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REGGE TRAJECTORIES AND THE QUARK-GLUON COUPLING CONSTANT*

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Abstract

It is argued that vector gluons interacting with quarks produce logarithmic behavior of Regge trajectories at large momentum transfer in agreement with proton-proton scattering data. It is found that $g^2/4\pi = 14$ where g is the gluonic charge (charm) of the quarks. Some suggestive relations between quark-gluon dynamics, duality, magnetic charge and Jacobi's imaginary transformation are discussed.

*Work supported in part by the U.S. Atomic Energy Commission under Contract # AT(11-1)-1764.

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Recently, Coon, Sukhatme and Tran Thanh Van¹ (called CST hereafter) showed that the non-Pomeranchukon part of the elastic proton-proton cross section can be fit remarkably well over a very wide range of energy s and momentum transfer $-t$ using two logarithmic trajectories of the form

$$\alpha(t) = \frac{1}{\ln q} \ln(b - at) \quad (0 < q < 1) \quad (1)$$

The logarithmic trajectory is associated with a modified version of the non-linear dual amplitude proposed earlier by Baker and Coon². For the purpose of the present note however, we can regard (1) as a purely phenomenological fit and try to deduce general implications of the logarithmic dependence of $\alpha(t)$ for $t \rightarrow -\infty$,

$$\alpha(t) \sim \frac{1}{\ln q} \ln(-t) \quad (-t \rightarrow \infty) \quad (2)$$

Specifically we show that (2) is consistent with the vector gluon model³ and further that the magnitude of the gluonic charge (or 'charm') of the quarks can be determined from the parameter q . The parametrization obtained by CST gives

$$q = .8 \quad (3)$$

for each of the two trajectories.

Quite generally, we may assume that $\alpha(t)$ satisfies a once subtracted dispersion relation

$$\alpha(t) = \alpha(t_1) + \frac{(t-t_1)}{\pi} \int_{t_0}^{\infty} dt' \frac{\text{Im} \alpha(t')}{(t'-t_1)(t'-t)} \quad (4)$$

The condition for the logarithmic behavior (2) for $t \rightarrow \infty$ is

$$\text{Im} \alpha(t) \rightarrow \text{const} \quad (t \rightarrow \infty)$$

and in such a case we have

$$\frac{1}{|\ln q|} = \frac{1}{\pi} \text{Im} \alpha(\infty) \quad (5)$$

The asymptotically constant $\text{Im} \alpha(\infty)$ is in fact, realized in the case of the positronium trajectory. Using an eikonal approximation, Lévy and Sucher⁴ inferred that the electron-positron Coulomb scattering amplitude, would be given by

$$T = \frac{1}{\lambda^2} \frac{\Gamma(1-f(s))}{\Gamma(f(s))} \left(\frac{-t}{4\lambda^2} \right)^{f(s)-1} \quad (6)$$

where t is the momentum transfer, λ is the photon mass and

$$f(s) = \frac{e^2}{8\pi} \frac{2s - 4m^2}{[s(4m^2 - s)]^{1/2}} \quad (7)$$

No electron loop contributions are included here.

As pointed out by Lévy and Sucher, there are different ways of identifying the positronium trajectory which lead to very small differences in the positronium spectrum. For larger e^2 and s far

from $4m^2$ these differences become important. If we identify $f(s)-1$ with the trajectory function as in potential scattering we have an unphysical left hand cut starting at $s = 0$. This identification would also imply that no bound state could have a mass less than $2^{\frac{1}{2}} m$, which would be disastrous for the quark-vector gluon model. We observe that within the eikonal framework it is natural to make the identification

$$f(s) = \alpha(s) - \alpha(4m^2-s) \quad (8)$$

where $\alpha(4m^2-s)$ enters as the small t approximation to $\alpha(u)$. Thus, s - u crossing is satisfied at $t = 0$. Note that $e^2 \rightarrow -e^2$ under s - u crossing since the u -channel interaction is repulsive. This identification is possible because $f(s)$ is an antisymmetric function of $\chi = s-2m^2$. We next observe that the problem of the left hand cut is immediately resolved, since the right hand cut in $\alpha(s)$ will produce the left and right hand cuts in $f(s)$.

If we determine the right hand cut of $\alpha(s)$ from (8) we find

$$\text{Im } \alpha(\infty) = \frac{e^2}{4\pi} \quad (9)$$

and upon dispersing $\text{Im } \alpha(s)$ we obtain

$$\alpha(s) = \alpha(2m^2) + \frac{e^2}{4\pi^2} \frac{s-2m^2}{(-s(4m^2-s))^{\frac{1}{2}}} \ln \left[\frac{(-s(4m^2-s))^{\frac{1}{2}} + s+2m^2}{2m^2} \right] \quad (10)$$

This expression appears to have a branch point at $s = 0$, but it is easy to verify that it does not. As $s \rightarrow -\infty$

$$\alpha(s) \sim - \frac{e^2}{4\pi^2} \ln(-s/m^2) \quad (11)$$

A possibly more rigorous treatment of high energy electron-positron scattering is due to Cheng and Wu,⁵ who obtained Eq. (6) with $f(s)$ replaced by its asymptotic form $ie^2/4\pi$. This result together with the assumption of no left hand cut in α is sufficient to give the logarithmic behavior (11).

The above result (11) is valid for massive as well as massless photons. However, the spin of the photon is crucial. According to Lévy and Sucher,⁴ $\text{Im } f(s) \rightarrow 0$ as $s \rightarrow \infty$ for a scalar photon. The difference is due to the absence of the factor $2s - 4m^2$ in Eq. (7). This factor is just the $t \rightarrow 0$ limit of $s-u$ which is the characteristic vertex factor of a vector exchange.

We next equate the coefficient of the phenomenological, non-Pomeronchukon hadronic trajectory (2) with the coefficient of a quark-antiquark trajectory of the form (11). Calling the vector gluon coupling constant g and using Eq. (3), we find

$$\frac{g^2}{4\pi} = - \frac{\pi}{\ln q} = 14 \quad (12)$$

which is a large and reasonable coupling constant for the vector gluon. This is in contrast to the perplexingly small value $g^2/4\pi \approx .3$ obtained by Cheng, Walker and Wu⁶ in their fit to the diffractive component of proton-proton scattering.

If our analysis is correct, we should expect a universal large $-t$

behavior given by (2) in any scattering dominated by quark-antiquark exchange. For elastic scattering, one must be able to separate the diffractive component (as in the CST fit) in order to observe this universal behavior.

We note that the identification

$$\ln q = - \frac{4\pi^2}{g^2} \quad (13)$$

relates q and g^2 in a way which makes sense over their entire range.

In the dual model² $0 < q \leq 1$, which is related by (13) to $0 < g^2 < \infty$.

The Veneziano model with linear trajectories is obtained² in a limit in which $q \rightarrow 1$ and a threshold point approaches infinity. We interpret this as the limit in which g^2 and the quark mass both go to infinity.³

We also observe that the parameter q in the dual model of Ref. 2 has a universal character in that all trajectories which participate in the same N -point function must have the same q . This can be understood as a simple consequence of hadrons having zero charm so that in an N meson quark duality graph all quarks have the magnitude of charm.

The mathematical functions which are associated with logarithmic trajectory dual models are the basic hypergeometric functions⁷ and the elliptic theta functions⁸ in which q is a parameter. One of the transformation properties of these functions involves $q \rightarrow 1/q$.⁹ We observe that this corresponds to a reversal of sign of one of the charges in Eq. (13). Another transformation involving q is Jacobi's imaginary transformation⁸ which relates parameters q and q' of two elliptic theta function as follows:¹⁰

$$(\ln q) (\ln q') = 4\pi^2. \quad (14)$$

Using Eq. (13) we can try to interpret (14) as a relation between two charges g and e . We find

$$(g^2/4\pi) (e^2/4\pi) = \frac{1}{4} \quad (15)$$

which is Dirac's relation¹¹ between the magnetic monopole charge and the electric charge. For unit electric charge, the magnetic charge $g^2/4\pi = 137/4 = 34$ which is larger than our experimentally determined $g^2/4\pi = 14$. (The value 34 would correspond to a more linear trajectory since $g^2 = \infty$ implies linearity.) The magnitude agreement and the existence of relation (15) are not so far from suggesting gluon = photon and quark = magnetic monopole.

An interpretation of Jacobi's imaginary transformation involving strings and dual models has been given by Brink and Nielsen.¹²

One of us (D.D.C.) would like to thank Professor Marshall Baker for numerous related discussions and the theoretical physics group at the University of Minnesota for their hospitality while this work was being carried out.

References and Footnotes

1. D. D. Coon, U. Sukhatme and J. Tran Thanh Van, Phys. Letters, to be published.
2. D. D. Coon, Physics Letters 29B, 669 (1969); M. Baker and D. D. Coon, Phys. Rev. D2, 2349 (1970); M. Baker and D. D. Coon, to be published.
3. H. Suura, Phys. Rev. D6, 3538 (1972); Phys. Letters 42B, 237 (1972).
4. M. Lévy and J. Sucher, Phys. Rev. D2, 1716 (1970); Phys. Rev. 186, 1656 (1969); M. Lévy, Phys. Rev. Letters 9, 235 (1962). See also E. Brezin, C. Itzykson and J. Zinn-Justin, Phys. Rev. D1, 2349 (1970).
5. H. Cheng and T. T. Wu, Phys. Rev. 186, 1611 (1969).
6. H. Cheng, J. K. Walker and T. T. Wu, Phys. Letters 44B, 97 (1973).
7. W. N. Bailey, Generalized Hypergeometric Series (Cambridge U.P., London, 1935); L. J. Slater, Generalized Hypergeometric Functions (Cambridge U.P., London, 1966).
8. E. T. Whittaker and G. N. Watson, Modern Analysis (Cambridge U.P., London, 1963), p. 474.
9. D. B. Sears, Proc. London Math. Soc. (2) 53, 158 (1951).
10. The parameter q of Refs. 2, 7 and 9 differs from the " q " of Ref. 8 as follows $q = (q_{\text{Ref8}})^2$.
11. P.A.M. Dirac, Phys. Rev. 74, 817 (1948).
12. L. Brink and H. B. Nielsen, Phys. Letters 43B, 319 (1973).