method of calculating heat generation due to gamma-ray absorption in structural members of reactors of rectanguloid geometry was devised. The calculated results checked experimental measurements in the Bulk Shielding Reactors to within $\pm 23\%$ for three different target metals. It is concluded that the model used represents the mechanisms involved to a fair approximation and should be adequate for design purposes.

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INTRODUCTION

One of the many problems associated with the design of nuclear reactors is determining the temperature distribution in the structural members outside the reactor core in order to estimate thermal stresses and cooling requirements. The temperature distribution is a function of the heat generation which is primarily dependent on the absorption of gamma rays escaping from the core. Calculation of the heat generation due to the absorption of monoenergetic gamma rays in homogeneous media of simple geometry is relatively easy. However, in more complex geometries composed of several materials with the energy spectrum of the gamma rays being of the continuous type, the calculation of gamma heating becomes quite involved and approaches the hopeless state when attempting to use rigorous methods. The attenuation of gamma rays in matter is actually quite complex and is accomplished by several mechanisms. A complete description of these attenuation processes and methods of calculation are given in articles by Fano,^(4,5) and a report by Goldstein and Wilkins.⁽⁷⁾

The object of this work was to develop a method of calculating gamma heating in a target material located outside the core of a reactor of rectanguloid geometry by utilizing a basically simple approach which is adaptable to calculation by the ORACLE. The criterion for judging the accuracy of the method is the comparison with the experimental data⁽¹⁾ for heat generation in target metals as measured in the Bulk Shielding Reactor (Swimming Pool type).⁽³⁾

DERIVATION OF BASIC EQUATIONS USED IN CALCULATION

Consider a monoenergetic point source radiating isotropically to points outside the reactor core. For exponential attenuation the fraction of source gammas striking a differential area at any point on a sphere with the source located at the center is

$$Se \frac{e^{-\mu_t s}}{A} \frac{(dA)}{A} \quad (1)$$

and the gamma current is

$$I_s = \frac{Se}{A_s} \frac{e^{-\mu_t s}}{(dA)} = \frac{Se}{4 \pi s^2} e^{-\mu_t s} \quad (2)$$

For a plane surface (see Fig. 1) parallel to a reactor face, the fraction of radiation "seen" by a differential area is decreased by $\cos \theta$.

To account for the contributions of the scattered gammas to heat generation in the target material located in the reflector, the current is multiplied by the energy buildup factor, which is a function of the number of relaxation lengths, $(\mu_t s)$. Therefore, the gamma current at any point on a plane parallel to the reactor face is

$$I_p = \frac{BSe}{4 \pi s^2} \frac{e^{-\mu_t s} \cos \theta}{\cos \theta} \quad (3)$$

Since the core and reflector are different materials, the product $Be^{-\mu_t s}$ is split into the core and reflector values. The final equation for the gamma current at the target material becomes

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$$I_P = \frac{S}{4\pi s^2} \left(B_c e^{-\mu_{tc} s_c} \right) \left(B_r e^{-\mu_{tr} s_r} \right) \cos \theta \quad (4)$$

This procedure is based on the assumptions that:

1. The partially attenuated beam of gammas leaving the core can be adequately expressed in terms of an equivalent gamma current of source energy by means of a buildup factor.
2. The gamma current leaving the core may be considered as the source term for the reflector.
3. The original direction is preserved in travel through the reflector.
4. The attenuation through the reflector is only a function of the reflector material and the original source energy.

The rate of heat generation in a unit mass of material exposed to an incident gamma current (see Figure 2) is

$$H = \frac{K E B_h I_P (1 - e^{-\mu_h x})}{\rho L} \quad (5)$$

$$\text{where } L = x \cos \theta \quad (6)$$

For thin slabs as $x \rightarrow 0$,

$$H = \frac{K E I_P \mu_h}{\rho \cos \theta} \quad (7)$$

which represents a maximum value for heat generation. Combining equations (3) and (5), the basic working equation for heat generation (contribution from only one point in the reactor core) in a target material located in the reflector becomes

$$H = \left(\frac{S K E B_c B_r e^{-\mu_{tc} s_c} e^{-\mu_{tr} s_r}}{4\pi s^2} \right) \left(\frac{\mu}{\rho} \right)_h \quad (8)$$

To obtain the total heat generation at any point, equation (8) must be applied to each element of volume (approximates a point) in the core for each energy group of gammas considered, and the individual contributions added up.

The calculation of the source term, buildup factors, and attenuation coefficients are described in the following sections.

SOURCE TERM

The number of gammas/sec generated from a point in the reactor is proportional to the thermal flux at that point since all significant gammas are produced as the result of thermal neutron capture.

Individual sources of the gamma spectrum are:

1. prompt fission gammas
2. fission product decay gammas
3. capture gammas from uranium
4. capture gammas from water
5. capture and decay gammas from aluminum

Combining the five methods of gamma production, the source term for each gamma energy group at a point or for a very small volume increment is

$$S = \phi(\Delta V) \left[N(p) \Sigma_p(U) + N(f.p.) \Sigma_{f.p.}(U) + N(C) \Sigma_C(U) + N(A) \Sigma(A) + N(W) \Sigma(W) \right] \quad (9)$$

Fission and Capture Gammas from Uranium

The first three methods of gamma production listed above create an energy spectrum of the continuous type. For use in the calculation the spectrum must be divided into convenient energy groups. The spectra for the prompt and fission product gammas have been measured experimentally, (6,10) and the spectrum for the capture gammas estimated. These spectra can be represented quite well by an equation of the form

$$\frac{dN}{dE} = A e^{-nE} \quad (10)$$

Integration of equation (10) over an energy range would give the number of gammas released per neutron capture at the true average energy of the range or group. For convenience, however, an equivalent number at the arithmetic mean of the energy interval was used and calculated from the following equation

$$N \left(\frac{E_1 + E_2}{2} \right) = a \int_{E_1}^{E_2} E e^{-nE} dE \quad (11)$$

and

$$N = \frac{2a}{(E_1 + E_2) n^2} \left[e^{-nE_1} (nE_1 + 1) - e^{-nE_2} (nE_2 + 1) \right] \quad (12)$$

where

$a = 10$ and $n = 1.4$ for fission product gammas,

$a = 7.7$ and $n = 1.03$ for prompt gammas, and

$a = 8.0$ and $n = 1.03$ for uranium capture gammas

Using equation (12) and the listed values of a and n , the equivalent numbers of gammas released in each of the six energy groups were calculated and shown in Table I, Appendix I.

Aluminum

The gamma emission spectrum for aluminum is partially given by Mittelman and Liedtke.⁽⁹⁾ Approximately 2 photons per capture are emitted as capture gammas and one 1.8 mev gamma from decay of Al-28. Only 1.46 capture gammas are accounted for; the remaining 0.54 apparently occur in the range of 1 to 3 mev. This was split between the 1.5 and 2.5 mev groups. The 1.8 mev decay gamma was included in the 1.5 mev group and the number increased by 1.8/1.5.

Water

Thermal neutron capture in water gives rise to a significant gamma of only one energy, 2.23 mev,⁽⁹⁾ which results from capture by hydrogen. This was included in the 2.5 mev group with the number decreased by 2.23/2.5.

Thermal Flux

The thermal flux used in the source term was based on the flux distribution measurements⁽⁸⁾ (see Appendix I) in the Bulk Shielding Facility for loading 33, the loading used for the heat generation determinations. Since flux measurements were made in only half the reactor, symmetry about the north-south axis of the reactor was assumed.

The experimental flux measurements were relatively few in number compared to the volume elements used (1485). By plots and cross plots of the data, the average thermal flux for each element of volume was determined.

Geometry and Composition

The Bulk Shielding Facility is composed of MTR type fuel elements (3 x 3 x 24 in.) in an aluminum lattice which is surrounded by a large volume of water. The active portion measures 18 x 15 x 24 inches (30 fuel elements). A more detailed description has been given elsewhere.⁽³⁾

For calculation purposes the lattice was considered a homogeneous mixture and composed of 1485 (11 x 9 x 15) volume elements, which results in approximately cubical elements with a volume of 71 c.c. For details on composition and cross sections, see Appendix I.

ATTENUATION COEFFICIENTS AND BUILDUP FACTORS

All attenuation coefficients and Buildup factor data used were those reported by or calculated by methods recommended by Blizard,⁽²⁾ which were based on reports by Fano,^(4,5) and by Goldstein and Wilkins.⁽⁷⁾ See Appendix IV for the values used.

Attenuation Coefficients

The linear attenuation coefficients for H_2O as a function of photon energy are listed in the Tables. The reactor core, however, is a mixture of uranium, aluminum and water and the average value was computed from the individual values. For each energy group, assuming that the core is a homogeneous mixture,

$$\mu_c = \sum_i \left(\frac{\mu}{\rho} \right)_i \rho_i \quad (13)$$

where ρ_i is the partial density of component i , the total mass of component i divided by the reactor core volume.

Buildup Factors

Buildup factors for the core were calculated by averaging the individual values on a mole per cent basis. This seems to be a reasonable procedure since buildup factors for elements are smooth functions of the atomic number.

For calculation purposes it was necessary to fit an equation to the tabulated buildup factor data as function of the number of mean free paths, μs . A very close fit was obtained with the polynomial,

$$B = 1 + a (\mu s) + b (\mu s)^2 + c (\mu s)^3 \quad (14)$$

NOTES ON THE CALCULATION ROUTINE

Numerical evaluation of the problem by hand would be straightforward, but tedious and lengthy requiring about one man-year of effort to calculate the heat generation at one point. The ORACLE took about 4 minutes to compute the contribution from one energy group, and thus 24 minutes for the complete calculation since 6 energy groups were used. The computing time could be reduced by about 50% by eliminating a section of the calculation which determines the particular core face a beam of gamma radiation will traverse. This particular part of the code is required only in calculating heat generation at points in the reflector that do not fall within the projection of the core faces.

A detailed description of the ORACLE code used, the conditions of applicability and method of use will be given in a future memorandum. The code is the "fixed point" type. Practical reasons limit the possible volume elements or source points to a maximum of 3375 (1485 used in this work). No real limit exists on the number of energy groups or methods of producing gamma rays in the core which are functions of the flux.

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RESULTS AND DISCUSSION

All results are for locations along a line perpendicular to the center of the north face of the Bulk Shielding Reactor. The calculated results are shown plotted in Figure 3 along with the experimental measurements of Binford et al⁽¹⁾ for three different metals as target materials. At the core face to about 6 in. away, all calculated and observed heat generation rates agree to within $\pm 23\%$; the lead is about 23% high, the iron 20% low and the aluminum varies from 4% low to 6% high. Both experimental and calculated results approximate a simple exponential function (straight line on semi-log paper); however, the calculated curve has a slight concavity upwards and the experimental curve a slight concavity downwards. This increases the error in the lead results to 39% at 9-in out from the core and apparently would raise the calculated value for iron higher than the experimental for equal and greater distances from the core.

Three controllable sources of error exist in the method of calculation aside from the uncontrollable errors resulting from inadequacies in the assumed model and possible errors in the fundamental parameters such as the number of gammas liberated per capture or fission, thermal flux, etc. These controllable error sources are: 1) insufficient number of energy groups; 2) insufficient number of volume elements; and 3) heat generation due to phenomena other than absorption of core gammas.

Obviously, errors due to the first two items can be virtually eliminated by using a large number of energy groups and volume elements of very small size. Practical considerations limit the number, however.

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The individual contributions from each energy group to the heat generation are shown in Table I. It is apparent that the main contributions come from the lower energy groups, and for lead about half of the total comes from the first energy group; the individual contributions for aluminum and iron are somewhat more evenly prorated. It does appear that a finer division of the low energy groups would be preferable and a coarser division in higher energy groups permissible. It does not appear, however, that significant changes in the results could occur by increasing the number of energy groups.

The obvious method of deciding whether sufficiently small volume elements were used is to repeat the calculation for a different size element and compare the results. The procedure used, however, was to consider the radiation emanating from an elemental volume to originate at some mean distance, $\bar{r}$, from the center and thus approximately correct for the presence of a uniform source instead of a point source as assumed in the derivation. For the approximately cubical volume elements about 4 cm on edge, $\bar{r} = 1$ was used. The difference in the calculated heat generation at the core face for $\bar{r} = 1$ and $\bar{r} = 0$ was only 4%, which is a good indication that sufficient volume elements were used.

Other possible methods of heat generation in the target metal not accounted for in the calculation method include: inelastic scattering gammas from the core, beta rays generated in the target, capture gammas generated in the target, capture gammas from the water surrounding the core, and slowing down of neutrons in the target. It is possible to estimate these contributions by a separate calculation fairly accurately if the flux distribution at the target is known. The contributions of the various mechanisms listed above were estimated and found to be insignificant compared to heating by the core gammas.

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TABLE I
Heat Generation Contribution* of Each Energy Group

Target Metal	Heat generation, watts/gm per Mw of Power					
	0.5	1.5	2.5	4.0	6.0	8.0
Pb	0.216	0.076	0.048	0.034	0.009	0.007
Al	0.0559	0.0598	0.0362	0.0207	0.0043	0.0033
Fe	0.0588	0.0581	0.0370	0.0232	0.0053	0.0043

*At core face

CONCLUSIONS

In view of the generally good agreement with experimental measurements, it seems reasonable to conclude that the model used represents the mechanism to a fair approximation and should be adequate for design purposes.

The same technique could be applied to one- and two-region homogeneous reactors of both spherical and cylindrical geometry.

FIGURE 1

FIGURE 2

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Fig. 3. Gamma Heating in Target Metals.

NOMENCLATURE

A	Area, cm^2
B	Buildup factor, dimensionless
E	Energy of incident gamma ray, mev
H	Heat generation, watt/gm
I	Gamma current, photons/ cm^2 sec
K	Conversion constant, 1.6×10^{-13} watt-sec/mev
L	Perpendicular distance into target, cm
N	Number of gammas/capture or fission
S	Source, number of gamma photons
V	Elemental volume, cm^3
a,b,c	Coefficients in Buildup factor equations, dimensionless
s	Length of path of travel of gamma ray, cm
x	Slant distance into target, cm
θ	Angle of gamma path with perpendicular, degrees
μ	Linear attenuation coefficient, cm^{-1}
ρ	Density, gm/c.c.
Σ	Macroscopic cross section, cm^{-1}
ϕ	Neutron flux, neutrons/ cm^2 sec

Subscripts:

c	Core
f	Fission
h	Target, energy attenuation coefficient
p	Plane
r	Reflector
s	Sphere
t	Total
C	Radiative capture

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NOMENCLATURE (Cont'd)

Parentetical Symbols:

A	Aluminum
U	Uranium
W	Water
f.p.	Fission product
p	Prompt

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APPENDIX I

SOURCE TERM DATAGamma Production Data

The equivalent number of gammas produced per fission or capture used in the calculations were computed by equation (12) and are shown below in Table II.

TABLE II

Equivalent Number of Gammas for Each Energy Group

Energy, mev	Gammas/fission or capture				
	Fission Product	Prompt	Uranium Capture	Aluminum Cap. and decay	Hydrogen
0.5	4.162	3.976	4.131	0	0
1.5	1.228	1.627	1.691	1.54	0
2.5	0.312	0.592	0.615	0.33	0.89
4.0	0.090	0.272	0.283	0.77	0
6.0	0.0057	0.0360	0.0374	0.21	0
8.0	0.00035	0.0046	0.0048	0.35	0

Thermal Flux

The thermal fluxes used in the calculations for each volume element were obtained from plots and cross plots of the flux data taken for loading 33 in the BSR. The measured fluxes as listed in Table III are for a power of 27.36 kilowatts and a loading of 3636 gm of U-235 of 90+ % enrichment. Since flux is directly proportional to power, the

values in Table III were put on a one-megawatt basis for use in the calculation by multiplying by $1000/27.36$.

The following diagrammatic cross-section view of the BSR identifies by lattice position number the active fuel elements used in loading 33.

NORTH
↑

3	4	5	6	7
13	14	15	16	17
23	24	25	26	27
33	34	35	36	37
43	44	45	46	47
	54	55	56	

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TABLE III

Flux Distribution in BSR for Loading 33

Lattice position	Face	Flux at Vertical Position, $\phi \times 10^{-11}$				
		+ 12"*	+ 6"*	Cen.	-6"	-12"
5	East	1.29	1.46	2.00	1.64	1.40
6	East	0.94	1.30	1.69	1.26	1.08
7	East	0.66	1.18	1.55	1.27	0.84
15	East	1.18	1.57	2.17	1.84	1.43
17	West	0.82	1.60	2.14	1.81	1.30
17	East	0.84	1.56	2.11	1.70	1.03
25	Center	1.35	2.18	3.35*	2.49*	2.06*
25	East	1.18	2.34	3.25	2.75	2.13
27	West	1.06	2.06	2.77	2.29	1.75
27	East	0.97	1.69	2.42*	1.86*	1.22*
36	West	1.50	2.31	3.03	2.58	1.83
36	East	1.16	1.80	2.50	2.15	1.48
37	East	1.12	1.81	2.38	1.93	1.26
45	East	1.53	2.37	3.03	2.55	1.86
46	East	1.18	1.48	1.97	1.62	1.25
47	East	0.88	1.41	1.82	1.50	1.07
54	West	0.68	1.23	2.02*	-	-
55	East	1.18	1.50	1.87*		

* Estimated from the fast flux measurement

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Geometry and Composition

Total volume of core = 99,120 c.c.
Vol. of Al to vol. of H_2O = 0.72
Mass of Uranium = 3,900 gm
Mass of Aluminum = 111,800 gm
Mass of Water = 57,500 gm
Core uranium density = 0.0392 gm/c.c.
Core Aluminum density = 1.128
Core Water density = 0.580
Average temperature of core = 30°C
Uranium fission cross section = 510 b
Uranium capture cross section = 94.1 b
Aluminum capture cross section = 0.202 b
Hydrogen capture cross section = 0.290 b

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APPENDIX II

ATTENUATION COEFFICIENTS AND BUILDUP FACTORS

TABLE IV

Attenuation Coefficients for Each Energy Group

Energy, mev.	Total linear atten. coeff., cm^{-1}		Energy abs. mass atten. coeff., cm^2/g		
	Core μ_{tc}	Reflec., μ_{tr}	Target metals - $(\mu/\rho)_h$		
			Pb	Al	Fe
0.5	0.1586	0.0967	0.107	0.0291	0.0306
1.5	0.0920	0.0576	0.0316	0.0248	0.0241
2.5	0.0701	0.0430	0.0294	0.0221	0.0226
4.0	0.0564	0.0339	0.0326	0.0200	0.0224
6.0	0.0479	0.0277	0.0383	0.0190	0.0232
8.0	0.0434	0.0242	0.0423	0.0185	0.0242

TABLE V

Coefficients for Buildup Factor Equation (14)

Energy, mev.	Core			Reflector		
	a	b	c	a	b	c
0.5	1.113	0.4300	-0.0033	0.9966	0.4426	0.0208
1.5	0.7750	0.1825	-0.0175	0.8240	0.1310	-0.0054
2.5	0.8100	-0.0300	-0.0100	0.7210	0.0537	-0.0046
4.0	0.5542	0.00625	-0.000417	0.5910	-0.00125	0.00042
6.0	0.4667	-0.0200	0.00333	0.4833	-0.0275	0.00417
8.0	0.3475	-0.00875	0.00125	0.3917	-0.0125	0.00083

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