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GEAP-10546
AEC RESEARCH AND
DEVELOPMENT REPORT
JANUARY 1972

1 RECEIVED BY TIC DEC 11 1973

**PART I,
THEORY REPORT
FOR CREEP-PLAST
COMPUTER PROGRAM:
ANALYSIS OF TWO-DIMENSIONAL PROBLEMS
UNDER SIMULTANEOUS CREEP
AND PLASTICITY**

Y. R. RASHID

PREPARED FOR UNION CARBIDE CORPORATION
NUCLEAR DIVISION
SUBCONTRACT NO. 3511
CONTRACT NO. W7405-ENG.-26

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Y. R. Rashid

Approved:

J. E. Wood
J. E. Wood, Manager

Fuel & Material Evaluation

Prepared for Union Carbide Corporation
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Subcontract No. 3511
Contract No. W7405-eng-26

NUCLEAR FUEL DEPARTMENT • GENERAL ELECTRIC COMPANY
SAN JOSE, CALIFORNIA 95114

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1. INTRODUCTION

Metal structures under elevated temperatures undergo significant creep and plastic deformations which generally occur simultaneously. In order to make useful quantitative analysis of current design problems one has to rely on existing material data which are generally limited to the time-dependent behavior of simple structures. In the particular problem of creep, for instance, simple extension data generated from single-step loading histories constitute the basis for most, if not all, current analytical predictions of the behavior of metal structures under variable load and temperature histories. However, the inelastic behavior of metal structures under such loading histories is highly nonlinear, requiring more extensive material data. Therefore many of the current analytical methods are mere generalizations of existing experimental data rather than fundamental models of material behavior.

The basic problem lies in generalizing the results of the single-step simple bar experiment to three-dimensional variable stress states. In doing this, two divergent views have been put forth: the first and most popular is fashioned after the plasticity theory and has hence been known as the equation-of-state approach.^{1,2} The other point of view treats creep as a memory process resulting in a general nonlinear theory of viscoelasticity.³ Various approximations have been proposed for this general theory with only limited applications to weakly nonlinear solids.^{4,5}

The plasticity theory, however, attracted the attention of a much larger number of workers over a longer period of time. The deformation theory originated by Hencky and Nadai and the flow, or incremental theory, proposed by Levy and von Mises are well developed.^{6,7} Further extensions to work-hardening materials by Melan and Prager⁸ and the subsequent introduction of Drucker's work-hardening postulate⁹ produced a unified treatment of incremental plasticity equations. Prager's kinematic-hardening rule¹⁰ offered an alternative to the earlier isotropic hardening postulate that failed to account for Bauehinger effect. Subsequent interpretations and modifications of Prager's rule were given by Shield and Ziegler¹¹ and by Ziegler.¹² More recently, Eisenberg and Phillips¹³ proposed an extension of the linear kinematic hardening rule to nonlinear stress-strain curves.

From an analysis point-of-view, the creep and plasticity problems have been treated separately. The literature is quite extensive on analytical solutions for axisymmetric and point-symmetric problems for the elastic-plastic and the elastic-creep conditions. Numerical solutions of practical problems were far less known until Mendelson, Hirschberg, and Manson¹⁴ put forth a general procedure for solving creep and plasticity problems. Their work formed the basis for many finite-difference and finite-element analyses that appeared in the recent literature. Because of the close relationship between the incremental plasticity and the equation-of-state equations, it was natural to follow Mendelson's approach for the solution of both creep and plasticity problems. The memory theory remained relatively unexplored numerically except for the case of linear viscoelasticity.¹⁵

Most of the analytical and numerical solutions treated creep and plasticity as two separate problems. The first analytical work on the combined problem is due to Fredrick, Chubb, and Bormley,¹⁶ in which they solved the thick-walled cylinder subjected to cyclic loading. Landau et al.,¹⁷ determined the transient and residual stress state in a quenched steel plate subjected to transient temperature distributions. Shinozuka¹⁸ analyzed an incompressible viscoelastic-plastic thick-walled cylinder subjected to an internal pressure using von Mises yield criterion and the standard solid viscoelastic model. Using a viscoelastic-plastic model, Crochet¹⁹ presented a general solution for a hollow cylinder under internal pressure and a solid cylinder under torsion. Jensen et al.²⁰ applied the finite element method to analyze the plastic and creep problem of a shear lag structure subjected to a two-step loading. More recently, Schultz and Van Fossen²¹ presented a finite element analysis of a pressure vessel subjected to a transient loading that produced short-time excursions into the plastic range. In addition to these analyses several others were published for the viscoelastic-plastic solids, where the material is represented by a linear-viscoelastic law and a temperature dependent yield criterion.

The present report deals with the solution of the combined creep and plasticity problem. Both creep theories, the equation-of-state and memory theories, are used within the framework of the finite element computational method. Because of the limited creep data, which is available only for single step simple extension experiments, the integral expansion in the memory theory formulation is limited to one nonlinear superposition integral. However, instead of using direct superposition as proposed by Findley and Stanley,⁵ the kernel function was derived as an integral transformation that reduced the time-temperature-stress relationship to a single quantity. The equation-of-state formulation is based on the monotonic strain hardening rule applied to the primary creep component only. The

uniaxial stress-strain law is first discussed with emphasis on the memory theory. Then the incremental stress-strain relations for the combined creep and plasticity problem are derived for both creep theories. The time-temperature-stress transformation is derived for a particular class of material which exhibits well-defined primary and secondary creep parts such as stainless steel. The properties of the resulting stress-strain integral are discussed in relation to the direct superposition method. Finally, example analyses are given.

2. UNIAXIAL STRESS-STRAIN LAW

Consider a simple extension bar under the uniaxial stress state:

$$\sigma(t) = \sigma_0 h(t) \quad (1)$$

where σ_0 = stress magnitude

$h(t)$ = unit step function

t = time.

The strain resulting from such a state of stress may be arbitrarily defined as the sum of three parts: elastic, plastic, and creep, respectively—

$$\epsilon(t) = \epsilon^e(t) + \epsilon^p(t) + \epsilon^c(t) \quad (2)$$

The elastic and plastic parts of the strain are generally assumed to occur instantaneously. Furthermore, equation (2) reflects no interdependence between the plastic and the creep components, implying that the strain hardening properties in creep and plasticity are separable. This particular point requires further analytical and experimental study, but in the meantime it may be regarded as a reasonable approximation to the true situation.

The separability assumption in equation (2) permits writing this equation in the incremental form:

$$\dot{\epsilon}(t) = \dot{\epsilon}^e(t) + \dot{\epsilon}^p(t) + \dot{\epsilon}^c(t) \quad (3)$$

The relationship between the stress and the instantaneous (elastic and plastic) strain offers no particular difficulty and will be left to a later discussion. The creep part, however, presents some fundamental considerations which are basic to the analysis given in this report. It is therefore discussed in this section on the basis of the uniaxial creep experiment.

In such an experiment the state of stress is represented by equation (2). The measured strain can be expressed in the following form:

$$\epsilon(t) = f(\sigma_0, T_0, t) \quad (4)$$

where T_0 = constant temperature of creep test

If one repeats the experiment for different stress and temperature levels, then the equation—

$$\epsilon(t) = f(\sigma, T, t) \quad (5)$$

represents a two-parameter family of curves for T and σ , which are varied over the entire range of interest. Equation (5), however, does not reflect the time variation of σ or T and could not be used to predict the creep strains in, say, multi-step or ramp stress histories. To do this one requires additional tests and further postulates on the material response to time-varying loading.

The analytical problem may be formulated on the basis of the well known hardening theories of which the strain-hardening equation of state theory has been most widely applied. The formulation of this theory is well known and the relevant equations are given without derivation in a later section.

The other analytical approach treats the material as nonlinear viscoelastic substance in which the current strain state can be made to depend on the loading history, namely—

$$\epsilon(t) = F \left[\dot{\sigma}(\tau) \right]_{\tau=-\infty}^{\tau=t} \quad (6)$$

This functional relation can be closely approximated by the following series:

$$\begin{aligned} \epsilon(t) = & \int_{-\infty}^t J_1(t-\tau_1) \frac{d\sigma(\tau_1)}{d\tau_1} d\tau_1 \\ & + \int_{-\infty}^t \int_{-\infty}^t J_2(t-\tau_1; t-\tau_2) \frac{d\sigma(\tau_1)}{d\tau_1} \frac{d\sigma(\tau_2)}{d\tau_2} d\tau_1 d\tau_2 \\ & + \int_{-\infty}^t \int_{-\infty}^t \int_{-\infty}^t J_3(t-\tau_1; t-\tau_2; t-\tau_3) \frac{d\sigma(\tau_1)}{d\tau_1} \frac{d\sigma(\tau_2)}{d\tau_2} \frac{d\sigma(\tau_3)}{d\tau_3} d\tau_1 d\tau_2 d\tau_3 \\ & + \dots \end{aligned} \quad (7)$$

Equation (7) is the counterpart of the time-hardening (or strain-hardening) equation in the equation-of-state creep theory. In a creep experiment where the stress σ does not vary with time, equation (7) reduces to—

$$\epsilon(t) = \sigma J_1(t) + \sigma^2 J_2(t, t) + \sigma^3 J_3(t, t, t) + \dots \quad (8)$$

which is a polynomial representation of a single-step creep test.

The use of equation (7) in its generality is not within current experimental and computational means at this time, since it requires several multi-step creep tests for the determination of the creep kernel functions $J_1, J_2, \dots$, which are not presently available. In order to make use of this general concept, however, within the limitation of single-step creep data, equation (7) is replaced by the following nonlinear superposition equation:

$$\epsilon(t) = \int_{-\infty}^t \frac{\partial C(\sigma, t-t')}{\partial \sigma(t')} \frac{d\sigma(t')}{dt'} dt' \quad (9)$$

where

$C(\sigma, t)$ = usual single-step creep formula.

Equation (9) can be written as—

$$\epsilon(t) = \int_{-\infty}^t J(\sigma, t-t') \frac{d\sigma(t')}{dt'} dt' \quad (10)$$

where $J(\sigma, t)$ = creep compliance at stress level σ .

Equation (10) predicts the strain due to time-varying σ by superposing single-step creep data. This equation is exact only for linear materials and it serves as a first approximation to equation (7). It has the advantage, however, of requiring the same creep test as the equation of state theory.

Instead of generalizing the single-step creep data to time-varying stresses by direct superposition, as indicated by equation (10), we first introduce the following transformation:

$$\xi(t, t') = \int_{t'}^t \phi[\sigma(\lambda); T(\lambda)] d\lambda \quad (11)$$

and then restate equation (10) in the following form:

$$\epsilon(t) = \int_{-\infty}^t J[\xi(t, t')] \frac{d\sigma(t')}{dt'} dt' \quad (12)$$

In this equation the physical time, t , is replaced by the equivalent time, $\xi(t)$, defined in equation (11), where $\phi(\sigma, T)$ is a continuous function that depends on the stress σ and temperature T . The basis for this formulation is the following: If one plots specific creep curves for various σ 's and/or T 's on a semi-log plot where J is plotted against $\log t$ one observes that all the curves can be obtained by simply displacing a suitable base curve rigidly parallel to the $\log t$ axis. The remaining derivations are given in Appendix B.

Equations (11) and (12) rest on the assumption that the material obeys the modified superposition stated in equation (12). Furthermore they imply that the transformation equation (11) always exists for all possible forms of the generalized creep formula (5). Other properties of equations (11) and (12) will be discussed later with reference to a particular form of equation (5).

3. INCREMENTAL STRESS-STRAIN RELATIONS

In the following derivation, matrix notation will be used, retaining indicial notation where appropriate. It is first necessary to state some of the usual assumptions governing the inelastic flow of the material. The first of these is the incompressibility of the plastic and creep strains, namely—

$$\epsilon_{kk}^p = 0 \quad (13)$$

$$\epsilon_{kk}^c = 0 \quad (14)$$

where the repeated index implies summation.

On the basis of these equations, the inelastic deviatoric strains become—

$$\begin{aligned} e_{ij}^p &= \epsilon_{ij}^p - \frac{1}{3} \epsilon_{kk}^p \delta_{ij} = \epsilon_{ij}^p \\ e_{ij}^c &= \epsilon_{ij}^c - \frac{1}{3} \epsilon_{kk}^c \delta_{ij} = \epsilon_{ij}^c \end{aligned} \quad (15)$$

where

$$\delta_{ij} = \text{Kronecker delta}$$

The second basic assumption is that the instantaneous plastic strains obey a flow rule, and the stresses satisfy a given yield function. We restrict attention to von Mises yield condition and its associated flow rule, which are stated as follows for the condition of initial yielding:

$$f = \frac{1}{2} S_{ij} S_{ij} - \kappa^2 = 0 \quad (16)$$

$$\dot{\epsilon}_{ij}^p = \lambda \frac{\partial f}{\partial \sigma_{ij}} \quad (17)$$

where the deviatoric stresses S_{ij} are given by—

$$S_{ij} = \sigma_{ij} - \frac{1}{3} \sigma_{kk} \delta_{ij} \quad (18)$$

In the above, λ is a scalar function and κ is a material parameter which may depend on the temperature.

The third basic assumption concerns the type of hardening law the material is postulated to obey. The kinematic hardening rule initially postulated by Prager constitutes the basis for the present development. In the interest of remaining theoretically consistent, the classical linear kinematic hardening formulation is used. On this basis, then, equations (16) and (17) remain of the same form, provided that the deviatoric stresses S_{ij} are replaced by S'_{ij} , namely—

$$S'_{ij} = (\sigma_{ij} - \alpha_{ij}) - \frac{1}{3} (\sigma_{kk} - \alpha_{kk}) \delta_{ij} \quad (19)$$

where

$$\alpha_{ij} = \text{stress coordinates of the origin of the translating yield surface}$$

The remaining derivations leading to the incremental stress-strain relations are included in Appendix A.

The end result is a relationship between the incremental plastic strains and incremental stresses, namely—

$$\Delta \underline{\epsilon} = \underline{H}^{-1} \Delta \underline{\sigma} + \Delta \underline{\theta} \quad (20)$$

where

$$\Delta \underline{\epsilon} = \text{incremental strain vector}$$

$$\Delta \underline{\sigma} = \text{incremental stress vector}$$

$$\Delta \underline{\theta} = \text{incremental initial strain vector due to thermal effects, defined in the appendix.}$$

$\underline{H}$ is the plasticity matrix defined by—

$$\underline{H} = \underline{H}_e - \eta \underline{H}_p \quad (21)$$

where

$$\underline{H}_e = \text{the elasticity matrix}$$

$$\eta = \text{zero for elastic state or unity for plastic state}$$

$\underline{H}_p$ is given below:

$$\underline{H}_p = (h_{ijkl})_p = \frac{2\mu}{2\kappa^2 \left(1 + \frac{E'}{3\mu}\right)} S'_{ik} S'_{jl} \quad (22)$$

In the above, $\kappa = \sigma_Y/\sqrt{3}$

where

σ_Y = uniaxial yield stress

μ = shear modules

E' = uniaxial hardening modules in a bilinear stress-strain curve.

It should be stated at this point that although this derivation is given for linear kinematic hardening, the computer program which is based on this development provides for a general stress-strain curve. The effect of history is introduced on an ad hoc basis by simply allowing E' to be a function of the stress state and a history parameter that defines the position on the true stress-strain curve.

It remains to derive the creep strain-stress relations. As was mentioned earlier the equation-of-state hardening theory is fairly well known and is fashioned after the plasticity theory. Therefore the basic assumptions stated above have similar implications leading to the following relationship:

$$\dot{\epsilon}_{ij}^c = \frac{3}{2} \frac{\dot{\epsilon}_e^c}{\sigma_e} S_{ij} \quad (23)$$

where σ_e is given by—

$$\sigma_e = \left(\frac{3}{2} S_{ij} S_{ij} \right)^{1/2} \quad (24)$$

and $\dot{\epsilon}_e^c$ is obtained from the experimental creep formula (5). It is generally a function of stress, temperature, and, depending upon the particular form of the hardening rule selected, the accumulated creep strains. The strain-hardening rule for the primary creep component is generally accepted as more representative of material behavior. The remaining derivations are given in Appendix B.

The alternative approach to the equation of state, as was discussed in the previous section, is the use of a modified superposition principle.

For this case, equation (23) is replaced by—

$$\epsilon_{ij}^c = \int_{-\infty}^t J[\xi(t, t')] \frac{\partial S_{ij}(t')}{\partial t'} dt' \quad (25)$$

where $J[\xi(t, t')] =$ creep compliance in shear.

Differentiating equation (25) with respect to time gives—

$$\dot{\epsilon}_{ij}^c(t) = \frac{\partial}{\partial t} \int_{-\infty}^t J[\xi(t, t')] \frac{\partial S_{ij}(t')}{\partial t'} dt' \quad (26)$$

The function $J[\xi(t, t')]$ is continuous in t and vanishes at $t' = t$. Hence, equation (26) can be differentiated under the integral sign with the following result:

$$\dot{\epsilon}_{ij}^c(t) = \int_{-\infty}^t \frac{\partial J[\xi(t, t')]}{\partial t} \frac{\partial S_{ij}(t')}{\partial t'} dt' \quad (27)$$

Combining equations (20), (27) and the elastic relations gives the incremental strain-stress relations in matrix form as follows:

$$\dot{\underline{\epsilon}}(t) = \underline{H}^{-1}(t) \dot{\underline{\sigma}}(t) + \dot{\underline{\theta}}(t) + \underline{R} \int_{-\infty}^t \frac{\partial J[\underline{\xi}(t, t')]}{\partial t} \underline{\sigma}(t') dt' \quad (28)$$

where

$$\begin{aligned} \underline{R} &= \text{matrix relating the deviatoric stresses } S_{ij} \text{ to the coordinate stresses } \sigma_{ij} \\ \underline{\theta}(t) &= \text{thermal strain vector} \end{aligned}$$

The vector $\underline{\theta}(t)$ may be generalized to include the incremental creep strains defined by equation (23) if the equation-of-state formulation is selected, in which case the integral in equation (28) is dropped.

Equation (28) includes the initial conditions which in the context of the present development is defined to be the elastic solution at $t = 0$. Extracting this condition from equation (28) gives—

$$\dot{\underline{\epsilon}}(t) = \underline{H}^{-1}(t) \dot{\underline{\sigma}}(t) + \dot{\underline{\theta}}(t) + \frac{\partial J[\underline{\xi}(t, 0)]}{\partial t} \underline{R} \underline{\sigma}(0) + \underline{R} \int_0^t \frac{\partial J[\underline{\xi}(t, t')]}{\partial t} \dot{\underline{\sigma}}(t') dt' \quad (29)$$

Recasting equation (29) in finite difference form with respect to time yields the following:

$$\Delta \underline{\epsilon}_n = \underline{H}_n^{-1} \Delta \underline{\sigma}_n + \Delta \underline{\theta}_n + J'_{n,0} \underline{R} \underline{\sigma}_0 + \underline{R} \sum_{i=1}^{n-1} \int_{t_i}^{t_{i+1}} J'[\underline{\xi}_n, \underline{\xi}(t')] \dot{\underline{\sigma}}(t') dt' \quad (30)$$

In this equation the subscript, n , refers to current time, t_n , and the prime superscript refers to the derivative with respect to t at t_n . The integral terms of the series in equation (30) can be evaluated using any suitable integration algorithm. In the present analysis Simpson's rule is used.

Dealing with the integral terms separately, we can write—

$$\int_{t_i}^{t_{i+1}} J'[\underline{\xi}_n, \underline{\xi}(t')] \dot{\underline{\sigma}}(t') dt' = \frac{\Delta \underline{\sigma}_{i+1}}{\Delta t_{i+1}} \int_{t_i}^{t_{i+1}} J'[\underline{\xi}_n, \underline{\xi}(t')] dt' \quad (31)$$

Defining—

$$J'_{n,i+1} \equiv \frac{1}{\Delta t_{i+1}} \int_{t_i}^{t_{i+1}} J'[\underline{\xi}_n, \underline{\xi}(t')] dt' \quad (32)$$

equation (30) can be rewritten as

$$\Delta \underline{\epsilon}_n = \underline{H}_n^{-1} \Delta \underline{\sigma}_n + \Delta \underline{\theta}_n + J'_{n,0} \underline{\sigma}_0 + \underline{R} \sum_{i=1}^{n-1} J'_{n,i+1} \Delta \underline{\sigma}_{i+1} \quad (33)$$

Extracting the n^{th} term from the series and rearranging terms we arrive at the final form of the incremental stress-strain relations as follows:

$$\Delta \underline{\sigma}_n = \underline{M}_n [\Delta \underline{\epsilon}_n - \underline{J}'_{n,0} \underline{R} \underline{\sigma}_0 - \Delta \underline{\theta}_n - \underline{R} \sum_{i=1}^{n-1} \underline{J}'_{n,i} \Delta \underline{\sigma}_i] \quad (34)$$

where

$$\underline{M}_n = [\underline{I} + \underline{J}'_{n,n} \underline{H}_n \underline{R}]^{-1} \underline{H}_n \quad (35)$$

Equations (34) and (35) are valid for the combined problem of elastic, plastic, and creep using either creep theory. If the equation-of-state creep theory is selected, the terms involving $\underline{J}'$ drop out and the vector $\Delta \underline{\theta}_n$ can be generalized to include the incremental creep strains calculated from equation (23). For elastic-plastic analysis only, i.e., zero Δt_i , the matrix, $\underline{M}_n$, reduces to the plasticity matrix, $\underline{H}_n$, and the terms involving the creep compliance, $\underline{J}$, are identically zero.

4. EXAMPLE ANALYSES

The analysis of three creep experiments under multi-step loading histories is shown in Figures 1, 2, and 3. The memory theory solutions are compared with strain hardening theory solutions and experimental data obtained by Oak Ridge National Laboratory (ORNL).²² The creep data are given in equations (4.12) to (4.15) of reference 22. The agreement between the two calculation methods is to be expected since, first, both the memory theory and the hardening theory are based on the same single-step creep data, and, secondly, no stress redistribution occurs in a simple extension problem. The difference in the two methods, as far as this particular problem is concerned, lies in the computational procedure. For instance, the memory theory solution is independent of the size of the time step when the stress state does not vary continuously with time. The intermediate points in the memory theory solution, as shown in the figures, were obtained to permit the drawing of a curve. In contrast, however, the strain-hardening theory requires sufficiently small time steps to permit the accumulated effective creep strain, which enters into the flow rule explicitly, to be calculated accurately.

Another example shown in Figures 4 and 5 is a thick-walled cylinder under a two-step internal pressure loading as described in the illustration. Both illustrations show the comparison between the memory and hardening theories. The strain-hardening solutions were obtained on the basis of the auxiliary hardening rules described in reference 22 by ORNL. The memory theory solution in Figure 4 did not include plasticity and therefore can be compared directly with ORNL analysis. As shown in Figure 5, however, plasticity was allowed to occur. The proportional limit for the material at the stated temperature is 10,060 psi. The effect of plasticity is, therefore, greater during the negative pressure phase. Comparing the memory theory solutions in Figures 4 and 5, it appears that the slower rate of relaxation, which occurred as a result of developing smaller peak stress, resulted in a higher final stress. The time steps in the memory theory solution ranged from 2 hours to 5000 hours, resulting in a total of 42 time steps for the entire 60,000 hour history.

Figure 1. Step-Load Creep Test of Type-304 Stainless Steel at 1200°F

Figure 2. Step-Load Creep Test of Type-304 Stainless Steel at 1200°F

Figure 3. Step-Load Creep Test of Type-304 Stainless Steel at 1200°F

AXISYMMETRIC, PLANE-STRAIN CREEP
ANALYSIS OF THICK-WALLED CYLINDER
SUBJECTED TO AN INTERNAL PRESSURE
OF 3650 psi FOR 30,000 hr, -3650 psi
FOR 30,000 hr

304 STAINLESS STEEL AT 1200°F
 $r_o = 0.25$ in., $r_i = 0.16$ in.

Figure 4. Predicted Response of Pressurized Thick-Walled Cylinder
(Elastic-Creep Analysis)

Figure 5. Predicted Response of Pressurized Thick-Walled Cylinder
(Elastic-Plastic-Creep Analysis)

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APPENDIX A

ELASTIC-PLASTIC INCREMENTAL STRESS-STRAIN RELATIONS

The instantaneous incremental strain $d\epsilon_{ij}$ is the sum of the elastic strain $d\epsilon_{ij}^e$ and the plastic strain $d\epsilon_{ij}^p$, namely—

$$d\epsilon_{ij} = d\epsilon_{ij}^e + d\epsilon_{ij}^p \quad (A-1)$$

where the elastic strains are related to the incremental stress by—

$$d\epsilon_{ij}^e = C_{ijkl} d\sigma_{kl} \quad (A-2)$$

or by the inverse relation—

$$d\sigma_{ij} = D_{ijkl} d\epsilon_{kl} \quad (A-3)$$

For isotropic material, both the material tensors C_{ijkl} and D_{ijkl} , respectively, reduce to—

$$C_{ijkl} = \frac{1-\nu}{E} \delta_{ik} \delta_{jl} - \frac{\nu}{E} \delta_{ij} \delta_{kl} \quad (A-4)$$

and

$$D_{ijkl} = \frac{E}{1+\nu} \delta_{ik} \delta_{jl} + \frac{\nu E}{(1+\nu)(1-2\nu)} \delta_{ij} \delta_{kl} \quad (A-5)$$

where

E = Young's modulus

ν = Poisson's ratio

δ_{ij} = Kronecker delta.

By Drucker's definition of work-hardening, the material is stable under plastic flow, i.e.—

$$(\sigma_{ij} - \sigma_{ij}^*) d\epsilon_{ij}^p \geq 0$$

or

$$d\sigma_{ij} d\epsilon_{ij}^p \geq 0$$

where

σ_{ij} = stress state lying on the current yield surface

σ_{ij}^* = previous stress state lying inside or on the yield surface

$d\sigma_{ij}$ = incremental stresses which occur as the stress state changes from σ_{ij}^* to σ_{ij} .

From the above, the incremental plastic strain vector must be normal to the loading surface, therefore—

$$d\epsilon_{ij}^p = \lambda \frac{\partial f}{\partial \sigma_{ij}} ; \lambda > 0 \quad (A-7)$$

when

$$\frac{\partial f}{\partial \sigma_{ij}} d\epsilon_{ij}^p > 0 \quad (A-8)$$

where

f = a suitable yield (or load) function to be defined later.

Equation (A-7) represents an associated flow rule since f is stipulated to be the yield function.

In view of equations (A-1), (A-3), and (A-7), the incremental stress-strain relations may be written as

$$d\sigma_{ij} = D_{ijkl} \left(d\epsilon_{kl} - \lambda \frac{\partial f}{\partial \sigma_{kl}} \right) \quad (A-9)$$

In the above equation, the free parameter λ remains to be determined.

Consider a material element subjected to a stress state, σ_{ij} , which lies on the yield surface. Under a subsequent load increment, we assume plastic flow continues, and therefore the new stress state satisfies a yield condition of the following form:

$$f = f(\sigma_{ij}, \epsilon_{ij}^p, \kappa) = 0 \quad (A-10)$$

where

$\kappa = \kappa(T)$ = a material parameter that depends on the temperature T .

Differentiating (A-10) gives—

$$df = \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} + \frac{\partial f}{\partial \epsilon_{ij}^p} d\epsilon_{ij}^p + \frac{\partial f}{\partial T} dT = 0 \quad (A-11)$$

This equation admits the dependence of the yield function on the temperature T through the parameter κ .

Eliminating $d\sigma_{ij}$ between (A-9) and (A-11) and substituting for $d\epsilon_{ij}^p$ from (A-7) yields—

$$\frac{\partial f}{\partial \sigma_{ij}} D_{ijkl} \left(d\epsilon_{kl} - \lambda \frac{\partial f}{\partial \sigma_{kl}} \right) + \lambda \frac{\partial f}{\partial \epsilon_{ij}^p} \frac{\partial f}{\partial \sigma_{ij}} + \frac{\partial f}{\partial T} dT = 0 \quad (A-12)$$

Solving for λ —

$$\lambda = \Lambda_{kl} d\epsilon_{kl} + \frac{\partial f}{\partial T} dT \left/ \left(D_{mnpq} \frac{\partial f}{\partial \sigma_{mn}} \frac{\partial f}{\partial \sigma_{pq}} - \frac{\partial f}{\partial \epsilon_{mn}^p} \frac{\partial f}{\partial \sigma_{mn}} \right) \right. \quad (A-13)$$

where

$$\Lambda_{kl} = D_{ijkl} \frac{\partial f}{\partial \sigma_{ij}} \left/ \left(D_{mnpq} \frac{\partial f}{\partial \sigma_{mn}} \frac{\partial f}{\partial \sigma_{pq}} - \frac{\partial f}{\partial \epsilon_{mn}^p} \frac{\partial f}{\partial \sigma_{mn}} \right) \right. \quad (A-14)$$

Finally, the incremental stress-strain law is found to be—

$$d\sigma_{ij} = D_{ijkl} \left(d\epsilon_{kl} - \eta \frac{\partial f}{\partial \sigma_{kl}} \Lambda_{mn} d\epsilon_{mn} \right) - d\theta_{ij} \quad (A-15)$$

where

$$d\theta_{ij} = \eta D_{ijkl} \frac{\partial f}{\partial \sigma_{kl}} \frac{\partial f}{\partial T} dT \left/ \left(D_{mnpq} \frac{\partial f}{\partial \sigma_{mn}} \frac{\partial f}{\partial \sigma_{pq}} - \frac{\partial f}{\partial \epsilon_{mn}^p} \frac{\partial f}{\partial \sigma_{mn}} \right) \right. \quad (A-16)$$

and

$$\begin{aligned} \eta &= 0 & \text{when } f < 0 \\ & & \text{or when } f = 0 \text{ and } \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} + \frac{\partial f}{\partial T} dT < 0 \end{aligned} \quad (A-17)$$

and

$$\eta = 1 \quad \text{when } f = 0 \text{ and } \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} + \frac{\partial f}{\partial T} dT \geq 0$$

We now apply the above results to the kinematic strain-hardening rule originally proposed by Prager.

The yield function is of the form—

$$f = f(\sigma_{ij} - \alpha_{ij}, \kappa) = 0 \quad (A-18)$$

where the tensor α_{ij} represents the translation of the yield surface. If the stress-strain curve is bilinear then α_{ij} is proportional to the plastic strain tensor, namely—

$$\alpha_{ij} = c \epsilon_{ij}^p \quad (A-19)$$

where c is a material constant which, for linear hardening materials, is equal to $\frac{2}{3} E'$, where E' is the hardening modulus in simple tension.

From (A-18) and (A-19) the yield surface becomes—

$$f(\sigma_{ij} - c \epsilon_{ij}^p, \kappa) = 0 \quad (A-20)$$

The above results are used in conjunction with von Mises yield condition, namely—

$$f = \frac{1}{2} S'_{ij} S'_{ij} - \kappa^2 = 0 \quad (A-21)$$

where $\kappa = \kappa(T)$ and

$$S'_{ij} = (\sigma_{ij} - \alpha_{ij}) - \frac{1}{3} (\sigma_{kk} - \alpha_{kk}) \delta_{ij} \quad (A-22)$$

From (A-21) and (A-22), making use of (A-19)—

$$\frac{\partial f}{\partial \epsilon_{ij}^p} = -c \frac{\partial f}{\partial \sigma_{ij}} \quad (A-23)$$

Equation (A-14) becomes—

$$\Lambda_{k\ell} = D_{ijkl} \frac{\partial f}{\partial \sigma_{ij}} \left/ \left(D_{mnpq} \frac{\partial f}{\partial \sigma_{mn}} \frac{\partial f}{\partial \sigma_{pq}} + c \frac{\partial f}{\partial \sigma_{mn}} \frac{\partial f}{\partial \sigma_{mn}} \right) \right. \quad (\text{A-24})$$

The substitution of (A-21) and (A-22) into (A-24) leads to—

$$\Lambda_{k\ell} = S'_{k\ell}/R \quad (\text{A-25})$$

with

$$R = 2\kappa^2 \left(1 + \frac{E'}{3\mu} \right) \quad (\text{A-26})$$

where μ is the shear modulus.

Finally, from equation (A-15), the incremental stress-strain relations are given by—

$$d\sigma_{ij} = 2\mu \left(d\epsilon_{ij} + \frac{\nu}{1-2\nu} \delta_{ij} d\epsilon_{kk} - \eta \frac{S'_{ij} S'_{mn}}{R} d\epsilon_{mn} \right) - \frac{2\eta S'_{ij} \kappa}{R} \frac{\partial \kappa}{\partial T} dT \quad (\text{A-27})$$

The translation of the yield surface is given by—

$$d\alpha_{ij} = (\sigma_{ij} - \alpha_{ij}) du \quad (\text{A-28})$$

The scalar du is determined by the condition that the stress point remains on the yield surface during plastic flow. This condition is stated as follows:

$$(d\sigma_{ij} - d\alpha_{ij}) \frac{\partial f}{\partial \sigma_{ij}} = 0 \quad (\text{A-29})$$

The scalar du is then determined from (A-28) and (A-29) as follows:

$$du = \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} \left/ \left[(\sigma_{k\ell} - \alpha_{k\ell}) \frac{\partial f}{\partial \sigma_{k\ell}} \right] \right. \quad (\text{A-30})$$

APPENDIX B

CREEP COMPLIANCE RELATIONS FOR THE MEMORY AND STRAIN-HARDENING THEORIES

B.1 DERIVATION OF THE TIME-TEMPERATURE-STRESS TRANSFORMATION

The derivation of the time-temperature-stress transformation, equation (11), is highly dependent upon the particular form of the creep formula. We restrict attention to stainless steel which is the prime material for liquid metal fast breeder reactors. The following creep equation has been proposed for this material:

$$C(\sigma, T, t) = C_1 (1 - e^{-rt}) + C_2 t \quad (B-1)$$

where C_1 , r , and C_2 are functions of stress and temperature. Differentiating this equation with respect to the stress gives an expression for the creep compliance, namely—

$$J(\sigma, T, t) = \frac{\partial C}{\partial \sigma} = C'_1 (1 - e^{-rt}) + C_1 r' t e^{-rt} + C'_2 t \quad (B-2)$$

where the prime indicates derivative with respect to stress.

In view of the fact that equation (B-1) is a fit of constant-stress creep data, the differentiation in equation (B-2) may not be a proper definition of the experimental creep compliance. In other words, equation (B-2) is not likely to represent the experimental creep compliance that could be determined from experiment directly.

A more suitable definition of the creep compliance, which is perhaps more consistent with equation (B-1), is the following:

$$J(\sigma, T, t) = \frac{C}{\sigma} = A (1 - e^{-rt}) + B t \quad (B-3)$$

where

$$A = A(\sigma, T) = C_1(\sigma, T)/\sigma \quad (B-4)$$

and

$$B = B(\sigma, T) = C_2(\sigma, T)/\sigma \quad (B-5)$$

Equation (B-3) can be written in the following form:

$$J = \frac{A}{A_0} J_1 + B t \quad (B-6)$$

where

$$J_1 = A_0 (1 - e^{-rt}) \quad (B-7)$$

and

$$A_0 = A(\sigma_0, T_0) \quad (B-8)$$

where σ_0 and T_0 are the reference stress and temperature respectively.

For a fixed value of J_1 , say J_1^* , equation (B-7) can be inverted to give t as function of σ , namely—

$$t = -\frac{1}{r} \log_e \left(1 - \frac{J_1^*}{A_0} \right) \quad (B-9)$$

Substituting for J_1^* from (B-7), keeping in mind that J_1^* is a single number, gives

$$t = \frac{r_0 t_0}{r} \quad (B-10)$$

where

$$r_0 = r(\sigma_0, T_0)$$

t_0 = a reference time point at which J_1^* was obtained.

Equation (B-10) gives the time at which J_1^* is reached for any given σ or T . If one plots J_1 versus $\log t$ in equation (B-7) for various values of σ , the horizontal displacement (parallel to $\log t$ axis) of any of the $J_1(\sigma, T, t)$ curves from a base curve $J_1(\sigma_0, T_0, t)$ is given by the following:

$$\log_e \frac{t_0}{t} = \log_e \phi = \log_e \frac{r}{r_0} \quad (B-11)$$

If

$$J_1(\sigma_0, T_0, t) = \hat{J}_1(\log_e t) = J_1(\xi) \quad (B-12)$$

then

$$J_1(\sigma, T, t) = \hat{J}_1(\log_e \phi + \log_e t) = \hat{J}_1(\log_e \phi t) \quad (B-13)$$

It follows from (B-12) and (B-13) that—

$$\xi(t) = \int_0^t \phi[\sigma(\lambda); T(\lambda)] d\lambda = \int_0^t \frac{r}{r_0} d\lambda \quad (B-14)$$

and

$$J_1(\sigma, T, t) = A_0(1 - e^{-r_0 \xi}) \quad (B-15)$$

Therefore,

$$J_1(\sigma, T, t, t') = A_0(1 - e^{-r_0(\xi - \xi')}) \quad (B-16)$$

where

$$\xi' = \int_0^{t'} \frac{r[\sigma(\lambda); T(\lambda)]}{r_0} d\lambda \quad (B-17)$$

From equation (B-6)—

$$J[\sigma(t'), T(t'); t, t'] = A[\sigma(t'), T(t')] (1 - e^{-r_0(\xi - \xi')}) + B[\sigma(t'), T(t')] (t - t') \quad (B-18)$$

It remains to eliminate $\sigma(t')$ and $T(t')$ as explicit arguments in equation (B-18). This is done by deriving a transformation between $\sigma(t')$, $T(t')$ and t .

Beginning—

$$\eta(t - t') = 1 - e^{-r_0 [\zeta(t) - \zeta(t')]} \quad (B-19)$$

In the above equation, t is a parameter to be treated as a constant in the following derivations. Following similar argument as before, equation (B-18) can be written as—

$$J(\sigma_0, T_0; t, t') = \hat{J}_1(\log_e \eta) + \hat{J}_2(\log_e t') \quad (B-20)$$

or

$$J(\sigma_0, T_0; t, t') = J_1(\bar{\xi}_1) + J_2(\bar{\xi}_2) \quad (B-21)$$

then

$$J(\sigma, T; t, t') = \hat{J}_1(\log_e \phi_1 \eta) + \hat{J}_2(\log_e \phi_2 t') \quad (B-22)$$

from which

$$\bar{\xi}_1 = \int_0^{\eta} \phi_1[\sigma(\lambda); T(\lambda)] d\eta' \quad (B-23)$$

and

$$\bar{\xi}_2 = \int_0^{t-t'} \phi_2[\sigma(\lambda); T(\lambda)] d\lambda \quad (B-24)$$

But

$$d\eta' = - \frac{\partial \eta(t-\lambda)}{\partial \lambda} d\lambda = r_0 e^{-r_0 [\zeta(t) - \zeta(\lambda)]} \frac{r}{r_0} d\lambda \quad (B-25)$$

or, simply,

$$d\eta' = r e^{-r_0 [\zeta(t) - \zeta(\lambda)]} d\lambda \quad (B-26)$$

In a manner similar to the derivations of ϕ in equations (B-9), (B-10), and (B-11), the functions ϕ_1 and ϕ_2 are obtained as follows:

$$\phi_1 = A/A_0 \quad (B-27)$$

and

$$\phi_2 = B/B_0 \quad (B-28)$$

Substituting (B-26), (B-27), and (B-28) in equations (B-23) and (B-24) and making use of (B-19) yields—

$$\bar{\xi}_1(t, t') = \int_0^{t-t'} \frac{Ar}{A_0} e^{-r_0 [\zeta(t) - \zeta(\lambda)]} d\lambda \quad (B-29)$$

and

$$\bar{\xi}_2(t, t') = \int_0^{t-t'} \frac{B}{B_0} d\lambda \quad (B-30)$$

By a simple change of variables, equation (B-29) can be reduced to the following form:

$$\bar{\xi}_1(t, t') = \xi_1(t, t) - \xi_1(t, t') \equiv \xi_1 - \xi_1' \quad (B-31)$$

where

$$\xi_1 = \xi_1(t, t) = \int_0^t \frac{Ar}{A_0} e^{-r_0 [\zeta(t) - \zeta(\lambda)]} d\lambda \quad (B-32)$$

and

$$\xi_1' = \xi_1(t, t') = \int_0^{t'} \frac{Ar}{A_0} e^{-r_0 [\zeta(t) - \zeta(\lambda)]} d\lambda \quad (B-33)$$

Similarly, equation (B-30) can be written directly as—

$$\bar{\xi}_2(t, t') = \xi_2(t) - \xi_2(t') \equiv \xi_2 - \xi_2' \quad (B-34)$$

where

$$\xi_2 = \int_0^t \frac{B}{B_0} d\lambda, \quad \xi_2' = \int_0^{t'} \frac{B}{B_0} d\lambda \quad (B-35)$$

Hence equation (B-18) becomes—

$$J(\xi, \xi') = A_0(\xi_1 - \xi_1') + B_0(\xi_2 - \xi_2') \quad (B-36)$$

where ξ stands for ξ_1 and ξ_2 .

This equation constitutes the creep compliance which appeared in equations (12) and (25).

B.2 PROPERTIES OF $J(\xi, \xi')$

It would be of interest to examine the properties of the creep compliance of equation (B-36) under variable loading. The following results from the uniaxial creep strain-stress relations using equations (B-31) and (B-36):

$$\epsilon(t) = \int_{-\infty}^t [A_0 \bar{\xi}_1(t, t') + B_0 \bar{\xi}_2(t, t')] \frac{d\sigma(t')}{dt'} dt' \quad (B-37)$$

Making use of equations (B-32) and (B-33), equation (B-37) can be written as—

$$\epsilon(t) = \int_{-\infty}^t \int_{t'}^t \left\{ A[\sigma(\lambda)] \cdot r[\sigma(\lambda)] e^{-r_0 [\xi(t) - \xi(\lambda)]} + B[\sigma(\lambda)] \right\} d\lambda \frac{d\sigma(t')}{dt'} dt' \quad (B-38)$$

The direct superposition integral, however, gives the following equation—

$$\epsilon(t) = \int_{-\infty}^t \left\{ A[\sigma(t')] (1 - e^{-r[\sigma(t')] \cdot (t-t')}) + B[\sigma(t')] \cdot (t-t') \right\} \frac{d\sigma(t')}{dt'} dt' \quad (B-39)$$

This equation can also be written as—

$$\epsilon(t) = \int_{-\infty}^t \int_{t'}^t \left(A[\sigma(t')] \cdot r[\sigma(t')] e^{-r[\sigma(t')] \cdot (t-t')} + B[\sigma(t')] \right) d\lambda \frac{d\sigma(t')}{dt'} dt' \quad (B-40)$$

Comparing equations (B-38) and (B-40) it is seen that both are of similar form. However, the kernel functions differ significantly. In equation (B-38) the material functions A, r, and B are nonconstants with respect to the inner integral, whereas they are treated as constants in equation (B-40). The physical implications of this can be argued as follows: Both equations represent superposition processes. Equation (B-40), however, is the usual superposition of unit step responses, but equation (B-38) is a superposition of histories. This implies that the creep strain at the observation time t, because of a stress increment at t', is affected by the variation of $\sigma(\lambda)$ in the interval $t' \leq \lambda \leq t$. This is a clear distinction from equation (B-40) which is formulated on the basis that $\sigma(\lambda)$ remains constant in the same interval.

B.3 STRAIN-HARDENING INCREMENTAL RELATIONS

The strain-hardening incremental relations, equation (23) in text, involves the term ϵ_e^c which is the effective creep strain increment. The derivation of this term is given below on the basis that the hardening property of the material is a function of the accumulated primary creep strain only.

The primary and secondary creep components, obtained from equation (B-1), follow:

$$\epsilon_1 = C_1(1 - e^{-rt}) \quad (B-41)$$

and

$$\epsilon_2 = C_2 t \quad (B-42)$$

Differentiating equation (B-41) with respect to time gives—

$$\frac{d\epsilon_1}{dt} = C_1 r e^{-rt} \quad (B-43)$$

By eliminating time between equations (B-41) and (B-43) and adding the secondary creep contribution, we obtain the total creep rate as follows:

$$\frac{d\epsilon^c}{dt} = C_1 r \left(1 - \frac{\epsilon_1}{C_1} \right) + C_2 \quad (B-44)$$

where ϵ^c stands for the total creep.

In terms of the effective strain, equation (B-44) becomes—

$$\dot{\epsilon}_e^c = C_1 r \left(1 - \frac{\epsilon_e^c}{C_1} \right) + C_2 \quad (B-45)$$

with the condition that ϵ_e^c/C_1 is not allowed to be greater than unity. In the above equation, the quantity—

$$\epsilon_e^c = \left(\frac{2}{3} \epsilon_{ij}^c \epsilon_{ij}^c \right)_{\text{primary}}^{1/2} \quad (B-46)$$

is the effective primary creep strain. The calculated quantity $\dot{\epsilon}_e^c$ is then entered into equation (23).

Implicit in equation (B-45) is the assumption that the calculated effective creep strain ϵ_e^c is monotonically increasing with time. Hence, this formulation is called the monotonic strain hardening rule. In actual applications, however, stress reversals generally occur and therefore equation (B-45) should be modified* by adjusting the origin of the accumulated creep strains. However, the initial version of the computer program CREEP-PLAST (April 1972) does not include those modifications.

*Corum, J. M., *Recommended Appendix to RDT Standard F9-1T*, Oak Ridge National Laboratory, January 20, 1972.

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