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ABSTRACT

Direct space-time diffusion theory codes in two and three dimensions have been applied to the problem of calculating the pulsed neutron experiment. Comparisons of two-group calculations and experiments on bare and reflected thermal reactors showed that, after suitable normalization of the production cross sections ($\nu\Sigma_f$), the overall prompt neutron response to a pulsed source was well represented by the codes. The source-induced harmonic response during and just after the source burst was less well represented with several approximate treatments for describing the neutron source.

The ability to calculate the pulsed neutron experiment has led to a new method of deriving subcritical reactivity from the measured data. This "space-time" method of analysis uses the normalized cross sections, found in fitting the source response calculation to the experimental data, in a static calculation of the reactivity. The derived reactivity from the space-time method is unaffected by the problems of kinetic distortion and prompt and delayed harmonics; hence, its use removes the necessity for deriving correction factors for these effects. Small errors in the neutron lifetime (through input velocities and absorption cross sections) may still be present. However, the variation of lifetime with subcritical reactivity is expressly included in the treatment.

CONTENTS

INTRODUCTION

One of the uses of the pulsed neutron experiment is to determine the reactivity of subcritical reactors. In published procedures, Sjöstrand,[1] Simmons and King,[2] Gozani,[3] and Garelis and Russell[4] have used the data collected in pulsed experiments to deduce the reactivity.

A more fundamental problem is to calculate the response of the reactor to a pulsed neutron source. However, until recently, realistic computations of the space and time response of the neutron flux to a time-dependent source have not been practical. The reason for this computational difficulty is the time and core demands placed on computing facilities by space- and time-dependent problems. Until a few years ago, point kinetics codes were the only time-dependent tools that could be core contained and run in reasonable computing time. However, the response to a pulsed source involves rapid and large changes in the flux shape that are not treated in the usual point kinetics codes.

Recent code developments have included the direct space and time solution, the quasistatic approximation, and the synthesis techniques.[5] All of these methods can provide solutions for the pulsed reactor, but this report will be concerned only with the direct space and time solutions. This type of solution has become practical for pulsed problems with increases in available computer storage capacity and decreases in the time required per operation.

At the Savannah River Laboratory, three direct space-time codes have been developed or modified to include a spatially distributed time-dependent source: WIGLE[6] in one dimension, DISCOTHEQUE[7] in two dimensions with orthogonal geometries, and TRIMHX[8] in three dimensions with hexagonal cells in layered planes, which will be called hex-z geometry. Realistic mockups of reactors usually require the two- or three-dimensional treatments.

In this report, pulsed neutron experiments and calculations for two different D_2O-moderated, thermal reactors are compared: one bare and the other reflected. The reflected reactor is of special interest because both kinetic distortion and harmonic distortions cause pronounced discrepancies in the conventional pulsed neutron analyses of Sjöstrand,[1] Gozani,[3] and Garelis-Russell.[4] Kinetic distortion results from spatial and spectral differences between the fundamental prompt and delayed neutron modes.[9]

The ability to calculate the pulsed neutron experiment with space-time codes gives rise to a new method of analyzing the pulsed reactor data for subcritical reactivity. This new procedure involves comparing calculated and measured pulsed responses recorded by one or more neutron detectors. Two basic assumptions are implicit in this method:

- Discrepancies between calculated and measured responses result primarily from small errors in the input parameters and not in the calculational method. These parameters can be altered (or normalized) to force agreement.

- If the space-time response of the reactor has been accurately calculated with these normalized parameters, then the static subcritical reactivity can be derived with the same neutronics model using the normalized parameters.

Application of this new procedure for extracting the subcritical reactivity from a pulsed neutron measurement can be described as follows. First, a detailed geometrical description of the lattice is assembled (as detailed as code limitations will allow). Second, a set of homogenized diffusion parameters is computed for each reactor region. Third, the response to the pulsed source is computed at least through the die-away of the prompt neutrons. Fourth, the computed response is compared to the measured response as recorded by a neutron detector. If the two prompt neutron responses do not agree over a substantial portion of the prompt decay including the fundamental decay, selected input diffusion parameters are changed, as described later, until the calculated response matches the measured response. Finally, a static computation is made with the normalized diffusion parameters, and the eigenvalue of that calculation is the reported reactivity. For the purposes of this report, the term "space-time method of analysis" will imply the foregoing steps.

SPACE-TIME MODELS

Diffusion theory models require as input information a description of the reactor geometry, group neutron velocities, diffusion coefficients, macroscopic cross sections, and delayed neutron fractions and decay constants. The neutronic parameters can usually be calculated with transport codes such as RAHAB[10] or HAMMER.[11] The initial calculation of the prompt neutron buildup and subsequent die-away using transport-calculated neutronic parameters generally does not match exactly the measurement. Improved fits are obtained by making small changes in the production cross sections (the normalizing process). Logically, a cross section change implies a spectrum or group velocity change. In practice, however, the velocities are not altered since the response is very sensitive to the cross section changes and relatively insensitive to velocity changes. The errors in the estimates of the velocity and absorption cross sections which determine the neutron lifetime are not removed by the normalization procedure.

Two different space-time diffusion theory JOSHUA[12] modules were available for modeling the experiments: DISCOTHEQUE in orthogonal geometries (XY or RZ) and TRIMHX in hex-z geometry. DISCOTHEQUE was used for the bare lattice calculations because RZ geometry was judged most applicable to the experiment which involved a uniform array of fuel in the relatively small, cylindrical facility. The reflected lattice experiments, however, used a hexagonal core surrounded by a hexagonal reflector and could be adequately modeled only with the TRIMHX code.

The DISCOTHEQUE module computes the solution to the space- and time-differenced, two-group neutron diffusion equations and delayed neutron precursor equations in two-dimensional geometry (either Cartesian or cylindrical coordinates). Zero through six delayed neutron groups are treated. Either arbitrary space- and time-dependent perturbations to any of the two-group diffusion parameters or a spatially distributed external neutron source with arbitrary time dependence is treated.

A backward time difference is used for subcritical, source-induced transients. The iterative procedure used is similar to that in the TWIGL[13] program. The spatial equations at each time step are solved iteratively using a cyclic Chebychev procedure. The inner iterations are performed using a line-overrelaxation procedure.

Initial flux conditions are computed by the auxiliary module DISCO as the solution to the static, two-group eigenvalue (k_{eff}) equations. The static calculation is performed using an outer power iteration with an alternating-direction, line-overrelaxation procedure for the inner iterations. Source extrapolation is used to accelerate convergence.

The TRIMHX module computes the solution to the space- and time-differenced, few-group neutron diffusion and delayed neutron precursor equations in three-dimensional geometry (hex-z coordinates). In the TRIMHX method, variables are changed by an exponential transformation. The transformed equations are time-differenced using a fully implicit approximation. The resulting difference equations are solved iteratively using a successive line-relaxation, block inversion technique for the inner iteration and a coarse-mesh rebalancing technique with fission source overrelaxation for the outer iteration. Initial conditions are computed by the static few-group neutron diffusion module GRIMHX.

TREATMENT OF THE SOURCE

The proper representation of pulsed sources in a few-group diffusion theory treatment is uncertain partly because such sources emit neutrons at a high energy (14 MeV in the case of $^{3}H(d,n)^{4}He$ accelerators). The transport calculations used to derive the cross sections do not extend to such a high energy. Further complicating the treatment in a two-group approximation is the fact that the fast energy group typically covers a very broad range extending from the fission energies to less than one eV. Another difficulty for our particular experiments was that the neutron source for both the bare and reflected lattices was external to the reactor, and diffusion theory is inherently incapable of dealing directly with this situation. The following fast and thermal group approximations were used to derive the source input for the diffusion theory calculations.

FAST SOURCE APPROXIMATIONS

The $^{3}H(d,n)^{4}He$ reaction does provide a nearly isotropic distribution of 14-MeV neutrons; thus, a first approximation would be to compute a fast neutron distribution resulting from $1/R^{2}$ dispersal to a mean first collision distance into the reactor. However, the energy of this source is still considerably above the mean energy of the fission distribution used in the transport cell computations for the cross section derivations. Some improvement might be made in calculating a distribution of the source density after slowing down to an average energy of approximately 2 MeV.

If one is using a few-group structure in the diffusion theory problems where the highest energy group is centered about the fission energy, a 2-MeV source distribution would be the most appropriate. However, diffusion theory calculations at Savannah River are usually limited to a two-group structure. The energy associated with the average fast group velocity is approximately 100 eV, not 2 MeV. Thus an alternative, and possibly more appropriate, fast source distribution would be that calculated in the same energy range as the fast group average.

One other consideration makes selection of the "proper" source distribution even more ambiguous. A neutron is a source neutron only during the time after it enters the reactor and before it initiates a fission event. This indicates that the source distribution should be calculated in a first generation fixed source problem, i.e., one where no neutron regeneration is allowed (accomplished by setting $\nu\Sigma_f$, the production cross section, to zero).

The static S_n theory code, DOT,[14] was used to compute the multigroup scalar flux distributions for the fixed source external to the reactors in RZ geometry. Cell calculations with ANISN[15] were used to generate the multigroup homogenized cross sections. Both codes used S_4 discrete ordinates and P_3 Legendre expansions. A 13-group structure commonly used in shielding problems was used for these calculations since its highest energy group included 14 MeV. Table I lists the group boundaries. At any point in the reactor, the source for the fast group of the diffusion theory input was defined as

$$S_1 \equiv \sum_{i=1}^{I} \sum_{j=i+I}^{13} \phi^i(\mathrm{DOT})\ \Sigma_s^{i\to j}$$

$$S_1 \approx \phi^I\ (\mathrm{DOT})\ \Sigma_r^I \qquad \mathrm{n/cm^3\text{-}sec}$$

where

$$\Sigma_r^I \equiv \sum_{i=I+1}^{13} \Sigma_s^{I\to i}$$

$\phi^i(\mathrm{DOT})$ = scalar flux from DOT in group i

$\Sigma_s^{i\to j}$ = scattering cross section of group i feeding group j

Implicit in this definition is the assumption that a statically derived source is instantaneously formed; i.e., the slowing-down time is ignored. This error is small since the slowing-down time to the energy represented by group I is very much smaller than the duration of the burst. The time dependence of the source is defined as an instantaneous step rise from zero to a constant amplitude for a duration equal to the source burst width followed by an instantaneous step return to zero. At all other times the source is zero.

Two arbitrary and different choices of the DOT group I were made for the fast source inputs. For a source at the fission energy, I = 7; for a source at the average energy of the diffusion fast group, I = 12.

TABLE I

13-Group Neutron Library

	Group Structure			
Upper Energy, eV	0.149E 08	0.100E 08	0.670E 07	0.549E 07
Group Number	*1*	*2*	*3*	*4*
Lower Energy, eV	0.100E 08	0.670E 07	0.549E 07	0.449E 07
	0.449E 07	0.368E 07	0.301E 07	0.202E 07
	5	*6*	*7*	*8*
	0.368E 07	0.301E 07	0.202E 07	0.907E 06
	0.907E 06	0.408E 06	0.111E 06	0.150E 05
	9	*10*	*11*	*12*
	0.408E 06	0.111E 06	0.150E 05	0.414E 00
	0.414E 00			
	13			
	0.0			

THERMAL SOURCE APPROXIMATIONS

Whenever a fast source input was defined as nonzero, the thermal source input was arbitrarily defined as zero. However, in two cases, the fast source was defined as zero, and the created thermal source was defined as $S_2 \approx \phi^{13}$(DOT) even though the units were not preserved.

COMPARISONS OF THE EXPERIMENTAL AND CALCULATED RESULTS FOR THE BARE LATTICE

PROMPT NEUTRON RESPONSE

The bare lattice, schematically shown in Figure 1, was formed of uranium-bearing fuel distributed uniformly in D_2O moderator within the Subcritical Experiment (SE) facility, and was pulsed from the top by an external $^3H(d,n)^4He$ source of 14-MeV neutrons. The reactor was pulsed with repeated 823-μsec-wide bursts, the beginning of each burst triggering a single sweep of a multichannel scaler in which the accumulated detector responses were stored simultaneously. Figure 2 shows the recorded prompt neutron responses (obtained from the raw data after deadtime correction and subtraction of average noise and delayed neutron tail).

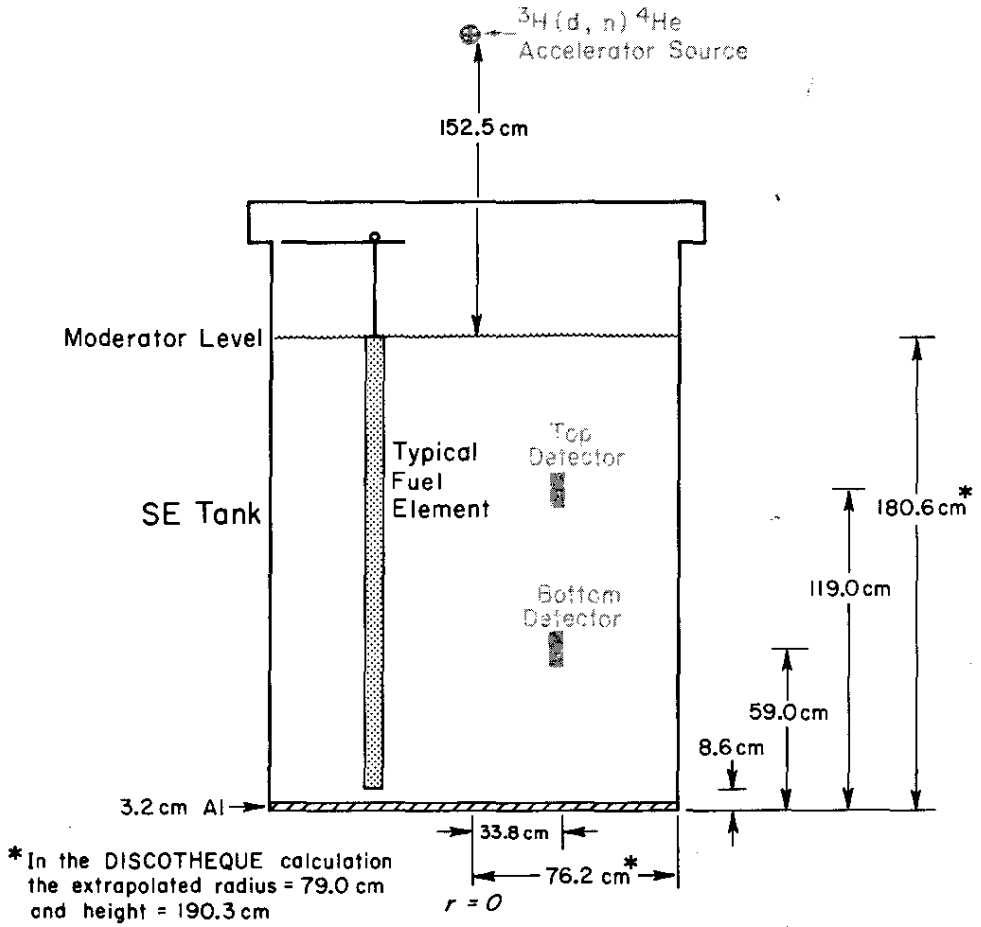

FIGURE 1 Experimental Arrangement for the Bare Lattice in the SE

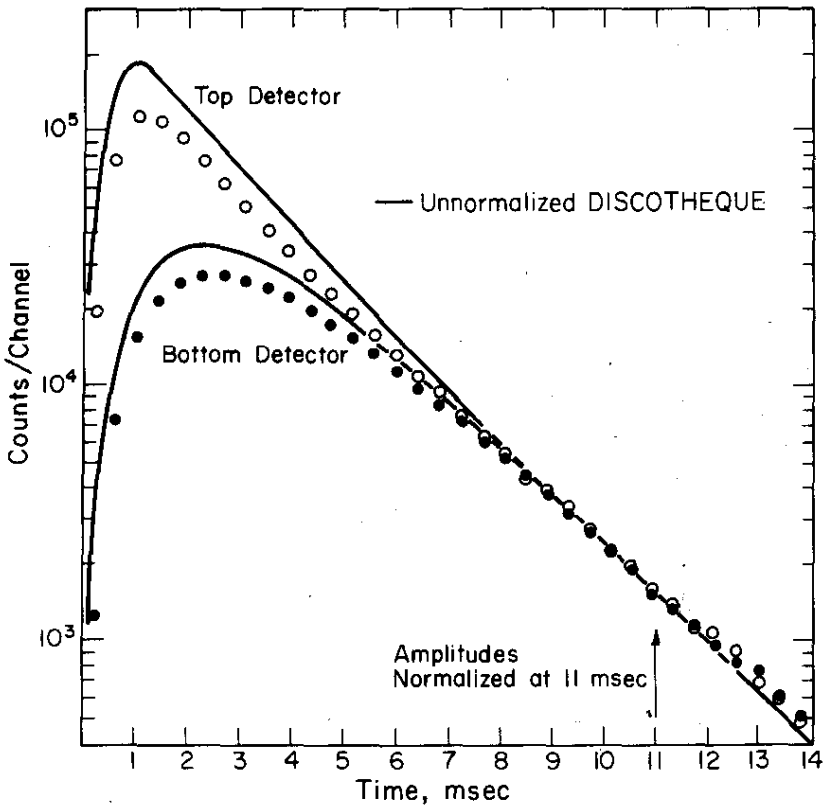

FIGURE 2 Measured and Calculated Prompt Neutron Responses in Bare Lattice - Unnormalized Diffusion Parameters

The calculations were performed with DISCOTHEQUE in RZ geometry where the top axial and radial boundaries were adjusted to include previously determined axial and radial extrapolation distances. The two-group initial diffusion parameter sets for the core, bottom D_2O reflector, and aluminum were computed with RAHAB and HAMMER (Table II). The source was defined with $S_1 = 0.0$ and $S_2 \approx \phi^{13}$(DOT). Because an excellent fit to the experiment in the source burst region was obtained with this thermal approximation, no other source choice was tried.

The calculated response with the unnormalized set of diffusion parameters is shown in Figure 2. The core group production cross sections were then changed by 0.90% to normalize the calculation to the experiment, and the final fit is shown in Figure 3. This cross section change caused a k_{eff} change of +0.0082 to the normalized value of 0.9204. The calculated responses match the measured responses very well over the entire range of the experiment.

TABLE II

Unnormalized Diffusion Parameters
for the Bare Lattice in the SE

Material	Group	D	$\Sigma_a + \Sigma_s$	$\nu\Sigma_f$	Σ_s [a]	1/v
Al	1	3.70960	6.30000×10^{-4}	0.0	2.33000×10^{-4}	Fixed same as for core
	2	3.74400	4.52700×10^{-3}	0.0		
D_2O	1	1.30360	1.19820×10^{-2}	8.30000×10^{-5} [b]	1.18960×10^{-2}	1.0590×10^{-7}
	2	0.817630	7.50000×10^{-5}	0.0		4.0265×10^{-6}
Core	1	1.19496	1.17939×10^{-2}	2.31160×10^{-3}	7.61350×10^{-3}	8.2772×10^{-8}
	2	0.856870	1.35873×10^{-2}	1.87360×10^{-2}		3.4541×10^{-6}

a. $\Sigma_s \equiv \Sigma$ downscatter $(1 \rightarrow 2)$

b. $\nu\Sigma_f$ includes photoneutrons

These problems used 16 radial mesh points, 36 axial mesh points, and a six-group delayed neutron structure corrected for effectiveness. A total of 283 time steps were used with durations varying from 1 to 500 μsec in going from the source burst to the delayed neutron tail. The CPU time on an IBM 360/65 facility was 99 minutes. The computer core required was 350 K bytes.

REACTIVITY

Two conventional methods of extracting the reactivity from the neutron die-away following a source burst are the Gozani[3] extrapolated area method and the Garelis-Russell[4] $k\beta/\ell$ method. Gozani has shown that the results of the Gozani method can be distorted by the effects of delayed neutron harmonics. The Garelis-Russell method is distorted from the combined effects of both prompt and delayed harmonics.[16] For reasonably uniform reactors where kinetic distortion is absent, the harmonic distortion of the two methods away from the true reactivity is in opposite directions. Thus Gozani recommends the "bracket procedure" where both methods are used and the results are averaged to obtain a bracket value that is closer to the true value than either alone.[16]

Both Gozani and Garelis-Russell methods are valid only when a fundamental mode has been established in the experimentally observed prompt neutron decay. Figure 3 shows that this has occurred within 10 msec after the burst initiation. The measured decay constants used in the Gozani and Garelis-Russell treatments are taken from data accumulated after 10 msec. The Gozani, Garelis-Russell, and bracket procedure values as well as those resulting from the Sjöstrand procedure, are listed in Table III.

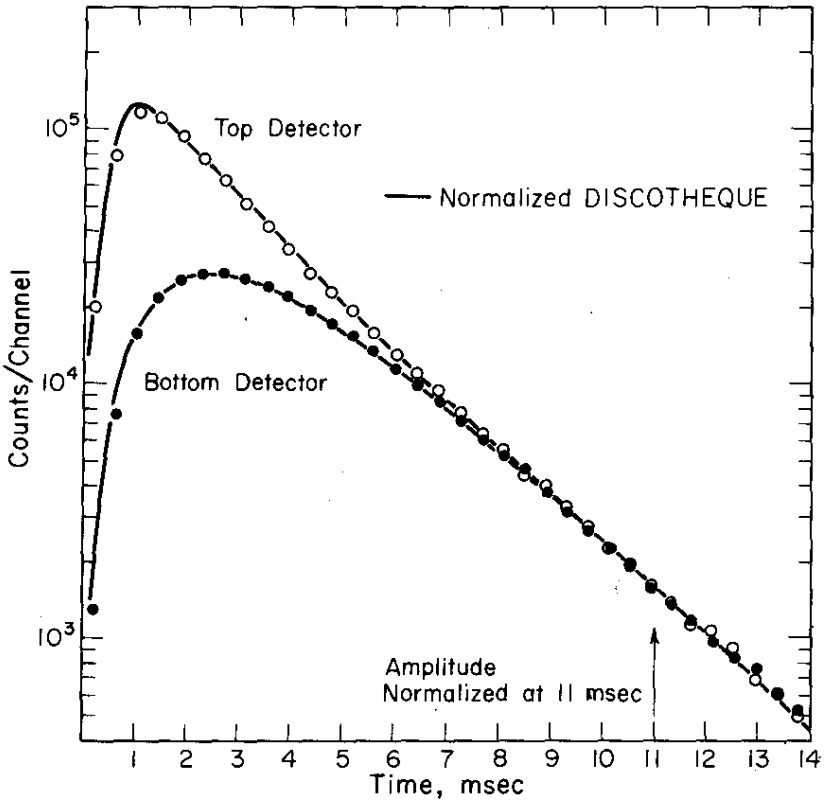

FIGURE 3 Measured and Calculated Prompt Neutron Responses in Bare Lattice - Normalized Diffusion Parameters

TABLE III

Conventional Method[a] and Space-Time Method[b] Reactivities for the Bare Lattice in the SE

Normalized DISCOTHEQUE	Detector	Bracket[c]	Gozani	Garelis-Russell	Sjöstrand
-0.0865 ±0.0020	Top	-0.0807	-0.0759	-0.0856	-0.0870
	Bottom	-0.0869	-0.0980	-0.0758	-0.0481
	Summed	-0.0828	-0.0832	-0.0824	-0.0746

a. Reactivity (ρ) = \$ β where β = 0.007982

b. Reactivity (ρ) = $(k_{eff} - 1)/k_{eff}$

c. ρ(bracket) = [ρ(Gozani) + ρ(Garelis-Russell)]/2

Table III also lists the conventional method and bracket reactivities for "summed detectors." The procedure for weighting the detector outputs to calculate a summed detector response is given in the appendix. The effects of source-induced axial harmonics in both prompt and delayed distributions are minimized by this procedure. The summed response (properly weighted) establishes the persisting mode of prompt decay more quickly than does either individual response, making it possible to extract the decay constant from earlier channels with increased statistical precision.

Figure 4 shows the summed data, both experimental and calculated. As expected, the persisting mode can be extracted from the summed data starting at earlier times, 4 msec in this case. The two methods of area analysis lead to the summed detector results listed in Table III clearly showing the reduced effects of prompt and delayed harmonics as seen in the closer agreement of the Gozani and Garelis-Russell methods.

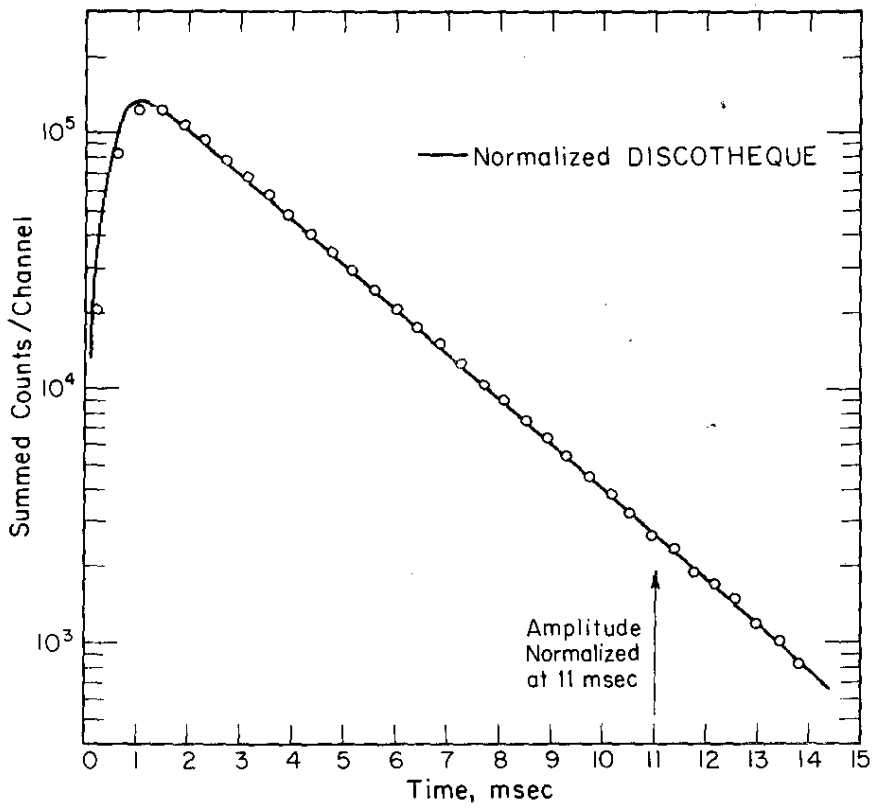

FIGURE 4 Measured and Calculated Summed Prompt Neutron Response in Bare Lattice - Normalized Diffusion Parameters

Table III lists the calculated reactivity as found from space-time analysis by normalizing to the experimental response. The precision of the calculated value is based on the subjective judgment of the variation of reactivity tolerable within the requirement of a good fit. The agreement of the bracket averages of the individual and summed data with the space-time calculated reactivity is good.

COMPARISONS OF THE EXPERIMENTAL AND CALCULATED RESULTS FOR THE REFLECTED LATTICE

PROMPT NEUTRON RESPONSE

Figures 5 and 6 show vertical and radial schematics of the experimental arrangement for the reflected lattice contained in the Process Development Pile (PDP). The core consisted of control assemblies, enriched uranium fuel assemblies, and depleted uranium target assemblies, all separated with a 7-inch center-to-center triangular spacing. The seven-assembly pattern consisting of one control assembly surrounded by three fuel and three target assemblies constitutes a cluster. Thus, the core consists of 19 clusters of seven cells (assemblies) each.

All fuel, target, and control assemblies were removed from the 18 cluster ring immediately surrounding the central core to form a heavy water reflector. The next ring of 24 clusters retained its control and target assemblies, but the fuel assemblies were removed. The outermost region was filled with target assemblies in an irregular pattern. All of the area outside the reflector can be considered a poison boundary.

Four different measurements were made on this lattice, one critical and three subcritical. The critical measurement established the critical water height at 241.50 cm. A gold pin

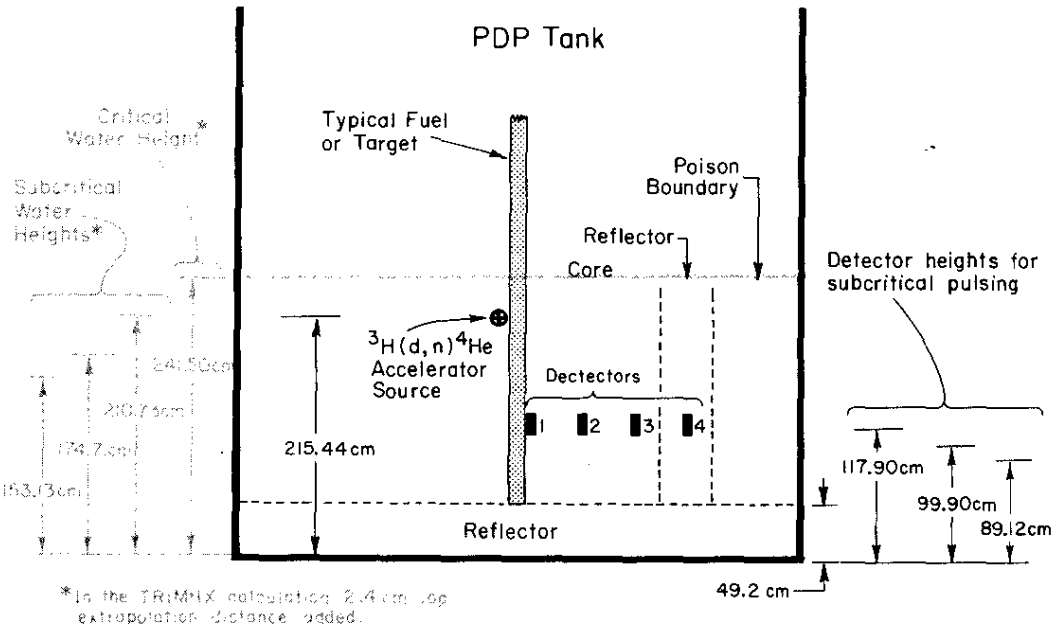

FIGURE 5 Experimental Arrangement for the Reflected Lattice in the PDP - Vertical Display

irradiation was also performed to find the vertical flux shape. These data established the top extrapolation distance at 2.4 cm. The subcritical pulsing experiments were performed at successively lower water heights to provide data with increasing subcriticality. Figure 5 also shows the vertical locations of the $^{3}H(d,n)^{4}He$ source, the neutron detectors, and the water levels for the three experiments. These detector positions were estimated to be close to the midplane of the static flux distributions. The radial position of the detectors is shown in Figure 6.

For the subcritical experiments, the reactor was pulsed with bursts of 2169 μsec at 210.73 cm water height, 2169 μsec at 174.71 cm, and 616 μsec at 153.13 cm. The data-recording sequence was the same as for the bare lattice experiments.

The calculations were performed with TRIMHX with the top axial boundary extended to include the measured vertical extrapolation distance. The unnormalized diffusion parameters were all computed with RAHAB and are listed in Table IV. The cross sections for the

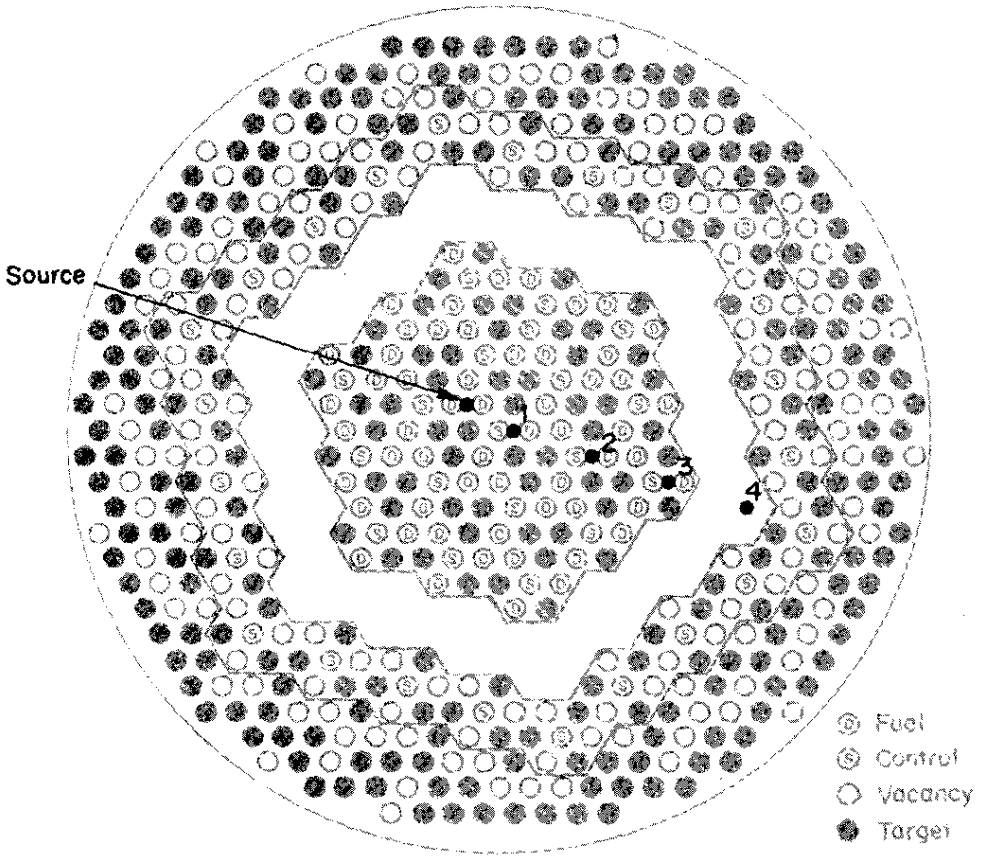

FIGURE 6 Experimental Arrangement for the Reflected Lattice in the PDP - Radial Display

core and inner poison boundary ring were computed with the "supercell" option of RAHAB. With this option the cross sections are homogenized over an entire seven-assembly cluster. The cross sections for the reflector and the irregular outer poison boundary ring were computed with the cell option of RAHAB.

The source distributions for the three cases were estimated with DOT. These estimates include two sources of error due to the limited calculational tools. First, the reactor geometry is hexagonal in the plane, and the closest approximation with DOT is cylindrical (RZ) geometry. Second, the source was not precisely on the axis of the reactor; the DOT calculations require a source centered about the axis. These errors affect only the flux buildup and initial die-away in the source-induced harmonic region; the fundamental mode decay is unaffected.

The data from the subcritical experiment at the water height of 210.73 cm were arbitrarily chosen as the basis of comparison in normalizing the cross sections for the TRIMHX calculations. The calculated eigenvalue with the unnormalized cross sections of Table IV was 0.97620. The source was arbitrarily defined with $S_1 = 0.0$ and $S_2 \approx \phi^{13}$(DOT). Figure 7 shows the unnormalized calculated response compared to the measured data. The production cross sections of all multiplying materials were then increased uniformly by 1.31% causing k_{eff} to increase to 0.98885; the resulting fit is shown in Figure 8. Part of the discrepancy between calculation and experiment in the prompt harmonic region (Figure 8) may be due to the use of a centered source in the calculations.

TABLE IV

Unnormalized Diffusion Parameters for the Reflected Lattice in the PDP

Material	Group	D	$\Sigma_a + \Sigma_s$	$\nu\Sigma_f$	$\Sigma_s{}^a$	1/v
Core (supercell)	1	1.28521	1.10589×10^{-2}	1.76751×10^{-3}	8.25901×10^{-3}	8.8818×10^{-8}
	2	8.71764×10^{-1}	1.37117×10^{-2}	1.66996×10^{-2}	-	3.4375×10^{-6}
D_2O Reflector and Vacancies in Outer Poison Boundary (cell)	1	1.29193	1.15596×10^{-2}	0.0	1.15582×10^{-2}	1.0402×10^{-7}
	2	8.19342×10^{-1}	7.50003×10^{-5}	0.0	-	4.2305×10^{-6}
Inner Poison Boundary (supercell)	1	1.25251	1.36568×10^{-2}	9.72144×10^{-4}	1.10905×10^{-2}	1.0687×10^{-7}
	2	8.35679×10^{-1}	6.10129×10^{-3}	2.22390×10^{-3}	-	4.0166×10^{-6}
Depleted Targets in Outer Poison Boundary (cell)	1	1.17449	1.41050×10^{-2}	1.56195×10^{-3}	8.72608×10^{-3}	9.9488×10^{-8}
	2	8.61634×10^{-1}	1.40208×10^{-2}	7.32649×10^{-3}	-	3.5173×10^{-6}

a. $\Sigma_s \equiv \Sigma$ downscatter $(1 \rightarrow 2)$

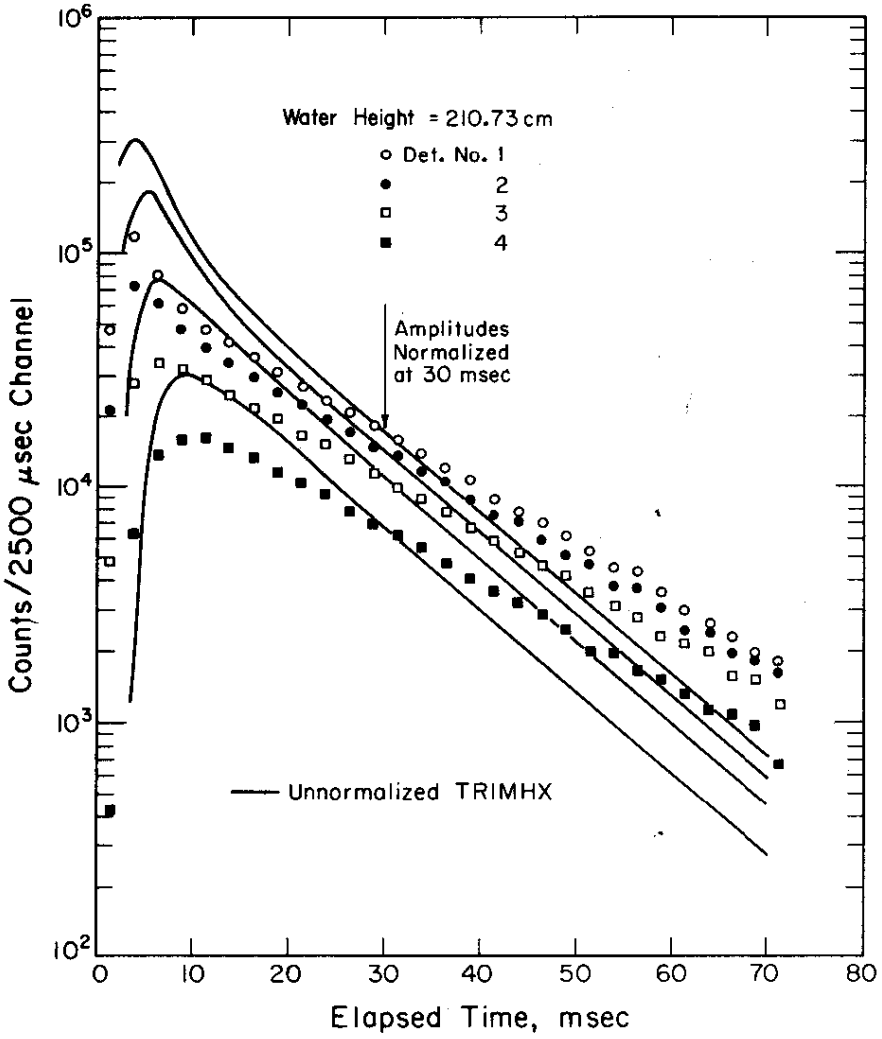

FIGURE 7 Measured and Calculated Prompt Neutron Responses in the Reflected Lattice at 210.73-cm Water Height - Unnormalized Diffusion Parameters

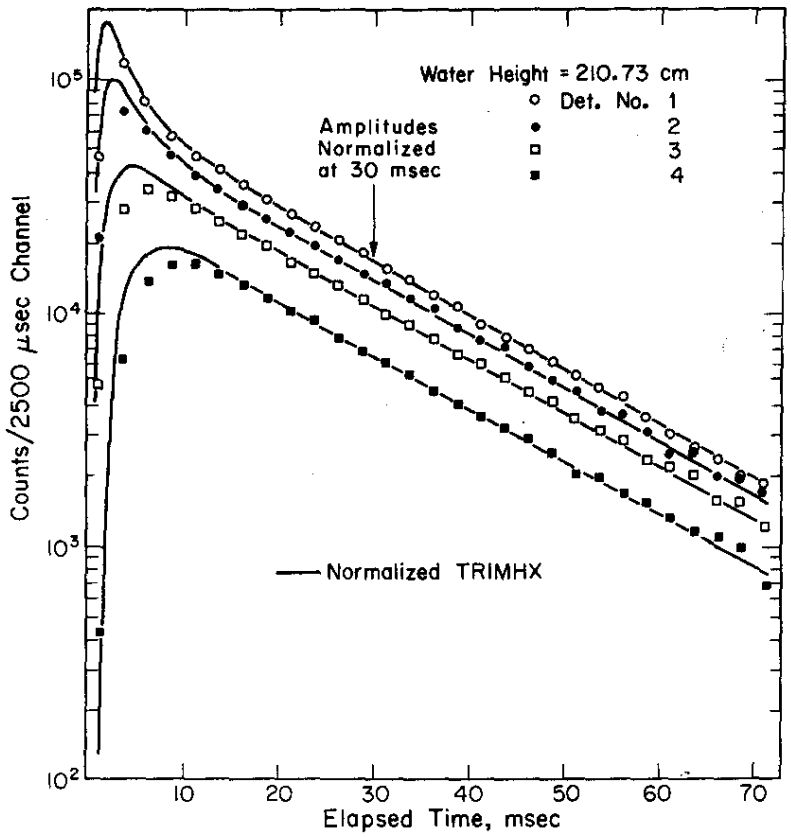

FIGURE 8 Measured and Calculated Prompt Neutron Responses in the Reflected Lattice at 210.73-cm Water Height - Normalized Diffusion Parameters

The sensitivity of the fit to the source description was then checked by redefining the source with $S_1 \approx \phi^7(DOT)\Sigma_r{}^7$ and $S_2 = 0.0$. No significant difference from the result with the thermal source was found; the influence of source energy on the response of this highly multiplying lattice appeared to be slight.

The same normalized cross section set, derived by fitting to the experiment at 210.73-cm water height, was used for the calculations at 174.71-cm and 153.13-cm water heights. For both cases, the source was arbitrarily defined as $S_1 \approx \phi^{12}(DOT)\Sigma_r{}^{12}$ and $S_2 = 0.0$. The comparisons between the experimental and calculated data are shown in Figures 9 and 10, respectively. Again, the source harmonic region has been poorly calculated. The fundamental decays, however, have been calculated reasonably well except for the innermost detectors in the 153.13 cm case. The eigenvalues were 0.96957 at 174.71 cm and 0.95026 at 153.13 cm.

The sensitivity of the calculated response to the choice of the source energy was checked for the 174.71 cm case. The thermal choice with $S_1 = 0.0$ and $S_2 \approx \phi^{13}(DOT)$ underestimated the severity of the source-induced harmonics. On the other hand, a fast source choice with $S_1 \approx \phi^7(DOT)\Sigma_r{}^7$ and $S_2 = 0.0$ overestimated slightly the harmonics. The best fit was obtained with the $S_1 \approx \phi^{12}\Sigma_r{}^{12}$ and $S_2 = 0.0$ choice. On balance, the group 12 fast source would appear to be the best choice, though the advantages of this choice are not marked.

These problems were solved with a 60° symmetry solution using 3 mesh points per cell, 20 axial mesh points, and a 6-group delayed neutron structure corrected for effectiveness. The core used was 750 K bytes. The time-step durations varied slightly in the different calculations. For the calculation of the 210.73-cm lattice, 83 minutes of CPU time on an IBM 360/195 facility were required to solve a 61 time-step problem with time durations varying from 0.1 to 25 msec.

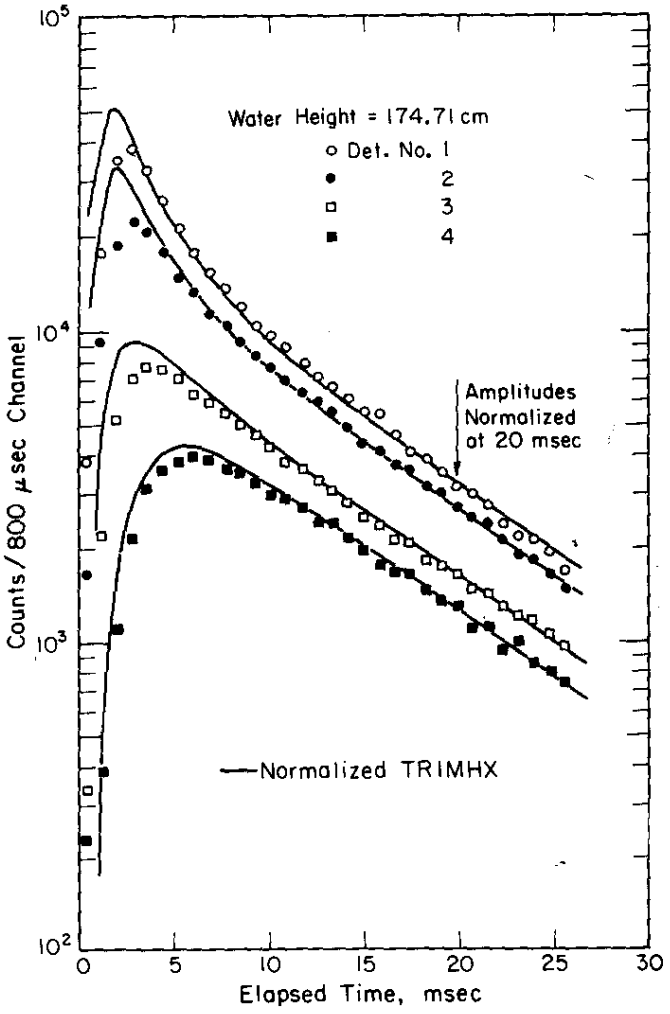

FIGURE 9 Measured and Calculated Prompt Neutron Responses in the Reflected Lattice at 174.71-cm Water Height - Normalized Diffusion Parameters

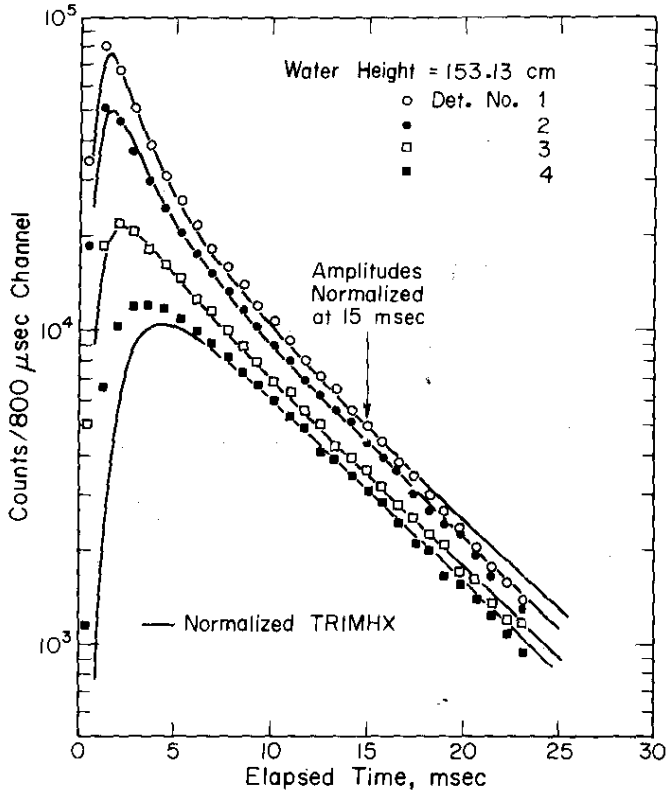

FIGURE 10 Measured and Calculated Prompt Neutron Responses in the Reflected Lattice at 153.13-cm Water Height - Normalized Diffusion Parameters

REACTIVITY

The validity of the space-time method of analysis for the reflected lattice was checked by computing the eigenvalue at the measured critical water height with the normalized cross sections. The calculated critical eigenvalue was 0.99899, which is in reasonable agreement with the anticipated value of unity. Uncorrected errors in the neutron lifetime may be responsible in whole or in part for this small discrepancy.

In these reflected lattice experiments, kinetic distortion as well as delayed neutron and prompt harmonics were present, invalidating Gozani's bracket procedure[16] as well as the conventional analyses. Nevertheless, it is instructive to compare the conventional results with the space-time method results. Table V lists the space-time reactivities and the conventional reactivities found using data only after the fundamental mode has been established. Figure 11 displays these values as a function of radial position. These results are in agreement with the findings of Kosály and his colleagues that the errors of the conventional analyses are least in the region of the core-reflector interface.[17] Moreover, they show the uncorrected Sjöstrand[1] method to have the least variation with radial position and to be generally somewhat more accurate than the uncorrected Gozani[3] and Garelis-Russell[4] methods.

TABLE V

Measured and Calculated Reactivities for the Reflected Lattice in the PDP

Water Height, cm	Normalized TRIMHX	Detector	Reactivity[a] by Gozani	Garelis-Russell	Sjöstrand
241.50[b]	-0.00101[c]				
210.73	-0.01128	1	-0.00918	-0.00855	-0.01038
		2	-0.00952	-0.00939	-0.00995
		3	-0.01043	-0.01049	-0.00939
		4	-0.01157	-0.01068	-0.00914
174.71	-0.03139	1	-0.02278	-0.02269	-0.02805
		2	-0.02430	-0.02344	-0.02669
		3	-0.03122	-0.02967	-0.02971
		4	-0.03818	-0.03515	-0.02972
153.13	-0.05234	1	-0.03737	-0.03937	-0.04857
		2	-0.03854	-0.04193	-0.04553
		3	-0.04673	-0.04675	-0.04412
		4	-0.07129	-0.06628	-0.05427

a. Reactivity (ρ) = \$ β_{eff} where β_{eff} = 0.0075762 or $(k_{eff} - 1)/k_{eff}$

b. Measured critical water height

c. Normalized GRIMHX

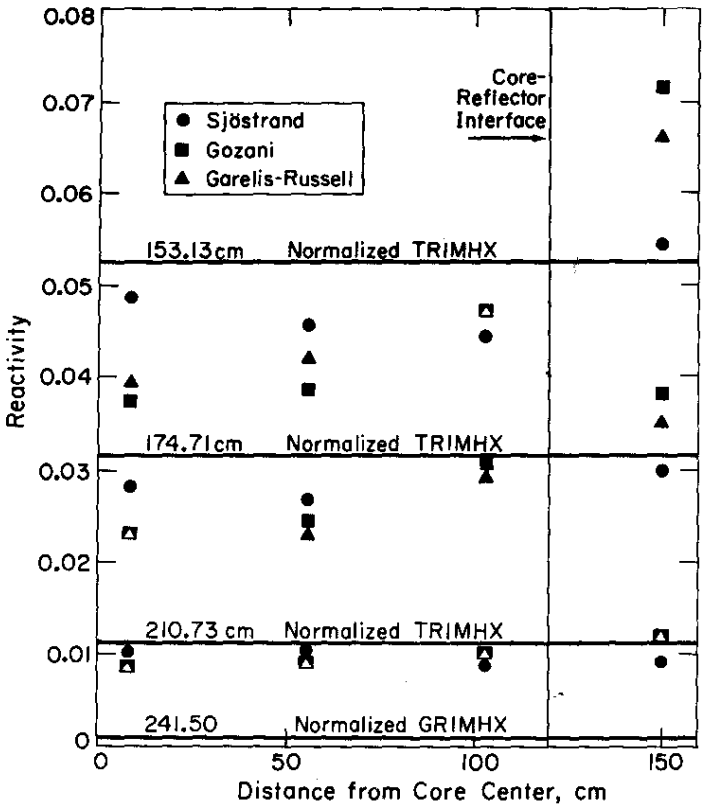

FIGURE 11 Reactivities for the Reflected Lattice Derived by Space-Time Method and Conventional Area Method

CONCLUSIONS

The pulsed neutron experiment on thermal reactors appears well modeled with two-group, space-time diffusion theory. This conclusion is supported by the good agreement of experiments and calculations for both a bare and a reflected reactor following small changes in the calculated diffusion parameters (normalization).

The DISCOTHEQUE-DISCO modules in RZ geometry adequately treated the uniform bare lattice experiment in the SE. The TRIMHX-GRIMHX modules gave good results for the hexagonal geometry of the reflected lattice experiments in the PDP. The weakest part of the calculational procedure involved the derivation of source input for the diffusion problems. However, the main characteristics of the flux response to the source bursts were duplicated reasonably well when the source input was approximated with S_n fixed source calculations.

For the small bare lattice in the SE, the effects of delayed and prompt neutron harmonics were evident in the conventional analyses, but kinetic distortion was not significant. For this case, the bracket procedure described by Gozani[16] provided a simple and effective correction for the harmonic distortion without recourse to calculated correction factors. The bracket reactivity so derived agreed well with the space-time calculated reactivity.

Both subcritical and critical measurements were made on the larger reflected lattice in the PDP. Extrapolation of the subcritical space-time calculations to the measured critical condition demonstrated the adequacy of the normalizing procedure in the space-time method. However, both harmonic distortion and kinetic distortion greatly affected the conventional reactivity analyses of Sjöstrand,[1] Gozani,[3] and Garelis-Russell[4] and destroyed the usefulness of the bracket procedure.

The space-time method should be among the most accurate of methods proposed to date for deducing the subcritical reactivity from pulsed neutron experiments. To the extent that a two-group treatment was adequate, the effects of delayed and prompt harmonics and kinetic distortion were directly included in the space-time calculations of the experiments; hence, the derived space-time reactivities need no external correction. However, a remaining small error from incorrect estimates of the neutron lifetime may be present.

Use of the space-time method requires significant investments of manpower time to set up the calculations and computer time to perform the calculations. For cases where both kinetic distortion and harmonic distortion are anticipated to be problems, as in reflected reactors, the required additional effort to use the space-time method should be expended. In unreflected lattices, the bracket procedure with conventional analyses appears adequate for coping with the harmonic distortion.

It was hoped that for those measurements where a fundamental mode is difficult to discern in the data, comparisons of experiment and calculation in the prompt harmonic region would suffice to establish the reactivity. However, this does not appear generally feasible in the absence of a rigorous derivation of the appropriate two-group source distribution. Presumably, the use of a larger number of neutron groups would allow the source response to be more accurately modeled.

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APPENDIX

Minimization of Source-Induced Harmonics in a Uniform Bare Reactor

The existence of the source induces large changes in the flux shape away from the asymptotic flux shape obtained when the source is off. In a uniform bare reactor, one can consider the flux shape at any time to consist of a summation of so-called harmonic shapes. For certain geometries, the transition between the flux shape just when the source is turned off and the final natural (or fundamental mode) flux shape can be approximated in terms of a harmonic expansion where all the terms are analytic. Space and time separability is assumed.

$$\phi(r,t) \equiv \sum_{k} \sum_{\ell} \sum_{m} R_{k\ell m}(r)\ T_{k\ell m}(t) \tag{1}$$

where $R_{k\ell m}(r)$ contains all the space dependence and $T_{k\ell m}(t)$ contains all the time dependence.

With a one-group model,

$$T_{k\ell m}(t) = \exp\left\{\left[v\Sigma_a k_\infty(1-\beta) - v\Sigma_a(1 + M^2 B^2_{k\ell m})\right]\ t\right\} \tag{2}$$

where

v = velocity

Σ_a = absorption cross section

k_∞ = infinite medium multiplication constant

β = delayed neutron fraction

M^2 = diffusion area

$B^2_{k\ell m}$ = harmonic geometric buckling

The harmonic flux shape eigenfunctions $R_{k\ell m}(r)$ have particularly simple analytical expressions for one-region reactors in cylindrical geometry when the source is on the central axis. There is no azimuthal dependence to the flux shape, and thus

only two harmonic indices are necessary. The harmonic eigenfunction is

$$R_{k\ell}(r) = C_{k\ell}\, J_o\left(\frac{\chi_k r}{R}\right)\, \sin\left(\frac{\ell\pi z}{Z}\right) \tag{3}$$

where

$J_o\left(\frac{\chi_k r}{R}\right)$ = zeroth order Bessel function

χ_k = 2.405 for k = 1, 5.520 for k = 2, etc.

R and Z = the extrapolated radius and extrapolated pile height, respectively

$C_{k\ell}$ = a constant reflecting the amplitude of the harmonic excited by the source

The harmonic geometric bucklings are calculated from

$$B^2_{k\ell} = \left(\frac{\chi_k}{R}\right)^2 + \left(\frac{\ell\pi}{Z}\right)^2 \tag{4}$$

The magnitude of the bucklings monotonically increases with increasing harmonic order. Thus the rate of decay of the harmonic, represented by the exponent of the time term (Equation 2), also increases as the harmonic order increases. As a consequence, the higher order harmonics disappear at a much faster rate than the fundamental mode (k = 1, ℓ = 1). This is the reason that pulsed neutron decay eventually assumes a single exponential form.

For the bare lattice experiment in this report, the detectors were placed at that position where $J_o(5.520r/R) = 0.0$; i.e., the second radial harmonic contribution was not recorded. Because the source was located high above the SE tank, the higher order radial harmonics were only weakly excited, and moreover, they should have died out quickly. Thus it is reasonable to assume that practically all the harmonic content was contained in a combination of the fundamental radial and the array of axial harmonics; i.e.,

$$\phi(\vec{r},t) \approx J_o\left(\frac{2.405r}{R}\right) \sum_{\ell} C_{1\ell}\, \sin\left(\frac{\ell\pi z}{Z}\right)\, T_{1\ell}(t) \tag{5}$$

The most important harmonics have low order axial index because the higher order harmonics die out very quickly. By placing detectors near $\frac{1}{3}Z$ and $\frac{2}{3}Z$, the third order axial harmonic is minimized. Finally, the second order axial harmonic is minimized by summing the outputs of the two detectors since the second order harmonic is in opposite directions at the two positions.

$$N\ (\text{summed}) = N(t) + N(b)\left[\frac{\sin\left(\frac{2\pi z_t}{Z}\right)}{\sin\left(\frac{2\pi z_b}{Z}\right)}\right] \tag{6}$$

The factor in the brackets multiplying the bottom detector count rate corrects for actual placement of the detectors.

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