EXPERIMENTAL DETERMINATION OF THE
CRITICAL POINT AND HIGH TEMPERATURE
SATURATED PHASE DENSITIES OF RUBIDIUM

by

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MASTER

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ABSTRACT

A cylindrical capsule partially filled with rubidium metal hangs from transverse knife edges in the plane normal to the axis of the capsule at its center. The equilibrium rest angle of the capsule is measured at various temperatures until it reaches the horizontal position, at which the rubidium inside the capsule is in one phase.

The rest angle at a given temperature is a function of the saturated vapor and liquid densities, and is solved simultaneously with the rest angle of another capsule containing a different mass of rubidium metal, at, or interpolated to, the same temperature.

The saturated phase densities are obtained, with slight extrapolation, up to the critical point. The critical temperature was obtained as $3331^{\circ}\text{F} \pm 10^{\circ}\text{F}$ and the critical density as $0.347 \text{ gm/cc} \pm 0.5\%$. Combined with the critical pressure of 1943 psia obtained previously in this laboratory, the critical compressibility factor is calculated as 0.189 ± 0.001 .

All computations were done with the IBM 360 digital computer of the Computing Center and the System Engineering Laboratories 810B computer of the Nuclear Science and Engineering Division of Columbia University.

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I. INTRODUCTION

Liquid metals have long been considered as coolant fluids in nuclear reactors, owing to their excellent heat transfer characteristics. Furthermore, in parallel with the tremendous increases in reactor temperatures, interest in liquid metals has equally increased as energy conversion fluids.

Specifically, the efficiencies of the possible power cycles increase the higher their maximum operating temperature and the lower their heat rejection temperature. However, the higher the heat rejection temperature the smaller and lighter is the optimum condenser or cooler, an important feature in space flight applications. Thus, such systems demand thermally stable working fluids with high heat transfer rates at high temperatures and pressures. In general, the lower the density of the fluids, the lighter the whole package, underlining the importance of the alkali metals. For the extremely important Liquid Metal Fast Breeder Reactor program sodium has already been adopted, for these various reasons, including low cost.

In the process design of such systems and in the detailed design of the plant equipment, it is essential to have accurate and reliable thermophysical properties, frequently not available at the higher temperature ranges of interest. In hazard analysis of these plants, and in research and design in plasmas and in fusion, reactions and power, properties must be estimated at very high temperatures and/or pressures. For all these cases it is very helpful, if not indispensable, to have reliable high temperature data, and in particular information as to the critical point properties, since then one can deduce a great deal of information utilizing the theory of corresponding states.

II. CRITICAL TEMPERATURE AND PHASE DENSITIES

The high temperature and the critical point properties of many substances have been measured, discussed, analyzed, and correlated for many years, developing a large number of semi-empirical and semi-theoretical procedures for verifying, interpreting and predicting such properties.

The present study is of the high temperature densities of the saturated vapor and liquid phases, up to the critical point, for the alkali metal rubidium, including the identification of the critical temperature and density. Accordingly, available literature methods for the measurement, correlation and prediction of these properties will be reviewed here, in view of their applicability in handling the result of this study.

Eyring(17) studied the concept of "free volume" in the saturated vapor phase, evaluating it at 3 % through compressibility data. He concluded that along an increase in saturation temperature, some active molecules in the liquid phase cross the interface boundary and fill this free volume, in order to stabilize the free energy of the system. Consequently they leave a molecular void in the liquid.

As the substance approaches its critical point, the "free volume" of molecular void in the liquid phase increases rapidly and finally equalizes with that of the vapor phase at the critical point. Thus, the vapor and liquid densities will be equal at that point.

Fisher(18) explains that below the critical temperature the vapor phase condenses at constant pressure to a liquid with a greater density, and that this density difference gets smaller as the temperature approaches the critical, finally disappearing altogether. When a gas is compressed at a temperature above the

critical temperature, transition from gas to liquid state can be achieved without discontinuity.

In fact, any of the derivatives of the free energy experiences the same transition without discontinuity. Thus, he. cited, one can observe a maximum in the isothermal compressibility on passing over the critical point, and this maximum will be close to the critical density. According to his estimation for a small hard-core with a weak attractive long range tail, the critical density is one-third of maximum density. If one lowers the temperature at constant critical density, the compressibility increases rapidly as it approaches critical temperature. Similar study by Egelstaff(12) defined the vapor-liquid critical point such that as temperature approaches the critical value,

$$\left(\frac{\partial P}{\partial V}\right)_T \rightarrow 0 \quad (1)$$

Since compressibility K_T is defined as;

$$K_T = -\frac{1}{V} \left(\frac{\partial V}{\partial P}\right)_T = \frac{1}{P} \left(\frac{\partial P}{\partial V}\right)_T \quad (2)$$

$$K_T \rightarrow \infty$$

as T approaches T_c .

Buckingham (20) has discussed that a singular entropy surface is consistent with a logarithmic singularity of the heat capacity (C_v), and that the critical transition (" λ -transition") of C_v at a coexistence point implies that,

$$\frac{X^2}{1 - \ln X} = A \left(1 - \frac{T}{T_c}\right) \quad (3)$$

where,

$$X = (\rho_L - \rho_v) / (\rho_L + \rho_v)$$

Relations involving the saturated phase densities

However, Guggenheim(23) discovered that for simple fluids, a simple power law governs the coexistence curve, namely, the "one-third law", with $\beta = 1/3$ in the following form:

$$\rho_L - \rho_V = A \left(1 - \frac{T}{T_c}\right)^\beta \quad (4)$$

This has been proved to be true for Xenon(45). However, the Ising model theory (18) of a lattice gas predicts $\beta = 5/16$. Also, Dillon and others (10) showed that for cesium β is 0.4 for the vapor and 0.5 to 0.7 for the liquid, with additional terms:

$$\frac{\rho}{\rho_c} = 1 + A(1 + T_r) \pm B(1 - T_r)^\beta \quad (5)$$

This agrees with Partington(36), who stated that the density difference between liquid and vapor is related to the latent heat of the transition, and that the transition latent heat vanishes at the critical point roughly as $|T - T_c|^{0.4}$.

Refining the exponent, Egelstaff (12) stated that generally the power law holds with β in the range of 0.33 - 0.36 for non-polar liquids and 0.45 for liquid metals. Based on the intrinsic symmetry of the thermodynamic system for fluids in the critical region, Piza (42) quoted the approximate expansion of Landau and Lifshitz (31) for $-(\partial p / \partial V)_T$, and by omitting higher order terms he obtained,

$$\rho_L - \rho_c = \rho_c - \rho_V = \left[-\frac{1}{\beta} (T - T_c) \right]^{\frac{1}{2}} \quad (6)$$

where, α and β are constants. According to the above equation, if $(\rho_c - \rho_v)^2$ or $(\rho_L - \rho_c)^2$ is plotted against $(1 - T_r)$ as abscissa, the resulting curve will be a straight line with slope of $3\alpha/\beta$, and particularly straight near the critical point.

A study on Helium reported by Tisza (42) indicated that the above straight line actually has a finite intercept on the abscissa, instead of going through the origin. This indicates flattening of the coexistence curve close to the critical point or measurement error in the temperature or critical temperature. Also, the above equation implies that the rectilinear equation is flat near the critical region.

The "rectilinear diameter" concept, originally claimed by Cailletet and Mathias (9), stated that the average of the saturated liquid and vapor densities is linear in temperature. This was particularly demonstrated by a study of sulfuric acid which they quoted. However, higher order terms of Landau and Lifshitz expansion are generally not negligible, and the following general form is widely recommended (9,18,33,35,36):

$$RD = \frac{\rho_L + \rho_v}{2} = A + B T + C T^2 + \dots \quad (7)$$

However, if the rectilinear diameter, defined as average density of the saturated vapor and liquid at a given temperature, is a smooth function of temperature, one can derive a simple equation,

$$|\rho - RD| = (\rho_L - \rho_v)/2 = c (T - T_c)^{1/2} \quad (8)$$

Details of this derivation are given in appendix. If the average density is indeed a smooth function of temperature, and is given by the equation:

$$RD = \rho_c + A (T_c - T) + B (T_c - T)^2 \quad (9)$$

where, at the critical temperature, RD, liquid and vapor densities equal to the critical density as expected. If this equation is substituted into the equation (8), then, liquid or vapor density can be written as,

$$\rho = \rho_c \pm C (T_c - T)^{\frac{1}{2}} + A (T_c - T) + B (T_c - T)^2 \quad (10)$$

and "+" sign represents for the liquid density. Therefore, if the equilibrium densities are known, a simple extrapolation would give the critical temperature.

Kordes (30) suggested that $\log(1/\bar{v}/\bar{\rho})$ is linear with $\log T$, and at the critical point the ratio $(1/\bar{v})/\bar{\rho}$ will be unity.

To present vapor density it is necessary to have a proper equation of state in order to find a corresponding state of a substance under a given pressure and temperature (12,27,29).

Alkali liquid metals are generally not too difficult to handle if air and moisture are kept out with proper containment materials. However, any slight exposure to air results in a oxide film on the exposed surface. With larger exposures one may expect violent reaction with the moisture and oxygen in air. Consequently, any experimental work involving alkali metals requires articulate method of precaution, and this has been a major bottle neck to obtain a reliable data at high temperature. Although mercury has opposite properties, and easier to handle, its corresponding thermodynamic state is quite similar to that of the alkali metals. Therefore, mercury data are often used to assist in interpolating thermophysical properties of alkali metals at high temperature.

Corresponding States

According to the principle of corresponding states (12,34,43,46), the state of gas for all substances is a universal function of reduced properties in the form of compressibility, assuming: first, the effect of quantization of the translational degrees of freedom is negligible, and thus, classical statistical mechanics hold. Furthermore, molecules are spherically symmetric allowing free rotation. Also, intermolecular degrees of freedom are independent of density, and consequently, intermolecular vibrations are same in the liquid and gas state. In addition to above, the intermolecular potential can be expressed as two parameters, pairwise additive functions, and the potential energy is a function of intermolecular distance only.

Certainly, alkali metals do not satisfy spherical symmetry but it can be assumed that they are effectively symmetrical by virtue of their free rotation. Also, their intermolecular potentials can not be explained by mere pairwise force, only a function of intermolecular distance. Also, the presence of polymers prevents effective analysis of data in any simple manner, preventing any neat universal function for alkali metals.

Hougen and others (27,32) introduced an additional parameter of critical compressibility into the universal functions, and the "corresponding state" can be written as;

$$Z = f(P_r, T_r, Z_c) \quad (11)$$

where,

$$Z_c = \frac{P_c V_c}{R T_c}$$

Accepting that alkali metals are effectively spherically symmetrical, by virtue of the free rotation of their molecules, the principle of corresponding states holds within the alkali metal group, the virial equation of state (27,29,31,32) and the compressibility factor can be written as follows:

$$\frac{PV}{RT} = Z = 1 + B \rho_v + C \rho_v^2 + D \rho_v^3 + E \rho_v^4 \quad (12)$$

where, B,C,D, and E are functions of temperature and were called second, third, etc., "virial coefficients". Comparing this equation with the equation of state by van der Waals (29,43),

$$P = \frac{RT}{V-b} - \frac{a}{V^2} = \frac{RT}{V} \left(1 + \frac{b}{V} + \frac{b^2}{V^2} + \dots \right) - \frac{a}{V^2} \quad (13)$$

At low densities, higher order terms of van der Waals equation will be negligible, and the second virial coefficient B can be written as:

$$B = b - \frac{a}{RT}$$

and B should therefore approach a constant value equal to b at high temperature.

The critical point properties of the alkali metals

Most of the critical data (1,2,10,11,21,41) are obtained either by data deduction, or by extrapolation of relatively low temperature data. Thus such data can't be expected to be accurate, particularly since these properties change rapidly as temperature approaches its critical value. The experimental and predicted critical values of alkali metals are summarized in table I.

Table I. Prior Critical Property Determinations for Liquid Metals

<u>metal</u>	<u>T_c, F</u>	<u>P_c, psia</u>	<u>ρ_c, g/cc</u>	<u>Z_c**</u>	<u>reference & comment</u>
Hg	2660	23329	5.07		(6,37)
Li	5342	9996	0.12	0.149	(10), <1200°C
Na	4172	5145	0.206	0.185	(33), <1930°C
K	3633*	2380*	0.194	0.175	(28*,33)
Rb*	3313	1943	0.349	0.188	(25,39)
Cs*	3231	1700	0.406	0.225	(25,39) <3219°F

* These values have been directly determined experimentally

** Z_c values are calculated from T_c, P_c and ρ_c quoted

Table II. Experimental Critical Properties of Cesium, determined in the Liquid Metals Research Laboratory, Columbia University.

<u>Investigator</u>	<u>T_c, °F</u>	<u>P_c, psia</u>	<u>ρ_c, g/cc</u>
Hochman* (25)	3240 ± 25	-	0.45
Silver (39)	3235 ± 4	1700 ± 6	-
Oster (35)	3227 ± 8	-	0.406

* These data were obtained by short extrapolation

Table II is summary of three independent investigations of cesium critical properties, carried out in this laboratory.

Table III shows that the rubidium critical temperature estimates are widely scattered, ranging from 3630°F to 2820°F depending on the method of evaluation.

Generally, these values are obtained by extrapolating the limited data at relatively low temperatures, with the exception of Dillon (10), who measured the saturated phase densities indirectly using an internal gamma radiation technique up to the critical region.

Table III. Critical Properties of Rubidium
(mostly estimates or averages
of estimates)

Reference	$T_c, ^\circ F$	$\rho_c, \text{gm/cc}$	P_c, psia	Z_c
Gross(21)	3472	0.32	-	-
ANL(33)	3436 $\pm$ 70	0.3314	2625.4	0.2594
Achener(1)	3290 $\pm$ 27	0.347 $\pm$ 0.006	2646 $\pm$ 300	-
Watson(43)	2820	-	-	-
Thodos(19)	3630	0.144	2240	-
Dillon(10)	3308 $\pm$ 63	0.346 $\pm$ 0.009	-	0.22
Hochman(26)	3340 $\pm$ 90	0.349**	-	-
Bhise(5)	3313*	0.348**	1942.9*	0.188**

* Direct experimental values

** Assuming linearity of average phase densities with temperature, obtained by author.

However, Dillon's data are not only widely scattered but also tend to give high vapor densities and low liquid densities, even at low temperatures, as observed with cesium(35).

Computation of Derived Properties

Once the equation of state is determined, other related properties of thermodynamics can be calculated. The heat of vaporization is given by the Clapeyron equation:

$$\Delta H_v = T \left(\frac{1}{T_v} - \frac{1}{T_L} \right) \left[\left(\frac{dP}{dT} \right)_v \frac{1}{P} \right] P \quad (14)$$

For this equation, the vapor pressure of rubidium previously measured in this laboratory (5) is available. The data were fitted with the equation:

$$\log_{10} P = A + \frac{B}{T} + C \log_{10} T + D T \quad (15)$$

with the coefficients of,

$$A = -114.771$$

$$B = 3558.75$$

$$C = 16.2416$$

$$D = -0.0032759$$

where, P is in psia and T in Kelvin degrees.

Also, the entropy and internal energy of vaporization are obtainable by,

$$\Delta S_v = \frac{H_v}{T} = \left(\frac{1}{T_v} - \frac{1}{T_L} \right) \left[\frac{1}{P} \left(\frac{dP}{dT} \right)_v \right] P \quad (16)$$

$$\Delta U_v = \Delta H_v - P \Delta V \quad (17)$$

Differentiating the vapor pressure equation, and substituting into the above,

$$\Delta H_v = \left(-\frac{B}{T} + C + D T \right) \left(\frac{1}{\rho_v} - \frac{1}{\rho_L} \right) P \quad (18)$$

$$\Delta S_v = \left(-\frac{B^2}{T^2} + \frac{C}{T} + D \right) \left(\frac{1}{\rho_v} - \frac{1}{\rho_L} \right) P \quad (19)$$

$$\Delta U_v = \left(\frac{1}{\rho_v} - \frac{1}{\rho_L} \right) \left[-\frac{B}{T} + C + D T + 10 \left(A + \frac{B}{T} + C \ln T + D T \right) \right] P \quad (20)$$

III. Phase Density and Critical Temperature Determination

Previous Methods

There are several possible methods of measuring the saturated phase densities of liquid metals at high temperatures, potentially up to the critical point.

The most common method is to locate the critical point by observing P-V-T space and tracing liquid-vapor equilibrium densities until the densities of the vapor and liquid form a continuous envelope, with the distinction between the two phases just disappearing. This method is applicable to any substance, but with liquid metals the experimental technique is particularly sophisticated and expensive, requiring refractory metal receivers of known volume with an elastic wall to measure the vapor pressure by measuring the balancing external inert gas pressure (14,16).

Dilatometric (41) and Pycnometer (2,38) methods are commonly used but only at relatively low temperatures, due to the difficulties of constructing proper apparatus at high temperatures.

Another method (3,4,10,11) has recently been applied to Na, K, Rb, and Cs to their critical regions. A fairly large cylindrical capsule of known volume is made of a refractory metal and partially charged with a known amount of the alkali metal. It is then irradiated with neutrons, inserted vertical in a high temperature radiation-shielded furnace, and the gamma radiation issuing through a low-level port and a high-level port counted at a convenient energy, corresponding to a convenient gamma activation. The ratio of the liquid and vapor phase densities is directly obtained, and the ratio with a low temperature density, but there are corrections for self-absorption, activity decay, etc., and evidently uncertainty as to where the liquid level is.

The reported results have shown large errors and inconsistencies, such as even a vapor density higher than the liquid density at the same temperature, and equal vapor and liquid densities at temperature well in the two-phase region according to the results from this laboratory. Coincidentally, however, the rubidium results by this method (10) are the most internally consistent of the four alkali metals tested.

Another, fairly direct method (25,26) has been developed in this laboratory. The level of the liquid metal in a sealed, partially filled, vertical cylindrical capsule is measured as a function of temperature at elevated temperatures. The capsule is made thin-walled, and preferably of a refractory metal alloy of low electrical conductivity. The liquid level corresponds to the break in a lengthwise IR plot, and repeating the test with a second capsule, or additional capsules, with a different mass of total charge yields two, or more, simultaneous equations, with which the unknown phase densities are calculated. Due to the decreasing electrical conductivity of the alkali metal as the temperature is raised, the break could only be pinpointed up to about 150° F below the critical temperature. On the other hand, additional valuable information is obtained from the experiments, in the form of the electrical conductivity of the alkali metal, useful for liquid structure theory.

Tilting Capsule Method

This new technique was devised by C. F. Bonilla, and first used in the dissertation of G. F. Oster (35). The objective was, similar to that by Bond(7), that this method would give a mechanical response dependent on the equilibrium phase densities all the way to the critical point, rather than only up to about 95 % of T_c , the range of the previous method.

The first new concept studied employed a vertical capsule swung as a pendulum, the period of which would depend on the location of the center of gravity, and would be measured electronically from outside of the furnace with great accuracy. However, the tilting capsule method is chosen over the dynamic pendulum method because of the design problems associated with the imprecise period caused by windage losses at high pressure and temperature.

A cylindrical capsule of refractory metal hangs by a transverse in the midplane normal to the axis, about one diameter away from the axis. Due to the difference in densities and volumes of the vapor and liquid, the horizontal moments of two phases will be different with respect to the knife-edge at the mid-point of the capsule. Naturally, the capsule would tilt to the direction of the heavier side until both moments are equalized.

If a capsule is charged with M gm of a liquid metal, at a certain temperature of T the capsule will attain an angle of θ . Then, the sum of the horizontal moments should be zero since they are in equilibrium. Mathematically,

$$f (M, D, \theta, \rho_L, \rho_v, T) = 0 \quad (21)$$

where, ρ_L, ρ_v are liquid, vapor density, D is dimension of capsule representing the capsule-yoke assembly. In this equation, all of the parameters except ρ_L and ρ_v are known either by their dimensions or by experimental determination, and the moment equation (21) has two unknowns of ρ_L and ρ_v , and it requires another equation in order to determine these unknowns.

An empirical correlation by Cailliet and Mathias (9) suggested that an average of vapor and liquid densities is linear with temperature. That is,

$$G (\rho_L, \rho_v) = (\rho_L + \rho_v) / 2 - (A + B T) = 0 \quad (22)$$

where, A and B are coefficients and defined by the average density equation. Therefore, two functions, f and G, can be solved for ρ_L and ρ_v as in the case of cesium study (35). However, it is necessary to have accurate information on the Rectilinear diameter line in order to do so, and the resulting values of ρ_L and ρ_v would not be self-consistent but rather depend on the average density equation.

Suppose two capsules, each charged with M_1 and M_2 gm of a liquid metal, give tilt angles of θ_1 and θ_2 respectively at a temperature T, then, the respective moment equations would be,

$$F_1(\theta_1, \rho_L, \rho_v) = f(M_1, \theta_1, D_1, \rho_L, \rho_v, T) = 0 \quad (23)$$

$$F_2(\theta_2, \rho_L, \rho_v) = 0 \quad (24)$$

Two broken lines in figure (1a) represent the above two equations, and the resulting angle/temperature curves would be as in the figure (1b). The moment equation and its derivation are given in appendix, and can be written as following.

$$F(\theta, \rho_L, \rho_v) = \tan^3\theta + X \tan^2\theta + Y = 0 \quad (25)$$

where, X and Y are function of ρ_L and ρ_v . This equation is valid only if the tilt angle θ is greater than an angle ϕ given by the equation (B-14) in appendix B, at which the saturated phase interface in the capsule falls on the corner of capsule ends. Based on the 6 inches long capsules of this experiment, ϕ is 1.23 degree angle.

However, to solve two moment equations with two different capsules, yokes and charges, it is extremely difficult to obtain accurate values of ρ_L and ρ_v because the respective coefficients X and Y of the moment equations (23) and (24) are approximately same, and the roots, ρ_L and ρ_v , are different in magnitude. Furthermore, the equation (25) is non-linear with several roots.

(a)

(b)

Figure 1. (a) P - V - T surface, (b) Equilibrium rest angle versus temperature curves.

Therefore, any error in angle measurement may give a small effect on liquid density but can results large error on vapor density. Since the magnitude of errors for two capsules is not same, it is decided to solve two moment equations separately with the equation (22) by guessing an initial starting value of RD for two capsules until ρ_L and ρ_v from two pairs of simultaneous equations are same or differences of ρ_L or ρ_v by two pairs of the equations are minimal. Details of numerical method is given in appendix D.

IV. EXPERIMENTAL DESIGN

A. Rubidium Capsule

Ten capsules, 6 inches long with both ends welded closed with 0.015 inch thick sheet, have been fabricated by the Nuclear Systems Programs of the General Electric Company. The alloy used has a weight analysis of 40% W - 30% Re - 30% Mo, and has strength equal to T-111 (Ta-90, W-8, Hf-2) at about 1000° F higher temperature, T-111 had been successfully used for cesium to over 3000° F.

These capsules were cut from nuclear quality, seamless tubing of 0.218" ID and 0.250" OD. A hole of 0.020" diameter had been provided in one end of each capsule for charging the liquid metal.

With the maximum defects of 10% in wall thickness, the tubing was specially extruded for this purpose and Eddy Current test has been subjected both longitudinally and transversely. The current flux disturbance shows not only the uniformity of the wall thickness but also the inner to outer diameter eccentricity. One out of the ten capsules was found questionable through EC testing.

Theoretically, thinner the wall and longer the tube length, the pivot moment will be greater and thus give better sensitivity. Particularly, when rubidium has lower density than cesium, for which 12 inches long T-111 tube was employed in this laboratory, it may sound logical to use even longer tube. However, it was extremely difficult to maintain uniform temperature and this non-uniform temperature distribution will offset the advantage of using longer tube.

High purity rubidium was transferred, here in Columbia University, into smaller glass ampoule by heating the large ampoule under vacuum. The residues (including Rb oxides) are eliminated by decantation under vacuum. The chemical analysis of the rubidium

by the supplier are shown in the table IV.

Table IV. Chemical Analysis of Rubidium

<u>Impurity</u>		<u>Purity, %</u>
Sodium		0.0070
Potassium		0.0500
Cesium		0.0700
Lithium	less than	0.0020
Calcium	less than	0.0005
Iron	less than	0.0005
Aluminum	less than	0.0005
Silicon	less than	0.0005
		0.1310 %

Based on the published critical density of 0.34 gm/cc and calculated inside capsule volume at room temperature, approximately 18% excess amount to critical loading of rubidium metal (1.49 gm) is charged. However, due to the difficulties of the dispensing operation by G. E., somewhat less or more of the metal is charged.

Four W-30Re-30Mo capsules were filled with rubidium using a heated hyperdermic syringe equipped with a #25 needle. The 0.020" D hole, through which the Rb is charged, is then welded by Electron Beam Welding. The quantity of rubidium charged to each capsule is shown in table V:

Table V. Rubidium Charges in Capsule

<u>Capsule No.</u>	<u>Rubidium Charge, gm</u>	<u>Capsule Height, empty, gm</u>
1	1.417	20.8810
2	1.431	20.7883
3	1.494	20.8202
4	1.563	20.9128

where, empty capsule was weighed with "Mettler" electronic balance. It is noticeable that the weights of the empty capsule are smaller than those shown in the summary table XII because of the metal charging hole at the one end. Each dimensions are carefully measured with micrometer before and after the experiments.

B. Yoke and Pivot Assembly

The most critical parts of this experiment are a pivoting yoke and cradle. Not only any appreciable friction between the yoke and the cradle alters the sensitivity of the pivot moment but also a perfect symmetry of the yoke itself and the pivot contact lines with the cradle should be maintained. Furthermore, the most optimum design of the pivot assembly should be such that any foreign influences on the pivot movements must be minimized, particularly at high temperature environment.

Due to the high temperature range of this experiment the expansion of each part is considerable, and consequently, yoke was made of same material as the capsule, W-30Re-30Mo. However, all of the stresses, due to the weights of yoke, capsule and rubidium metal is concentrated at the knife blade line. Based on the actual experimental parts and condition, the estimated fiber stress along this knife edge was less than 400 psi, which would give safety margin of better than two, assuming more than 20 hours of continuous heating at 2200°C.

The linear thermal expansion coefficient of W-30Re-30Mo is tabulated in table VI and the data is plotted in figure 2. Also, the expansion coefficient is fitted to polynomial by linear regression analysis yielding the following equation;

$$\alpha = -0.9173 \times 10^{-4} + 0.4575 \times 10^{-5}T + 0.1022 \times 10^{-8}T^2$$

FIGURE 2
LINEAR THERMAL EXPANSION
POWDER METALLURGY W-25Re-30Mo
MEASURED IN HELIUM

where, T is given as degree $^{\circ}\text{C}$ and the expansion coefficient as fraction,

$$\alpha = (L_T - L_{25^{\circ}\text{C}}) / L_{25^{\circ}\text{C}}$$

and the standard deviation is 0.00020 fractional expansion.

Table VI. Linear Thermal Expansion Coefficient of W-30Re-30Mo in fraction

<u>Temperature, $^{\circ}\text{C}$</u>	<u>Expansion Coefficient</u>	
	<u>Observed</u>	<u>Calculated</u>
25.	.00000	.00002
1085.	.00630	.00607
1330.	.00760	.00780
1555.	.00950	.00949
1755.	.01100	.01108
1950.	.01260	.01271
2110.	.01440	.01411
2275.	.01550	.01560

The yoke was grinded from 0.183 inch thick W-30Re-30Mo plate, also fabricated by G.E., with the accuracy better than ± 5.0 mil except the quarter inch bore through which the Rb capsule is snugly fitted. For this bore the milling tolerance was maintained with ± 0.2 mil accuracy. This bore diameter is one mil bigger than the Capsule ϕ .D., and by the time the temperature is raised, the fitting between the capsule and yoke hole will be tightened. The pivoting distance between the knife edge and center of gravity of the capsule, which is on the axis of the capsule cylinder, was chosen to be same as outside diameter of the capsule ; 0.250 inch. The corner of the knife edge was right angle in order to give the required strength of the knife to sustain the concentrated fiber stress.

At tip of the yoke, a 0.015" diameter and 1/8" deep hole is drilled by Electric Discharging Machine, through which a flexible 0.005" diameter W-26/Re wire is loosely fit-in to function as a common electric tap. Not only the wire is thin enough but also 2 inches of the end is left bare so that stiffness of the wire is minimized. Also, the hole is bigger than the wire itself and located at the top center of the yoke. Since whole assembly will tilt together, any practical effect on the rest angle is not detectable. The schematic diagram is shown in figure (3a).

It would be much easier to use an electrically conductive material for the cradle since this can be utilized as an electric probe of angle sensor. Preliminary test with T-111 tube have shown that as temperature increases the contact point of the cradle become softer or even semi-welded with the knife point, and sensible results were impossible.

Furthermore, small amount of oxygen presence may decrease sensitivity considerably due to oxidation with the cradle material. It is decided to use triangular R. R. Morganite tube as a cradle since it is made of 99.7+ percent of Al_2O_3 with the maximum operating temperature of 1950°C, which is believed to be about 100°C above the critical temperature of rubidium.

A morganite tube of 7/8" O.D., 11/16" I.D., and 6" in length is used. Approximately one half an inch from one end, U - shaped groove, ~0.4" width and ~0.2" deep, is grinded at right angle to tube axis on diamond wheel, and about 0.1" of the tube top between the groove and tube end is also grinded off, shown in figure (3b), so that at the vertical position with this end-up, the yoke with the rubidium capsule can be dropped through the end slot and fit into the U-groove. When the assembly is tilted to horizontal position, the knife-edge will sit freely on the bottom of the U-groove. The farther end from the slot is astro-cemented to the Flange surface inside the heater and its supporting stainless steel tube,

FIGURE 3. (a) Yoke, (b) Pivot Assembly

which is attached to the flange by three set-screws.

C. Thermocouples and Heater Assembly

To sustain the high temperature for the prolonged period, it was decided to use 0.010" diameter W-5%Re/W-26%Re thermocouple wires, which was calibrated within 1% by Hoskins Manufacturing Company from 200°F to 4200°F.

The calibrated data is fitted into the second degree polynomial up to 3500°F as shown in table VII.

Table VII. Calibration table of W-5%Re/W-26%Re Thermocouple Wires.

EMF, mv	Temperature, F	
	Observed	Calculated
1.292	200	206.86
3.119	400	394.45
5.141	600	594.93
7.248	800	797.73
9.412	1000	1001.63
11.583	1200	1203.91
13.719	1400	1402.84
15.819	1600	1600.40
17.904	1800	1800.47
19.916	2000	1999.08
21.865	2200	2198.35
23.740	2400	2397.96
25.521	2600	2596.05
27.307	2800	2804.26
28.974	3000	3008.31
30.442	3200	3196.60
31.911	3400	3393.79
33.352	3600	3596.39
34.676	3800	3791.13
36.319	4000	4044.97
37.095	4200	4169.81

These data is plotted in figure 4, where the line represents the fitted equation given as following;

$$T = 69.7643 + 107.682 E - 1.26092 E^2 + 0.0360607 E^3$$

T is in degree F and E in mv. The standard deviation is 13.977⁹ F. The effective temperature of the capsule is obtained by taking arithmetic average of three thermocouples.

Cold junction is furnished with ice cubes in Thermo Dewar, in which copper wires were soldered to the thermocouple wires, then led to potentiometer and eight-points Leeds & Northrup temperature recorder as shown in figure 5. This copper wires will eliminate any external potentials other than those by hot junction.

At hot junction the relatively flexible W-26% Re wire is twisted about less flexible W-5% Re for an inch long in order to allow enough room to expand at high temperature without splitting the tip. The tips of an inch long twists are spot-welded with high voltage Heliarc Welder.

Each thermocouple wires inside the heater are insulated with double-bore BeO₂ thermocouple tubes. Four thermocouples are arranged as shown in the schematic diagram of figure 5, to give accurate presentation of capsule temperature.

The thermocouples are directly led through 3/8" diameter, center hole of the blind flange, where TG-20-A 8-T conax gland is welded to the outer face of the flange.

Heater tube is also Morganite Al₂O₃ tube, 1 1/2" OD, 31/32" ID, 8" long. A two inches long stainless steel heater support, 1" ID, 1 1/4" OD, is firmly attached to pressure flange by three set-screw, and Morganite cradle tube is acetrocmented concentrically to s.s.

tube and heater. Opposite end of s.s. tube is one inch long, one-end closed Morganite cap, which closes the heater tube. Of 8" heater tube Molybdenum heater wire, 0.040" diameter, is wound 7 turns to 10 turns per inch density covering 7.5" of the heater, where both ends are densely wound than the middle portion in order to insure proper heat distribution eventhough both ends of the heater openings are enclosed. High grade Morganite Al_2O_3 cement is uniformly covered the Moly windings to maintain the winding spacings.

It is decided to use triple leads of 0.040" diameter moly wires from the heater to conax gland in order to cut down the electric resistant and thus prevent any substantial heat generation. PL-14-A6 conax gland is pressure-fitted, then welded to the blind flange, and each three No.12 copper wires of the gland are led through $\frac{1}{2}$ " diameter hole into the heater and tightly binded with the triple moly wires. Similarly, three 0.030" D molybdenum wires of angle sensing probes are connected through $\frac{1}{4}$ " hole to TG-20-A 4-F conax gland. All these wires are insulated with alumina ceramic tubes inside the pressure vessel.

Since radiation heat loss is dominating factor at high temperature due to the proportionality of fourth power of temperature, Six layers of 0.001" thick tantalum foil are wrapped around the heater tube as radiation shields. The tantalum foil is dimpled to maintain the spacings between the layers. Over these layers all of the ceramic protected wirings are extended to the assigned holes of the blind flange, thus giving additional convective shielding space. Again, six layers of same tantalum sheet are provided, where between the last two outer layers, Fiberfrax is inserted to give conductive protection. Fiberfrax blanket is consist of 50.9% Al_2O_3 and 46.8% SiO_2 , and is rated up to 2300° F.

When the assembly is heated to 3400° F for several hours, the temperature of the welding neck cylinder reaches approximately

300 ~ 400°P with a fan blowing cold air continuously from outside.

Zirconium turnings are inserted inside the pressure vessel at the inlet of the balancing argon gas near the Morganite heater cap, and thus, any trace of Oxygen along the gas line can be removed.

D. Angle Sensors and Measurement of Rest Angle

The rest angle of the capsule is measured remotely due to high temperature-pressure environment. The capsule serves as an electric switch or bridge as it pivots, and the circuit is consisting of capsule-yoke, 0.005" W-26%Re wire inserted into 15 mil hole at the center top of the yoke, then to one of the external circuit as shown in figures 5 and 6. The other end of the external line is extended into two molybdenum strips through which the capsule makes the contacts to complete the circuit. The moly strips are fixed to the heater-cap end of the capsule, parallelly aligned to the pressure vessel, and the capsule pivots freely between the strips.

At high temperature, ordinary ohm-meter can sense the circuit 'on-off' without difficulties. However, it is decided to use high impedance Vacuum Tube Volt Meter. Also, D.C. circuit is employed in order to avoid stray current in A.C. circuit. When the circuit is opened by breaking the moly contacts, VTVM will read 6 volt. The closed circuit via top contact gives 3 volts, and via bottom one results 9 volts. When the circuit is closed the pressure vessel will be pivotted to the direction of opening the circuit by watching the VTVM reading, whether capsule touches top strip or bottom one as shown in the figure 5.

The angle of the pressure vessel at the open circuit will be the rest angle of the capsule at the given temperature provided that the capsule and the external vessel is in perfect alignment each other.

FIGURE 5.

A 180 degree semi-circular protractor is attached to the pressure vessel aligned with the axis of the pressure cylinder, and a surveyor transit enables to read the protractor within 1/10 of a degree. The schematic diagram is shown in figure 7.

Comparing the rest angles of the capsule at room or low temperature before and after the assembly, the misalignment between two is pre-calibrated at room temperature, accepting the errors caused by the spacing between the woly strips. However, one phase "zero-angle" point may give this differences. Also, at a given temperature measurements are done both ways by tilting the capsule to clock-wise and anticlock-wise. Thus, the average of two would give the same results as "zero-angle" point.

E. External Pressure System and Power Supply

It is decided to use Argon gas for balancing the external pressure rather than Helium since convective heat transfer properties of argon is better than helium by a margin of better than 4 to 1 based on the simulated experimental conditions.

Ultra-high purity Linde gas is choosen, and the chemical analysis of the impurity contents is tabulated in the following table as given by the supplier.

Table VIII. Purity Analysis of Argon Gas

<u>Impurities</u>	<u>ppm</u>
N ₂	< 5
O ₂	< 1
H ₂	< 1
total hydrocarbon as CH ₄	< 2
H ₂ O	< 1

FIGURE 6. Power Vessel in Final Assembly

All of the external pressure lines is connected by Swagelok unions through $\frac{1}{2}$ inch O.D. seamless stainless steel tube except vacuum line, for which copper tubing is used.

Pressure regulation is done with high pressure needle valve, rated up to 3000 psig at 450° F. The required argon pressure is supplied directly by the fresh 330 cu.ft. argon cylinder with 2400 psia cylinder pressure. The volume inside the heater assembly is so small that the argon pressure of the cylinder can be maintained without exhausting the cylinder appreciably.

In order to ease the stress on the line joints, the pressure tubing leading to the vessel is hellically wounded around the pivot arm of the vessel to absorb the strain caused by the repeated pivotings. Pressure regulation is monitored through 3000 psig gauge with 25 psi subdivisions, and 30 inch Hg vacuum gauge with an half inch subdivision is connected to $\frac{1}{4}$ HP, 1725 RPM vacuum pump. Schematic diagram of the this arrangement is given in figure 7.

Power is fed through step-down transformer (208 to 115 volts), capable of supplying more than 30 amperes of A.C. current to give 3 KW of power for the heating element. A 150 volt variac controls main power line and a series of Rheostats enables to regulate fine controls.

F. Pressure Vessel and Final Assembly.

By design it was necessary to choose thin-walled rubidium capsule to increase the sensitivity. However, this caused the problem of compensating vapor pressure of rubidium in order to prevent the possible capsule rupture or bulging at high temperature, where this balancing pressure goes up to 2000 psia near the critical region.

A double extra strong welding neck, made of stainless steel, is

used as pressure hull with normal size of $2\frac{1}{2}$, 2500 pounds capacity. The hemispheric end cap is welded enclosing the welding neck as in figure 6, where at the top of the end cap, $\frac{1}{4}$ " swagelok tube union is welded for the inlet of argon gas.

Bottom closure of the vessel is also 2500 pounds, xx-strong, size $2\frac{1}{2}$ blind flange, and this pressure vessel provides 2.3" inner diameter, 13 inches long spacing for the heater assembly. The welding neck and blind flange are separated by Flexitallic gasket sandwiched between two $1/8$ " thick Teflon sheet, which eases the leak problems, and eight $1\frac{1}{8}$ " diameter bolts hold two flanges together.

The pressure vessel is aligned on $\frac{1}{4}$ " thick $6" \times 10\frac{1}{2}"$ steel plate and held with two Ω -shaped 2" wide $\frac{1}{4}$ " thick strips by four $\frac{1}{2}"$ diameter bolts. The supporting steel plate is welded to $1\frac{7}{16}"$ ID $1\frac{11}{16}"$ OD pipe, which will pivot on A-frame as shown in figure 6.

A hook is threaded into a side of the blind flange, and $\frac{1}{2}$ ton capacity chain fall is extended to pulley type arrangement, thus allowing the assembly to pivot by lowering or lifting the flange. The individual loop of the chain is about an half-inch long and capable of pivoting less than two tenth of a degree at a time. Final assembly is completed under vertical position by placing the blind flange face up and conax glands extensions down position on a heavy steel frame covered with a $\frac{1}{2}"$ thick $2" \times 2"$ steel plate with $7\frac{1}{2}"$ hole at the center.

The stainless steel heater support is screwed on the blind flange face, and the cradle morganite tube is cemented inside the s.s. support tube with U-groove end at top, aligning the slot and knife seat with the chain hook precisely. After thermocouples are placed into proper position, then, the heater is mounted with alumina cement, and capsule-yoke is lowered from the upper opening of the heater into the cradle slot seating the knife-edge into the pivot groove. The angle-sensing wires are firmly cemented at inside wall of the

FIGURE 7.

heater tube but leaving enough space for capsule movement as in figure 6. The heater end-cap is pre-fabricated with two electric contacts with 15/16" spacing, made of 1 3/4" long, 3/4" wide, 1/16" thick molybdenum sheet.

This end cap is placed at the top of the heater by snugly fitting the molybdenum contacts into the heater opening. After connecting necessary wirings, insulations and shields, Teflon sandwiched flexitalllic gasket is placed. By this time the assembly is 12" long cylinder with approximately 2.2" diameter. Any over-size or under-size of the assembly can be checked when the gasket is lowered.

Outside the end closure of the welding neck flange, a chain-lip is machined around the cap to mount $\frac{1}{2}$ " D twisted steel-wire, and the chain fall is hooked to this steel wire, which allows to lower the neck flange vertically. Thus, the heater assembly can be slid into the bottom opening of the flange.

V. EXPERIMENTAL PROCEDURE

In operation of this experiment, whole assembly of the cylinder is kept at horizontal position throughout the heating period, and temperature is constantly monitored to recorder. When the temperature reaches steady state, chain-fall is either lowered or lifted until VTVM signals open or closed circuit through one of the electric taps. Then, potentials of the thermocouples are obtained through the potentiometer, followed by the rest-angle measurement on the protractor through surveyor.

As one may expect, the rest angle at a given temperature varies due to the friction between the knife-edge and the cradle, depend on the way the rest position is attained; either during lifting the cylinder or lowering the vessel clock-wise. Also, the spacing of the circuit breaker taps virtually allows a certain variation. If the tap spacing is large, then it may be easy to locate the capsule rest position but may cause greater variation of the resulting rest angles as in the case of the initial runs with the capsule #4.

Also, it is not expected to have complete alignment of the cradle-capsule with the angle-measuring protractor outside of the vessel, and when the capsule rests at horizontal position, the surveyor reading on the protractor may be other than zero angle.

This difference can be determined accurately by cross-checking with three independent methods: First, a capsule with a known rest angle can be measured within the cylinder by surveyor. Also, at a given temperature the capsule can be tilted to both clock-wise positive angle and anticlock-wise negative angle. One-half of this difference between the positive and negative tilt angles would be the misalignment of the protractor with respect to the capsule. The last method is direct observation of one phase with surveyor, at which the capsule should be in horizontal position. However,

this last method is impractical and extremely difficult, because the capsule vibrates vigorously near the one-phase region.

The rest angle of capsule is visually observed at room temperature on several different cradles with the accuracy better than ± 0.01 degree angle, and the capsule is assembled inside the pressure cylinder. Then, the rest angles are determined at low temperature by protractor-surveyor under four different conditions: The capsule rest position is obtained while increasing and decreasing the cylinder tilt angles, using either top or bottom "circuit breaker" molybdenum taps. The calibration data is tabulated in appendix D and statistics are shown in the following table IX.

Table IX. Alignment Calibration of the Rubidium Capsule with Angle Protractor.

	Tilt Angle, degree				degree of freedom	
	average		std. dev.			
	run#1	run#2	run#1	run#2	run#1	run#2
<u>Exp.1 (capsule#1)</u>						
nominal	23.3					
combined	24.595		.710		49	
bottom tap	24.604		.657		26	
top tap	24.585		.769		22	
increasing angle	24.607		.659		26	
decreasing angle	24.580		.766		22	
<u>Exp.2 (capsule#4)</u>						
nominal	24.8	24.8				
combined	26.086	26.014	.865	.486	64	42
bottom tap	25.972	25.950	.840	.418	29	22
top tap	26.184	26.087	.874	.544	34	19
increasing angle	26.730	26.137	.769	.441	29	26
decreasing angle	25.581	25.756	.604	.443	34	15

For Experiment #1 with capsule #1, the statistics of the above table shows that all four cases are within the range of one standard deviation, and therefore, it is decided to give 1.3 degree of angle correction for the misalignment regardless of how the rest angle is obtained. The reason for this relatively invariance is believed to be responsible on close spacing of 15/32 inch between bottom and top electric taps. However, due to this narrow spacing of the circuit breaker taps, additional time and precaution are necessary to locate the resting position of the capsule.

In lieu of this experience, the spacing of these taps are widened for Exp#2, run#1 by $\frac{1}{4}$ inch and the resulting effect is obvious as shown in the above table, where rest angles obtained while increasing the cylinder angle is given 1.8 degree of correction and 0.7 degree of misalignment correction is given separately for the case of decreasing angle.

During this runs with capsule #4 and yoke#2, power lines are short-circuited inside of the conax gland caused by the failure of electrical insulation. Consequently, all of the wirings are replaced with new ones including thermocouple wires from same lots and spool. However, the spacing of the circuit breakers are narrowed back to approximately one-half of an inch in order to obtain more stable results. As a result, the corrections of misalignment are all within one standard deviation with a single correction of 1.2 degree angle.

As mentioned previously, balancing external argon pressure is applied closely following the temperature - vapor pressure chart of figure 8. However, when the capsule #2 is ran, this balancing pressure is accidentally neglected, and thus giving only 1200 to 1400 psia of argon pressure at the temperature range of 3200°F or higher, where the vapor pressure of rubidium inside the capsule may reach as high as 1900 psia. Later, when this capsule #2 is removed from the pressure cylinder, the capsule was bulged outward to

FIGURE 8. Vapor Pressure of Rubidium.

compensate the pressure differential. This expansion left the outside diameter of the capsule to 0.280 inch from the original value of 0.250". Thereafter, external compensating pressure of argon is closely followed the vapor pressure of rubidium and in fact somewhat higher than the vapor pressure but always within the range as estimated for the capsule wall stress. Also, as an additional precaution the capsule outer diameter is always rechecked after each experiment.

Temperature is adequately raised by turning the variacs once at a time allowing enough time to reach steady temperature. At low temperature, between 1000°F to 2500°F, it takes approximately 20 to 40 minutes to raise 100°F, and at the temperature higher than 3200 F, more than 2 hours are required in order to increase the temperature by 50°F and maintain a steady temperature.

Summary of Assembly Calibration

Dimensions of each parts are measured by precision "Optical Comparator" with the accuracy of better than one-tenth of a mil.

The part to be measured is optically enlarged by ten times, and projected on a movable glass screen which is finely scaled. This glass screen can be moved manually parallel to the object, and a fixed scale marks on the screen can be brought into focus on one end of the projected shadow and then move to the other end of the projection. This travel distance of the glass screen from one end to the other end of the projection is measured by a high precision micrometer attached to the comparator. The accuracy of the micrometer is 10^{-5} inch in actual projected scale and 10^{-6} inch of the original object.

The yoke dimensions are measured twice, once for each side by turning the yoke over. The capsule bore of the yoke is measured two ways, measuring the diameter parallel to and vertical to the knife edge for each side of the yoke. Outside diameter of each capsule is measured approximately at the middle of the capsule and an inch far from both ends. The following of the measurements are done at $72^{\circ}\text{P} \pm 3^{\circ}\text{P}$.

Also, measured by Optical Comparator is distance between the center of gravity of the yoke and knife-edge, designated as $\bar{P}_y$ in appendix B for the moment equation. The center of gravity is located by balancing the yoke on a dull knife, then projected on the comparator.

Table X. Yoke Dimensions by Optical Comparator.

	<u>yoke #1</u>	<u>yoke #2</u>
knife-edge to bore, inch*	.1282 $\pm$.0002	.1265 $\pm$.0003
Bore diameter, inch**	.2514 $\pm$.0005	.2518 $\pm$.0001
Bore-to-outside diameter	.0647 $\pm$.0055	.0645 $\pm$.0048
one blade length, inch	.1300 $\pm$.0053	.1316 $\pm$.0033
Total knife-edge length, in.	.5613 $\pm$.0002	.5623 $\pm$.0001
Yoke width, inch	.1822 $\pm$.0002	.1826 $\pm$.0003
Distance from C.G. of yoke to knife edge, inch	.4453 $\pm$.0010	.5087 $\pm$.0020

* Pivot length $\bar{P}$ will be this plus outside radius of capsule

** these values should be compared with the capsule O.D.

Table XI. Capsule Dimensions by Optical Comparator.

<u>Capsule #</u>	<u>outside diameter</u>	<u>Length, inch</u>	
	<u>inch</u>	<u>outside</u>	<u>inside**</u>
#1	0.2502 $\pm$.0001 (0.2507)*	6.072	6.042
#2	0.2502 $\pm$.0001 (0.2510)*	6.061	6.031
#3	0.2504 $\pm$.0000 (0.2504)	6.058	6.028
#4	0.2502 $\pm$.0001 (0.2502)	6.062	6.032

* measured with micrometer

** calculated with the wall thickness of 0.016 " $\pm$ 5% as checked by Eddy Current Test.

Weights of capsules and yokes are measured with "Mettler" electronic balance with the accuracy better than 0.0001 gm. The following is the summary of the weights.

Table XII. Summary of Weights by Electronic Balance.

<u>Capsule #</u>	<u>Weights, gm</u>		
	<u>yoke (W_y)</u>	<u>Capsule + Rb</u>	<u>empty capsule</u>
1	4.6230	22.3228	20.9058
2	4.6320	22.2567	20.8257
3	-	22.3620	20.8680
4	-	22.5117	20.9489

The external volume of capsule is determined by displacement technique using a Burette filled with distilled water. Yoke volume is similarly obtained in Pycnometer. Since wall thickness of the capsule and its uniformity are checked by Eddy Current test as

mentioned previously, internal volume of the capsule should be reasonably accurate, if calculated based on the other dimensions and external volume.

Table XIII. Volumes of Capsule and Yoke,
measured at 80°F.

	Volume, cm ³		
	<u>yoke</u>	<u>capsule</u>	
		<u>external</u>	<u>internal</u> *
1	0.2212	4.98	3.6956
2	0.2526	4.89	3.6889
3	-	4.88	- 3.6870
4	-	4.90	3.6895

* based on I.D., internal capsule length, and measured external volume.

VI. RESULTS

The rest angles of two capsules, #1 and #4, are obtained as a function of temperature as tabulated in appendix D, where the corrections on misalignment is applied, and temperatures are averaged from three thermocouple readings.

As mentioned previously, it is extremely difficult to obtain the rest angles of two capsules at same temperature, and consequently, as many points are obtained as possible.

The equilibrium rest angles as a function of temperature are plotted in figure 9, obtained with the capsule #1, and figure 10 from the capsule #4.

When temperature approaches one-phase region, the capsule was vibrating and it was not quite difficult to locate the zero-angle, one-phase point. However, in order to pinpoint this zero-angle point accurately, several points including the points farther from this point are fitted into the least squares polynomial, and the zero-angle point is extrapolated.

$$\theta = A_0 + A_1 T + A_2 T^2 + A_3 T^3 \quad (26)$$

where, θ is in angle and T in degree F. The coefficients of the polynomial and zero angle points are tabulated in table XIV, in which the angle is slightly higher than zero because the zero angle points are obtained by successive approximations.

In general, the rest angles are widely scattered because of several factors; first, rubidium density is lower than that of cesium, with which we had experience in this laboratory, and thus

the tilting moment of the capsule is expected to be lower than that of the cesium. Secondly, the linear dimensions of this experiment is approximately an half of those for cesium study. Consequently, the ratio of capsule weight to the metal charge will be larger than the previous cesium study (40), thus giving lower tilting moment at a given temperature.

Also, it is essential to have both rest angles of two capsules at same temperature. Therefore, it is decided to smooth the angle-temperature curves using a set of polynomials, ranging from second to sixth degrees. To give the best presentation of the experimental points, the experimental points are divided into several temperature ranges, where several points outside of the temperature range are included for each zone in order to give a smooth transition from one region to another. The selection of a particular curve for a given temperature zone is based on the total variance and distribution of deviation. The coefficients of polynomials are tabulated in table XV and XVI, satisfying the equation (26). These fitted curves are also plotted in figures 9 and 10 along with the experimental points.

As mentioned before, pivot length is critical for density result since any small error would affect the final rest angle enormously. However, pivot arm is arbitrary weighted from the center-of-gravity of the combined weights of yoke and capsule, given by the equation (E-5) of appendix. Therefore, the effective pivot arm is calibrated using generally accepted values of saturated phase densities at low temperatures.

Using the moment equation as derived in appendix B and collecting terms for the pivot arm, the effective pivot length of capsule $\bar{P}_0$ can be written as,

$$\bar{P}_{c,eff} = \frac{1}{W_c} \left[-\frac{X'(W_y+W_c)}{\tan \theta} - \frac{Y'(W_y+W_c)}{\tan^3 \theta} - W_y \bar{P}_y \right] \quad (27)$$

and $X' = \bar{P}X$, $Y' = \bar{P}Y$ in appendix B.

The saturated phase densities for this calibration are the values reported by Achener (1) to 2000°F, and the resulting capsule pivot arms are tabulated in table XVII for two capsules, #1 and #4. From these pivot arms and rest angles, complete phase densities are obtained up to the last experimental point with the capsule #4, which is only 16°F away from the estimated critical point. Of course, the zero angle point is just 1°F lower than the critical temperature.

Generally, the liquid densities are fairly accurate but the accuracy begins to loose as temperature approaches the critical point. Near 2000°F they are within 0.5 % accuracy and this error steadily increases to 5 % near the critical point. The complete tabulation of the saturated phase densities of rubidium is given in table XVIII.

In figure 11, the liquid density values are in good agreement with the accepted values at the lower temperatures but higher than those reported by Dillon (10), and similar trend has been observed also in cesium study (35).

Based on the estimated critical temperature of 3331°F and critical density of 0.347 gm/cc, the reduced densities are calculated in table XIX, and are plotted in figure 12 along with other data of alkali metals. An interesting feature in this figure is that the reduced densities of rubidium data are well within the range of the experimental error with cesium data, particularly at high temperature.

Table XIV. Zero-angle, One-phase Points.

a). Capsule #4, Yoke #2.

ANGLE, DEG. = -0.0000019
 TEMPERATURE, F = 3324.83
 DENSITY, GM/CC = 0.40911

Y = 0.768357E+02
 -0.219772E-02*X
 0.501701E-05*X**2
 -0.340061E-08*X**3
 0.000000E+00*X**4
 0.000000E+00*X**5
 0.000000E+00*X**6

b). Capsule #1, Yoke #1.

ANGLE, DEG. = -0.0000081
 TEMPERATURE, F = 3330.86
 DENSITY, GM/CC = 0.37025

Y = -0.167966E+03
 0.127192E+00*X
 -0.432263E-05*X**2
 0.458444E-08*X**3
 -0.960048E-12*X**4
 -0.539126E-15*X**5
 -0.277769E-19*X**6

Table XV. Coefficients of Smoothed Curves
for Angle-Temperature Points,
Capsule #4, Yoke #2.

$T \leq 2000^\circ F$

POLYNOMIAL COEFFICIENTS

FIRST ONE IS THE HIGHEST ORDER

$$A_2 = -.724504E-06$$

$$A_1 = -.796552E-03$$

$$A_0 = 0.270704E-02$$

DEGREE OF FREEDOM= 40.

VARIANCE= 4.79063

STANDARD ERROR ESTIMATED= 0.34607

$2000^\circ F < T \leq 2500^\circ F$

POLYNOMIAL COEFFICIENTS

FIRST ONE IS THE HIGHEST ORDER

$$A_4 = -.7291661E-11$$

$$A_3 = 0.218928E-07$$

$$A_2 = -.607704E-04$$

$$A_1 = 0.703751E-01$$

$$A_0 = -.364064E-01$$

DEGREE OF FREEDOM= 38.

VARIANCE= 4.53661

STANDARD ERROR ESTIMATED= 0.34552

2

$$2500^{\circ}\text{F} < T \leq 2900^{\circ}\text{F}$$

FIRST ONE IS THE HIGHEST ORDER

$$A_4 = -.147449\text{E}-12$$

$$A_3 = 0.161630\text{E}-09$$

$$A_2 = -.345769\text{E}-06$$

$$A_1 = 0.390423\text{E}-02$$

$$A_0 = 0.162126\text{E} 02$$

DEGREE OF FREEDOM = 19.

VARIANCE = 2.18097

STANDARD ERROR ESTIMATED = 0.33880

$$2900^{\circ}\text{F} < T \leq 3050^{\circ}\text{F}$$

FIRST ONE IS THE HIGHEST ORDER

$$A_2 = -.132232\text{E}-04$$

$$A_1 = 0.613370\text{E}-01$$

$$A_0 = -.486429\text{E} 02$$

DEGREE OF FREEDOM = 15.

VARIANCE = 2.67945

STANDARD ERROR ESTIMATED = 0.42265

$$3050^{\circ}\text{F} < T \leq 3200^{\circ}\text{F}$$

FIRST ONE IS THE HIGHEST ORDER

$$A_6 = -.705319\text{E}-19$$

$$A_5 = 0.213139\text{E}-16$$

$$A_4 = 0.282611\text{E}-12$$

$$A_3 = -.226153\text{E}-09$$

$$A_2 = 0.691156\text{E}-05$$

$$A_1 = 0.490911E-02$$

$$A_0 = -.309323E-02$$

DEGREE OF FREEDOM= 31.

VARIANCE= 10.05638

STANDARD ERROR ESTIMATED= 0.56956

$$3200^{\circ}F < T < 3280^{\circ}F$$

POLYNOMIAL COEFFICIENTS

FIRST ONE IS THE HIGHEST ORDER

$$A_2 = -.432549E-04$$

$$A_1 = 0.227233E-00$$

$$A_0 = -.233435E-03$$

DEGREE OF FREEDOM= 23.

VARIANCE= 8.31506

STANDARD ERROR ESTIMATED= 0.60127

$$T > 3280^{\circ}F$$

POLYNOMIAL COEFFICIENTS

FIRST ONE IS THE HIGHEST ORDER

$$A_3 = -.903070E-08$$

$$A_2 = 0.114140E-04$$

$$A_1 = 0.119164E-00$$

$$A_0 = -.191459E-03$$

DEGREE OF FREEDOM= 6.

VARIANCE= 2.18366

STANDARD ERROR ESTIMATED= 0.60328

Table XVI. Coefficients of Smoothed Curves
for Angle-Temperature Points,
Capsule #1, Yoke #1.

$T \leq 2500^\circ F$

POLYNOMIAL COEFFICIENTS

FIRST ONE IS THE HIGHEST ORDER

$$A_2 = -.658955E-06$$

$$A_1 = 0.172664E-03$$

$$A_0 = 0.242319E 02$$

DEGREE OF FREEDOM = 29.

VARIANCE = 5.07936

STANDARD ERROR ESTIMATED = 0.41851

$2500^\circ F < T \leq 2900^\circ F$

POLYNOMIAL COEFFICIENTS

FIRST ONE IS THE HIGHEST ORDER

$$A_5 = -.2827E9E-15$$

$$A_4 = 0.816457E-12$$

$$A_3 = 0.120128E-08$$

$$A_2 = -.222214E-05$$

$$A_1 = -.176111E-01$$

$$A_0 = 0.491938E 02$$

DEGREE OF FREEDOM = 25.

VARIANCE = 4.52361

STANDARD ERROR ESTIMATED = 0.42538

2900° F < T < 3050° F

FIRST ONE IS THE HIGHEST ORDER

$$A_4 = -.249136E-12$$

$$A_3 = 0.557505E-09$$

$$A_2 = -.403978E-05$$

$$A_1 = 0.184300E-01$$

$$A_0 = 0.290753E 01$$

DEGREE OF FREEDOM= 24.

VARIANCE= 7.70892

STANDARD ERROR ESTIMATED= 0.56675

3050° F < T < 3200° F

FIRST ONE IS THE HIGHEST ORDER

$$A_2 = -.641676E-05$$

$$A_1 = 0.556391E-02$$

$$A_0 = 0.465131E 02$$

DEGREE OF FREEDOM= 42.

VARIANCE= 21.45021

STANDARD ERROR ESTIMATED= 0.71465

3200° F < T < 3280° F

FIRST ONE IS THE HIGHEST ORDER

$$A_2 = -.834262E-05$$

$$A_1 = 0.735261E-02$$

$$A_0 = 0.731657E 02$$

DEGREE OF FREEDOM= 41.

VARIANCE= 18.57582

STANDARD ERROR ESTIMATED= 0.67310

$3280^{\circ}\text{F} < T \leq 3320^{\circ}\text{F}$

POLYNOMIAL COEFFICIENTS

FIRST ONE IS THE HIGHEST ORDER

$$A_2 = -.185071\text{E}-04$$

$$A_1 = 0.702681\text{E}-02$$

$$A_0 = 0.183471\text{E} 03$$

DEGREE OF FREEDOM= 34.

VARIANCE= 52.74266

STANDARD ERROR ESTIMATED= 1.24549

$T > 3320^{\circ}\text{F}$

POLYNOMIAL COEFFICIENTS

FIRST ONE IS THE HIGHEST ORDER

$$A_6 = -.277789\text{E}-19$$

$$A_5 = -.539126\text{E}-15$$

$$A_4 = -.960048\text{E}-12$$

$$A_3 = 0.453444\text{E}-08$$

$$A_2 = -.432263\text{E}-05$$

$$A_1 = 0.127192\text{E} 00$$

$$A_0 = -.167966\text{E} 03$$

DEGREE OF FREEDOM= 11.

VARIANCE= 24.04926

STANDARD ERROR ESTIMATED= 1.47861

Capable #1

TEMPERATURE F	ANGLE	LIO. DEN.	VAP. DEN.	PIVOT
1081.37598	24.45000	1.249013	.000370	.652250
1082.95898	23.70000	1.248631	.000380	.670703
1090.72168	23.70000	1.246756	.000400	.678205
1093.82520	22.89999	1.246004	.000410	.704360
1146.57520	24.09999	1.233109	.000540	.660069
1289.21875	23.09999	1.197967	.001190	.686533
1320.25977	23.99998	1.190187	.001370	.651257
1529.28027	22.70000	1.136620	.003300	.670634
1530.86035	22.59999	1.136208	.003310	.683343
1532.42969	22.45000	1.135799	.003380	.588546
1592.32227	22.74998	1.120028	.004150	.671394
1624.86328	22.49998	1.111376	.004660	.677110
1850.49707	22.79999	1.049599	.009710	.637155
1974.15820	22.09999	1.014235	.013650	.643982
1997.48047	21.29999	1.007430	.014710	.670378

AVERAGE= .671154
STANDARD ERROR EST= .1903E-01

Table XVIII. Saturated Phase Densities of Rubidium.

TEMPERATURE DEG F	ANGLE DEG	DENSITY, GM/CC LIQUID	VAPOUR	WEIGHT, GM LIQUID	VAPOR
1115.556	23.60	1.246750	0.00230642	1.557293	0.005706
1124.854	23.59	1.237129	0.00017889	1.562559	0.000440
1160.689	23.52	1.222091	0.00045351	1.561890	0.001110
1222.541	23.46	1.211608	0.00068357	1.561332	0.001668
1230.292	23.45	1.205656	0.00072216	1.561240	0.001760
1244.244	23.43	1.206101	0.00052188	1.560755	0.002244
1248.893	23.42	1.204923	0.00054516	1.560650	0.002309
1272.156	23.39	1.199149	0.00058127	1.560617	0.002382
1326.470	23.30	1.185455	0.00134055	1.559761	0.003238
1419.814	23.15	1.161914	0.00200545	1.558189	0.004809
1451.066	23.09	1.154002	0.00227641	1.557568	0.005431
1460.444	23.08	1.151628	0.00234813	1.557403	0.005597
1617.631	22.79	1.111745	0.00255503	1.557017	0.005983
1630.453	22.76	1.111732	0.00350126	1.554752	0.008207
1642.990	22.74	1.111680	0.00454206	1.552340	0.010659
1790.532	22.43	1.074741	0.00457681	1.551520	0.011478
1839.113	22.32	1.061255	0.00476658	1.552067	0.010913
1891.432	22.20	1.047684	0.00480483	1.552084	0.010915
1891.596	22.20	1.046729	0.00435724	1.553019	0.009980

1939.320	22.09	1.035370	0.00476273	1.552254	0.010747
1954.288	22.05	1.031408	0.00480663	1.552181	0.010819
1967.563	22.02	1.027966	0.00480819	1.552197	0.010801
1990.805	21.56	1.021371	0.00512300	1.551521	0.011467
2009.240	21.52	1.015658	0.00556699	1.550575	0.012424
2091.986	21.71	0.998150	0.00635935	1.548945	0.014053
2107.363	21.67	0.994346	0.00627346	1.549169	0.013829
2155.387	21.54	0.982491	0.00597673	1.549929	0.013070
2167.466	21.51	0.975473	0.00593967	1.550025	0.012963
2170.940	21.50	0.976615	0.00591440	1.550097	0.012900
2172.676	21.50	0.978191	0.00590256	1.550128	0.012871
2289.767	21.17	0.950842	0.00574894	1.550698	0.012300
2289.767	21.17	0.950842	0.00574895	1.550698	0.012300
2379.288	20.91	0.928577	0.00546158	1.551510	0.011489
2379.288	20.91	0.928577	0.00546155	1.551509	0.011489
2441.043	20.73	0.913097	0.00546652	1.551640	0.011359
2446.539	20.71	0.911717	0.00547776	1.551630	0.011369
2466.746	20.65	0.906584	0.00561274	1.551397	0.011602
2474.146	20.63	0.904718	0.00564104	1.551355	0.011643
2511.127	20.54	0.895473	0.00603806	1.550628	0.012371
2513.000	20.54	0.894946	0.00607034	1.550567	0.012431
2624.170	20.14	0.867024	0.00600665	1.551017	0.011982
2628.016	20.13	0.866014	0.00603946	1.550949	0.012048
2660.635	19.98	0.857183	0.00661754	1.549899	0.013099
2666.349	19.95	0.855630	0.00672270	1.549711	0.013287
2728.467	19.61	0.836255	0.00719546	1.549045	0.013954
2744.197	19.52	0.831703	0.00777281	1.547995	0.015003
2751.556	19.47	0.829419	0.00808392	1.547427	0.015570
2791.638	19.19	0.817475	0.01000108	1.543946	0.019052
2795.549	19.16	0.816292	0.01019501	1.543599	0.019400
2807.521	19.07	0.812572	0.01088720	1.542353	0.020645
2819.440	18.98	0.808834	0.01161136	1.541056	0.021942
2875.809	18.48	0.790381	0.01580932	1.533636	0.029364
2908.362	18.23	0.778235	0.01970728	1.526767	0.036212
2924.709	17.97	0.770293	0.02351445	1.520041	0.042956
2936.949	17.78	0.764289	0.02642226	1.514947	0.048051

2937.083	17.77	0.764257	0.02641575	1.514954	0.048044
2966.012	17.30	0.745865	0.03345531	1.502769	0.060229
2969.956	17.23	0.747831	0.03453130	1.501008	0.061991
3011.638	16.50	0.726466	0.04535260	1.482040	0.079959
3024.159	16.28	0.719938	0.04871410	1.477619	0.085379
3024.247	16.28	0.719875	0.04875481	1.477552	0.085446
3053.809	15.88	0.703867	0.05447155	1.469362	0.093637
3057.874	15.76	0.702068	0.05803493	1.463010	0.099989
3072.706	15.32	0.689253	0.06435276	1.453700	0.109299
3081.362	15.06	0.682418	0.06856746	1.446568	0.116430
3087.587	14.87	0.677400	0.07234365	1.441433	0.121566
3091.855	14.74	0.673959	0.07470536	1.437865	0.125133
3100.482	14.48	0.666885	0.07947659	1.430755	0.132203
3108.963	14.22	0.659850	0.08421546	1.423918	0.139080
3111.161	14.16	0.658013	0.08545732	1.422149	0.140849
3119.669	13.50	0.650838	0.09026912	1.415356	0.147643
3119.727	13.90	0.650769	0.09022160	1.415311	0.147688
3128.328	13.63	0.643419	0.09528518	1.408523	0.154476
3139.032	13.31	0.632610	0.09928375	1.405172	0.157826
3143.402	13.17	0.630334	0.10406357	1.397044	0.165955
3149.561	12.57	0.624541	0.10757119	1.392150	0.170849
3152.148	12.50	0.622600	0.10927248	1.390559	0.172439
3156.480	12.77	0.618735	0.11185616	1.387457	0.175541
3162.558	12.57	0.612861	0.11581624	1.382816	0.180182
3169.357	12.37	0.607121	0.11962658	1.378563	0.184435
3171.564	12.30	0.605116	0.12056661	1.377100	0.185898
3178.147	12.10	0.599100	0.12458545	1.372845	0.190153
3182.411	11.56	0.595188	0.12760520	1.370151	0.192807
3186.745	11.63	0.591173	0.13029146	1.367560	0.195439
3191.146	11.69	0.589450	0.13699025	1.355636	0.207363
3193.309	11.62	0.587476	0.13841677	1.354151	0.208808
3195.477	11.55	0.585495	0.13984931	1.352764	0.210234
3199.852	11.41	0.581514	0.14272237	1.349969	0.213009
3199.974	11.41	0.581390	0.14281541	1.349856	0.213102
3204.240	11.06	0.578440	0.14465725	1.348356	0.214602
3213.098	10.65	0.564945	0.15026256	1.348418	0.214581

3219.583	10.35	0.558558	0.16068828	1.333641	0.229358
3221.786	10.25	0.555723	0.16256631	1.331822	0.231177
3226.217	10.04	0.550052	0.16751760	1.328436	0.234563
3235.001	9.63	0.538899	0.17644721	1.322839	0.240160
3235.032	9.63	0.538851	0.17648673	1.322816	0.240183
3237.215	9.53	0.536108	0.17868406	1.321665	0.241334
3239.417	9.43	0.532346	0.18088931	1.320622	0.242376
3239.444	9.43	0.532317	0.18091214	1.320614	0.242385
3243.844	9.22	0.527819	0.18529618	1.318863	0.244136
3246.064	9.12	0.525062	0.18749124	1.318157	0.244842
3250.482	8.91	0.519617	0.19181865	1.317128	0.245870
3254.931	8.70	0.514209	0.19610101	1.316635	0.246364
3257.129	8.60	0.511559	0.19819397	1.316554	0.246404
3281.576	7.23	0.481892	0.22167712	1.327052	0.235946
3290.553	6.20	0.465109	0.22220650	1.368052	0.194947
3299.459	5.18	0.461262	0.23681462	1.353420	0.209568
3312.959	3.62	0.451047	0.31250125	1.202264	0.360737

Table XIX. Reduced Densities of Rubidium.

TEMPERATURE DEGREE F		SPECIFIC VOLUME LIQUID VAPOR		REDUCED DENSITY LIQUID VAPOR	
1115.56	0.41560	0.80209	433.5715	3.61377	0.00669
1124.85	0.41806	0.80898	5590.0195	3.56298	0.00052
1180.69	0.43279	0.81827	2205.0088	3.54229	0.00131
1222.54	0.44383	0.82535	1462.0552	3.51151	0.00198
1230.29	0.44587	0.82668	1384.7334	3.50625	0.00209
1244.24	0.44955	0.82912	1084.7427	3.49595	0.00267
1248.89	0.45078	0.82993	1013.5583	3.49253	0.00275
1272.16	0.45691	0.83392	1019.0825	3.47579	0.00284
1326.47	0.47124	0.84356	745.5631	3.43610	0.00389
1419.81	0.45586	0.86065	497.6494	3.36787	0.00582
1451.07	0.50411	0.86655	439.2888	3.34453	0.00660
1460.44	0.50658	0.86834	425.8699	3.33805	0.00681
1617.83	0.54810	0.89949	391.3850	3.22245	0.00741
1630.45	0.55143	0.89950	285.6028	3.22241	0.01015
1642.99	0.55473	0.89954	220.1644	3.22226	0.01317
1790.53	0.59365	0.93046	200.8514	3.11519	0.01443
1839.11	0.60647	0.94228	209.7060	3.07610	0.01382
1891.43	0.62027	0.95449	208.1240	3.03676	0.01393
1891.60	0.62031	0.95536	227.4154	3.03400	0.01275
1939.32	0.63290	0.96584	209.9197	3.00107	0.01381
1954.29	0.63685	0.96955	208.0461	2.98959	0.01393
1967.56	0.64035	0.97279	207.9786	2.97961	0.01394
1990.80	0.64648	0.97968	155.1580	2.96049	0.01485

2009.24	0.65134	0.98439	179.6302	2.94452	0.01614
2091.99	0.67317	1.00185	157.2489	2.89319	0.01843
2107.36	0.67723	1.00569	159.4016	2.88216	0.01818
2155.39	0.68989	1.01782	167.3155	2.84780	0.01732
2167.47	0.69308	1.02056	168.3554	2.83905	0.01722
2170.94	0.69400	1.02185	169.0788	2.83658	0.01714
2172.68	0.69445	1.02229	169.4179	2.83524	0.01711
2289.77	0.72534	1.05170	173.9451	2.75606	0.01666
2289.77	0.72534	1.05170	173.9447	2.75606	0.01666
2379.29	0.74895	1.07692	183.1073	2.69153	0.01583
2379.29	0.74895	1.07692	183.0982	2.69153	0.01583
2441.04	0.76524	1.09517	182.9317	2.64666	0.01584
2446.54	0.76669	1.09683	182.5565	2.64266	0.01588
2466.75	0.77202	1.10304	178.1661	2.62778	0.01627
2474.15	0.77358	1.10532	177.1273	2.62237	0.01635
2511.13	0.78373	1.11673	165.6161	2.55558	0.01750
2513.00	0.78423	1.11738	164.7355	2.59405	0.01760
2624.17	0.81355	1.15337	166.6485	2.51311	0.01739
2628.02	0.81456	1.15472	165.5776	2.51019	0.01751
2660.64	0.82317	1.16661	151.1043	2.48459	0.01918
2666.35	0.82468	1.16873	148.7498	2.48005	0.01949
2728.47	0.84106	1.19581	138.8994	2.42353	0.02087
2744.20	0.84521	1.20235	128.6536	2.41073	0.02253
2752.00	0.84727	1.20566	123.7023	2.40411	0.02343
2791.64	0.85773	1.22328	99.9892	2.36945	0.02899
2795.55	0.85876	1.22505	98.0872	2.36606	0.02955
2807.52	0.86192	1.23066	91.8510	2.35528	0.03156
2819.44	0.86506	1.23635	86.1225	2.34445	0.03366
2875.81	0.87993	1.26521	63.2538	2.29056	0.04582
2908.36	0.88852	1.28456	50.7427	2.25575	0.05712
2924.71	0.89283	1.29821	42.5270	2.23273	0.06816
2936.95	0.89606	1.30840	37.8469	2.21523	0.07659
2937.08	0.89609	1.30846	37.8505	2.21524	0.07658
2966.01	0.90372	1.33357	29.6549	2.17352	0.09709
2969.96	0.90476	1.33720	28.9592	2.16763	0.10009
3011.64	0.91576	1.37653	22.0489	2.10570	0.13146

3024.16	0.91906	1.38901	20.5279	2.08678	0.14120
3024.25	0.91908	1.38913	20.5108	2.08659	0.14132
3053.81	0.92688	1.42072	18.3582	2.04019	0.15789
3057.87	0.92795	1.42436	17.2210	2.03498	0.16822
3072.71	0.93187	1.45084	15.5391	1.99784	0.18653
3081.36	0.93415	1.46538	14.4996	1.97802	0.19991
3087.59	0.93579	1.47621	13.8229	1.96350	0.20969
3091.86	0.93692	1.48377	13.3659	1.95350	0.21654
3100.48	0.93919	1.49951	12.5623	1.93300	0.23037
3108.96	0.94143	1.51550	11.8737	1.91261	0.24411
3111.16	0.94201	1.51973	11.7017	1.90728	0.24770
3119.67	0.94425	1.53648	11.0755	1.88649	0.26171
3119.73	0.94427	1.53660	11.0715	1.88634	0.26180
3128.33	0.94654	1.55420	10.4948	1.86498	0.27619
3139.03	0.94936	1.58075	10.0721	1.83365	0.28778
3143.40	0.95051	1.58646	9.6095	1.82705	0.30163
3149.96	0.95224	1.60118	9.2617	1.81026	0.31296
3152.15	0.95282	1.60617	9.1514	1.80464	0.31673
3156.48	0.95356	1.61620	8.9401	1.79344	0.32422
3162.96	0.95567	1.63169	8.6244	1.77641	0.33570
3169.36	0.95736	1.64712	8.3593	1.75977	0.34674
3171.56	0.95794	1.65258	8.2667	1.75396	0.35063
3178.15	0.95968	1.66917	8.0007	1.73652	0.36229
3182.41	0.96080	1.68014	7.8367	1.72518	0.36987
3186.74	0.96195	1.69155	7.6751	1.71354	0.37766
3191.15	0.96311	1.69650	7.2598	1.70855	0.39707
3193.31	0.96368	1.70220	7.2246	1.70283	0.40121
3195.48	0.96425	1.70796	7.1506	1.69709	0.40536
3199.85	0.96541	1.71965	7.0066	1.68555	0.41369
3199.97	0.96544	1.72001	7.0020	1.68519	0.41396
3204.24	0.96656	1.72879	6.9110	1.67664	0.41941
3213.10	0.96890	1.77008	6.6541	1.63752	0.43560
3219.58	0.97061	1.79032	6.2232	1.61901	0.46576
3221.79	0.97119	1.79946	6.1362	1.61079	0.47237
3226.22	0.97236	1.81801	5.5695	1.59435	0.48556
3235.00	0.97468	1.85563	5.6674	1.56203	0.51144

3235.03	0.97469	1.85580	5.6661	1.56189	0.51156
3237.21	0.97526	1.86529	5.5565	1.55394	0.51792
3239.42	0.97584	1.87495	5.5282	1.54593	0.52432
3239.44	0.97585	1.87506	5.5275	1.54585	0.52438
3243.84	0.97701	1.89459	5.3568	1.52951	0.53709
3246.06	0.97760	1.90454	5.3336	1.52192	0.54345
3250.48	0.97876	1.92449	5.2133	1.50614	0.55600
3254.93	0.97993	1.94473	5.0994	1.49046	0.56841
3257.13	0.98051	1.95481	5.0456	1.48278	0.57448
3281.58	0.98696	2.07516	4.5111	1.39679	0.64254
3290.55	0.98933	2.15004	4.5003	1.24814	0.64408
3299.46	0.99168	2.16797	4.2227	1.33699	0.68642
3312.96	0.99524	2.21706	3.2000	1.26728	0.90580

A. Liquid Density

The zero-angle, one-phase points are directly observed for both capsules #1 and #4, and therefore, the liquid density equation must pass through these two points, where the liquid density equation can be written as,

$$\rho_L = \rho_0 + A_1(T_0 - T)^\beta + A_2(T_0 - T)^\gamma + A_3(T_0 - T)^2$$

To fit this curve for the liquid densities, the coefficients are obtained by step-wise regression analysis with the constraints.

However, the zero-angle points have several degrees of temperature uncertainty, and the following three cases are investigated.

case 1: The liquid density equation passing through both constraint points, $\rho = 0.37025$ gm/cc at $T = 3330.86^\circ\text{F}$ from capsule #1, and $\rho = 0.40911$ gm/cc at $T = 3324.83^\circ\text{F}$ from capsule #4.

case 2: The equation passing through only one zero-angle point from capsule #1, $\rho = 0.37025$ gm/cc at $T = 3330.86^\circ\text{F}$.

case 3: The equation passing through only one zero-angle point from capsule #4, $\rho = 0.40911$ gm/cc at $T = 3324.83^\circ\text{F}$.

The fitted curves of all three cases are fairly good with standard deviations of less than 0.01 gm/cc, and the coefficients of the fitted equations are summarized in the following table XX.

Table XX. The coefficients of Density Equation for Rubidium Liquid.

	<u>case 1</u>	<u>case 2</u>	<u>case 3</u>
$T_c, ^\circ P$	3331	3331	3331
$\rho_c, \text{gm/cc}$	0.333698	0.348224	0.34689
θ	0.76	0.76	0.76
A_1	0.117473×10^{-1}	0.109306×10^{-1}	0.109978×10^{-1}
A_2	-0.178233×10^{-2}	-0.162425×10^{-2}	-0.163696×10^{-2}
A_3	0.155806×10^{-6}	0.138995×10^{-6}	0.140259×10^{-6}
Standard Error, g/cc	0.00632	0.00581	0.00582

By graphical observation of the density envelope, figure 11, the critical density of 0.347 gm/cc is obtained, which does agree with the value obtained from the case 3. Therefore, it is decided to choose the coefficients from the case 3, and the fitted and observed density values are tabulated in table XXI.

Comparing this results with others at different temperature ranges, the following table is tabulated.

Table XXII. Saturated Liquid Densities of Rubidium (quoted from Ref.1)

<u>Observer</u>	<u>Method</u>	<u>T^oC</u>	<u>Deviation from recommended value, %</u>
Achener	Pycnometer	88 - 910	+0.4 - +0.2
Cohen	Buoyancy	39 - 400	+3.4 - -0.1
Dillon	Radioisotope	870 - 1720	-2.1 - +4.3
Shpil'rain	Pycnometer	50 - 800	-0.5 - -2.7
Tepper	Dilatometer	70 - 1090	+1.2 - +2.0
Hackspill	Dilatometer	38 - 220	+0.1 - -0.9
This study	Tilt Angle	580 - 1820	

It is difficult to see that which values should be the best acceptable data. However, the results from this study are consistently lower by 0.5 % than those by Tepper for the over-laping temperature range, and if the recommended values in the above table are acceptable, then, the results in this study are approximately 1 % off from the acceptable values as quoted in the table (1).

B. Vapor Density

The vapor density is generally inaccurate at low temperatures because the moment equation is incapable of giving the accurate values of the vapor densities at low temperatures. In the moment equation, density terms are either in the form of $(\rho_L + \rho_v)$ or $(\rho_L - \rho_v)$, and unless ρ_v is in the same order of magnitude with ρ_L , one can not expect an accurate estimation of vapor densities.

The vapor densities of rubidium is fitted into the virial equation using the points at temperatures higher than 3020° F.

$$\frac{PV}{RT} = A_0 + A_1 \rho_v + A_2 \rho_v^2 + \dots + A_n \rho_v^n$$

where, $n = 4$ and 5 are tested and found that standard deviation is 0.01116 from $n = 5$ compared with 0.00756 from $n = 4$. The coefficients are,

$n = 4$

$$\begin{aligned} A_0 &= 2.20837 \\ A_1 &= -32.1118 \quad \text{cm}^3/\text{gm} \\ A_2 &= 222.211 \quad (\text{cm}^3/\text{gm})^2 \\ A_3 &= -714.938 \quad (\text{cm}^3/\text{gm})^3 \\ A_4 &= 855.308 \quad (\text{cm}^3/\text{gm})^4 \end{aligned}$$

$$\begin{aligned}
 \underline{n = 5} \quad & A_0 = 1.96823 \\
 & A_1 = -22.4494 \quad \text{cm}^3/\text{gm} \\
 & A_2 = 80.9039 \quad (\text{cm}^3/\text{gm})^2 \\
 & A_3 = 234.270 \quad (\text{cm}^3/\text{gm})^3 \\
 & A_4 = -2101.81 \quad (\text{cm}^3/\text{gm})^4 \\
 & A_5 = 3440.66 \quad (\text{cm}^3/\text{gm})^5
 \end{aligned}$$

Observing the variance, the virial equation with $n = 4$ is recommended for the temperature range from 2700°F to 3320°F . The fitted values are tabulated in table XXIII along with the observed vapor density values.

C. Rectilinear Diameter

Average densities of saturated liquid and vapor, defined as rectilinear diameter, are fitted to polynomials by the least square method.

$$RD = \frac{\rho_L + \rho_V}{2} = A + B T + C T^2 + \dots + N T^n$$

Eventhough the vapor densities at low temperatures are not reliable, their values are too low to give any significant contribution on RD. The resulting coefficients are as following;

$$\begin{aligned}
 \underline{n = 2} \quad & A = 0.750956 \\
 & B = -0.115086 \times 10^{-3} \\
 & C = -0.202801 \times 10^{-8} \\
 \\
 \underline{n = 3} \quad & A = 0.755934 \quad D = -0.519248 \times 10^{-12} \\
 & B = -0.122596 \times 10^{-3} \\
 & C = 0.149040 \times 10^{-8}
 \end{aligned}$$

Standard deviations are both comparable with the values of 0.0367 and 0.0370 gm/cc respectively. However, their deviations are better distributed for the higher order polynomial, and the fitted and observed values are given in table XXIV.

D. Critical Properties of Rubidium

1. Graphical and analytical study of the saturated liquid densities yields, as given in section A, critical temperature of 3331°F and critical density of 0.347 gm/cc. These values are within the experimental errors of the values given by Hochman (26) and Bhise (5) in table III.
2. Using the above critical values, simple power law is tested as proposed by Tisza (42), Fisher (18) and Guggenheim (23). $(\rho_L - \rho_v)/2$ is plotted against $(T_c - T)$ in log-log scale, and a straight line is obtained in figure 13. Using the points at temperatures higher than 3000°F , the slope of this line is 0.55.
3. Guggenheim's correlation (23) for simple fluids is plotted in figure 14; Z versus $T^{\circ}\text{F}$. In this figure, the line becomes fairly straight as temperatures approach its critical value, and intersects at $T_c = 3329^{\circ}\text{F}$. Where, Z is defined in figure 14.
4. Kordes plot (30) for covalent substances is shown in figure 15 by plotting $(1/\bar{v})/\bar{\rho}$ against $\ln(T)$ in semi-log paper. Again, a straight line is observed with the critical temperature of 3335°F by extrapolating $(1/\bar{v})/\bar{\rho}$ to 1.
5. Buckingham correlation (20,35) shows that when temperatures approach the critical value, $X = Z^2/(1 - \ln Z) \rightarrow 0$,

FIGURE 13. Correlation of power law

FIGURE 14. Guggenheim Correlation

where, $Z = (\rho_L - \rho_V) / (\rho_L + \rho_V)$. Figure 16 is a straight line correlation; X versus $(1 - T/T_c)$ with the extrapolated value of $T_c = 3313^\circ\text{F}$. The straight line begins to level off for temperatures below 3000°F .

6. Entalpy, internal energy and entropy of vaporization are calculated according to the equations (18), (19) and (20) near the critical region, and are tabulated in table XXV. Their extrapolations give the critical temperature of rubidium, $T_c = 3328 \pm 5^\circ\text{F}$. The figures 17 and 18 are their plots.
7. Critical compressibility, $Z_c = 0.189$ is obtained based on the critical values of $T_c = 3331^\circ\text{F}$, $\rho_c = 0.347 \text{ gm/cc}$ and $P_c = 1943 \text{ psia}$. Where, the critical pressure of rubidium was previously obtained in this laboratory (5).

E. Conclusion

1. Critical temperature of rubidium is $3331^\circ\text{F} \pm 10^\circ\text{F}$.
2. Critical density of rubidium is $0.347 \text{ gm/cc} \pm 0.5\%$.
3. Critical compressibility of rubidium is 0.189 ± 0.001 .

Table XXI. Fitted and Observed Saturated Liquid
Densities of Rubidium. $\beta = 0.76$,
X in P, Y in gm/cc.

NO. POINT	VARIABLE X	RESPONSE, Y		DIFFERENCE
		OBSERVED	CALCULATED	
1	2215.443	1.24675	1.24448	.00225
2	2206.146	1.23613	1.24171	-.00559
3	2150.311	1.22209	1.22523	-.00313
4	2108.459	1.21161	1.21313	-.00152
5	2100.707	1.20935	1.21093	-.00128
6	2086.756	1.20610	1.20597	-.00087
7	2082.107	1.20432	1.20555	-.00073
8	2056.844	1.19915	1.19910	.00005
9	2004.530	1.19545	1.18409	.00137
10	1911.186	1.16191	1.15910	.00282
11	1879.935	1.15400	1.15093	.00307
12	1870.557	1.15153	1.14052	.00111
13	1713.170	1.11174	1.10910	.00265
14	1700.548	1.11173	1.10674	.00500
15	1608.011	1.11158	1.10300	.00860
16	1540.469	1.07474	1.05823	.00651
17	1491.886	1.05125	1.05757	-.00418
18	1439.566	1.04758	1.04520	.00240
19	1439.404	1.04673	1.04515	.00158
20	1391.679	1.03537	1.03439	.00098
21	1376.712	1.03141	1.03174	-.00037
22	1363.436	1.02797	1.02806	-.00009
23	1340.194	1.02137	1.02297	-.00150
24	1321.760	1.01596	1.01074	.00288
25	1239.016	.99815	1.00028	-.00213

26	1223.637	.99415	.99693	-.00249
27	1175.611	.98249	.98607	-.00358
28	1163.533	.97947	.98335	-.00389
29	1160.061	.97842	.98257	-.00395
30	1158.324	.97819	.98219	-.00397
31	1041.232	.95042	.95543	-.00499
32	1041.232	.95034	.95543	-.00499
33	951.713	.92858	.93424	-.00566
34	951.713	.92858	.93424	-.00566
35	889.957	.91310	.91905	-.00595
36	884.463	.91172	.91758	-.00596
37	864.254	.90658	.91256	-.00597
38	856.855	.90472	.91067	-.00595
39	819.871	.89547	.90106	-.00559
40	817.996	.89475	.90057	-.00562
41	706.828	.86772	.86976	-.00273
42	702.982	.86601	.86843	-.00242
43	670.363	.85718	.85898	-.00179
44	664.648	.85533	.85715	-.00182
45	602.531	.83625	.83740	-.00114
46	586.803	.83170	.83214	-.00044
47	579.006	.82942	.82949	-.00007
48	539.359	.81747	.81537	.00211
49	535.451	.81659	.81416	.00243
50	523.477	.81257	.80976	.00281
51	511.559	.80843	.80531	.00312
52	455.189	.79038	.78303	.00735
53	422.639	.77823	.76913	.00910
54	406.291	.77029	.76143	.00886
55	394.049	.76429	.75520	.00907
56	394.049	.76426	.75520	.00905
57	364.988	.74916	.74028	.00887
58	361.045	.74733	.74032	.00701
59	319.363	.72647	.71057	.01590
60	306.038	.71944	.71151	.00793
61	306.752	.71937	.71156	.00781

62	277.191	70397	69426	.00961
63	273.127	70297	69178	.01029
64	258.295	68925	68248	.00677
65	249.639	68242	67698	.00554
66	243.410	67741	67276	.00465
67	239.143	67396	66990	.00406
68	230.518	66680	66400	.00289
69	222.035	65995	65805	.00180
70	219.838	65801	65648	.00153
71	211.330	65094	65031	.00053
72	211.271	65079	65027	.00052
73	202.674	64312	64396	-.00044
74	191.967	63231	63552	-.00301
75	187.600	63033	63217	-.00184
76	181.039	62454	62639	-.00235
77	175.852	62250	62510	-.00250
78	174.518	61873	62152	-.00278
79	168.041	61286	61606	-.00320
80	161.645	60712	61053	-.00341
81	159.436	60512	60859	-.00347
82	152.854	59910	60270	-.00360
83	148.588	59519	59841	-.00362
84	144.256	59117	59478	-.00361
85	139.854	58945	59061	-.00115
86	137.687	58728	58853	-.00105
87	135.523	58519	58613	-.00094
88	131.148	58151	58213	-.00062
89	131.023	58139	58201	-.00062
90	126.762	57844	57773	.00071
91	117.900	56474	56356	-.00362
92	111.414	55856	56130	-.00304
93	109.213	55572	55718	-.00345
94	104.781	55015	55424	-.00418
95	95.996	53820	54408	-.00519
96	95.967	53835	54404	-.00519
97	93.783	53611	54144	-.00533

98	91.582	53335	53678	-.00543
99	91.557	53332	53375	-.00543
100	87.156	52792	53333	-.00551
101	84.936	52576	53354	-.00548
102	80.518	51962	52497	-.00525
103	76.070	51421	51900	-.00479
104	73.871	51156	51603	-.00447
105	49.420	48189	47948	.00241
106	40.445	46511	46394	.00117
107	31.541	46126	44671	.01435
108	18.039	45135	41650	.03455

Table XXIII. The Fitted and Observed Rubidium Densities
of Saturated Liquid and Vapor.
X in gm/cc, Y in PV/RT.

NO. POINT	VARIABLE X	RESPONSE, Y		DIFFERENCE
		OBSERVED	CALCULATED	
1	.049	1.11497	1.09294	.02193
2	.054	1.02140	1.01050	.01090
3	.058	.96173	.95313	.00860
4	.064	.87754	.88675	-.00921
5	.069	.82374	.83547	-.01173
6	.072	.78005	.80099	-.02094
7	.075	.76610	.77814	-.01204
8	.079	.72494	.73505	-.01011
9	.084	.66834	.69600	-.02766
10	.085	.67936	.68610	-.00674
11	.090	.64688	.65112	-.00424
12	.090	.64679	.65089	-.00410
13	.095	.61671	.61800	-.00129
14	.099	.59631	.59400	.00231
15	.104	.57058	.56769	.00289
16	.108	.55239	.54805	.00434
17	.109	.54664	.54185	.00479
18	.112	.53553	.53004	.00549
19	.116	.51944	.51313	.00631

20	.120	.50514	.49814	.00695
21	.121	.50016	.49313	.00703
22	.125	.48620	.47893	.00727
23	.128	.47748	.47028	.00720
24	.130	.46932	.46188	.00704
25	.137	.44727	.44239	.00459
26	.138	.44325	.43830	.00435
27	.140	.43934	.43521	.00413
28	.143	.43151	.42805	.00355
29	.143	.43138	.42793	.00355
30	.145	.42642	.42335	.00357
31	.150	.41326	.41030	.00245
32	.161	.38833	.38975	.00191
33	.163	.38315	.38555	.00251
34	.168	.37373	.37747	.00374
35	.176	.35657	.35215	.00548
36	.176	.35655	.35204	.00548
37	.179	.35264	.35346	.00582
38	.181	.34891	.35476	.00595
39	.181	.34678	.35458	.00590
40	.185	.34135	.34748	.00613
41	.187	.33791	.34332	.00612
42	.192	.33134	.33677	.00573
43	.196	.32462	.32978	.00517
44	.198	.32150	.32540	.00479
45	.222	.29146	.28692	.00464
46	.222	.29221	.28597	.00634
47	.237	.27543	.26078	.01465
48	.313	.21023	.21249	.00225

Table XXIV. Correlation of Rectilinear Diameter of
Rubidium Densities into Parabolic
Equation.

X in degree F, Y in gm/cc.

DEGREE OF FREEDOM= 105.

VARIANCE= .00141

STANDARD ERROR ESTIMATED= .00367

NO. POINT	VARIABLE X	RESPONSE, Y	
		OBSERVED	CALCULATED
1	1115.556	.62453	.62005
2	1124.854	.61615	.61894
3	1180.688	.61127	.61225
4	1222.540	.60615	.60723
5	1230.292	.60519	.60530
6	1244.243	.60351	.60452
7	1248.893	.60294	.60406
8	1272.155	.60036	.60127
9	1726.470	.59340	.59473
10	1419.814	.58196	.58347
11	1451.065	.57814	.57959
12	1460.443	.57699	.57856
13	1617.830	.56715	.55946
14	1630.452	.55752	.55792
15	1642.989	.55811	.55640
16	1790.531	.53986	.53839
17	1839.114	.53301	.53244
18	1891.434	.52624	.52673
19	1891.596	.52556	.52501
20	1939.321	.52057	.52014
21	1954.288	.51811	.51830
22	1967.564	.51639	.51657
23	1990.806	.51325	.51381

24	2009.240	51071	51154
25	2031.934	50225	50133
26	2107.363	50071	49942
27	2155.389	49423	49348
28	2167.467	49271	49199
29	2170.939	49227	49156
30	2172.676	49235	49134
31	2249.768	47830	47691
32	2269.768	47830	47691
33	2379.287	46752	46566
34	2379.287	46752	46566
35	2441.043	45928	45795
36	2446.537	45860	45726
37	2466.746	45610	45473
38	2474.145	45518	45391
39	2511.129	45075	44918
40	2513.004	45051	44894
41	2624.172	43651	43499
42	2628.018	43603	43451
43	2660.637	43190	43040
44	2666.352	43118	42968
45	2722.469	42173	42135
46	2744.197	41974	41987
47	2751.994	41875	41899
48	2791.641	41374	41398
49	2795.549	41324	41339
50	2807.523	41173	41187
51	2819.441	41022	41036
52	2875.611	40309	40322
53	2908.361	39827	39910
54	2924.709	39690	39702
55	2936.951	39536	39547
56	2936.951	39534	39547
57	2966.012	39168	39177
58	2969.955	39118	39127
59	3011.637	38591	38597
60	3024.162	38433	38438
61	3024.248	38431	38436
62	3053.809	37917	38060
63	3057.873	38005	38008
64	3072.705	37680	37619
65	3081.361	37569	37708
66	3087.590	37498	37629
67	3091.857	37433	37574
68	3100.482	37318	37464
69	3103.965	37203	37356
70	3111.162	37173	37328
71	3119.670	37056	37219
72	3119.729	37055	37219
73	3128.326	36935	37109
74	3139.033	36595	36972
75	3143.400	36720	36916
76	3149.961	36626	36832
77	3152.148	36594	36804

78	3156.482	36530	35749
79	3162.959	36434	35655
80	3169.355	36337	35594
81	3171.564	36304	35555
82	3178.146	36204	35472
83	3182.412	36140	35417
84	3186.744	36073	35362
85	3191.146	36022	35305
86	3193.312	36295	35278
87	3195.477	36257	35250
88	3199.852	36212	35194
89	3199.977	36210	35192
90	3204.238	36157	35138
91	3213.100	35751	35024
92	3219.586	35952	35941
93	3221.787	35934	35913
94	3226.219	35878	35856
95	3235.004	35757	35743
96	3235.033	35757	35743
97	3237.217	35740	35715
98	3239.418	35712	35697
99	3239.443	35711	35586
100	3243.844	35656	35610
101	3246.064	35628	35602
102	3250.482	35572	35545
103	3254.930	35515	35498
104	3257.129	35498	35450
105	3261.580	35178	35146
106	3290.555	34356	35031
107	3299.459	34904	34916
108	3312.961	38177	34743

Table XXV. Thermodynamic Properties of Vaporization
for Rubidium in British Units.

TEMPERATURE R	REDUCED TEMP.	PRESSURE PSI.	ENTHALPY	ENTROPY	INTERNAL ENERGY
.308387E 04	.813536E 00	.865353E 03	.211943E 04	.607262E 00	.169428E 04
.308772E 04	.814550E 00	.871795E 03	.211418E 04	.684706E 00	.168913E 04
.312034E 04	.823155E 00	.918064E 03	.199155E 04	.638249E 00	.158338E 04
.312605E 04	.824653E 00	.925356E 03	.197121E 04	.630574E 00	.156582E 04
.313817E 04	.841049E 00	.101746E 04	.194484E 04	.610018E 00	.152939E 04
.320390E 04	.845199E 00	.104107E 04	.182370E 04	.569214E 00	.143025E 04
.321169E 04	.847255E 00	.105280E 04	.176381E 04	.549185E 00	.138140E 04
.325134E 04	.857714E 00	.111341E 04	.146605E 04	.450905E 00	.113997E 04
.325525E 04	.858745E 00	.111942E 04	.144185E 04	.442930E 00	.112033E 04
.326722E 04	.861974E 00	.113783E 04	.135035E 04	.416366E 00	.105461E 04
.327914E 04	.865048E 00	.115548E 04	.128495E 04	.391855E 00	.993853E 03
.333551E 04	.879919E 00	.124509E 04	.972175E 03	.291452E 00	.743314E 03
.336806E 04	.888505E 00	.122673E 04	.789505E 03	.234409E 00	.599334E 03
.338441E 04	.892818E 00	.132301E 04	.663645E 03	.196089E 00	.501903E 03
.339665E 04	.896048E 00	.134276E 04	.591545E 03	.174185E 00	.446163E 03
.339665E 04	.896048E 00	.134276E 04	.591702E 03	.174201E 00	.446206E 03
.342571E 04	.903714E 00	.138956E 04	.467731E 03	.136550E 00	.350263E 03
.347965E 04	.904754E 00	.139588E 04	.453751E 03	.132302E 00	.339420E 03
.347134E 04	.915750E 00	.145330E 04	.344950E 03	.993450E -01	.255162E 03
.348386E 04	.919054E 00	.148361E 04	.320545E 03	.920144E -01	.236369E 03
.348395E 04	.919077E 00	.148364E 04	.320252E 03	.919250E -01	.236139E 03
.351351E 04	.926875E 00	.153171E 04	.286091E 03	.814232E -01	.209153E 03
.351757E 04	.927948E 00	.153915E 04	.267230E 03	.759700E -01	.195136E 03
.353240E 04	.931850E 00	.155228E 04	.230000E 03	.676649E -01	.173755E 03

354106E 04	934144E 00	157531E 04	221545E 03	625648E-01	160622E 03
354729F 04	935787E 00	158540E 04	210121E 03	592342E-01	152041E 03
355156E 04	936913E 00	159339E 04	202729E 03	570810E-01	146494E 03
356018E 04	939198E 00	160717E 04	189042E 03	530988E-01	136225E 03
356866E 04	941425E 00	162094E 04	175931E 03	495789E-01	127144E 03
357086E 04	942005E 00	162451E 04	173991E 03	487225E-01	124934E 03
357937E 04	944250E 00	163829E 04	163196E 03	455935E-01	116856E 03
357943E 04	944255E 00	163839E 04	163127E 03	455734E-01	116804E 03
358803E 04	946533E 00	165193E 04	153092E 03	426695E-01	109303E 03
359873E 04	949358E 00	165925E 04	145577E 03	404522E-01	103546E 03
360310E 04	950510E 00	167645E 04	137619E 03	381942E-01	977344E 02
360966E 04	952240E 00	168670E 04	131449E 03	364156E-01	931341E 02
361185E 04	952817E 00	169037E 04	129503E 03	358551E-01	916837E 02
361618F 04	953951E 00	169734E 04	125741E 03	347719E-01	888604E 02
362266E 04	955669E 00	170756E 04	120269E 03	331990E-01	848096E 02
362905E 04	957357E 00	171799E 04	115320E 03	317769E-01	811265E 02
363126E 04	957940E 00	172149E 04	113643E 03	312956E-01	790800E 02
363785E 04	959676E 00	173200E 04	108877E 03	299096E-01	762895E 02
364211F 04	960801E 00	173990E 04	105814E 03	290530E-01	740696E 02
364644E 04	961944E 00	174544E 04	102925E 03	281986E-01	718560E 02
365085E 04	963105E 00	175277E 04	963035E 02	263798E-01	671851E 02
365301E 04	963676E 00	175528E 04	949199E 02	259840E-01	661607E 02
365518E 04	964248E 00	175948E 04	935343E 02	255097E-01	651393E 02
365955E 04	965402E 00	175538E 04	908515E 02	248259E-01	631599E 02
365968E 04	965435E 00	175552E 04	907639E 02	248010E-01	630958E 02
366394E 04	966559E 00	177325E 04	890378E 02	243011E-01	617892E 02
367280E 04	968896E 00	178742E 04	838950E 02	228422E-01	580089E 02
367929E 04	970608E 00	179759E 04	761213E 02	206691E-01	524914E 02
368149E 04	971199E 00	180097E 04	744530E 02	202236E-01	512933E 02
368592E 04	972357E 00	180793E 04	712504E 02	193304E-01	489944E 02
3689470E 04	974675E 00	182183E 04	553773E 02	176949E-01	447855E 02
369473E 04	974683E 00	182176E 04	553435E 02	176869E-01	447652E 02
369692E 04	975259E 00	182508E 04	539745E 02	173048E-01	437822E 02

.369912E 04	.975840E 00	.182875E 04	.525333E 02	.169320E-01	.428228E 02
.369914E 04	.975846E 00	.182881E 04	.525204E 02	.169283E-01	.428134E 02
.370354E 04	.977007E 00	.183551E 04	.500133E 02	.162043E-01	.409509E 02
.370576E 04	.977593E 00	.183918E 04	.537494E 02	.158535E-01	.400407E 02
.371018E 04	.978758E 00	.184514E 04	.563190E 02	.151796E-01	.783156E 02
.371463E 04	.979931E 00	.185295E 04	.539994E 02	.145343E-01	.366563E 02
.371683E 04	.980512E 00	.185546E 04	.528529E 02	.142279E-01	.358608E 02
.374128E 04	.986962E 00	.189406E 04	.415124E 02	.110958E-01	.278315E 02
.375025E 04	.989329E 00	.190791E 04	.399849E 02	.106619E-01	.266886E 02
.375916E 04	.991679E 00	.192159E 04	.348929E 02	.928211E-02	.231850E 02
.377266E 04	.995240E 00	.194215E 04	.166393E 02	.441041E-02	.109784E 02

VII. RECOMMENDATION FOR FUTURE WORK

It has been recognized that the tilting tube method is applicable for the substances with reasonably high density values, such as mercury. However, zero-angle, one-phase point can be located without any difficulty even for K or Na since the capsule tends to vibrate near such point.

If several capsules are used with different mass of a substance in the capsule, one can directly locate the critical point accurately. Furthermore, each capsule is only for locating one phase point, less sensitive pivot arrangement could be used.

For mercury this method is particularly attractive since the critical density of mercury is approximately ten times higher than that of alkali metal rubidium, and thus one can construct a heavy wall capsule without losing the pivot moment.

VIII. APPENDIX

A. Conceptual Designs of Apparatus

Heat Loss

It is desirable to choose a proper gas as external balancing gas to compensate the rubidium vapor pressure, since a minimum loss of heat through the gas would be the prime factor in choosing such gas. Helium and Argon gases are considered mainly because of their inert nature. Based on the fact that the balancing gas will occupy the annulus inside of the heater surrounding rubidium capsule, heat transfer properties of both argon and helium gases are compared under the experimental conditions of following:

1. I.D. of heater tube is 1 inch, capsule O.D. as $\frac{1}{4}$ inch with 500°F temperature gradient as extreme case.
2. Heat is transferred by both conduction and natural convection, and radiation transfer is identical for both cases. Also, both gases behave like ideal gas.

The following table is summarized to show the heat transfer properties of two gases.

	<u>Argon</u>	<u>Helium</u>
$Nu,$	5.6	2.195
$K, \text{ btu/hr-ft}^2\text{-}^{\circ}\text{F/ft}$	0.01022	0.086
$h, \text{ btu/hr-ft}^2\text{-}^{\circ}\text{F}$		
capsule surface	2.745	9.05
inside heater surface	4.35	24.85
Heat Loss, $\text{btu/hr-}\Delta T^{\circ}\text{F}$	0.178	0.735
watt/ $\Delta T^{\circ}\text{F}$	0.0522	0.2154

Fiber-Stress of Knife-Edge

The fiber stress of W-30Mo-30Re is compared with the maximum stress on the knife blade.

$$S_{\text{max.}} = \frac{12 Wc}{d^2 b}$$

where, c is blade length, b is width, and the thickness from the tip of the blade to the top is d , as shown in figure 3a. Based on 50 gms of assembly weight W , the fiber-stress on the blade would be 460 psi, at most, and W-30Mo-30Re can sustain more than 20 hours of continuous heating under 800 psi of stress at 4000°F.

Argon Pressure Balance

For six inches long capsule with O.D. of $\frac{1}{4}$ " and I.D. of 0.218", rupture pressure of the capsule is estimated with the thin wall theory.

$$P_{\text{rupt.}} = 0.132 S_{\text{wall}}$$

During 10 hours of operation at 3300°F, S_{wall} is approximately 15000 psi for W-30Mo-30Re. Allowing a safety factor of two, permissible pressure differential between inside and outside of the capsule is 99 psi. Thus, the external argon pressure is maintained at 50 psi higher than the vapor pressure of rubidium near the critical temperature.

B. Moment Equation

When a cylindrical capsule is resting at an angle of θ as shown in figure 19b, the vapor and liquid in the capsule would be in equilibrium, only separated by an interface. Neglecting the

Figure 19. Moment Balance and Equilibrium Angle of Rubidium Capsule.

meniscus due to the interfacial tension, their respective "center of gravity" can be located from the figure 19a of a truncated-right circular cylinder with an average height of h .

$$X = \frac{h}{2} + \frac{R^2 \cot^2 \theta}{8h} \quad (B-1)$$

$$Y = \frac{R^2 \cot \theta}{4h}$$

Denoting subscripts L and v as liquid and vapor respectively, a rubidium capsule with the inside volume of $V \text{ cm}^3$ is charged with $M \text{ gm}$ of rubidium under vacuum.

$$\begin{aligned} M &= m_L + m_v \\ V &= V_L + V_v \end{aligned} \quad (B-2)$$

and,

$$\begin{aligned} \rho_L &= \frac{m_L}{V_L} \\ \rho_v &= \frac{m_v}{V_v} \end{aligned} \quad (B-3)$$

under the equilibrium condition, the vertical moment of the capsule should be zero, and the horizontal moment, S_x , should be same for both moment arms. Referring to the figure 19c,

$$S_{m_L} = S_{m_v} + S_{W_y} + S_{W_c} \quad (B-4)$$

Since S_{W_c} is comparatively smaller than S_{W_y} , two terms can be combined together into one single term of S_W . Using the subscripts c and y as capsule and yoke respectively, total pivot distance $\bar{P}$ can be weighed as following.

$$\bar{P} = \frac{\bar{P}_c W_c + \bar{P}_y W_y}{W_c + W_y} \quad (B-5)$$

X-moment of the tilted capsule with the internal length of L is,

$$S_{m_L} = \left[\left(\frac{1}{2} - x_L \right) - (\bar{P} + Y_L) \tan \theta \right] m_L \cos \theta \quad (B-6)$$

$$S_{m_V} = \left[\left(\frac{1}{2} - x_V \right) + (\bar{P} - Y_V) \tan \theta \right] m_V \cos \theta \quad (B-7)$$

$$S_W = W \bar{P} \tan \theta \cdot \cos \theta \quad (B-8)$$

Substituting these equations into (B-4) and rearranging the resulting equation using the equation (B-1),

$$\begin{aligned} & m_L \left[\frac{L}{2} - \frac{LR^2 \cot^2 \theta}{8h_L h_V} - \frac{LR^2 \cot \theta \cdot \tan \theta}{4h_L h_V} \right] - M \left[\frac{h_L}{2} - \right. \\ & \left. - \frac{R^2 \cot^2 \theta}{8h_V} - \frac{R^2 \cot \theta \cdot \tan \theta}{4h_V} \right] - (M+W) \bar{P} \tan \theta = 0 \end{aligned} \quad (B-9)$$

If $\tan^2 \theta$ is multiplied and collect the terms,

$$\tan^3 \theta + X \tan^2 \theta + Y = 0 \quad (B-10)$$

liquid and vapor densities are,

$$\rho_L = \frac{m_L}{\pi R^2 h_L} \quad (B-11)$$

$$\rho_V = \frac{m_V}{\pi R^2 h_V}$$

their respective weights are,

$$\begin{aligned} m_L &= \left(\frac{\rho_L}{\rho_L - \rho_v} \right) (M - \rho_v V) \\ m_v &= \left(\frac{\rho_v}{\rho_L - \rho_v} \right) (V \rho_L - M) \end{aligned} \quad (B-12)$$

where,

$$W = W_c + W_y$$

$$x = \frac{1}{2\bar{P}(M+W)} \left[\left(\frac{M - V\rho_v}{\rho_L - \rho_v} \right) \left(\frac{M}{V} - \rho_L \right) L + \frac{R^2 V}{2L} (\rho_L - \rho_v) \right] \quad (B-13)$$

$$y = \frac{R^2 V (\rho_L - \rho_v)}{8\bar{P}L(M+W)}$$

Minimum Allowable Angle

If the liquid and vapor interface is either to fall on the corner of the capsule or to touch the end plug, the moment balance equation (B-10) would not be valid, and this limiting angle ϕ is given by the following:

$$\tan \phi = \frac{R}{h_L}$$

However, when this interface line is on the corner, $h_L = h_v = L/2$. Thus,

$$\phi = \arctan \left(\frac{2R}{L} \right) \quad (B-14)$$

Buoyancy and Expansion Correction

Since test temperature is in the range above 2000°F, each linear dimensions are corrected for their thermal expansion by multiplying Z as defined in the following:

$$Z = 1 + \alpha \quad (B-15)$$

where, α is linear expansion coefficient in fraction and their values are tabulated in table VI.

Due to the high pressure, as high as 2000 psi near the critical point, the buoyancy force exerted by argon gas would be considerable since the density of the argon gas is not negligible. Denoting the subscripts a and b as for argon gas and buoyancy, weights and density are,

$$W_b = (V_c + V_y) P_a$$

$$\rho_a = \frac{P_a M_a}{RT}$$

Total effective weight, W, in the moment equation should be,

$$W = W_c + W_y - \frac{(V_c + V_y) P_a M_a}{RT} \quad (B-16)$$

P_a will be directly read from the pressure gauge during the experimental runs, and,

M_a = molecular weight of argon (39.95)

R = gas constant (82.054 cc-atm/mole-°K)

Recti-Diameter Equation

Defining Recti-Diameter, RD, as average density of the saturated vapor and liquid densities at a given temperature,

$$RD = \frac{\rho_L + \rho_v}{2} \quad (B-17)$$

Then, it would be easy to show the equality of $\overline{AB}$ and $\overline{BD}$ in the figure 20a and the above equation.

$$\overline{AB} = \overline{BD} = RD - \rho_v = \rho_L - RD$$

If both coordinates are transformed to $\rho' = \rho - \rho_c$, and $T = T_c - T$, and all of the density points are subtracted by $\overline{BC} = \overline{B'C'}$. Then, the dotted line, resulted from the coordinate transformation, will move to the solid line in figure 20b.

$$B' - \overline{BC} = B'', \quad C' - \overline{BC} = C'', \quad A' - \overline{BC} = A'', \quad D' - \overline{BC} = D''$$

and new coordinates of ρ'' and T'' become,

$$\begin{aligned} \rho'' &= \rho' - (RD - \rho_c) = \rho - RD \\ T'' &= T' = T_c - T \end{aligned}$$

Hence, the solid line will be a parabola with respect to ρ'' and T'' coordinate system.

$$T'' = T_c - T = (\text{constant}) (\rho_L - RD)^2 = (\text{constant}) (RD - \rho_v) \quad (B-18)$$

(a)

(b)

FIGURE 20

C. Numerical Methods

Moment Equation

The moment equations at a constant temperature is given by,

$$F_i(\theta_i, \rho_L, \rho_v) = \tan^3 \theta_i + x_i \tan^2 \theta_i + I_i = 0 \quad (C-1)$$

The subscript i represents each capsule, i=1 or 2. Where,

$$x \equiv \frac{1}{2\bar{P}_i(M_i + W_i)} \left[\frac{V_i L_i (\bar{P} - \rho_v)(\bar{P} - \rho_L)}{(\rho_L - \rho_v)} + \frac{(R_i^2 V_i)}{2L_i} (\rho_L - \rho_v) \right]$$

$$I \equiv \frac{\pi R_i^4 (\rho_L - \rho_v)}{8\bar{P}_i^3 (M_i + W_i)}$$

$$\bar{\rho} = \frac{M_i}{V_i}$$

and,

L = inside length of capsule

R = inside radius capsule

θ = angle in degree at a given temperature

M = rubidium charge in the capsule

V = inside volume of capsule

W = effective weight of yoke and capsule

$\bar{P}$ = total effective pivot length

Observing the moment equation (C-1), it is non-linear with respect to ρ_L and ρ_v , and is linearized in order to solve for ρ_L and ρ_v by expanding it into Taylor's series but truncating the higher

order terms.

$$\begin{aligned}
 F_i(\rho_L, \rho_V) = F_i(\rho_L^0, \rho_V^0) + (\rho_L - \rho_L^0) \frac{\partial F_i}{\partial \rho_L} \bigg|_{\substack{\rho_L = \rho_L^0 \\ \rho_V = \rho_V^0}} \\
 + (\rho_V - \rho_V^0) \frac{\partial F_i}{\partial \rho_V} \bigg|_{\substack{\rho_L = \rho_L^0 \\ \rho_V = \rho_V^0}} \quad (C-2)
 \end{aligned}$$

Representing two capsules at a constant temperature, a pair of equations (C-1) for $i=1$ and 2 is solved simultaneously by successive iterations, starting with a pair of initial values of ρ_L^0 and ρ_V^0 . Thus, the unknowns ρ are transformed to $\Delta\rho = \rho - \rho^0$ in the equation (C-2) and successive iterations are continued until the moment equation F_i is equal or less than 10^{-6} . Otherwise, a new set of ρ^0 is evaluated from $\Delta\rho$, obtained from the previous ρ^0 .

The partial derivatives in the expanded equation (C-2) are,

$$\begin{aligned}
 \frac{\partial F_i}{\partial \rho_L} = \frac{\tan^2 \theta_i}{2\bar{\rho}_i(M_i + W_i)} \left[\frac{M_i(\rho_L V_i - M_i)}{\pi R_i^2 (\rho_L - \rho_V)^2} + \frac{L_i \rho_V (M_i - \rho_V V_i)}{(\rho_L - \rho_V)^2} \right. \\
 \left. + \frac{\pi R_i^4}{2} \right] + \frac{\pi R_i^4}{8\bar{\rho}_i(M_i + W_i)} \quad (C-3)
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial F_i}{\partial \rho_V} = \frac{\tan^2 \theta_i}{2\bar{\rho}_i(M_i + W_i)} \left[\frac{M_i(M_i - \rho_L V_i)}{\pi R_i^2 (\rho_L - \rho_V)^2} - \frac{L_i \rho_L (M_i - \rho_L V_i)}{(\rho_L - \rho_V)^2} \right. \\
 \left. - \frac{\pi R_i^4}{2} \right] - \frac{\pi R_i^4}{8\bar{\rho}_i(M_i + W_i)} \quad (C-4)
 \end{aligned}$$

Liquid Density Equation

The experimental points of the saturated rubidium liquid densities are subjected to the least squares curve fitting. The liquid density equation to be fitted is a non-linear equation, given in the following form, and expect to pass through two points: zero-angle points from two capsules.

$$\rho_L = \rho_c + a_1 (T_c - T)^{\beta} + a_2 (T_c - T) + a_3 (T_c - T)^2 \quad (C-5)$$

These two constraint points of single phase are directly observed and should be the most accurate experimental points. Designating these two points by the subscripts, r and s, for two capsules,

$$\begin{aligned} \rho_L &= \rho_{Lr} & \text{at } T &= T_r \\ \rho_L &= \rho_{Ls} & \text{at } T &= T_s \end{aligned}$$

The equation (C-5) must satisfy the above points .

The equation (C-5) is rearranged into a form, which is readily applicable to the least squares method. Subscribing each density point by a subscription k,

$$G_k = \rho_{Lk} - \rho_c - a_1 (T_c - T)^{\beta} - a_2 (T_c - T) - a_3 (T_c - T)^2 \quad (C-6)$$

Again, this equation is expanded into Taylor's series, also truncating higher order terms. Thus, when $T_k = T_r$, or T_s , the above equation (C-6) must be satisfied too.

$$\begin{aligned}
G_k(\rho_c, \beta, T_c, a_1, a_2, a_3) &= G_k(\rho_c^0, \beta^0, T_c^0, a_1^0, a_2^0, a_3^0) \\
&+ (\rho_c - \rho_c^0) \frac{\partial G_k^0}{\partial \rho_c} + (\beta - \beta^0) \frac{\partial G_k^0}{\partial \beta} + \\
&+ (T_c - T_c^0) \frac{\partial G_k^0}{\partial T_c} + \sum_{q=1}^3 (a_q - a_q^0) \frac{\partial G_k^0}{\partial a_q} \quad (C-7)
\end{aligned}$$

for the constraints,

$$G_r(\rho_c, \beta, T_c, a_1, a_2, a_3) = 0$$

$$G_s(\rho_c, \beta, T_c, a_1, a_2, a_3) = 0$$

Thus, Lagrangian Multiplier, α , is introduced in the variance equation for the least squares fitting of the data points.

$$\begin{aligned}
\text{Variance} &= \sum_{k=1}^{np} G_k(\Delta \rho_c, \Delta \beta, \Delta T_c, \Delta a_1, \Delta a_2, \Delta a_3)^2 \\
&+ \alpha_r G_r(\Delta \rho_c, \Delta \beta, \Delta T_c, \Delta a_1, \Delta a_2, \Delta a_3) \\
&+ \alpha_s G_s(\Delta \rho_c, \Delta \beta, \Delta T_c, \Delta a_1, \Delta a_2, \Delta a_3) \quad (C-8)
\end{aligned}$$

where, total number of the experimental points is represented by integer k with the total of np points. If each parameters, ρ_c , β , T_c , a_1 , a_2 , a_3 and α by x_i , then,

$$\Delta x_i = x_i - x_i^0$$

Thus, the variance, V , should be minimum with respect to the new parameters ΔX_i :

$$\frac{\partial V}{\partial (\Delta X_i)} = 0 \quad (C-9)$$

where,

<u>i</u>	<u>X_i</u>
1	ρ_c
2	β
3	T_c
4,5,6	a_1, a_2, a_3
7,8	α_r, α_s

By substituting the equation (C-8) into (C-9), the following set of linear simultaneous equations would be resulted. In matrix notation,

$$A \cdot X = B \quad (C-10)$$

and X can be solved by matrix inversion. Where,

$$X = \begin{bmatrix} \Delta \rho_c \\ \Delta \beta \\ \Delta T_c \\ \Delta a_1 \\ \Delta a_2 \\ \Delta a_3 \\ \Delta \alpha_r \\ \Delta \alpha_s \end{bmatrix} \quad B = \begin{bmatrix} - \sum_{k=1}^{np} G_k^o \left(\frac{\partial G_k}{\partial \rho_c} \right)_{X_i=X_i^o} \\ \vdots \\ - \sum_{k=1}^{np} G_k^o \left(\frac{\partial G_k}{\partial a_3} \right)_{X_i=X_i^o} \\ - G_r^o \\ - G_s^o \end{bmatrix}$$

Iteration starts with a set of initial values of ρ_c^0 , β^0 , T_c^0 , a_1^0 , a_2^0 , a_3^0 and multipliers. When the equation (C-7) is satisfied or within a permissible error (10^{-6}), then, the iteration will be terminated.

D. Raw DataTable D-1. Low Temperature Calibration of
Misalignment. Capsule #1.

I	OBSERVED DIFFERENCE	
1	25.100	0.505
2	26.000	1.405
3	25.150	0.555
4	24.000	-0.595
5	25.100	0.505
6	25.600	1.005
7	24.300	-0.295
8	23.700	-0.895
9	24.400	-0.195
10	24.000	-0.595
11	25.350	0.755
12	24.600	0.005
13	25.400	0.805
14	24.000	-0.595
15	24.000	-0.595
16	24.500	-0.095
17	23.250	-1.345
18	25.500	0.905
19	25.350	0.755
20	24.250	-0.345
21	24.250	-0.345
22	24.900	0.305
23	24.000	-0.595
24	24.800	0.205
25	24.500	-0.095
26	23.600	-0.995
27	25.000	0.405
28	24.250	-0.345
29	24.900	0.305
30	25.500	0.905
31	24.300	-0.295
32	25.500	0.905
33	24.100	-0.495
34	24.300	-0.295
35	25.750	1.155
36	25.350	0.755
37	23.950	-0.645
38	24.300	-0.295
39	24.800	0.205
40	24.300	-0.295
41	23.800	-0.795
42	23.900	-0.695
43	24.600	0.005
44	24.000	-0.595
45	24.000	-0.595

I	OBSERVED DIFFERENCE	
46	23.500	-1.095
47	24.800	0.205
48	26.750	2.155
49	24.350	-0.245
50	24.150	-0.445

DEGREE OF FREEDOM = 49
 MEAN = 24.595
 STANDARD DEVIATION = 0.710

Table D-2. Low Temperature Calibration of Misalignment. Capsule #4. Run#1, increasing angle.

I	OBSERVED DIFFERENCE	
1	25.500	-1.230
2	25.150	-1.580
3	25.950	-0.780
4	27.900	1.170
5	27.900	1.170
6	26.900	0.170
7	25.100	-1.630
8	27.000	0.270
9	26.150	-0.580
10	27.000	0.270
11	27.300	0.570
12	25.900	-0.830
13	26.900	0.170
14	26.900	0.170
15	27.900	1.170
16	26.750	0.020
17	26.150	-0.580
18	25.600	-1.130
19	27.750	1.020
20	27.300	0.570
21	26.700	-0.030
22	26.250	-0.480
23	26.800	0.070
24	26.700	-0.030
25	26.850	0.120
26	27.400	0.670

I OBSERVED DIFFERENCE

27	27.000	0.270
28	26.400	-0.330
29	27.500	0.770
30	27.300	0.570

DEGREE OF FREEDOM = 29
 MEAN = 26.730
 STANDARD DEVIATION = 0.769

Table D-3. Low Temperature Calibration of Misalignment. Capsule #4. Run #1, decreasing angle.

I OBSERVED DIFFERENCE

1	26.450	0.869
2	24.900	-0.681
3	24.900	-0.681
4	26.600	1.019
5	24.500	-1.081
6	26.250	0.669
7	24.700	-0.881
8	25.800	0.219
9	26.200	0.619
10	25.750	0.169
11	25.700	0.119
12	25.200	-0.381
13	26.000	0.419
14	25.300	-0.281
15	25.400	-0.181
16	25.300	-0.281
17	24.400	-1.181
18	25.200	-0.381
19	25.600	0.019
20	26.200	0.619
21	24.800	-0.781
22	25.900	0.319
23	25.250	-0.331
24	25.500	-0.081
25	25.750	0.169
26	25.250	-0.331

I	OBSERVED DIFFERENCE	
27	26.750	1.169
28	25.250	-0.331
29	25.100	-0.481
30	26.750	1.169
31	25.750	0.169
32	25.850	0.269
33	25.500	-0.081
34	26.300	0.719
35	25.300	-0.281

DEGREE OF FREEDOM = 34
 MEAN = 25.581
 STANDARD DEVIATION = 0.604

Table D-4. Low Temperature Calibration of Misalignment. Capsule #4. Run #2, combined.

I	OBSERVED DIFFERENCE	
1	26.150	0.136
2	25.600	-0.414
3	26.300	0.286
4	26.700	0.686
5	26.700	0.686
6	25.800	-0.214
7	26.900	0.886
8	25.500	-0.514
9	26.500	0.486
10	26.800	0.786
11	26.500	0.486
12	26.400	0.386
13	26.200	0.186
14	25.200	-0.814
15	25.750	-0.264
16	25.000	-1.014
17	26.100	0.086
18	25.750	-0.264
19	25.800	-0.214

I OBSERVED DIFFERENCE

20	25.400	-0.614
21	26.250	0.236
22	26.250	0.236
23	26.000	-0.014
24	26.100	0.086
25	25.500	-0.514
26	26.000	-0.014
27	26.200	0.186
28	26.200	0.186
29	25.250	-0.764
30	26.500	0.486
31	26.650	0.636
32	26.500	0.486
33	26.000	-0.014
34	25.800	-0.214
35	25.700	-0.314
36	25.500	-0.514
37	25.600	-0.414
38	25.200	-0.814
39	25.750	-0.264
40	26.700	0.686
41	26.000	-0.014
42	26.500	0.486
43	25.400	-0.614

DEGREE OF FREEDOM = 42
 MEAN = 26.014
 STANDARD DEVIATION = 0.486

Table D-5. Capsule #1, Experimental Series 1.

ANGLE DEG.	THERMOCOUPLE READINGS MV			ARGON PRES. PSIA
24.200	2.000	2.120	2.280	200.000
24.800	4.300	4.500	4.900	200.000
24.450	9.800	10.000	11.000	200.000
22.500	17.000	17.200	14.000	250.000
20.700	24.100	24.700	25.500	680.000
20.400	24.500	25.050	25.900	680.000
20.300	24.700	25.300	26.100	635.000
17.550	28.400	28.800	29.300	1125.000
17.000	28.100	28.450	29.050	1400.000
16.750	28.300	28.600	29.450	1570.000
17.100	28.300	28.600	29.400	1570.000
16.250	28.700	29.100	29.800	1600.000
15.500	29.000	29.250	29.900	1625.000
23.450	3.200	3.500	3.400	200.000
21.300	19.800	20.000	19.900	320.000
21.300	22.700	22.950	22.700	425.000
17.800	27.800	28.050	28.200	1240.000
17.300	28.700	29.000	29.000	1650.000
15.800	29.000	29.200	29.500	1655.000
-15.700	29.400	29.500	29.750	1700.000
14.700	29.200	29.500	29.850	1720.000
13.250	30.000	30.050	30.500	1740.000
13.450	30.000	30.100	30.550	1780.000

-13.300	30.050	30.100	30.500	1780.000
11.450	30.300	30.350	30.300	1825.000
11.700	30.100	30.250	30.700	1825.000
10.500	30.500	30.500	30.750	1910.000
10.200	30.550	30.500	30.900	1935.000
-9.700	30.700	30.700	30.900	1955.000
9.200	30.900	30.800	30.800	1970.000
11.200	30.600	30.600	30.600	1970.000
9.600	30.500	30.500	30.600	1980.000
10.250	30.500	30.500	30.800	1980.000
10.450	30.500	30.500	30.900	1975.000
9.050	30.600	30.550	31.000	1975.000
-9.300	30.850	30.800	31.000	1980.000
-9.300	31.000	30.900	31.000	1990.000
6.900	31.000	31.000	31.050	1990.000
8.000	31.000	30.900	31.000	2000.000
21.500	22.700	22.750	23.700	560.000
20.650	23.400	23.500	24.500	570.000
19.400	26.900	26.900	27.800	1230.000
16.200	28.800	28.800	29.500	1490.000
15.000	29.500	29.500	29.750	1500.000
13.650	29.800	29.800	30.000	1750.000
-14.700	29.900	30.000	30.000	1780.000
12.700	29.900	30.100	30.200	1800.000
14.500	30.100	29.800	30.300	1800.000
-12.900	30.000	30.000	30.400	1780.000
10.000	30.500	30.400	30.500	1900.000
-11.100	30.450	30.450	30.500	1900.000
9.000	30.800	30.600	30.900	1910.000
-9.600	30.800	30.700	30.900	1920.000
8.200	30.950	30.950	30.950	1950.000
-6.650	31.250	31.200	31.250	1980.000
-3.200	31.350	31.200	31.350	1980.000
-2.800	31.300	31.200	31.350	1990.000
-0.300	31.450	31.400	31.500	2000.000
19.700	26.250	26.300	26.500	1130.000
20.600	25.000	25.100	25.300	1100.000
21.950	21.500	21.500	21.600	600.000

Table D-6. Capsule #1, Experimental Series 2.

ANGLE, DEG.	THERMOCOUPLE READINGS, MV			ARGON PRES. PSIA
23.100	12.100	13.100	12.300	290.000
24.000	12.500	13.450	12.550	300.000
22.750	15.500	16.200	15.500	300.000
23.700	10.000	10.600	10.250	130.000
23.700	10.100	10.750	10.250	135.000
22.900	10.100	10.800	10.300	140.000
24.100	10.600	11.600	10.700	145.000
22.700	14.950	15.300	14.950	185.000
22.450	14.900	15.400	15.000	200.000
22.600	14.900	15.400	14.950	200.000
22.800	18.300	18.600	18.350	220.000
22.100	19.500	20.000	19.500	310.000
20.900	22.100	22.500	22.100	475.000
21.200	22.200	22.700	22.200	490.000
21.700	22.150	22.800	22.200	490.000
20.700	23.500	24.000	23.500	605.000
20.300	24.200	25.000	24.200	845.000
20.200	25.250	26.200	25.250	850.000
-19.400	25.700	26.400	25.800	850.000
18.950	27.450	28.100	27.450	1230.000
19.400	27.400	28.100	27.400	1230.000
19.100	27.300	28.300	27.300	1240.000
18.900	27.200	28.300	27.200	1330.000
19.450	27.350	28.500	27.350	1340.000

16.850	28.250	29.250	28.250	1500.000
-18.300	28.400	29.150	28.400	1500.000
17.200	28.600	29.150	28.600	1500.000
15.300	28.600	29.750	29.600	1620.000
16.100	28.600	29.900	28.600	1620.000
-17.800	29.200	29.200	29.200	1600.000
14.500	29.550	29.550	29.550	1600.000
15.450	29.500	29.500	29.500	1620.000
15.300	29.400	29.400	29.400	1630.000
14.600	29.550	29.550	29.550	1635.000
14.400	29.600	29.600	29.600	1635.000
-14.300	30.000	30.000	30.000	1635.000
-13.300	30.200	30.300	30.300	1635.000
12.000	30.500	30.500	30.450	1670.000
10.600	30.700	30.700	30.550	1675.000
6.600	31.000	31.550	31.000	2000.000
6.700	31.000	31.550	31.000	2000.000
5.950	31.000	31.500	30.950	2000.000
-5.500	31.100	31.100	31.050	1900.000
4.700	31.300	31.300	31.100	1900.000
-0.200	31.350	31.350	31.300	1900.000
-2.600	31.450	31.450	31.300	1900.000
-2.550	31.500	31.250	31.500	1900.000
1.500	31.500	31.200	31.500	1910.000
1.300	31.300	31.500	31.300	1925.000
0.950	31.400	31.500	31.400	1925.000
-0.300	31.450	31.450	31.400	1930.000

Table D-7. Capsule #1, Experimental Series 3.

ANGLE, DEG.	THERMOCOUPLE READINGS, MV			ARGON PRES. PSIA
19.700	26.550	26.500	26.550	1100.000
18.900	27.100	27.050	27.450	1350.000
17.900	28.100	28.100	28.600	1480.000
17.700	28.150	28.100	28.800	1480.000
16.400	29.000	28.950	29.250	1510.000
15.800	29.000	28.900	29.300	1510.000
15.900	29.000	29.000	29.400	1520.000
16.450	28.950	28.800	29.450	1520.000
15.300	29.450	29.200	29.950	1720.000
15.550	29.400	29.400	30.000	1720.000
12.950	29.900	29.800	30.450	1720.000
13.100	29.900	29.750	30.450	1725.000
12.050	30.050	30.000	30.700	1730.000
12.100	30.050	30.000	30.700	1730.000
12.000	30.000	30.000	30.550	1930.000
12.200	30.000	29.950	30.650	1935.000
12.000	29.950	29.950	30.650	1940.000
11.900	30.100	30.050	30.750	1950.000
11.950	30.200	30.200	30.750	1950.000
11.950	30.450	30.350	30.950	1950.000
9.400	30.500	30.500	31.000	1955.000
10.000	30.500	30.450	31.050	1965.000
10.350	30.500	30.450	31.100	1970.000

9.000	30.550	30.500	31.300	1965.000
9.100	30.550	30.550	31.300	1970.000
9.400	30.600	30.600	31.300	1975.000
9.150	30.650	30.650	31.300	1980.000
8.950	30.700	30.700	31.300	1980.000
9.400	30.750	30.700	31.350	1985.000
9.050	30.800	30.750	31.400	2000.000
7.850	30.800	30.800	31.400	2000.000
8.100	30.850	30.850	31.400	2000.000
7.700	30.850	30.800	31.400	2000.000
7.600	30.850	30.800	31.400	2000.000
7.600	30.900	30.900	31.400	2000.000
7.200	31.000	31.000	31.300	2000.000
6.850	31.000	31.000	31.300	2000.000
7.100	31.000	30.950	31.400	2000.000
6.700	31.000	31.000	31.400	2000.000
6.700	31.000	31.000	31.400	2000.000
6.550	31.000	31.000	31.400	2000.000
6.200	31.000	31.000	31.450	2000.000
6.200	31.050	31.000	31.450	2000.000
6.000	31.050	31.000	31.500	2010.000
5.700	31.050	31.050	31.450	2010.000
5.700	31.250	31.200	31.450	2010.000
5.600	31.200	31.100	31.400	2010.000
4.700	31.250	31.200	31.450	2010.000
-4.900	31.300	31.200	31.500	2010.000
-4.900	31.350	31.250	31.400	2010.000
-5.100	31.250	31.300	31.500	2010.000
-4.050	31.250	31.350	31.500	2010.000
-3.700	31.250	31.350	31.550	2010.000
-2.900	31.250	31.400	31.550	2010.000
-2.200	31.250	31.450	31.600	2010.000
-1.300	31.250	31.400	31.550	2010.000
0.200	31.250	31.350	31.550	2010.000
0.200	31.300	31.400	31.500	2010.000
-0.050	31.300	31.400	31.550	2010.000

Table D-8. Capsule #4, Run #1.

ANGLE DEG.	THERMOCOUPLE READINGS MV			ARGON PRES. PSIA
25.000	10.100	11.150	10.950	260.000
25.650	10.800	11.700	11.500	260.000
24.900	11.200	12.200	11.950	260.000
25.400	11.300	12.300	12.000	260.000
25.000	11.700	12.800	12.450	260.000
24.700	12.300	13.400	13.000	265.000
24.150	13.300	14.400	14.000	270.000
25.500	13.400	14.500	14.800	290.000
24.400	13.500	14.600	14.900	290.000
23.300	15.600	15.400	17.000	290.000
24.600	15.600	15.800	17.000	290.000
23.000	17.400	17.500	18.500	300.000
22.900	18.500	18.500	19.500	300.000
22.900	19.100	19.100	19.750	300.000
19.300	27.000	27.000	27.600	1200.000
19.000	27.000	27.400	27.600	1200.000
19.600	27.200	27.500	27.600	1200.000
18.300	28.000	28.000	28.500	1420.000
17.700	28.100	28.200	28.600	1440.000
17.900	28.200	28.200	28.800	1450.000
15.900	29.000	28.900	29.400	1620.000
14.900	29.150	29.100	29.750	1620.000
14.300	29.400	29.300	29.950	1620.000

14.250	29.600	29.500	30.000	1620.000
13.300	29.700	29.600	30.050	1620.000
-13.000	29.800	29.900	29.850	1620.000
-12.800	29.950	30.000	29.600	1620.000
13.350	29.700	29.700	29.900	1625.000
12.200	29.900	29.900	30.450	1620.000
12.000	29.900	29.900	30.500	1625.000
12.500	30.100	30.150	30.500	1750.000
-11.550	30.150	30.150	30.400	1750.000
8.950	30.300	30.200	30.500	1765.000
9.900	30.300	30.300	30.600	1770.000
9.600	30.400	30.500	30.550	1770.000
10.100	30.300	30.400	30.550	1770.000
8.950	30.400	30.500	30.600	1770.000
-8.350	30.600	30.600	30.650	1775.000
-7.850	30.800	30.700	30.700	1780.000
-7.150	30.900	30.800	30.600	1785.000
7.900	30.800	30.800	30.700	1785.000
7.450	30.700	30.700	30.850	1790.000
6.900	30.700	30.800	30.900	1800.000
6.700	30.700	30.800	30.950	1800.000
6.100	30.750	30.850	31.050	1810.000
5.800	30.800	30.900	31.000	1810.000
3.950	31.100	31.050	31.100	1820.000
2.100	31.200	31.200	31.150	1820.000
1.900	31.200	31.250	31.200	1820.000
-0.050	31.250	31.250	31.300	1840.000
10.250	30.400	30.300	30.400	1820.000
-9.550	30.500	30.400	30.500	1600.000
22.500	19.700	19.800	20.000	1000.000

Table D-9. Capsule #4, Run #2.

ANGLE, DEG.	THERMOCOUPLE READINGS, mV			ARGON PRES. PSIA
25.100	10.500	10.300	11.100	310.000
24.550	11.800	11.750	12.500	320.000
24.800	11.900	11.800	12.500	320.000
23.300	16.200	16.000	16.600	360.000
23.000	18.300	18.000	18.600	380.000
22.500	18.800	18.700	19.000	385.000
22.800	19.300	19.100	20.000	390.000
22.450	19.400	19.300	20.100	400.000
22.300	20.200	19.700	20.150	405.000
22.300	21.000	20.500	21.000	415.000
22.050	21.150	20.550	21.250	415.000
22.100	21.600	21.000	21.750	425.000
22.100	21.800	21.100	21.900	425.000
22.600	21.800	21.100	21.800	430.000
22.350	21.850	21.100	21.900	430.000
21.800	22.800	22.550	22.850	470.000
21.050	22.800	22.550	22.850	470.000
21.100	23.600	23.150	23.950	630.000
21.200	23.600	23.150	23.950	630.000
20.800	24.100	23.850	24.450	650.000
20.800	24.150	23.900	24.500	650.000
20.700	24.300	24.100	24.700	800.000

20.650	24.400	24.100	24.800	800.000
20.500	24.800	24.500	25.050	805.000
20.400	24.800	24.500	25.000	805.000
19.900	25.800	25.500	26.000	825.000
19.850	25.800	25.500	26.100	825.000
19.800	26.100	25.750	26.400	970.000
19.850	26.200	25.900	26.300	970.000
19.400	26.800	27.000	26.800	1150.000
19.500	26.900	26.500	27.000	1150.000
18.500	26.800	26.500	26.700	1155.000
18.800	27.200	27.000	27.500	1165.000
18.300	28.000	27.600	28.100	1415.000
17.600	28.700	28.200	29.000	1440.000
17.300	28.500	28.300	28.400	1550.000
17.300	28.700	28.600	28.700	1550.000
15.000	29.400	29.300	29.400	1550.000
-14.000	29.600	29.600	29.700	1560.000
16.500	29.000	29.200	28.800	1560.000
17.600	29.100	29.100	29.100	1565.000
15.300	29.500	29.450	29.500	1660.000
-11.900	29.900	30.150	29.700	1675.000
12.400	30.100	30.200	29.800	1680.000
14.350	29.600	29.600	29.600	1700.000
-12.400	29.950	30.000	30.050	1725.000
-10.300	30.100	29.800	30.500	1670.000
11.200	30.200	29.850	30.500	1795.000
9.950	30.300	30.000	30.600	1790.000
8.950	30.400	30.450	30.450	1760.000
-9.700	30.400	30.200	30.800	1820.000
-8.100	30.550	30.300	30.850	1840.000
-8.000	30.600	30.550	30.850	1850.000
-7.000	30.800	30.550	30.850	1850.000
-5.950	30.850	30.750	30.950	1850.000
1.950	31.200	30.950	31.300	1840.000
-0.950	31.300	31.150	31.500	1840.000
8.300	30.600	30.600	30.700	1800.000

Table D-10. Capsule #1, Experimental Series 1.

ANGLE, DEG.	T.C.1	TEMPERATURE, F		AVERAGE
		T.C.2	T.C.3	
24.200	280.373	292.727	309.152	294.08
24.800	512.350	532.086	571.375	538.60
24.450	1037.895	1056.559	1149.700	1081.38
22.500	1713.150	1732.389	1429.140	1624.89
20.700	2437.394	2503.737	2593.782	2511.64
20.400	2481.512	2542.903	2639.518	2554.64
20.300	2503.737	2571.096	2662.569	2579.13
17.550	2937.087	2986.712	3049.580	2991.13
17.000	2900.249	2943.258	3018.029	2953.85
16.750	2924.772	2961.826	3068.625	2985.07
17.100	2924.772	2961.826	3062.267	2982.96
16.250	2974.251	3024.320	3113.403	3037.32
15.500	3011.747	3043.251	3126.284	3060.43
23.450	402.617	432.751	422.724	419.36
21.300	1987.503	2007.572	1997.529	1997.53
21.300	2286.285	2312.908	2286.285	2295.16
17.800	2863.732	2894.141	2912.493	2890.12
17.300	2974.251	3011.747	3011.747	2999.25
15.800	3011.747	3036.931	3074.993	3041.22
-15.700	3062.267	3074.993	3106.977	3081.41
14.700	3036.931	3074.993	3119.839	3077.25
13.250	3139.205	3145.681	3204.410	3163.10
13.450	3139.205	3152.166	3210.986	3167.45
-13.300	3145.681	3152.166	3204.410	3167.42
11.450	3178.207	3184.743	3178.207	3180.39
11.700	3152.166	3171.682	3230.776	3184.87
10.500	3204.410	3204.410	3237.393	3215.40
10.200	3210.986	3204.410	3257.306	3224.23
-9.700	3230.776	3230.776	3257.306	3239.62
9.200	3257.306	3244.020	3244.020	3248.45
11.200	3217.572	3217.572	3217.572	3217.57
9.600	3204.410	3204.410	3217.572	3208.80
10.250	3204.410	3204.410	3244.020	3217.61
10.450	3204.410	3204.410	3257.306	3222.04
9.050	3217.572	3210.986	3270.634	3233.06
-9.300	3250.658	3244.020	3270.634	3255.10
-9.300	3270.634	3257.306	3270.634	3266.19
6.900	3270.634	3270.634	3277.314	3272.36
8.000	3270.634	3257.306	3270.634	3266.19
21.300	2286.285	2291.597	2393.707	2323.86

20.650	2361.217	2372.021	2481.512	2404.92
19.400	2756.044	2756.044	2863.732	2791.94
16.200	2986.712	2986.712	3074.993	3016.14
15.000	3074.993	3074.993	3106.977	3085.65
13.850	3113.403	3113.403	3139.205	3122.00
-14.700	3126.284	3139.205	3139.205	3134.90
12.700	3126.284	3152.166	3165.166	3147.87
14.500	3152.166	3113.403	3178.207	3147.93
-12.900	3139.205	3139.205	3191.288	3156.57
10.000	3204.410	3191.288	3204.410	3200.04
-11.100	3197.844	3197.844	3204.410	3200.03
9.000	3244.020	3217.572	3257.306	3239.63
-9.800	3244.020	3230.776	3257.306	3244.03
8.200	3263.965	3263.965	3263.965	3263.96
-6.650	3304.137	3297.415	3304.137	3301.90
-3.200	3317.612	3297.415	3317.612	3310.88
-2.800	3310.869	3297.415	3317.612	3308.63
-0.300	3331.130	3324.366	3337.905	3331.13
19.700	2679.940	2685.746	2709.050	2691.58
20.600	2537.286	2548.527	2571.096	2552.30
21.950	2160.514	2160.514	2170.873	2163.97

Table D-11. Capsule #1, Experimental Series 2.

ANGLE, DEG.	T.C.1	TEMPERATURE, F		AVERAGE
		T.C.2	T.C.3	
23.100	1252.000	1345.094	1270.604	1289.23
24.000	1289.214	1377.740	1293.867	1320.27
22.750	1570.208	1636.637	1570.208	1592.35
23.700	1056.559	1112.473	1079.870	1082.97
23.700	1065.886	1126.437	1079.870	1090.73
22.900	1065.886	1131.090	1064.529	1093.84
24.100	1112.473	1205.503	1121.783	1146.59
22.700	1518.304	1551.307	1518.304	1529.31
22.450	1513.597	1560.754	1523.013	1532.45
22.600	1513.597	1560.754	1518.304	1530.88
22.800	1839.112	1868.508	1844.002	1850.54
22.100	1957.531	2007.572	1957.531	1974.21
20.900	2222.993	2265.094	2222.993	2237.03
21.200	2233.484	2286.285	2233.484	2251.08
21.700	2228.235	2296.916	2233.484	2252.88
20.700	2372.021	2426.433	2372.021	2390.16
20.300	2448.383	2537.286	2448.383	2478.02
20.200	2565.442	2674.142	2565.442	2601.68
-19.400	2616.589	2697.382	2628.038	2647.34
18.950	2821.526	2900.249	2821.526	2847.77
19.400	2815.531	2900.249	2815.531	2843.77
19.100	2803.566	2924.772	2803.566	2843.97
18.900	2791.635	2924.772	2791.635	2836.01
19.450	2809.544	2949.438	2809.544	2856.18
16.850	2918.628	3043.251	2918.628	2960.17
-18.300	2937.087	3030.621	2937.087	2968.26
17.200	2961.826	3030.621	2961.826	2984.76
15.300	2961.826	3106.977	3087.757	3052.19
16.100	2961.826	3126.284	2961.826	3016.65
-17.800	3036.931	3036.931	3036.931	3036.93
14.500	3081.370	3081.370	3081.370	3081.37
15.450	3074.993	3074.993	3074.993	3074.99
15.300	3062.267	3062.267	3062.267	3062.27
14.600	3081.370	3081.370	3081.370	3081.37
14.400	3087.757	3087.757	3087.757	3087.76
-14.300	3139.205	3139.205	3139.205	3139.21
-13.300	3165.166	3178.207	3178.207	3173.86
12.000	3204.410	3204.410	3197.844	3202.22
10.600	3230.775	3230.776	3210.986	3224.18
6.600	3270.634	3344.691	3270.634	3295.32

6.700	3270.634	3344.691	3270.634	3295.32
5.950	3270.634	3337.905	3263.965	3290.83
-5.500	3284.004	3284.004	3277.314	3281.77
4.700	3310.869	3310.869	3284.004	3301.91
-0.200	3317.612	3317.612	3310.869	3315.36
-2.600	3331.130	3331.130	3310.869	3324.38
-2.550	3337.905	3304.137	3337.905	3326.65
1.500	3337.905	3297.415	3337.905	3324.41
1.300	3310.869	3337.905	3310.869	3319.88
0.950	3324.366	3337.905	3324.366	3328.88
-0.300	3331.130	3331.130	3324.366	3328.88

Table D-12. Capsule #1, Experimental Series 3.

ANGLE DEG.	T.C.1	TEMPERATURE, F		AVERAGE
		T.C.2	T.C.3	
19.700	2714.895	2709.050	2714.895	2712.95
18.900	2779.738	2773.802	2821.526	2791.69
17.900	2900.249	2900.249	2961.826	2920.78
17.700	2906.367	2900.249	2966.712	2931.11
16.400	3011.747	3005.474	3043.251	3020.16
15.800	3011.747	2999.211	3049.580	3020.18
15.900	3011.747	3011.747	3062.267	3028.59
16.450	3005.474	2986.712	3068.625	3020.27
15.300	3068.625	3036.931	3132.740	3079.43
15.550	3062.267	3062.267	3139.205	3087.91
12.950	3126.284	3113.403	3197.844	3145.84
13.100	3126.284	3106.977	3197.844	3143.70
12.050	3145.681	3139.205	3230.776	3171.89
12.100	3145.681	3139.205	3230.776	3171.89
12.000	3139.205	3139.205	3210.986	3163.13
12.200	3139.205	3132.740	3224.169	3165.37
12.000	3132.740	3132.740	3224.169	3163.22
11.900	3152.166	3145.681	3237.393	3178.41
11.950	3165.166	3165.166	3237.393	3189.24
11.950	3197.844	3184.743	3263.965	3215.52
9.400	3204.410	3204.410	3270.634	3226.48
10.000	3204.410	3197.844	3277.314	3226.52
10.350	3204.410	3197.844	3284.004	3228.75
9.000	3210.986	3204.410	3310.869	3242.09

9.100	3210.986	3210.986	3310.869	3244.28
9.400	3217.572	3217.572	3310.869	3248.67
9.150	3224.169	3224.169	3310.869	3253.07
8.950	3230.776	3230.776	3310.869	3257.47
9.400	3237.393	3230.776	3317.612	3261.93
9.050	3244.020	3237.393	3324.366	3268.59
7.850	3244.020	3244.020	3324.366	3270.80
8.100	3250.658	3250.658	3324.366	3275.23
7.700	3250.658	3244.020	3324.366	3273.01
7.800	3250.658	3244.020	3324.366	3273.01
7.800	3257.306	3257.306	3324.366	3279.66
7.200	3270.634	3270.634	3310.869	3284.05
6.850	3270.634	3270.634	3310.869	3284.05
7.100	3270.634	3263.965	3324.366	3286.32
6.700	3270.634	3270.634	3324.366	3288.54
6.700	3270.634	3270.634	3324.366	3288.54
6.550	3270.634	3270.634	3324.366	3288.54
6.200	3270.634	3270.634	3331.130	3290.80
6.200	3277.314	3270.634	3331.130	3293.03
6.000	3277.314	3270.634	3337.905	3295.28
5.700	3277.314	3277.314	3331.130	3295.25
5.700	3304.137	3297.415	3331.130	3310.89
5.600	3297.415	3284.004	3324.366	3301.93
4.700	3304.137	3297.415	3331.130	3310.89
-4.900	3310.869	3297.415	3337.905	3315.40
-4.900	3317.612	3304.137	3324.366	3315.37
-5.100	3304.137	3310.869	3337.905	3317.64
-4.050	3304.137	3317.612	3337.905	3319.88
-3.700	3304.137	3317.612	3344.691	3322.15
-2.900	3304.137	3324.366	3344.691	3324.40
-2.200	3304.137	3331.130	3351.487	3328.92
-1.300	3304.137	3324.366	3344.691	3324.40
0.200	3304.137	3317.612	3344.691	3322.15
0.200	3310.869	3324.366	3337.905	3324.38
-0.050	3310.869	3324.366	3344.691	3326.64

Table D-13. Capsule #4, Run #1.

ANGLE DEG.	T.C.1	TEMPERATURE, F		AVERAGE
		T.C.2	T.C.3	
25.000	1065.886	1163.654	1145.048	1124.86
25.650	1131.090	1214.802	1196.204	1180.70
24.900	1168.304	1261.302	1238.050	1222.55
25.400	1177.605	1270.604	1242.700	1230.30
25.000	1214.802	1317.143	1284.561	1272.17
24.700	1270.604	1373.073	1335.774	1326.48
24.150	1363.743	1466.616	1429.140	1419.83
25.500	1373.073	1476.000	1504.188	1451.09
24.400	1382.407	1485.389	1513.597	1460.46
23.300	1579.671	1560.754	1713.150	1617.86
24.600	1579.671	1598.623	1713.150	1630.48
23.000	1751.677	1761.339	1858.694	1790.57
22.900	1858.694	1858.694	1957.531	1891.64
22.900	1917.804	1917.604	1922.497	1939.37
19.300	2767.874	2767.874	2839.562	2791.77
19.000	2767.874	2815.531	2839.562	2807.66
19.600	2791.635	2827.529	2839.562	2819.58
18.300	2888.042	2868.042	2949.438	2908.51
17.700	2900.249	2912.493	2961.826	2924.86
17.900	2912.493	2912.493	2986.712	2937.23
15.900	3011.747	2999.211	3062.267	3024.41
14.900	3030.621	3024.320	3106.977	3053.97
14.300	3062.267	3049.580	3132.740	3081.53
14.250	3087.757	3074.993	3139.205	3100.65
13.300	3100.561	3087.757	3145.681	3111.33
-13.000	3113.403	3126.284	3119.839	3119.84
-12.800	3132.740	3139.205	3087.757	3119.90
13.350	3100.561	3100.561	3126.284	3109.14
12.200	3126.284	3126.284	3197.844	3150.14
12.000	3126.284	3126.284	3204.410	3152.33
12.500	3152.166	3158.661	3204.410	3171.75
-11.550	3158.661	3158.661	3191.288	3169.54
8.950	3178.207	3165.166	3204.410	3182.59
9.900	3178.207	3178.207	3217.572	3191.33
9.600	3191.288	3204.410	3210.986	3202.23
10.100	3178.207	3191.288	3210.986	3193.49
8.950	3191.288	3204.410	3217.572	3204.42
-8.350	3217.572	3217.572	3224.169	3219.77
-7.650	3244.020	3230.776	3230.776	3235.19

-7.150	3257.306	3244.020	3217.572	3239.63
7.900	3244.020	3244.020	3230.776	3239.61
7.450	3230.776	3230.776	3250.658	3237.40
6.900	3230.776	3244.020	3257.306	3244.03
6.700	3230.776	3244.020	3263.965	3246.25
6.100	3237.393	3250.658	3277.314	3255.12
5.800	3244.020	3257.306	3270.634	3257.32
3.950	3264.004	3277.314	3284.004	3261.77
2.100	3297.415	3297.415	3290.704	3295.18
1.900	3297.415	3304.137	3297.415	3299.66
-0.050	3304.137	3304.137	3310.869	3306.38
10.250	3191.288	3178.207	3191.288	3186.93
-9.550	3204.410	3191.288	3204.410	3200.04
22.500	1977.495	1987.503	2007.572	1990.86

Table D-14. Capsule #4, Run #2.

ANGLE-DEG.	T.C.1	TEMPERATURE-F		AVERAGE
		T.C.2	T.C.3	
25.100	1103.161	1084.529	1159.003	1115.56
24.550	1224.101	1219.451	1289.214	1244.26
24.800	1233.400	1224.101	1289.214	1248.90
23.300	1636.637	1617.611	1674.807	1643.02
23.000	1839.112	1809.845	1868.508	1839.15
22.500	1868.180	1878.336	1907.914	1891.48
22.800	1937.635	1917.804	2007.572	1954.34
22.450	1947.574	1937.635	2017.634	1967.61
22.300	2027.714	1977.495	2022.672	2009.29
22.300	2109.037	2058.065	2169.037	2092.05
22.050	2124.426	2063.140	2134.711	2107.43
22.100	2170.873	2109.037	2186.451	2155.45
22.100	2191.655	2119.291	2202.078	2171.01
22.600	2191.655	2119.291	2191.655	2167.53
22.350	2196.864	2119.291	2202.078	2172.74
21.800	2296.916	2270.383	2302.241	2289.85
21.050	2296.916	2270.383	2302.241	2289.85
21.100	2382.851	2334.316	2420.962	2379.38
21.200	2382.851	2334.316	2420.962	2379.38
20.800	2437.394	2410.040	2475.973	2441.14
20.800	2442.865	2415.498	2481.512	2446.63
20.700	2459.398	2437.394	2503.737	2466.84
20.650	2470.441	2437.394	2514.891	2474.24
20.500	2514.891	2481.512	2542.903	2513.10
20.400	2514.891	2481.512	2537.236	2511.23

19.900	2628.038	2593.782	2651.028	2624.28
19.850	2628.038	2593.782	2662.569	2628.13
19.800	2662.569	2622.310	2697.382	2660.75
19.850	2674.142	2639.518	2685.746	2666.47
19.400	2744.246	2767.874	2744.246	2752.12
19.500	2756.044	2709.050	2767.874	2744.32
18.500	2744.246	2709.050	2732.482	2728.59
18.800	2791.635	2767.874	2827.529	2795.68
18.300	2888.042	2839.562	2900.249	2875.95
17.600	2974.251	2912.493	3011.747	2966.16
17.300	2949.438	2924.772	2937.087	2937.10
17.300	2974.251	2961.826	2974.251	2970.11
15.000	3062.267	3049.580	3062.267	3058.04
-14.000	3087.757	3087.757	3100.561	3092.03
16.500	3011.747	3036.931	2986.712	3011.80
17.600	3024.320	3024.320	3024.320	3024.32
15.300	3074.993	3068.625	3074.993	3072.87
-11.900	3126.284	3158.661	3100.561	3128.50
12.400	3152.166	3165.166	3113.403	3143.58
14.350	3087.757	3087.757	3087.757	3087.76
-12.400	3132.740	3139.205	3145.681	3139.21
-10.300	3152.166	3113.403	3204.410	3156.66
11.200	3165.166	3119.839	3204.410	3163.14
9.950	3178.207	3139.205	3217.572	3178.33
8.950	3191.288	3197.844	3197.844	3195.66
-9.700	3191.288	3165.166	3244.020	3200.16
-8.100	3210.986	3178.207	3250.658	3213.28
-8.000	3217.572	3210.986	3250.658	3226.41
-7.000	3244.020	3210.986	3250.658	3235.22
-5.950	3250.658	3237.393	3263.965	3250.67
1.950	3297.415	3263.965	3310.869	3290.75
-0.950	3310.869	3290.704	3337.905	3313.16
8.300	3217.572	3217.572	3230.776	3221.97

Table D-15. Summary of Computer Input.

Capsule #1

NCAT = 2	
INCXA = 1	
INCXB = 2	
EPSIL = 0.100000E-04	
PC = 0.671154E 00	
PY = 0.451104E 00	
P = 0.644906E 00	
ROC1 = 0.125000E C1	
ROC2 = 0.400000E-03	
EXTERNAL CAPSULE VOL, CL, CM, =	4.98000
INTERNAL RADIUS OF CAPSULE, CM=	0.27666
EXTERNAL RADIUS OF CAPSULE, CM=	0.31775
INTERNAL LENGTH OF CAPSULE, CM=	15.34668
EXTERNAL LENGTH OF CAPSULE, CM=	15.42288
LIQUID METAL CHARGE IN GM=	1.41700
WEIGHT OF CAPSULE ASSEMBLY, GM=	25.52879
WEIGHT OF CAPSULE ALONE, GM=	20.90579
WEIGHT OF YOKE ALONE, GM=	4.62300
VOLUME OF YOKE ALONE, CU. CM. =	0.22120

Capsule #4

NCAT = 2	
INCXA = 1	
INCXB = 2	
EPSIL = 0.100000E-04	
PC = 0.646060E 00	
PY = 0.486156E 00	
P = 0.641056E 00	
ROC1 = 0.125000E C1	
ROC2 = 0.400000E-03	
EXTERNAL CAPSULE VOL, CU. CM. =	4.90000
INTERNAL RADIUS OF CAPSULE, CM=	0.27686
EXTERNAL RADIUS OF CAPSULE, CM=	0.31775
INTERNAL LENGTH OF CAPSULE, CM=	15.32128
EXTERNAL LENGTH OF CAPSULE, CM=	15.39748
LIQUID METAL CHARGE IN GM=	1.56300
WEIGHT OF CAPSULE ASSEMBLY, GM=	25.58089
WEIGHT OF CAPSULE ALONE, GM=	20.94890
WEIGHT OF YOKE ALONE, GM=	4.63200
VOLUME OF YOKE ALONE, CU. CM. =	0.25260

E. Computer Programs

Table E-1.

FURTRAN IV G LEVEL 19

MAIN

DATE = 72056

13/11/58

```

0001      DIMENSION RO1(200),RO2(200),TEMP(3,200),THET(3,200),VYY(3),VOL(3),
      1EL(3),R(3),ROO(3,200),AA(9,9),WCC(3),WYY(3),Q(10),ELINT(3),
      2RADIA(3),EM1(200),EM2(200),VT(4,3,200),C(10),PR(3,200)
      3,CP(3),YP(3),PP(3),ANP(3)
0002      COMMON AS,ELL,VY,RA,TEM,T,O(9,9),NECUP,PRES
0003      REAL MP(3),MASS(3)

C
C      INPUT CARDS STATEMENTS
C      .....
C
C      NRUN IS NUMBER OF RUNS WITH NEW INLET DATA
C      NPDUP=0 FOR DIRECT PRESSURE READING INPUT
C      NRD=C FOR NCNT CONTROL, =1 FOR RC VARIATION WITH NCNT=2
C      NCUM=1 FOR DIRECT TEMPERATURE INPUT,=2 FOR THERMOCOUPLE INPUT IN E.V.
C      DDC IS NUMBER OF THERMOCOUPLE READINGS AT EACH TEMPERATURE
C      NCNT IS NO. OF MASS LOADINGS
C      P IS PIVOT TO C.M. DISTANCE
C      RCO1 + 2 ARE LIQUID AND VAPOR DENSITIES
C      NP IS NO. OF TEMPERATURE-ANGLE POINTS
C      AN + BN ARE RECTILINEAR DIAMETER CONSTANTS (HOCHMAN DATA)
C      A AND B ARE RECTILINEAR DIAMETER CONSTANTS (EWINGS DATA)
C      MP IS PIPE MASS
C      VY IS YOKE VOLUME
C      WY IS YOKE WEIGHT, WC IS PIPE WEIGHT
C      M IS THE NUMBER OF EQUATIONS FOR SIMULTANEOUS SOLUTIONS
C      VOLI IS INTERNAL VOLUME, VOL IS EXTERNAL VOLUME
C      INDXA = 1 FOR W-30MG-30RE      INDXA = 2 FOR TANTALUM-121
C      INCXE = 1 WHEN YOKE AND CAPSULE WGT. ARE UNKNOWN -- SET WC,WY,VY=0.
C      INCXE = 2 WHEN YOKE AND PIPE VOL. AND WGT. ARE KNOWN-- THEN SET MP AS S

```

```

C
C   ALL OF THE UNITS ARE IN C.G.S. UNITS EXCEPT P IN PSIA, T IN T F
C .....
C
0004      REAC(5,88) NRUN,NFCUP,NRC
0005      554 READ(5,1) NCNT,INDXA,INCXE,NDUP
0006      READ(5,7) (ANP(I),I=1,NCNT)
0007      READ(5,3) A,B,EPSIL,      ROC1,FCE2,AN,BN
0008      DO 20 I=1,NCNT
0009      NP=ANP(I)
0010      READ(5,11) VOL(I),MASS(I),EL(I),R(I),MP(I),WCC(I),WYY(I),      VYY
      *(I),ELINT(I),RADIN(I),CP(I),YP(I),PP(I)
0011      READ(5,14) QDD
0012      DO 20 J=1,NP
0013      IF(NCUP.EQ.1) GO TO 10
0014      READ(5,17) THET(I,J), (VT(L,I,J),L=1,3), PR(I,J)
0015      IF(I.EQ.1) GO TO 8
0016      THET(I,J)=THET(I,J)-1.30
0017      8 CONTINUE
0018      GO TO 20
0019      18 READ(5,15) TEMPI(I,J),THET(I,J),PR(I,J)
0020      20 CONTINUE
0021      READ(5,600) D,E,TC
      C
      C   SET INITIAL DENSITY VALUE ARBITRARILY
      C
0022      DO 2 I = 1,6
0023      DO 2 J = 1,6
0024      2 D(I,J) = 0.
0025      IF(NCNT.EQ.1) ANP(2)=ANP(1)
0026      NP=ANP(2)
0027      DO 10 J = 1, NP
0028      RC1(J) = ROC1
0029      RC2(J) = ROC2
0030      10 CONTINUE
0031      WRITE(6,5)

```

```

0032      DO 12 I = 1, NCNT
0033      NF=ANP(I)
0034      PC=CF(I)
0035      PY=YF(I)
0036      P=PP(I)
0037      WRITE(6,4)NP,NCNT,INDXA,INDXB,EPSIL,PC,PY,P,RD01,RCO2
0038      WRITE(6,6)VOL(I),RADIN(I),RI(I),ELINT(I),EL(I),MASS(I),MPI(I),WCC(I)
      *,WYY(I),VYY(I)
0039      12      CCNTINLE
0040      EPI = 3.1415926
0041      44      DO 25 I = 1, NCNT
0042      NF=ANP(I)
0043      DO 25 J = 1, NP
0044      VT(4,I,J)=0.
      C
      C      CONVERSION OF POTENTIOMETER READINGS TO TEMPERATURE.
      C
0045      IF(NCUM.EQ.1)GO TO 25
      C
      C      COEFFICIENTS OF QUADRATIC EQUATION FOR TEMPERATURE VS. VOLTAGE
      C
      C      THERMOCOUPLE SET NO. 2 ( 1971)
      C
0046      X1=0.0360607
0047      X2=-1.26092
0048      X3=107.682
0049      X4=65.7643
0050      SM=0.
0051      DO 41 M=1,3
0052      41      SM=SM+X1*VT(M,I,J)**3+X2*VT(M,I,J)**2+X3*VT(M,I,J)+X4
0053      TEMP(I,J)=SM/DDD
0054      25      CCNTINLE
0055      WRITE(6,13)
0056      KKK=1
0057      NF=ANP(1)
0058      BXB=8

```

0059		DERD=0.001
0060		DO 100 J = 1,NP
0061		NEG=C
0062		CCEF=0.005
0063		IF(NEG.EQ.1) KKK=KKK-1
0064		IF(KKK.GT.1) RO1(J)=ROO(1,KKK-1)
0065		IF(KKK.GT.1) RO2(J)=ROO(2,KKK-1)
0066		IF(KKK.EQ.1) RO1(J)=ROO1
0067		IF(KKK.EQ.1) RO2(J)=ROO2
0068		IF(NEG.EQ.1) KKK=KKK+1
0069		B=DXB
0070		IF(TEM.LT.1500.) B=0.7618C
0071		DLN=C.
0072		LAM=1
0073	110	CONTINUE
0074		MMF=0.
0075	30	CONTINUE
0076		MMM = MMF + 1
0077		SSS=C.
0078		M = 2
0079		IF(NRD.GT.0) NCNT=1
0080		DO 40 A = 1, NCNT
0081		I=N
0082		IF(NRD.GT.0) I=NRD
0083		TEM=TEMP(1,J)
0084		IF(TEM.LT.1000.) GO TO 100
0085		IF(NRD.GT.0) GO TO 53
0086		IF (NCNT.EQ.1.AND.NRUN.EQ.1) GO TO 87
0087	53	IF(I.EQ.1) GO TO 29
0088	87	CONTINUE
0089		IF(TEM.GT.2500.) GO TO 21
0090		B0= C.242319E+02
0091		B1= C.172684E-03
0092		B2=-C.658955E-06
0093		B3= C.
0094		B4= C.
0095		B5= C.

0096		B6= C.
0097		GO TO 71
0098	21	IF(TEM.GT.2900.) GO TO 22
0099		B0= 0.491938E+02
0100		B1=-0.176111E-01
0101		B2=-0.222214E-05
0102		B3= 0.160128E-08
0103		B4= C.616497E-12
0104		B5=-0.262789E-15
0105		B6= C.
0106		GO TO 71
0107	22	IF(TEM.GT.3050.) GO TO 23
0108		B0= C.290753E+01
0109		B1= C.184300E-01
0110		B2=-0.403978E-05
0111		B3= C.557509E-09
0112		B4=-0.249136E-12
0113		B5= C.
0114		B6= C.
0115		GO TO 71
0116	23	IF(TEM.GT.3200.) GO TO 24
0117		B0= 0.465121E+02
0118		B1= C.956391E-02
0119		B2=-0.641676E-05
0120		B3= C.
0121		B4= 0.
0122		B5= C.
0123		B6= C.
0124		GO TO 71
0125	24	IF(TEM.GT.3280.) GO TO 26
0126		B0= C.731657E+02
0127		B1= C.735261E-02
0128		B2=-0.834362E-05
0129		B3= C.
0130		B4= 0.
0131		B5= 0.

0132		B6= C.
0133		GO TC 71
0134	26	IF(TEM.GT.3320.) GO TO 27
0135		B0= C.183471E+03
0136		B1= 0.702681E-02
0137		B2=-C.185071E-04
0138		B3=0.
0139		B4=0.
0140		B5=0.
0141		B6=0.
0142		GO TC 71
0143	27	B0=-C.167966E+03
0144		B1= C.127192
0145		B2=-C.432263E-05
0146		B3= 0.458444E-08
0147		B4=-C.960048E-12
0148		B5=-C.539126E-15
0149		B6=-C.277789E-19
0150		GO TC 71
0151	29	IF(TEM.GT.2000.) GO TO 84
0152		B0= C.270704E+02
0153		B1=-C.796552E-03
0154		B2=-C.724504E-06
0155		B3=0.
0156		B4=0.
0157		B5= C.
0158		B6= 0.
0159		GO TC 71
0160	84	IF(TEM.GT.2500.) GO TO 32
0161		B0=-C.364064E+01
0162		B1= C.703751E-01
0163		B2=-C.607704E-04
0164		B3= C.218928E-07
0165		B4=-C.291661E-11
0166		B5= C.
0167		B6= 0.

0168		GO TC 71
0169	32	IF(ITEM.GT.2900.) GO TO 33
0170		B0= 0.162126E+02
0171		B1= 0.390423E-02
0172		B2=-0.345769E-06
0173		B3= 0.161630E-09
0174		B4=-0.147449E-12
0175		B5= 0.
0176		B6= 0.
0177		GO TC 71
0178	33	IF(ITEM.GT.3050.) GO TO 34
0179		B0=-0.486429E+02
0180		B1= 0.613370E-01
0181		B2=-0.132232E-04
0182		B3= 0.
0183		B4= 0.
0184		B5= 0.
0185		B6= 0.
0186		GO TC 71
0187	34	IF(ITEM.GT.3200.) GO TO 35
0188		B0=-0.309323E+02
0189		B1= 0.496911E-02
0190		B2= 0.691156E-05
0191		B3=-0.226193E-09
0192		B4= 0.282611E-12
0193		B5= 0.213139E-16
0194		B6=-0.705319E-19
0195		GO TC 71
0196	35	IF(ITEM.GT.3280.) GO TO 36
0197		B0=-0.233435E+03
0198		B1= 0.227233E+00
0199		B2=-0.472549E-04
0200		B4=0.
0201		B5=0.
0202		B6=0.
0203		GO TC 71
0204	36	IF(ITEM.GT.3320.) GO TO 37

0205		B0=-C.191459E+03
0206		B1= 0.119164
0207		B2= 0.114140E-04
0208		B3=-C.503070E-08
0209		B4= 0.
0210		B5= C.
0211		B6= C.
0212	37	CONTINUE
0213	71	AS=MASS(I)
0214		T=B0+B1*TEM+B2*TEM**2+B3*TEM**3+B4*TEM**4+B5*TEM**5+B6*TEM**6
0215		T=ABS(T)
0216		AS = .MASS(I)
0217		W = MP(I)
0218		REXT = R(I)
0219		ELEXT = EL(I)
0220		ELL = ELINT(I)
0221		RA = RADIN(I)
0222		VCL=EPI*RA**2*ELL
0223		VUL=VOL(I)
0224		VY = VYY(I)
0225		WY = WYY(I)
0226		WC = WCC(I)
0227		IF(I.EC.1) GO TO 9
0228		L=ANP(2)
0229		DO 16 K=1,L
0230		IF(TEM.GT.TEMP(2,K)) GO TO 16
0231		PRES=(PR(2,K)+PR(2,K-1))/2
0232		GO TO 19
0233	16	CONTINUE
0234	9	PRES=PR(I,J)
0235	19	P=PP(I)
0236		PC=CP(I)
0237		PY=YP(I)
0238		PA=P
0239		CALL CCREC(W,WC,WY,PN,PC,PY,INCLX,INCLB,VUL,VOL,I)
0240		ROCI = ROI(J)

```

0241      RCC2 = RQ2(J)
0242      IF(RQ01.EQ.RQ02)GO TO 70
0243      CALL FUNCT(F,DFDR1,CFOR2,PN,W,RQ01,RQ02,VOL1)
      C
      C CONVERGENCE CRITERION IS WHEN THE FUNCTION F IS SMALLER THAN EPS
      C
0244      IF(NRD.GT.0) I=1
0245      O(I,1) = DFDR1
0246      O(I,2) = DFDR2
0247      O(I,3) = F
0248      O(3,1)=O(I,3)
0249      O(3,2)=F*F
0250      THET(I,J)=T
0251      SSS=SSS+ABS(F)
0252      40  CONTINUE
0253      F=SSS
0254      IF(NRD.GT.0) GO TO 50
0255      IF(NCAT.EQ.1)GO TO 50
0256      GO TO 60
0257      50  C(2,1) = 0.5
0258      C(2,2) = 0.5
0259      C(2,3) = (RQ01 + RQ02)/2. - (A*TEM + B)
0260      C(3,2)=O(2,3)
0261      60  CALL INV(M)
      C
      C RELAXATION CONSTANT OF BETA =0.6
      C
0262      DELT1=-0.6*O(1,3)
0263      DELT2=-0.6*O(2,3)
0264      IF(ABS(DELT1).LT.EPSIL.AND.ABS(DELT2).LT.EPSIL.AND.ABS(F).LT.EPSIL
      *) GO TO 51
0265      RQ1(J) = RQ1(J) + DELT1
0266      RQ2(J) = RQ2(J) + DELT2
0267      IF(M+M.GT.200) GO TO 51
0268      GO TO 30
0269      51  IF(NRD.EQ.0) GO TO 64

```

0270		ROO(ARC,J)=RO1(J)
0271		IF(NRD.EQ.1) ROO(3,J)=RO2(J)
0272		IF(NRD.EQ.2) GO TO 52
0273		NRD=2
0274		GO TO 110
0275	52	CCNV=AES(RCO(1,J)-RCO(2,J))
0276		DEL=CDEF*ROO(1,J)
0277		GAM=C.1
0278		IF(LAM.GT.50) GAM=0.2
0279		IF(DERC.LT.0.) GO TO 55
0280		DERC=GAM*CCNV
0281		GO TO 56
0282	55	DERD=-GAM*CONV
0283	56	CONTINUE
	C	
	C	CONVERGENCE CRITERIA BY DEL FOR LESS THAN 1 %
	C	
0284		IF(RC1(J).LT.RC2(J)) GO TO 79
0285		DEN1=(ROO(1,J)+ROO(2,J))/2.
0286		DEN2=(ROO(3,J)+ RC2(J))/2.
0287		IF(INEG.EQ.1) KKK=KKK-1
0288		IF(KKK.GT.1) PDEN2=RCG(2,KKK-1)*0.9
0289		IF(TEM.GT.3150.) PDEN2=RCG(2,KKK-1)
0290		IF(KKK.GT.1) PDEN1=ROO(1,KKK-1)
0291		IF(KKK.EQ.1) PDEN1=1.28
0292		IF(KKK.EQ.1.AND.TEM.LT.1500.) PDEN2=0.00001
0293		IF(INEG.EQ.1) KKK=KKK+1
0294		IF(DEN1.GT.PDEN1) GO TO 79
0295		IF(DEN2.LT.PDEN2) GO TO 79
0296		IF(TEM.LT.1500..AND.DEN2.GT.0.004) GO TO 79
0297		IF(CCNV.LT.DEL) GO TO 58
0298	79	CONTINUE
0299		IF(LAM.EQ.1) GO TO 54
0300		IF(CCNV.GT.DUM) DERC=-DERC
0301	54	CONTINUE
0302		LAM=LAM+1
0303		DUM=CCNV

0304		NRD=1
0305		B=8-CERD
0306		IF(LAM.EQ.3) COEF=C.01
0307		IF(LAM.EQ.6) COEF=0.02
0308		IF(LAM.EQ.10) COEF=0.03
0309		IF(LAM.EQ.15) COEF=0.05
0310		IF(LAM.EQ.100) COEF=0.06
0311		IF(LAM.EQ.145) COEF=0.10
0312		IF(LAM.EQ.4) B=8XB
0313		IF(LAM.EQ.7.OR.LAM.EQ.12.OR.LAM.EQ.20) B=8XB
0314		IF(LAM.EQ.110) B=8XB
0315		IF(LAM.EQ.150) B=8XB
0316		IF(LAM.EQ.200) NEG=1
0317		IF(LAM.EQ.200) GO TO 100
0318		GO TO 110
0319	58	RO1(J)=(ROO(1,J)+ROO(2,J))/2.
0320		IF(TEM.LT.1500.) GO TO 78
0321		RC2(J)=(RO2(J)+ROO(3,J))/2.
0322		GO TO 64
0323	78	IF(RC2(J).GT.ROO(3,J)) RO2(J)=ROO(3,J)
0324	64	WRITE(6,68)KMM,LAM,J,RO1(J),RC2(J),VWL,W
0325		IF(NRD.GT.0) NRD=1
0326		ROO(1,KKK)=RO1(J)
0327		ROO(2,KKK)=RO2(J)
0328		TEMP(1,KKK)=TEMP(1,J)
0329		THET(1,KKK)=THET(1,J)
0330		IF(INCNT.EQ.1) THET(2,J)=THEI(1,J)
0331		THET(2,KKK)=THET(2,J)
0332		KKK=KKK+1
0333	70	CONTINUE
0334	100	CONTINUE
0335		KLM=KKK-1
0336		WRITE(6,75) NP,KLM

C
C
C

```

C
0337      NP=KLM
0338      DO 73 I=1,NP
0339      R01(I)=R00(1,I)
0340      R02(I)=R00(2,I)
0341      73  TEMP(2,I)=TEMP(1,I)
0342      IF(INCNT.LT.3)GO TO 119

C
C      DETERMINATION OF RECTILINEAR DIAMETER
C
0343      CALL RECT(R01,R02,TEMP,NP)
0344      119  CONTINUE

C
C      LEAST SQUARE FIT FOR (M-1) DEGREE POLYNOMIAL
C
0345      M = 4
0346      120  CONTINUE
0347      WRITE(6,222)
0348      DO 210 I = 1,2
0349      KK=M
0350      LL = M + 1
0351      DO 130 K = 1,KK
0352      DO 125 L = 1,LL
0353      125  D(K,L)=0.
0354      130  CONTINUE
0355      DO 170 J = 1, NP
0356      R00(1,J) = R01(J)
0357      R00(2,J) = R02(J)
0358      DO 180 L=1,4
0359      180  C(L)=(TEMP(1,J)/100.)**(4-L)
0360      C(5)=R00(1,J)
0361      DO 160 L=1,LL
0362      DO 160 K=1,LL
0363      160  D(K,L)=D(K,L)+C(K)*C(L)
0364      170  CONTINUE
0365      CALL INV(M)

```


0366		DO 200 NK = 1, KK
0367	200	AA(I, NK) = D(NK, LL)
0368	210	CONTINUE
0369		DC 212 I = 1, 2
0370		DO 201 LM = 1, 4
0371	201	Q(IM) = AA(I, IM) / 100. ** (4 - IM)
0372	212	WRITE(6, 213) Q(1), Q(2), Q(3), Q(4)
	C	
	C	DETERMINATION OF CRITICAL TEMPERATURE BY NEWTON-RAPHSON ITERATIONS
	C	
0373		UL = 0.
0374		UUU = 0.
0375		DC 400 LM = 1, 2
0376		TT = 32.
0377		DELT = 0.
0378		DC 215 J = 1, 100
0379		GPR = AA(LM, 1) * 3. * TT ** 2 + 2. * AA(LM, 2) * TT + AA(LM, 3) - A
0380		G = AA(LM, 1) * TT ** 3 + AA(LM, 2) * TT ** 2 + AA(LM, 3) * TT + AA(LM, 4) - A * * TT - B
0381		DELT = G / GPR
0382		IF (ABS(DELT) .GT. 0.01) GC TC 215
0383	216	CONTINUE
0384		TCRIT = TT * 100.
0385		GC TC 219
0386	215	TT = TT - DELT
0387	219	DCRIT = A * TCRIT + B
0388		WRITE(6, 218) TCRIT, DCRIT
0389		UU = UL + DCRIT
0390		UUU = UUL + TCRIT
0391	400	CONTINUE
0392		PCCA = (AN - A) * 100. / A
0393		PCDB = (BN - B) * 100. / B
0394		WRITE(6, 255) A, B, AN, BN, PCCA, PCDB
0395		DC 265 I = 1, NCNT
0396		WRITE(6, 42)
0397	42	FORMAT (/ /)

```

0398      AS=MASS(I)
0399      DO 265 J = 1, NP
0400      TEM=TEMP(I,J)
0401      VOLI=EPI*ELINT(I)*RADIN(I)**2
0402      CALL CCEC(W,WC,WY,PN,PC,PY,INXA,INXB,VUL,VOLI)
0403      VOL(I)=VOL I
0404      XYZ=RO1(J)-RC2(J)
0405      IF(XYZ.EQ.0.)GO TO 96
0406      EM1(J)=RO1(J)*(AS-RO2(J)*VOL(I))/XYZ
0407      EM2(J)=RO2(J)*(VOL(I)*RC1(J)-AS)/XYZ
0408      GO TO 265
0409      96      EM1(J)=MASS(I)
0410      EM2(J)=EM1(J)
0411      265      WRITE(6,270)TEMP(I,J),T-ET(I,J),RC1(J),RO2(J),EM1(J),EM2(J)
0412      WRITE(6,320)
0413      CTT=31.60
0414      TCRIT=X1*CTT**3 +X2*CTT**2+ X3*CTT +X4
0415      TCRIT=3331.
0416      DCRIT=C.345
0417      DO 310 J=1,NP
0418      VOV=1./RO2(J)
0419      VAL=1./RO1(J)
0420      RTP=(TEMP(I,J) +460.)/ITCRIT +460.)
0421      ROL=RO1(J)/DCRIT
0422      RDV=RO2(J)/DCRIT
0423      310      WRITE(6,330) TEMP(I,J),RTP,VAL,VOV,ROL,RDV
0424      500      CONTINUE
0425      WRITE(6,300) TCRIT,DCRIT
C
C      DETERMINATION OF CRITICAL POINT BY TESTING ONE-THIRD LAW
C      (RL-RV)=0*(TC-T)**E
C
0426      IF(NCNT.LT.3) GO TO 452
0427      M=6
0428      MN=M+1
0429      WRITE(6,451)

```

0430		DC 599 I=1,M
0431		Q(I)=1.0
0432		Q(4)=0.5
0433	599	Q(3)=TCRIT
0434	602	DC 601 I=1,MN
0435		DC 601 J=1,MN
0436	601	D(I,J)=0.
0437		KCCUNT=0
0438		DC 650 K=1,NP
0439		Y=ROC(1,K)
0440		X=TEMP(1,K)
0441		CALL ZACOB1(X,Y,C,MN,Q,F)
0442		DC 650 I=1,MN
0443		DC 650 J=1,MN
0444	650	O(I,J)=D(I,J)+C(I)*C(J)
0445		DF=NP-M
0446		STER=SCRT(C(MN,MN)/DF)
0447		CALL INV(M)
0448		DC 620 I=1,M
0449		Q(I)=Q(I)+C(I,MN)
0450		EPS=C.C0001*Q(I)
0451		IF(AES(D(I,MN)),GT.EPS) KCCUNT=1
0452	620	CONTINUE
0453		IF(KCCUNT.EQ.1) GO TO 6C2
0454		WRITE(6,603) (Q(I),I=1,M),STER
0455		WRITE(6,610)
0456		DC 651 K=1,NP
0457		X=TEMP(1,K)
0458		Y=ROC(1,K)
0459		CALL ZACOB1(X,Y,C,MN,Q,F)
0460	651	WRITE(6,611) K,X,Y,F,C(7)
0461	452	CONTINUE
0462		NRUN=NRUN-1
0463		IF(NRUN.GT.0)GO TO 554
	C	
	C	FORMAT STATEMENTS

	C	FORMAT STATEMENTS FOR INPUT
	C	
0464	88	FORMAT(3I5)
0465	1	FORMAT(4I5)
0466	3	FORMAT(2F12.8/3F10.8/2E12.6)
0467	7	FORMAT(2F10.2)
0468	11	FORMAT(6F10.5/7F10.5)
0469	14	FORMAT(F5.0)
0470	15	FORMAT(3F10.5)
0471	17	FORMAT(5F10.5)
0472	600	FORMAT(3F10.5)
	C	
	C	FORMAT STATEMENTS FOR OUTPUT
	C	
0473	4	FORMAT(11////9X,5HNF = ,15/30X,7HNCAI = ,12/30X,8HINOXA = ,12/30X,8 *HIKX8 = ,12/30X,8*EPSIL = ,E12.6/30X,5HPC = ,E12.6/30X,5HPY = ,E1 *2.6/30X,4HP = ,E12.6/30X,7HROO1 = ,E12.6/30X,7HROO2 = ,E12.6////)
0474	5	FORMAT(1H1,20X, 22*INPUT DATA INFORMATION///)
0475	6	FORMAT(1H ,29X,28*EXTERNAL CAPSULE VOL,CU.CM.=F13.5/30X,31HINTERNA *L RADII OF CAPSULE, CM=F10.5/30X,31*EXTERNAL RADII OF CAPSULE, C *M=F10.5/30X,31HINTERNAL LENGTH OF CAPSULE, CM=F10.5/30X,31*EXTERNA *L LENGTH OF CAPSULE, CM=F10.5/30X, 26*LIQUID METAL CHARGE IN GM=F1 *4.5/30X,31*WEIGHT OF CAPSULE ASSEMBLY, GM=F10.5/30X,28*WEIGHT OF C *APSULE ALONE, GM=F13.5/30X,25*WEIGHT OF YOKE ALONE, GM=F16.5/30X,2 *9*VOLUME OF YOKE ALONE, CU.CM.=F12.5)
0476	13	FORMAT(1H11//////10X,15HITERATION CCLK1,10X,5HPOINT,10X,3HRO1,17X,3 *HRO2,16X,3*VUL,16X,1HW/15X,1HM,6X,3HLM//)
0477	75	FORMAT(11////2I5////)
0478	222	FORMAT(1H1)
0479	68	FORMAT(1H ,9X,15,4X,15,9X,15,15X,F7.4,13X,F7.5,12X,F8.4,14X,F8.4)
0480	213	FORMAT(11////40X,6HRO = ,E11.5,7H*1**3 ,E11.5,7H*1**2 ,E11.5,4H*1 * ,E11.5////)
0481	218	FORMAT(1H ////,30X,23*CRITICAL TEMPERATURE = ,F7.2,10X,2HRO,2X,3H *= ,F7.4/)
0482	255	FORMAT(11////20X,89*HEWINGS RECTILINEAR DIAMETER EQUATION CALCULATED

```

*FROM LEAST SQUARES FIT OF HIS DATA POINTS///50X,2H= ,F10.7,5H T +
*,F12.6///20X,69HTHE RECTILINEAR DIAMETER EQUATION OF TEPPER AS QU
30TED BY ROCHMAN      /// 50X, 2H= , F10.7, 3H T +,      F1
*2.6///20X,32HPERCENT DIFFERENCE FOR TERM A IS,F6.3,2H %///20X,32
*HPERCENT DIFFERENCE FOR TERM B IS,F9.3,2H %/1H1,10X,11HTEMPERATURE
*,4X,5HANGLE,11X,13HDENSITY,GM/CC,18X,5HWEIGHT,GM/13X,5HDEG F,
*8X,3HDEG,8X,6HLIQLID,9X,5HVAPOR,10X,6HLIQUID,7X,5HVAPOR//)
0483      300  FORMAT(///10X, 24CRITICAL TEMPERATRE F = , F10.3//10X,
*24HCRITICAL DENSITY GM/CC = , F10.5//)
0484      320  FORMAT(1H1,4X,11HTEMPERATRE,12X,1HNSPECIFIC VOLUME ,
*12X,15HREDUCED DENSITY/1X,8HDEGREE F,4X,7HREDUCED,
*8X,6HLIQUID,4X,5HVAPOR,12X,6HLIQUID,4X,5HVAPOR///)
0485      330  FORMAT(F9.2,F11.5,F14.5,F11.4,F16.5,F11.5)
0486      270  FORMAT(12X,F9.3,3X,F6.2,4X,F10.6,4X,F11.8,6X,F9.6,4X,F9.6)
0487      603  FORMAT(/10X,40DEN=Q1+Q2(Q3-T)**C4+C5(Q3-T)+Q6(Q3-T)**2//
*10X,4HC1 =, E12.4/10X,4HC2 =, E12.4/10X,4HC3 =,E12.4/10X,4HC4 =,
*E12.4/10X,4HC5 =, E12.4/10X,4HC6 =,E12.4///,10X,5HS.O.=,E12.4///)
0488      451  FORMAT(1H1,10X,14HLIQUID DENSITY///)
0489      610  FORMAT(///4X,3HNO.8X,8HVARIALE,14X,11HRESPONSE, Y/3X,5HPOINT,10X
*1HX,13X,8HOBERVED,3X,1HCALCULATED,5X,10HDIFFERENCE///)
0490      611  FORMAT(3X,15,8X,F10.5,4X,F10.5,3X,F10.5,5X,F10.5)
0491      STOP
0492      END
0001      SUBROUTINE COREC(W,WC,WY,F,PC,PY,      INDXA,INDXB,VUL,VOLI)
0002      COMMON AS,ELL,VY,RA,TEM,T,Q(9,9),NFEUN,PRES.
C
C      BUOYANCY AND EXPANSION CORRECTION
C
0003      XTF = ITEM - 32.)/1.8 + 273.15
0004      XTF8 = TEM + 459.6
0005      TC = XTF - 273.15
0006      IF(INDXA-1)500,500,600
C
C      INDXA=1 FOR THE LINEAR THERMAL EXPANSION COEFFICIENT OF W-30MO-30RE
C
0007      500  ALPHA=(.1022E-8)*TC**2 +(.4575E-5)*TC - .9173E-4

```

```

0008      Z = 1. + ALPHA
0009      Z3 = Z**3
0010      GC TC 700
      C
      C      INDXA=2 FOR THE LINEAR THERMAL EXPANSION COEFFICIENT OF TANTALUM-111
      C
0011      600      ALPHA = -.734E-2 + (.536E-3)*TC + (.126E-6)*TC**2
0012      Z = 1. + ALPHA/100.
0013      Z3 = Z**3
0014      700      VOL = VOL * Z3
0015      ELL = ELL * Z
0016      P=P*Z
0017      PC=PC*Z
0018      PY=PY*Z
0019      RA = RA * Z
      C
      C      BASED ON THE VAPOR PRESSURE DATA BY EWINGS
      C
0020      VOLI = VOLI * Z3
0021      IF(INPOLM.EQ.0) GO TO 710
0022      XLNP=8.2817 - 7680.3/XTFR - 1.2*ALCG(XTFR)
      *+4502.60/XTFR*EXP(-7182.5/XTFR)
0023      XP = 2.71828**XLNP
      C
      C      BUOYANCY CONTRIBUTED BY ARGON(39.95) GAS
      C
0024      XDEN = XP*39.95/(82.054*XTFR)
0025      GC TC 740
0026      710      XDEN =PRES * 39.95/(82.054*XTFR * 14.696)
0027      740      XBUOY=XDEN*(VOL+VY)
0028      IF(INDXB.EQ.1)GC TC 750
0029      P = (WY*PY + WC*PC)/(WC + WY)
0030      XBUOY = XDEN*(VOL + VY)
0031      W = WC + WY
0032      750      W = W - XBUOY
0033      RETURN
0034      END

```

```

0001 SUBROUTINE FUNCT(F,DFCR1,CFOR2,P,N,ROO1,ROO2,VUL)
0002 CCMCN AS,ELL,VY,RA,TEM,T,0(9,9),NEQUM,PRES
C
C MOMENT EQUATION AND DERIVATIVES
C
0003 Z = ROO1 - ROO2
0004 EPI = 3.141592
0005 RAV=AS/VUL
0006 SML=C.1000E-10
0007 B=VUL/ELL
0008 BB = B*RA**2
0009 A1=RAV-ROO1
0010 A2=RAV-ROO2
0011 A3=0.5/1P*(AS+W)
0012 X=A3*(VUL*ELL*A1*A2/Z+0.5*BB*Z)
0013 Y=A3*0.25*BB*Z
0014 ANG = T/57.29
0015 TA =TAN(ANG)
0016 F = TA**3 + X*TA**2 + Y
0017 IF(ABS(F).LT.SML) F=0.
0018 AA = TA**2/(2.*P*(AS+W))
0019 DFCR1 = AA*(AS*(ROO2*VUL-AS)/(B*Z**2) + ELL*ROO2*(AS-ROO2*VUL)/Z**
*Z + BB/2.) + BB/(8.*P*(AS+W))
0020 DFCR2 = AA*(AS*(AS-ROO1*VUL)/(B*Z**2) - ELL*ROO1*(AS-ROO1*VUL)/Z**
*Z - BB/2.) - BB/(8.*P*(AS+W))
0021 RETURN
0022 END
0001 SUBROUTINE INV(M)
0002 CCMCN AS,ELL,VY,RA,TEM,T,0(9,9),NEQUM,PRES
0003 DIMENSION B(9,9)
C
C MATRIX INVERSION AND SOLUTION OF SIMULTANEOUS LINEAR EQUATIONS
C
0004 MN = M + 1
0005 DO 8 K = 1,M
0006 DO 1 I = 1,MN

```

```

0007      DO 1 J = 1,MN
0008      1      B(I,J) = Q(I,J)
0009      DO 8 I = 1,MN
0010      DO 8 J = 1,MN
0011      IF(I-K)2,5,2
0012      2      IF(J-K)3,4,3
0013      3      Q(I,J) = (B(I,J)*B(K,K) - B(I,K)*B(K,J))/B(K,K)
0014      GO TO 8
0015      4      Q(I,J) = -B(I,K)/B(K,K)
0016      GO TO 8
0017      5      IF(J-K)6,7,6
0018      6      Q(I,J) = B(K,J)/B(K,K)
0019      GO TO 8
0020      7      Q(I,J) = 1./B(K,K)
0021      8      CONTINUE
0022      RETURN
0023      END
0001      SUBROUTINE ZACOB(X,Y,C,MN,Q,F)
0002      COMMON AS,ELL,VY,RA,TEX,T,Q(S,S),ACCUM,PRES
0003      DIMENSION C(10),Q(10)
0004      C
0005      C(1) IS PARTIAL DERIVATIVES OF FUNCTION WITH RESPECT TO COEFFICIENTS
0006      C TO BE DETERMINED
0007      C
0008      C(1)=1.
0009      DIFF=ABS(Q(3)-X)
0010      C(2)= DIFF**Q(4)
0011      C(3)=Q(2)*Q(4)*DIFF**Q(4)-1./2.*C(6)*DIFF+Q(5)
0012      C(4)=Q(2)*DIFF**Q(4)
0013      C(5)=DIFF
0014      C(6)=DIFF*C(4)
0015      F=Q(1)+C(4)+Q(5)*DIFF+Q(6)*DIFF**2
0016      C(4)=F*ALOG(Q(3)-X)
0017      C(7)=Y-F
0018      RETURN
0019      END

```


0001		SUBROUTINE RECT(R01,R02,TEMP,NP)
0002		COMMON AS,ELL,VY,RA,TEM,T,D(9,9),RECUM,PRES
0003		DIMENSION RECTLN(200),SUM(5),TEMP(3,200)
	C	
	C	CALCULATION OF RECTILINEAR DIAMETER
	C	
0004		DO 1 N=1,5
0005	1	SUM(N)=0.
0006		DO 2 J=1,NP
0007		RECTLN(J)=(R01(J)+R02(J))/2.
0008		SUM(1)=SUM(1)+RECTLN(J)
0009		SUM(2)=SUM(2)+TEMP(1,J)
0010		SUM(3)=SUM(3)+TEMP(1,J)**2.
0011	2	SUM(4)=SUM(4)+TEMP(1,J)*RECTLN(J)
0012		AAA=NP
0013		A1=(AAA*SUM(4)-SUM(1)*SUM(2))/(AAA*SUM(3)-SUM(2)**2.)
0014		A2=(SUM(1)-A1*SUM(2))/AAA
0015		DO 3 I=1,NP
0016	3	SUM(5)=SUM(5)+(RECTLN(I)-A1*TEMP(1,I)-A2)**2.
0017		SE=SQRT(SUM(5)/(AAA-2.))
0018		WRITE(6,4)A1,A2,SE
0019		WRITE(6,5)
0020		DO 6 J=1,NP
0021		DUM=A1*TEMP(1,J)+A2
0022		DEL=RECTLN(J)-DUM
0023	6	WRITE(6,7)TEMP(1,J),RECTLN(J),DUM,DEL
0024	4	FORMAT(1H1,19X,16HCOEFFICIENT A1 *E12.6/20X,16HCOEFFICIENT A2 *E1 *2.6/20X,25HSTANDARD ERROR ESTIMATED=E12.6/////15X,6HWHERE,/20X, *27HRECT. LIN. DIAM.= A1*T + A2////)
0025	5	FORMAT(1H1,30X,16HRECT. LIN. DIAM./5X,14HTEMPERATURE, F,5X,8HOBSE *VED,6X,6HFITTED,10X,10HDIFFERENCE//)
0026	7	FORMAT(9X,F10.4,1X,2E12.6,8X,E12.6)
0027		RETURN
0028		END

Table E-2.

```

C .....
C
C NON-LINEAR EQUATIONS
C NRUN=NUMBER OF RUN
C NTRAN=VARIABLE TRANSFORMATIONS
C NCASE= THE FOLLOWING EQUATIONS TO FIT.
C NCASE=0, ANGLE VS. TEMPERATURE POLYNOMIAL
C NCASE=1 AND 2, GUGGENHEIM FOR SIMPLE FLUIDS
C NCASE=3,  $\eta$  POLYNOMIAL WITH MD DEGREES
C NCASE=4, BUCKINGHAM CORRELATION
C NCASE=5,  $(1/V)/\rho$  VS.  $\log T$ 
C NCASE=6, RECTILINEAR DIAMETER
C NCASE=7, VIRIAL EQUATION FOR VAPOR DENSITY
C NCASE=8, LIQUID DENSITY CORRELATION
C .....
C DIMENSION AF(250),X(250),Y(250),A(10,10),B(10,10),DJM(250),CC(10)
C ,RL(250),RV(250),THET(250),V1(3,250),T(250)
C DATA NRUN/1/
C DATA PO/0.44/
C DATA TC/3331.7,RC/0.347/
C
C CALCULATION OF FUNCTIONS, DERIVATIVES AND ITERATION MATRIX
C
53 CONTINUE
   LMN=0
   LLL=0

```

```

199 READ(5,99) NCASE, NP, NTRAN
    LLL=LLL+1
    IF(NCASE.EQ.3) GO TO 80
    NCNTN=1
    NNN=NP
1   DO 2 I=1,10
    DO 2 J=1,10
2   A(I,J)=0.
    SUM1=0.
    SUM2=0.

C
C   CORRELATION OF FLUID DENSITIES NEAR THE CRITICAL REGION
C   AND NP IS NUMBER OF POINTS.
C
    IF(NCASE.EQ.0) GO TO 5
    IF (NCASE.GT.1) GO TO 16
    WRITE(6,401)
    GO TO 16
5   WRITE(6,104)
    READ(5,102) (THET(M), (VT(L,M), L=1,3), T(M), M=1, NP)
16  CONTINUE
    IF(NCASE.EQ.6) NP=NNN
    IF(NCASE.EQ.7) NP=NNN-40
    DO 3 M=1, NP
    IF(NCASE.EQ.1) GO TO 4
    IF(LMN.EQ.0) GO TO 4
    IF(NCASE.GT.1) GO TO 9
C.....
C   TO SOLVE THE EQUATION 0
C.....
    WRITE(6,105) THET(M), (VT(L,M), L=1,3)
    Y(1)=THET(M)
    Y(4)=ABS(Y(1))
    CS1=VT(1,M)
    CS2=VT(2,M)
    CS3=VT(3,M)

```

```

SUM1=69.7643+107.682*CS1 -1.26092*CS1*CS1+0.0360507*CS1**3
SUM2=69.7643+107.682*CS2 -1.26092*CS2*CS2+0.0360507*CS2**3
SUM3=69.7643+107.682*CS3 -1.26092*CS3*CS3+0.0360507*CS3**3
X(M)=(SUM1+SUM2+SUM3)/3.

```

```
MD=2
```

```
MDM=6
```

```
GO TO 3
```

```
4 READ(5,103) T(M),RL(M),RV(M)
```

```
WRITE(6,402) T(M),RL(M),RV(M)
```

```
9 CONTINUE
```

```
GO TO (20,21,22,23,24,25,30), NCASE
```

```

C .....
C LEAST SQUARE FITTING OF EQUATION 1
C .....

```

```
20 Y(M)=ALOG(RL(M)-RV(M))/ALOG(TC-T(M))
```

```
X(M)=TC-T(M)
```

```
MD=1
```

```
MDM=3
```

```
GO TO 3
```

```
21 Z=(RL(M)-RV(M))/(RL(M)+RV(M))
```

```
IF(NCASE.EQ.3) GO TO 22
```

```
Y(M)=Z/(1.-ALOG(Z))
```

```
X(M)=T(M)
```

```
MD=1
```

```
MDM=3
```

```
GO TO 3
```

```
22 X(M)=1.-(T(M)+460.)/(TC+460.)
```

```
Z=(RL(M)-RV(M))/(RL(M)+RV(M))
```

```
Y(M)=Z*Z/(1.-ALOG(Z))
```

```
SUM1=X(M)+Y(M)+SUM1
```

```
SUM2=X(M)*X(M)+SUM2
```

```
GO TO 3
```

```
23 Y(M)=2.*RV(M)/(RV(M)+RL(M))
```

```
X(M)=ALOG(T(M))
```

```

MD=2
MDM=3
GO TO 3
24 Y(M)=(RV(M)+RL(M))/2.
X(M)=T(M)
MD=2
MDM=4
GO TO 3
25 L=M+40
XTR=T(L)+759.6
XLNP=-11.771+3558.75/XTR+.16.2416*ALOG(X)+R)-0.0032759*XTR
XP=2.71828**XLNP*85.48*144+.01602/1545.
Y(M)=XP/(RV(L)*XTR)
X(M)=RV(L)
MD=4
MDM=5
GO TO 3
30 Y(M)=RL(M)
X(M)=TC-T(M)
3 CONTINUE
LM=1
IF (NCASE.EQ.0) GO TO 32
GO TO (82,82,80,83,82,82,82,35), NCASE
33 COF=SUM1/SUM2
SUM=0.
DO 34 I=1,NP
DUM(I)=Y(I)-COF*X(I)
34 SUM=SUM+DUM(I)**2
STD=SQRT(SUM/FLOAT(NP-1))
WRITE(6,200) COF,STD
GO TO 400
35 DO 6 M=1,NP
CALL FUNC(M,CC,X,Y,PO)
DO 6 I=1,5
DO 6 J=1,5
6 A(I,J)=A(I,J)+CC(I)*CC(J)

```

C
C CRL,CT,CCRL,CCT ARE CONSTRAINT POINTS OF DENSITIES AND TEMPERATURE
C

CRL=0.37025
CCRL=0.40911
CT=3330.86
CCT=3324.83
CX=TC-CT
CCX=TC-CCT
IF(NCNTN.EQ.1) ML=6
IF(NCNTN.EQ.2.OR.NCNTN.EQ.3) ML=5
MN=ML+1
DO 7 I=1,MN
A(I,MN)=A(I,5)
7 A(MN,1)=A(1,MN)
IF(NCNTN.EQ.3) GO TO 45
A(1,5)=1.
A(2,5)=CX**P0
A(3,5)=CX
A(4,5)=CX*CX
A(5,5)=1.
IF(NCNTN.EQ.2) GO TO 42
A(1,6)=1.
A(2,6)=CCX**P0
A(3,6)=CCX
A(4,6)=CCX*CCX
A(6,6)=1.
A(5,7)=CRL
A(6,7)=CCRL
A(7,7)=CRL*CCRL
GO TO 47
42 CONTINUE
A(5,6)=CRL
A(6,6)=CRL**2
GO TO 47
45 CONTINUE

```

A(1,5)=1.
A(2,5)=CCX**PO
A(3,5)=CCX
A(4,5)=CCX*CCX
A(5,5)=1.
A(5,6)=CCRL
A(6,6)=CCRL*CCRL
47 DO 8 I=1,MN
A(5,I)=A(1,5)
A(MN,I)=A(1,MN)
8 A(ML,I)=A(1,ML)
CALL INV(A,ML)
I=4
SUM=0.
DO 27 I=1,NP
AF(I)=A(1,MN)+A(2,MN)*X(I)**PO
DO 26 J=3,4
26 AF(I)=AF(I)+A(J,MN)*X(I)**(J-2)
DUM(I)=Y(I)-AF(I)
27 SUM=SUM+ DUM(I)**2
WRITE(6,50)
DO 49 I=1,M
49 WRITE(6,48) A(I,MN)
C
C STATISTICS
C
DNP=NP-M
STER=SQRT(SUM/DNP)
WRITE(6,60) DNP,SUM,STER
GO TO 400

```

```

C.....
C POLYNOMIAL CURVE FITTING OF DEGREE MD
C INPUT FORMAT IS,
C NCASE=3, NP=IN 15
C X AND Y IN F10,5 FOR NP POINTS
C.....

```

```

80  READ(5,100) (X(I),Y(I), I=1,NP)
    CALL TRANS(NP,NTRAN,X,Y)
    READ(5,97) MD
82  CONTINUE
    WRITE(6,94)
94  FORMAT(1H1)
    WRITE(6,302)MD.
    CALL POLY(X,Y,AF,DUM,A,NP,MD)
400  WRITE(6,201)
    WRITE(6,202)(I,X(I),Y(I),AF(I),DUM(I),I=1,NP)

C
C  FORMAT STATEMENTS FOR INPUT
97  FORMAT(15)
99  FORMAT(315)
100  FORMAT(2F10.5)
102  FORMAT(5F10.5)
103  FORMAT(3F10.5)
C
C  FORMAT STATEMENTS FOR OUTPUT
48  FORMAT(10X,3HA =E12.6/)
50  FORMAT(///5X,23HCOEFFICIENT OF LIQ.DEN.//5X,29HFIRST ONE IS THE LO
    2WEST ORDER//)
50  FORMAT(///5X,18HDEGREE OF FREEDOM=F5.0/5X,9HVARIANCE=F15.5/5X,25HST
    1ANDARD ERROR ESTIMATED=F10.5/////)
104  FORMAT(1H1,6X,5HANGLE, 13X, 17HW/RE THERMOCOUPLE/ 4X,9HIN DEGREE,
    113X, 15HREADINGS, IN MV///)
105  FORMAT(F12.2,6X,3F10.3)
401  FORMAT(1H1,6X, 11HTEMPERATURE, 8X, 14HDENSITY, 6M/CC/10X,
    18HDEGREE F, 8X, 6HLIQUID, 6X,5HVAPOR///)
402  FORMAT(5X,F12.2,4X, 2F10.5)
200  FORMAT(1H1,5X, 14HCOEFFICIENT = , E12.5/5X,16HSTANDARD DEV. = .
    *E12.6///)
302  FORMAT(1H1,5X,31HLEAST SQUARES FIT OF POLYNOMIAL//10X,8HDEGREE =15
    *////)
201  FORMAT(///4X,3HNO., 8X,9HVARIALE,15X,11HRESPONSE, Y/3X,5HPPOINT,10X
    *,1HX,14X,8HOBSERVED,4X,10HCALCULATED,7X,10HDIFFERENCE///)

```



```

202  FORMAT(3X,15,6X,F10.3,4X,F10.5,6X,F10.5,5X,F10.5)
      IF(NCASE.EQ.3) GO TO 300
      IF(NCASE.EQ.8) NCNTN=NCNTN+1
      IF(NCASE.EQ.8.AND.NCNTN.EQ.4) GO TO 300
      IF(NCASE.EQ.8) GO TO 1
      IF(MD.EQ.MDM) GO TO 41
      MD=MD+1
      GO TO 42
41  IF(NCASE.EQ.0) GO TO 300
      IF(NCASE.EQ.7) GO TO 73
      GO TO 75
73  WRITE(6,74)
74  FORMAT(/// 5X,34HVIRIAL EQUATION FOR Rb310IUM VAPOR 141)
75  CONTINUE
      IF(NCASE.EQ.2) NCASE=NCASE+1
      NCASE=NCASE+1
      GO TO 1
300  IF(LLL.EQ.NRUN) GO TO 1000
      GO TO 199
1000 CONTINUE
      STOP
      END

      SUBROUTINE TRANS(NP,NTRAN,X,Y)
      DIMENSION X(250), Y(250)
      .....
C .....
C      NTRAN=1 FOR X=ALOGX
C      NTRAN=2 FOR Y=ALOGY
C      NTRAN=3 FOR X AND Y IN LOGARITHMIC TRANSFORMATION
C      NTRAN=4 FOR X=X/(1-ALOGX)
C .....
      DO 100 I=1,NP
      IF(NTRAN.EQ.0) GO TO 100
      IF(NTRAN.EQ.1) GO TO 1
      IF(NTRAN.EQ.2) GO TO 2
      IF(NTRAN.EQ.3) GO TO 3

```

```

      IF (NTRAN.EQ.4) GO TO 4
      GO TO 100
1     X(I)=ALOG(X(I))
      GO TO 100
2     Y(I)=ALOG(Y(I))
      GO TO 100
3     X(I)=ALOG(X(I))
      Y(I)=ALOG(Y(I))
      GO TO 100
4     X(I)=X(I)/(1.-ALOG(X(I)))
10    X(I)=X(I)/(1.-ALOG(X(I)))
100   CONTINUE
      RETURN
      END
      SUBROUTINE INV(A,M)
      DIMENSION A(10,10),B(10,10)
      MN = M + 1
      DO 8 K = 1,M
      DO 1 I = 1,MN
      DO 1 J = 1,MN
1     B(I,J) = A(I,J)
      DO 8 I = 1,MN
      DO 8 J = 1,MN
      IF(I-K)2,5,2
2     IF(J-K)3,4,3
3     A(I,J) = (B(I,J)*B(K,K) - B(I,K)*B(K,J))/B(K,K)
      GO TO 8
4     A(I,J) = -B(I,K)/B(K,K)
      GO TO 8
5     IF(J-K)6,7,6
6     A(I,J) = B(K,J)/B(K,K)
      GO TO 8
7     A(I,J) = 1./B(K,K)
8     CONTINUE
      RETURN
      END

```

```

SUBROUTINE FUNC(M,CC,X,Y,P0)
DIMENSION CC(15),Y(250),X(250)
CC(1)=1.
CC(2)=X(M)**P0
DO 1 I=3,4
1 CC(I)=X(M)**(I-2)
CC(5)=Y(M)
RETURN
END
SUBROUTINE POLY(X,Y,AF,DUM,A,NP,MD)
DIMENSION X(250),Y(250),AF(250),DUM(250),A(10,10),B(10,10),C(10)
C
C POLYNOMIAL DEGREE MD IN 15
C
M=MD+1
MN=MD+2
DO 1 I=1,MN
DO 1 J=1,MN
1 A(I,J)=0.
DO 4 K=1,NP
DO 10 I=1,M
C(I)=(X(K)/100.)**(M-I)
10 C(MN)=Y(K)
DO 4 I=1,MN
DO 4 J=1,MN
4 A(I,J)=A(I,J)+C(I)*C(J)
CALL INV(A,M)
SUM=0.
DO 6 I=1,NP
S=0.
DO 5 J=1,M
5 S=S+(X(I)/100.)**(M-J)*A(J,MN)
AF(I)=S
DUM(I)=Y(I)-S
6 SUM=SUM+DUM(I)**2
WRITE(6,50)

```

```

DO 49 I=1,M
A(1,MN)=A(1,MN)/100.** (M-1)
49 WRITE(6,48) A(1,MN)
48 FORMAT(10X,3HA =E12.6/)
50 FORMAT(///5X,23HPOLYNOMIAL COEFFICIENTS//5X,30HFIRST ONE IS THE HI
*GHEST ORDER//)
DNP=NP-M
STER=SQRT(SUM/ONP)
WRITE(6,50)DNP,SUM,STER
60 FORMAT(///5X,18HDEGREE OF FREEDOM=F5.0/5X,9HVARIANCE=F15.5/5X,25HST
*ANDARD ERROR ESTIMATED=F10.5////)
RETURN
END

```

Table E-3.

```

      DIMENSION T(200),RHL(200),RHV(200),TR(200),P(200),HV(200),
      *SV(200),TRED(200),XX(200),U(200)
      DATA DOT,ASTER,PLUS,X/1H.,1H*,1H+,1HX/
1000  READ(5,100)N
100   FORMAT(15)
      DO 1 I=1,N
1    READ(5,101) T(I),RHL(I),RHV(I)
101   FORMAT(3F10.5)
3-----CONVERT T TO DEG RANKINE
      DO 2 I=1,N
2    TR(I)=T(I)+459.7
      CF=.061023744/(12.*.0022046226*777.9)
C-----CALCULATE PRESSURES
      A=-114.771
      B=3558.75
      C=16.2416
      D=-0.0032759
      DO 3 I=1,N
      XX(I)=A+B/TR(I)+C*ALOG(TR(I))+D*TR(I)
      XTFR=XX(I)
3    P(I)=2.71828**XTFR
C-----CALCULATE ENTHALPY OF VAPORIZATION
      DO 4 I=1,N
4    HV(I)=(1./RHV(I)-1./RHL(I))*P(I)*(-B/TR(I)+C+D*TR(I))*CF
C-----CALCULATE ENTROPY OF VAPORIZATION
      DO 5 I=1,N
5    SV(I)=HV(I)/TR(I)

```

```

C-----CALCULATE REDUCED TEMP IN DEG RANKINE
DO 6 I=1,N
6 TR(I)=TR(I)/(3331.+459.7)
CF2=144./((777.900*62.427961)
DO 111 I=1,N
111 U(I)=HV(I)-P(I)*(1./RHH(I)-1./RHL(I))*CF2
WRITE(6,500)
500 FORMAT(1H1,1X,13HTEMPERATURE F, 1X,13HTEMPERATURE R,1X,13HREDUCED
*TEMP., 1X,13HPRESSURE PSI.,4X,9HENTHALPY,5X,7HENTROPY, 3X,
*15HINTERNAL ENERGY///)
DO 10 I=1,N
10 WRITE(6,501) I(I),TR(I),TREO(I),P(I),HV(I),SV(I),J(I)
501 FORMAT(7E14.6)
DO 8 I=1,N
8 TR(I)=TR(I)/1000.
CALL RANGE(TR, 1,N,XL,XS)
CALL RANGE(HV, 1,N,YL,YS)
AC=PLUS
CALL PLOT(TR,HV,1,N,48,XL,XS,YL,YS,AC)
WRITE(6,201)
201 FORMAT(/8H YSCALE=,1X,24HENTHALPY OF VAPORIZATION/8H XSCALE=,
*1X, 13HTEMPERATURE R)
CALL RANGE(SV, 1,N,YL,YS)
AC=ASTER
CALL PLOT(TR,SV,1,N,48,XL,XS,YL,YS,AC)
WRITE(6,202)
202 FORMAT(/8H YSCALE=,1X, 7HENTROPY/8H XSCALE=,1X,13HTEMPERATURE R)
AC=DOT
CALL PLOT(TREO,SV, 1,N,48,1.,.5,YL,YS,AC)
WRITE(6,203)
203 FORMAT(/8H YSCALE=,1X,7HENTROPY/8H XSCALE=, 1X, 19HREDUCED TEMPER
*ATURE)
CALL RANGE(U, 1,N,YL,YS)
AC=X
CALL PLOT(TR,U, 1,N,48,XL,XS,YL,YS,AC)
WRITE(6,204)
204 FORMAT(/8H YSCALE=, 1X,15HINTERNAL ENERGY/8H XSCALE=,1X,
*13HTEMPERATURE R)

```

STOP

END

SUBROUTINE RANGE(R,JJ,II,RL,RS)

RAN10

C.....
C R, ARRAY OF NUMBER, FOR WHICH RANGE WILL BE DETERMINED
C JJ, STARTING INTEGER
C II, TOTAL NUMBER OF POINTS, OR LAST INTEGER
C RL, LARGEST OF R
C RS, SMALLEST OF R

C.....
C DIMENSION R(500) RAN20
C RL=R(JJ) RAN30
C RS=R(JJ) RAN40
C DO I=JJ,II RAN50
C RL=AMAX1(RL,R(I)) RAN60
C RS=AMIN1(RS,R(I)) RAN70
C RETURN RAN80
C END RAN90

SUBROUTINE PLOT(X,Y,J1,KI,IYMAX,XL,XS,YL,YS,AC)

C.....
C
C KI-J1=(TOTAL NUMBER OF POINTS - 1)
C
C.....

C DIMENSION X(500),Y(500),IX(500),IY(500),AFX(200),ZX(200)
C DATA BLX/1H /
C XSCALE=(XL-XS)*0.01
C YSCALE=(YL-YS)/FLOAT(IYMAX)
C J=0
C DO 105 I=J1,KI
C IF (XL.LT.X(I),OR,X(I).LT.XS,OR,YL.LT.Y(I),OR,Y(I).LT.YS)
C 1GO TO 105
C J=J+1
C IX(J)=(X(I)-XS)/XSCALE+0.5
C IY(J)=(Y(I)-YS)/YSCALE+0.5

```

105  CONTINUE
      JMAX=J
      FORMAT (1H1)
      FORMAT (10H Y VALUES ,4X,1H1,21(5H....1))
      FORMAT (1H .F10.3,2X,1H1,101A1,4X,1H1)
      FORMAT (14X,1H1,21(5H....1))
      FORMAT (8X,11(2X,F8.3))
      FORMAT(8H YSCALE=,F8.3/8H XSCALE=,F8.3)
      WRITE (6,1)
      WRITE (6,2)
      IF (IYMAX.GT.55) GO TO 11
      LL=IYMAX
      GO TO 13
10    LL=LL-1
      GO TO 13
11    LL=0
      GO TO 13
12    LL=LL+1
13    DO 14 I=1,101
14    AFX(I)=BLX
      DO 16 J=1,JMAX
      IF (IY(J).NE.LL) GO TO 16
15    II=IX(J)+1
      AFX(II)=AC
16    CONTINUE
      ZY=FLOAT(LL)*YSCALE+YS
      WRITE (6,4) ZY,(AFX(I),I=1,101)
      IF (IYMAX.GT.55) GO TO 18
17    IF (LL) 19,19,10
18    IF (LL.LT.IYMAX) GO TO 12
19    WRITE (6,6)
      DO 20 K=1,11
20    ZX(K)=10.0*FLOAT(K-1)*XSCALE+XS
      WRITE (6,7) (ZX(K),K=1,11)
      WRITE (6,8) YSCALE,XSCALE
      RETURN
      END

```



```

C.....
C  CALCULATION OF CAPSULE PIVOT FROM LOW TEMPERATURE DATA
C  DENSITY DATA FROM AGN REPORT BY ACHENER
C  RI = INTERNAL RADIUS IN CM
C  ALE = INTERNAL CAPSULE LENGTH IN CM
C  AM = LIQUID METAL CHARGE IN GM
C.....
C  DIMENSION ANG(50),RO1(50),RO2(50),ALPHA(50),P(50),T(50),PRE(50)
1000 NCNT=1
99  READ(5,1) NP,R,AL,AM,VV,CV,WC,MY,CP,YP
1   FORMAT(15,4F10.5/5F10.5)
   READ(5,2) (RO2(I),T(I),PRE(I),ANG(I),I=1,N5)
2   FORMAT(4F10.5)
   PEI = 3.1415926
   SUM = 0.0
   WRITE(6,80)
80  FORMAT(1H1)
   WRITE(6,40)
40  FORMAT(9X,13HTEMPERATURE F, 7X,5HANGLE,8X,9HLD, DEN.,5X,9HVAP. DE
   *N., 9X,5HPIVOT////)
   DO 100 I=1,NP
   XTFR=T(I)+459.6
   T(I)=(T(I)-32.)/1.8
   RO1(I)=0.347+0.02105*(1810.-T(I))*(.5)+0.0001342*(1810.-T(I))
   ALPHA(I)=(-.1022E-8)*T(I)**2+(-.4575E-5)*T(I)-.9173E-4
   T(I)=T(I)+1.8+32.
   ALE = AL *(1.+ALPHA(I))
   RI = R *(1.+ALPHA(I))
   VOL = PEI*RI**2*ALE
   AEP=1.+ALPHA(I)
   VV=VV*AEP**3
   VC=CV*AEP**3
   PC=CP*AEP
   PY=YP*AEP
   XDEN=PRE(I)*39.95/(82.054*XTFR*14.7)
   XBUOY=XDEN*(VC+VY)

```

```

N=WC+WY-XBUOY
RDUM = AM/VOL
CONST = AM + W
Y = RI**2*VOL*(R01(1)-R02(1))/(ALE*8.*CONST)
ANUM = VOL*ALE*(RDUM-R02(1))*(RDUM-R01(1))
ANUM = ANUM/((R01(1)-R02(1))*2.*CONST)
X = ANUM + 2.*Y
IF(NCNT.EQ.2) ANG(1)=ANG(1)-1.30
THETA = ANG(1)/57.29
TA=SIN(THETA)/COS(THETA)
P(1) = -(X+TA**2*Y)/(TA**3)
P(1)=(P(1)*(WC+WY)-WY*Y)/WC
P(1)=P(1)/(1.+ALPHA(1))
SUM = SUM + P(1)
100 WRITE(6,50) T(1),ANG(1),R01(1),R02(1),P(1)
50  FORMAT(10X,F10.5,5X,F10.5,5X,F10.6,5X,F10.5,5X,F10.6)
ABC = NP
AVER = SUM/ABC
SOS = 0.0
DO 200 I = 1,NP
200 SOS = (P(I)-AVER)**2/(ABC+1.) + SOS
SOS = SQRT(SOS)
81  WRITE(6,81)
    FORMAT(//////)
    WRITE(6,51)AVER,SOS
51  FORMAT(5X,8HAVERAGE=,F10.6/20H STANDARD ERROR EST=,E10.4)
    NCNT=NCNT+1
    IF(NCNT.EQ.2) GO TO 99
    STOP
END

```

```

C.....
C      NCNT = 0: T.C. CONVERSION
C      = 1: CORRECTION ANGLE STATISTICS
C      = 2: BISECTION METHOD FOR POLYNOMIAL ROOT
C      CONVERSION OF THERMOCOUPLE READINGS IN MV TO DEGREE FARRENHEIT
C      NR = NUMBER OF RUN FOR THE GIVEN SETTING
C      NP = NUMBER OF EXPERIMENTAL POINTS FOR THE GIVEN RUN
C      NAX = NUMBER OF THERMOCOUPLES
C.....
C      DIMENSION AX(4,200),FT(200),THETA(200)
C
C      COMMON INPUT FOR ALL OPTIONS
C
C      READ(1,9) NRUN
99      READ(1,1) NP,NAX,NCNT
C      NDUM = 0
C      IF(NCNT.EQ.1) GO TO 11
C      IF(NCNT.EQ.2) GO TO 70
C.....
C      CORRECTION ANGLE STATISTICS
C.....
C      READ(1,2) (THETA(I),(AX(J,I),J=1,NAX),FT(I), I=1,NP)
C      WRITE(2,100)
C      DO 60 I=1,NP
60      WRITE(2,300) THETA(I),(AX(J,I),J=1,NAX),FT(I)
C      GO TO 12
C.....
C      THERMOCOUPLE CONVERSION
C.....
11      READ(1,3) (THETA(I), I=1,NP)
C      SUM=0.
C      DO 40 I=1,NP
40      SUM=SUM+THETA(I)

```

```

AVER=SUM/FLOAT(NP)
WRITE(2,45)
SUM=0.
DO 41 I=1,NP
DIF=THETA(I)-AVER
WRITE(2,46) I,THETA(I),DIF
41 SUM=SUM+ABS(DIF)**2
DEV=SUM/FLOAT(NP)
ST=SQRT(DEV)
MM=NP-1
WRITE(2,47) MM,AVER,ST
GO TO 50
12 CONTINUE
WRITE(2,400)
DO 5 I=1,NP
SUM=0.
DO 4 J=1,NAX
AVER=AX(J,I)
TF =.036067*AVER**3-1.26092*AVER**2+107.682*AVER+69.7643
AX(J,I)=TF
SUM=SUM+AX(J,I)
AN=FLOAT(NAX)
AV=SUM/AN
4 CONTINUE
5 WRITE(2,200) THETA(I),(AX(J,I),J=1,NAX),AV
GO TO 50
C.....
C BISECTION METHOD FOR POLYNOMIAL ROOT
C.....
70 DELX=10.
NDN=1
EPS=0.00001
61 IF(NDN.EQ.2) GO TO 62
X=3310.
A0=0.768337E+02
A1=-0.219772E-02

```

```

A2=0.501701E-05
A3=-0.340061E-08
A4=0.
A5=0.
A6=0.
VOL=3.6894808
AMET=1.563
GO TO 65
62 X=3320.
A0=-0.167966E+03
A1=0.127192
A2=-0.432263E-05
A3=0.458444E-08
A4=-0.960048E-12
A5=-0.539126E-15
A6=-0.277789E-19
VOL=3.6955973
AMET=1.417
65 CONTINUE
71 Y=A0+A1*X+A2*X*X+A3*X**3+A4*X**4+A5*X**5+A6*X**6
IF(ABS(Y).LT.EPS) GO TO 80
IF(Y.LT.0.) GO TO 75
72 X=X+DELX
GO TO 71
75 X=X-DELX
DELX=DELX/2.
GO TO 72
80 CONTINUE
AA=(X-32.)/1.8
ALP=1.022E-9 *AA*AA + 4.575E-6 *AA -0.9173E-4
V=(1.+ALP)**3*VOL
DEN=AMET/V
WRITE(2,510) Y,X,DEN,A0,A1,A2,A3,A4,A5,A6
IF(NDN.EQ.2) GO TO 50
NDN=2
DELX=10.

```

```

        GO TO 61
50      CONTINUE
        NDUM=NDUM+1
        IF (NDUM.LT.NRUN) GO TO 99
C      FORMAT STATEMENTS
9        FORMAT(I5)
1        FORMAT(3I5)
2        FORMAT(5F10.4)
100     FORMAT(1H1///5X,10HANGLE,DEG. 7X, 24HTHERMOCOUPLE READINGS,MV,
          * 6X, 16HARGON PRES. PSIA////)
200     FORMAT(F10.3,3F12.3,F14.2)
300     FORMAT(F14.3, 4F12.3)
45      FORMAT(1H1, 2X, 1H1, 13X, 8HOBSERVED, 1X, 10HDIFFERENCE//)
46      FORMAT(14,10X, 2F10.3)
400     FORMAT(1H1///33X, 13HTEMPERATURE,F,/5X, 10HANGLE,DEG.,
          *5X, 5HT.C.1, 6X, 5HT.C.2, 7X, 5HT.C.3, 8X, 7HAVERAGE////)
47      FORMAT(/////10X, 20HDEGREE OF FREEDOM = 15/10X, 7HMEAN = ,10X,
          *F10.3/10X, 21HSTANDARD DEVIATION = , F10.3//)
3        FORMAT(6F10.3)
510     FORMAT(1H1, 5X, 12HANGLE,DEG. = F12.7/6X,15HTEMPERATURE,F = F10.2
          (/6X, 15HDENSITY,GM/CC = F10.5
          *///6X, 4HY = E14.6/10X, E14.6, 2H*X/10X, E14.6, 5H*X**2/10X,E14.6,
          25H*X**3/10X,E14.6, 5H*X**4/10X,E14.6, 5H*X**5/10X,E14.6, 5H*X**6/)
          STOP
          END

```

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      AVER=SUM/FLOAT(NP)
      WRITE(2,45)
      SUM=0.
      DO 41 I=1,NP
      DIF=THETA(I)-AVER
      WRITE(2,46) I,THETA(I),DIF
41    SUM=SUM+ABS(DIF)**2
      DEV=SUM/FLOAT(NP)
      ST=SQRT(DEV)
      MM=NP-1
      WRITE(2,47) MM,AVER,ST
      GO TO 50
12    CONTINUE
      WRITE(2,400)
      DO 5 I=1,NP
      SUM=0.
      DO 4 J=1,NAX
      AVER=AX(J,I)
      TF =.036067*AVER**3-1.26092*AVER**2+107.682*AVER+69.7643
      AX(J,I)=TF
      SUM=SUM+AX(J,I)
      AN=FLOAT(NAX)
      AV=SUM/AN
4    CONTINUE
5    WRITE(2,200) THETA(I),(AX(J,I),J=1,NAX),AV
      GO TO 50
C.....
C    BISECTION METHOD FOR POLYNOMIAL ROOT
C.....
70    DELX=10.
      NDN=1
      EPS=0.00001
61    IF(NDN.EQ.2) GO TO 62
      X=3310.
      A0=0.768337E+02
      A1=-0.219772E-02

```

```

A2=0.501701E+05
A3=-0.340061E-08
A4=0.
A5=0.
A6=0.
VOL=3.6894808
AMET=1.563
GO TO 65
62 X=3320.
A0=-0.167966E+03
A1=0.127192
A2=-0.432263E-05
A3=0.458444E-08
A4=-0.960048E-12
A5=-0.539126E-15
A6=-0.277789E-19
VOL=3.6955973
AMET=1.417
65 CONTINUE
71 Y=A0+A1*X+A2*X*X+A3*X**3+A4*X**4+A5*X**5+A6*X**6
IF(ABS(Y).LT.EPS) GO TO 80
IF(Y.LT.0.) GO TO 75
72 X=X+DELX
GO TO 71
75 X=X-DELX
DELX=DELX/2.
GO TO 72
80 CONTINUE
AA=(X-32.)/1.8
ALP=1.022E-9 *AA*AA + 4.575E-6 *AA -0.9173E-4
V=(1.+ALP)**3*VOL
DEN=AMET/V
WRITE(2,510) Y,X,DEN,A0,A1,A2,A3,A4,A5,A6
IF(MOD(EQ,2) GO TO 50
NDN=2
DELX=10.

```

```

      GO TO 61
50    CONTINUE
      NDUM=NDUM+1
      IF (NDUM.LT.NRUN) GO TO 99
C     FORMAT STATEMENTS
9     FORMAT(15)
1     FORMAT(3I5)
2     FORMAT(5F10.4)
100   FORMAT(1H1///5X,10HANGLE,DEG. 7X, 24HTHERMOCOUPLE READINGS,MV,
* 6X, 16HARGON PRES. PSIA////)
200   FORMAT(F10.3,3F12.3,F14.2)
300   FORMAT(F14.3, 4F12.3)
45    FORMAT(1H1, 2X, 1H1, 13X, 8HOBERVED, 1X, 10HDIFFERENCE//)
46    FORMAT(I4,10X, 2F10.3)
400   FORMAT(1H1///33X, 13HTEMPERATURE,F,/5X, 10HANGLE,DEG.,
*5X, 5HT.C.1, 6X, 5HT.C.2, 7X, 5HT.C.3, 8X, 7HAVERAGE////)
47    FORMAT(/////10X, 20HDEGREE OF FREEDOM = I5/10X, 7HMEAN = ,10X,
*F10.3/10X, 21HSTANDARD DEVIATION = , F10.3//)
3     FORMAT(6F10.3)
510   FORMAT(1H1, 5X, 12HANGLE,DEG. = F12.7/6X,15HTEMPERATURE,F = F10.2
(/6X, 15HDENSITY,GM/CC = F10.5
*///6X, 4HY = E14.6/10X, E14.6, 2H*X/10X, E14.6, 5H*X**2/10X,E14.6,
25H*X**3/10X,E14.6, 5H*X**4/10X,E14.6, 5H*X**5/10X,E14.6, 5H*X**6/)
      STOP
      END

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