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RESEARCH MEMORANDUM

POINT SOURCE EXPLOSION IN AIR

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RM-1824-AEC

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ABSTRACT

Using a high speed JOHNNIAC computer and a program which allows the solution of the partial differential equations governing hydrodynamic motion in spherical geometry, the problem of the blast wave from a point source in air has been carried out. The first section of the report describe the methods and the equation of state for air. The second section covers the results of the particular solution for a point source, giving shock values of pressure, density, particle velocity, temperature and dynamic pressure as functions of the shock radius and of the peak overpressure. The time histories of the same variables are given for a number of distances from the source, as well as values of the durations of positive phases and impulses from overpressure and from dynamic pressure.

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AECU-3517

I. INTRODUCTION

The blast wave from an instantaneous release of energy at a point has been solved numerically in considerable detail for an ideal gas⁽¹⁾. The same source in real air will result in a blast that is different in a number of important ways. This paper is a report on a calculation of the point source explosion in real air. The method is similar to that used in previous problems,⁽¹⁾ relying on the basic technique of the artificial viscosity⁽²⁾, but the equation of state of air makes it a technically more complicated problem.

The first portions of this paper will deal with the details of the method and of the equation of state fits since much of this information is of general usefulness and does not appear elsewhere. The differential equations, the corresponding difference equations, and the computation procedures all are described so as to be useful both in evaluating the various problems that have been run and in setting up similar problems where desired. The fits to the equation of state were obtained empirically through a series of tedious plottings of data in different forms and with many guesses and adjustments of parameters. The resulting fit over the two thermodynamic variables is quite satisfactory however and represents a convenient analytical expression for the equation of state of air over the entire range of pressures and densities necessary to treat shocks of arbitrary strength in air at standard sea level conditions to conditions at sixty thousand feet or better. The fit as given here does not include corrections for the more recently accepted binding energy of the nitrogen molecule. Subsequent calculations did use an improved fit which corrected for this difference and also included some further refinements.⁽³⁾

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The remaining sections of this report present a summary of calculational results for the blast from a point source in air. This is a rather classic problem in that two numerical solutions already exist for a point source in an ideal gas ^(1,4) and several analytical solutions exist under the limitations of a strong shock and an ideal gas ^(5,6,7). A comparison between these ideal gas results and the results for real air is therefore interesting. The point source is not a physically sensible beginning, of course, since in the regions of very high pressures and temperatures other phenomena will play an important role (e.g., radiation transport). Some of the subsequent problems that have been carried out are based on more realistic initial conditions ⁽³⁾, but the point source problems have the advantage of a simple flow field involving only one shock wave at the same time possessing the most salient features of other more complex spherical blast wave problems.

The summary given in this report includes most of the results of analysis of the numerical data, including the behavior of the various shock parameters, the time dependences of overpressures, particle velocity, density, temperature, and dynamic pressure at a series of distances from the source, and including the behavior of such integral qualities as the positive and negative impulses of overpressure and dynamic pressure. Time of shock arrival, durations of positive and negative phases of pressure and velocity, balance of energy between kinetic and internal all are illustrated. For most applications this data is sufficiently detailed and complete so that further reference to the numerical results is unnecessary; however, for those cases where further information about gradients and values of pressure, velocity and density within the flow field at various times is desired, a complete set of plots of these variables as functions of the radius are available in a supplement to this paper ⁽⁸⁾.

II. EQUATIONS OF MOTION AND STATE

EQUATION OF MOTION

The equations of motion are expressed in a Lagrangean form with the independent variables being the time (t) and a mass coordinate (X) where X is defined as $X = \rho_1 r_0^3 / 3$ and ρ_1 is the initial density of the gas and r_0 its initial position. The hydrodynamic equations may then be written as follows:

$$\frac{1}{\rho} = R^2 \frac{\partial R}{\partial X}, \quad \text{conservation of mass;} \quad (1)$$

$$\frac{\partial u}{\partial t} = -R^2 \frac{\partial P}{\partial X}, \quad \text{conservation of momentum;} \quad (2)$$

$$\frac{\partial E}{\partial t} = \frac{P}{\rho^2} \frac{\partial \rho}{\partial t}, \quad \text{conservation of energy;} \quad (3)$$

$$P(E, \rho), \quad \text{caloric equation of state;} \quad (4)$$

$$T(E, \rho), \quad \text{thermal equation of state;} \quad (5)$$

$$u = \frac{\partial R}{\partial t}, \quad \text{definition of particle velocity} \quad (6)$$

It is convenient to reduce these equations to a dimensionless set which then may be applied to any initial total energy (w) and any initial pre-shock conditions (P_0, ρ_0, T_0, E_0).

For this we define

$$\alpha^3 = \frac{w}{P_0}, \quad (7)$$

$$\delta = \frac{P_0}{\rho_0 E_0} = (\gamma_0 - 1) = 0.4, \quad (8)$$

$$\gamma = \frac{c_o^2 \rho_o}{P_o} = \gamma_o = 1.4 \quad (9)$$

Substituting the following variables

$$\left. \begin{aligned} \pi &= P/P_o \\ \eta &= \rho/\rho_o \\ \beta &= u/c_o \\ \epsilon &= E/E_o \\ \theta &= T/T_o \\ \lambda &= R/\alpha \\ x &= X/\rho_o \alpha^3 = \frac{\eta_1 \lambda_o^3}{3} = \frac{1}{3} \frac{\rho_1 r_o^3}{\rho_o \alpha^3} \\ \lambda_o &= r_o/\alpha \\ \tau &= t c_o/\alpha \end{aligned} \right\} \quad (10)$$

the equations of motion become

$$\frac{1}{\eta} = \lambda^2 \frac{\partial \lambda}{\partial x} = \frac{1}{3} \frac{\partial \lambda^3}{\partial x} \quad (11)$$

$$\frac{\partial \beta}{\partial \tau} = - \frac{1}{\tau_o} \lambda^2 \frac{\partial \pi}{\partial x} \quad (12)$$

$$\frac{\partial \epsilon}{\partial \tau} = \delta \frac{\pi}{\eta^2} \frac{\partial \eta}{\partial \tau} \quad (13)$$

$$\frac{\partial \lambda}{\partial \tau} = \beta \quad (14)$$

It will be convenient to write the equations of state in still another form.

$$\theta = z S(z, \eta) , \quad \text{where } z = \pi/\eta , \quad (15)$$

and

$$\varepsilon = \delta z \left(\frac{\mu-1}{2} \right) ; \quad \mu(z, \eta) . \quad (16)$$

EQUATION OF STATE

The quantities $S(z, \eta)$, $\mu(z, \eta)$ are then derived from equation of state data as some analytical or tabular functions. These two fits are not entirely independent. In order to satisfy the second law of thermodynamics a reciprocity relation exists which places a constraint on these functions. For purely hydrodynamical considerations, however, there is no need to observe this restraint since the thermal equation is superfluous to the equation of motion. In such cases where the temperature is coupled to the hydrodynamics, e.g. as by radiation, heat conduction, or heat diffusion, it would be necessary to observe the reciprocity restriction. Consequently the following derivation points to an approximation which makes such a constraint less difficult to comply with in fitting the equation of state data analytically.

Choosing as our independent variables $z = \pi/\eta$, and η the second law in the form

$$ds = \frac{Dq}{T} = \frac{dE + p dv}{T} , \quad \text{where } v = \frac{1}{\rho} , \quad (17)$$

becomes

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$$\begin{aligned} \frac{\rho_o T_o}{P_o} ds &= \frac{1}{2S(z, \eta)} \left\{ \left(\frac{\partial \mu}{\partial \eta} \right)_z d\eta + \left[\left(\frac{\partial \mu}{\partial z} \right)_\eta + \frac{\mu-1}{z} \right] dz \right\} - \frac{d\eta}{\eta S} \\ &= \left[\frac{1}{2S} \left(\frac{\partial \mu}{\partial \eta} \right)_z - \frac{1}{\eta S} \right] d\eta + \frac{1}{2S} \left[\left(\frac{\partial \mu}{\partial z} \right)_\eta + \frac{\mu-1}{z} \right] dz, \end{aligned} \quad (18)$$

and the reciprocity relation is

$$\frac{\partial}{\partial z} \left[\frac{1}{2S} \left(\frac{\partial \mu}{\partial \eta} \right)_z - \frac{1}{\eta S} \right] = \frac{\partial}{\partial \eta} \left[\frac{1}{2S} \left(\frac{\partial \mu}{\partial z} \right)_\eta + \frac{\mu-1}{2zS} \right]. \quad (19)$$

To within 10% in the density range of interest the function S can represent the thermal properties of air as a function of z alone (independent of η). Furthermore, the function μ can fit the caloric properties to the same accuracy with the form

$$\mu = \mu_o(z) + \mu_1(z) \ln \eta. \quad (20)$$

In this case, then, the reciprocity relation requires that

$$\mu_1(z) = \frac{2zS'}{S+zS}. \quad (21)$$

The equation of state of air data were taken from references 9, 10 and 11 and fitted prior to the publication of the later information of references 12 and 13. Subsequent problems were carried out with a fit corrected to these latter data.

These functions as first fitted are the following:

$$S(z) = \frac{485}{1000+z^2} + \frac{3860+z}{7500+16.5z} \quad (22)$$

$$\mu_o(z) = 1 + \frac{17+3z}{3+z} + \frac{800(z-1)}{2000+z^2} + \frac{2725z}{16 \times 10^4 + z^2} + \frac{45,000z}{16 \times 10^6 + z^2} \quad (23)$$

$$zS' = z \left(\frac{dS}{dz} \right) = - \frac{2(485)z^2}{(1000+z^2)^2} - \frac{(16.5 \times 3860 - 7500)z}{(7500 + 16.5z)^2} \quad (24)$$

It will now be convenient to rewrite the energy conservation equation (13)

$$\frac{\partial \xi}{\partial \tau} = \xi \frac{\pi}{2} \frac{\partial \eta}{\partial \tau} = \delta z \frac{\partial \xi}{\partial \tau}, \quad (13)$$

in terms of z and η which after some algebra may be written

$$\frac{\partial z}{\partial \tau} = \frac{(2 - \mu_1)z}{\mu - 1 + z(\mu_0 + \mu_1 \xi)} \frac{\partial \xi}{\partial \tau}, \quad (25)$$

where $\xi = \ln \eta$.

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III. NUMERICAL PROCEDURE

In order to solve the set of partial differential equations (11), (12), (14), and (25), we resort to the progressive solution (from a time τ to a new time $\tau + \Delta\tau$) of a set of difference equations which in the limit of infinitesimal differences converges to the desired differential equations. Before such a scheme is workable, however, several conditions must be carefully met.

- (1) The difference equations, besides converging in the limit to the differential equations, should approximate them to as high an order in the difference quantities (Δx and $\Delta\tau$) as is practicable, since we strive to reproduce numerically the solution throughout several thousand time steps.
- (2) Stability of the numerical procedure must be guaranteed, i.e. an upper bound dependent on some of the variables must be imposed on the size of time increments ($\Delta\tau$) used.
- (3) Discontinuities as arise in shock fronts must be avoided. To this end the artificial viscosity technique of Von Neumann and Richtmyer⁽²⁾ is employed.
- (4) Economy of computation must be balanced with the desire for numerical accuracy and detail, i.e. the problem must be completable in a reasonable number of machine hours and yet contain as much detail as possible or as is necessary.

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DIFFERENCE EQUATIONS

The following set of difference equations was chosen with the above conditions in mind. A space grid defined by $\Delta x_{l+1/2}$ between points (l and $l+1$) is carried with the variables β_l and λ_l defined at l -points and the variables $\pi_{l-1/2}$, $l_{-1/2}$, $z_{l-1/2}$, $\eta_{l-1/2}$ defined at the half- l points. An additional variable is introduced which is the artificial viscosity (ξ).^{*} For convenience this is sometimes carried as $\sigma = \xi/\eta$. Time is indicated in units of $\Delta \tau$ as $\tau = n \Delta \tau$.

$$\beta_l^{n+1/2} = \beta_l^{n-1/2} - k_1 (\lambda_l^n)^2 \frac{\Delta \tau}{\Delta x_l} \left[\pi_{l+1/2}^n - \pi_{l-1/2}^n + \xi_{l+1/2}^{n-1/2} - \xi_{l-1/2}^{n-1/2} \right], \quad (26)$$

$$\text{where } \Delta x_l = \frac{1}{2} (\Delta x_{l+1/2} + \Delta x_{l-1/2}) \text{ and } k_1 = \gamma_0^{-1} \quad (27)$$

$$\lambda_l^{n+1} = \lambda_l^n + \beta_l^{n+1/2} \Delta \tau, \quad (28)$$

$$\sigma_{l-1/2}^{n+1/2} = k_2 (\beta_l^{n+1/2} - \beta_{l-1}^{n+1/2})^2 \text{ for } \beta_{l-1} > \beta_l,$$

$$\text{and } \sigma_{l-1/2}^{n+1/2} = 0 \text{ for } \beta_l \geq \beta_{l-1},$$

$$k_2 = \frac{9}{2} (\gamma_0 + 1) \gamma_0 \left(\frac{m}{3\pi} \right)^2 \simeq 6 \text{ for } m = 6. \quad (28)^{**}$$

$$\eta_{l-1/2}^{n+1} = \frac{3 \Delta x_{l-1/2}}{(\lambda_l^{n+1} - \lambda_{l-1}^{n+1}) \left[(\lambda_l^{n+1})^2 + \lambda_l^{n+1} \lambda_{l-1}^{n+1} + (\lambda_{l-1}^{n+1})^2 \right]} \quad (29)$$

^{*}This viscosity is discussed more fully in references (1) and (2).

^{**}The number m represents the number of space points over which a shock is spread, i.e., for $m = 6$ the variables will rise from pre-shock conditions to shock values in a space of approximately six l -points.

$$\xi_{l-1/2}^{n+1} = \ln \eta_{l-1/2}^{n+1} \quad (30)$$

$$z_{l-1/2}^{n+1} = \frac{(A+B\xi + C\Delta\xi)^* z_{l-1/2}^n + 2\sigma_{l-1/2}^{n+1/2} \Delta\xi}{(A+B\xi - C\Delta\xi)^*} \quad (31)$$

where $(A+B\xi \pm C\Delta\xi)^*$ is first computed with

$$z_{l-1/2}^n, \xi_{l-1/2}^n, \Delta\xi = \xi_{l-1/2}^{n+1} - \xi_{l-1/2}^n$$

With this a new value of z is obtained and the above quantities can be recomputed with

$$z_{l-1/2}^{n+1/2}, \xi_{l-1/2}^{n+1/2}, \Delta\xi = \xi_{l-1/2}^{n+1} - \xi_{l-1/2}^n$$

This single iteration leaves us with an accuracy of the same order as in the other difference equations, namely good to second order in $\Delta\tau$ or Δx . The other variables may then be computed in terms of these preceding ones.

$$\pi_{l-1/2}^{n+1} = \eta_{l-1/2}^{n+1} z_{l-1/2}^{n+1}, \quad (32)$$

$$\bar{\sigma}_{l-1/2}^{n+1/2} = \left(\frac{\eta_{l-1/2}^{n+1} + \eta_{l-1/2}^n}{2} \right) \sigma_{l-1/2}^{n+1/2} \quad (33)$$

And some not needed in the computation are also worth noting.

$$\xi_{l-1/2}^{n+1} = \delta z_{l-1/2}^{n+1} \left(\frac{\mu_{l-1/2}^{n+1} - 1}{2} \right), \quad (34)$$

$$\theta_{l-1/2}^{n+1} = z_{l-1/2}^{n+1} s_{l-1/2}^{n+1}, \quad (35)$$

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$$q_{l-1/2}^{n+1/2} = \left(\beta_l^{n+1/2} + \beta_{l-1}^{n+1/2} \right) \left| \beta_l^{n+1/2} + \beta_{l-1}^{n+1/2} \right| \quad (36)$$

DISTRIBUTION OF SPACE POINTS

As the shock waves or hydrodynamical disturbances advance through undisturbed material more space points must be computed over in the numerical procedure. The machine memory is limited, however, and for high speed computation it is desirable to carry a more or less fixed number of space points. Accordingly, the following scheme is particularly appropriate for spherical shock wave problems.

As the shock begins to move the last zone (at $l = L$) a new zone is invented in front of it and a zone further back is eliminated.

When β_L is greater than zero (or some small constant) the following arithmetic is accomplished.

$$\Delta r = 2^{-k_{15}} \lambda_{L-1} \quad (37)$$

$$\text{or } \Delta r = k_{16} \quad (38)$$

$$\lambda_L = \lambda_{L-1} + \Delta r \quad (39)$$

$$\Delta x_{L-1/2} = \Delta r \eta_I \left(\lambda_L \lambda_{L-1} + \frac{\Delta r^2}{3} \right) \quad (40)$$

$$\xi_{L-1/2} = \beta_L = 0 \quad (41)$$

$$z_{L-1/2} = z_I$$

$$\eta_{L-1/2} = \eta_I \quad (\text{e.g. } z_I = \eta_I = \pi_I = 1.0) \quad (42)$$

$$\xi_I = 0$$

$$\pi_{L-1/2} = \pi_I$$

$$\xi_{L-1/2} = \xi_I$$

After the new zone is invented one previous zone is to be eliminated.

This zone is determined as follows:

Beginning at some point near the origin called l_0 the variables at $l_0 + 1$ and $l_0 + 2$ are combined with

$$\begin{aligned}\Delta x'_{l_0 + 1/2} &= \Delta x_{l_0 + 1/2} + \Delta x_{l_0 + 3/2} , \\ u'_{l_0 + 1} &= u_{l_0 + 2} , \\ \lambda'_{l_0 + 1} &= \lambda_{l_0 + 2} ,\end{aligned}\tag{43}$$

and with

$$z'_{l_0 + 1/2}, \xi'_{l_0 + 1/2}, \zeta'_{l_0 + 1/2}, \eta'_{l_0 + 1/2}, \pi'_{l_0 + 1/2}$$

computed by

$$z'_{l_0 + 1/2} = \frac{z_{l_0 + 1/2} \Delta x_{l_0 + 1/2} + z_{l_0 + 3/2} \Delta x_{l_0 + 3/2}}{\Delta x_{l_0 + 1/2} + \Delta x_{l_0 + 3/2}} .\tag{44}$$

After this, all the data at $l_0 + 3$ and greater l are shuffled down one l value.

Provided $l_0 + 1$ is not larger than some fixed l -value near the front, l_0 is increased by one for the next zone doubling. If $l_0 + 1$ is too big, then l_0 is reset to some value l_∞ which in turn is determined by the smallest l -value for which

$$\lambda_{l+1} - \lambda_l < 2^{-k_{17}} \lambda_l \quad (\text{e.g., } k_{17} = 5) .\tag{45}$$

In order to avoid doubling some zones near the origin, however, it is

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also arranged that $l_{oo} > l_{op}$ where l_{op} defines the zones not to be doubled.

STABILITY

There are two different conditions on stability to be satisfied, one, the usual Courant condition on the sound speed and mesh speed, the other, a diffusion-type term due to the artificial viscosity. These conditions impose the following restrictions:

$$\Omega \equiv \left[\frac{\lambda^4 \pi \eta}{(\Delta x)^2} \right]_{\max} (\Delta \tau)^2 = \left[\frac{z}{(\Delta \lambda)^2} \right]_{\max} (\Delta \tau)^2 < 1 \quad (46)$$

$$\Sigma = \left[\frac{4\lambda^2 \gamma \Delta \tau}{\gamma_0 \left| \frac{\partial u}{\partial x} \right| (\Delta x)^2} \right]_{\max} = \left[\frac{4 \gamma \Delta \tau}{\gamma \Delta u \Delta \lambda} \right]_{\max} = K_{12} \left[\frac{\Delta u}{\Delta \lambda} \right]_{\max} \Delta \tau < 1 \quad (47)$$

$$K_{12} = \frac{4K_2}{\gamma_0} = \frac{2(\gamma_0 + 1)M^2}{(3.1416)^2} = 17.5 \text{ (for } M=6)$$

Since the second condition is generally the more stringent, we arrange to change $\Delta \tau$ on the basis of

$$\Delta \tau = \frac{2/3}{K_{12} \left| \frac{\Delta u}{\Delta \lambda} \right|_{\max}} = \frac{2/3 \Delta \tau}{\Sigma},$$

$$\text{where } \Delta u = u_l^{n+1/2} - u_{l-1}^{n+1/2} \text{ and } \Delta \lambda = \lambda_l^{n+1} - \lambda_{l-1}^{n+1}.$$

If this value of $\Delta \tau$ satisfies the first condition (Eq. 46) then the $\Delta \tau$ is changed. However, this will be attempted or tested only at certain intervals which correspond to the times when we expect to print out the list of variables.

OUTPUT

It was planned to store in the electrostatic memory all the variables used in the computation plus some which are only desired at the time of printing.

<u>Variable</u>	<u>Carried in Compute</u>	<u>Print</u>	(no. of digits)
$\beta_l^{n+1/2}$	yes	yes	6(+ sign)
λ_l^{n+1}	yes	yes	7(+ sign)
$\Delta x_{l-1/2}$	yes	yes	7(+ sign)
$\zeta_{l-1/2}^{n+1}$	yes	yes	7(+ sign)
$q_{l-1/2}^{n+1/2}$	no	yes	7(+ sign)

$\eta_{l-1/2}^{n+1}$	yes	yes	6(no sign)
$\pi_{l-1/2}^{n+1}$	yes	yes	7(no sign)
$\lambda_{l-1/2}^{n+1}$	no	yes	7(no sign)
$\theta_{l-1/2}^{n+1}$	yes	yes	7(no sign)
$\varepsilon_{l-1/2}^{n+1}$	yes	yes	7(no sign)

$z_{l-1/2}^{n+1}$	yes	no	
$\xi_{l-1/2}^{n+1}$	yes	no	
$\sigma_{l-1/2}^{n+1}$	yes	no	

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Not all these variables need to be carried for the computation, nor are they all important for the print-out, but as the problems attempted with this code may become more detailed and may include more variables, this extravagance may be cashed in on and the superfluous space re-assigned in a tighter code.

It will be desirable to display several 'constants' and characteristic numbers at the time of output, also.

$$\begin{aligned}\Delta\tau &= \text{time increment (previous or current)} \\ \tau &= \text{time} = \sum_{i=1}^n \Delta\tau_i\end{aligned}\quad (49)$$

$$\begin{aligned}n &= \text{number of cycles of } \Delta\tau \text{ (time steps)} \\ m &= \text{number of space zones (unit for each added zone)} \\ l_o &= l \text{ point for doubling} \\ l_{oo} &= \text{last restart } l\text{-value for doubling} \\ l_{op} &= \text{minimum allowed for doubling}\end{aligned}$$

$$\Omega = \text{Courant stability} = \frac{z}{(\Delta\lambda)^2} \Bigg]_{\max} \Delta\tau^2 \quad (46)$$

$$\Sigma = \text{viscosity stability} = \frac{K_{12} \Delta u}{\Delta\lambda} \Bigg)_{\max} \Delta\tau \quad (47)$$

$$E = \text{net internal energy } k_{13} \sum_{l=1}^L \left(\epsilon_{l-1/2}^{n+1} - \frac{1}{\eta_{l-1/2}^{n+1}} \right) \Delta x_{l-1/2} \quad (50)$$

$$k_{13} = 4\pi/\delta = 31.416$$

$$K = k_{14} \sum_{l=1}^L q_{l-1/2} \Delta x_{l-1/2} = \text{total kinetic energy } k_{14} = \frac{\pi}{2} \gamma \quad (51)$$

$$\begin{aligned} W &= K + E = \text{total energy (normalized)} \\ &= \text{unity (if all is well.)} \end{aligned} \quad (52)$$

N = output number

THE RANKINE-HUGONIOT CONDITIONS

In the units of this paper the general shock conditions (expressing conservation of mass, momentum and energy) may be written as

$$\frac{\beta_s}{B} = 1 - \frac{1}{\eta_s}, \quad (53)$$

$$B^2 = \frac{\pi_s^{-1}}{\gamma_1} \frac{\eta_s}{\eta_s^{-1}}, \quad \text{or} \quad \beta_s^2 = \left(\frac{\eta_s^{-1}}{\eta_s} \right) \left(\frac{\pi_s^{-1}}{\gamma_1} \right), \quad (54)$$

$$1 + \pi_s = \frac{2}{(\gamma_1 - 1)} \frac{\eta_s}{\eta_s^{-1}} (\epsilon_s^{-1}), \quad \text{or} \quad B^2 = \frac{\pi_s^{-1}}{\epsilon_s^{-1}} \left(\frac{\gamma_1^{-1}}{2\gamma_1} \right) \quad (55)$$

where

$$B = \frac{u}{c_1}, \quad \beta_s = \frac{u_s}{c_1}, \quad c_1^2 = \frac{P_1 \gamma_1}{\rho_1} \quad (56)$$

$$E_1 = \frac{P_1}{\rho_1 (\gamma_1 - 1)}, \quad \epsilon_s = \frac{E_s}{E_1}, \quad \pi_s = \frac{P_s}{P_1}, \quad \eta_s = \frac{\rho_s}{\rho_1}.$$

These expressions are useful in establishing the scaling of velocities and densities for certain shock pressures, (one needs also to refer to the equation of state to determine a relation between π_s , η_s , ϵ_s). Some maximum velocities and densities are listed for the following shock pressures:

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π_B	β_B	B	η_B	ϵ_B
6×10^6	1800	2380	4.1	$.905 \times 10^6$
10^4	252	283	9	1780
200	11.3	12.6	9.4	37
10	2.17	2.95	3.8	2.7
2	0.523	1.36	1.625	1.23

SCALING AND NUMERICAL SIGNIFICANCE

The equation of state fits are intended to be valid for all temperatures above normal, and for densities from twenty times normal to less than one hundredth normal. However, radiation pressure and energy are not included in the current forms, so that for near normal densities we shall be restricted to temperatures less than a thousand times normal.

In a fixed-point code, as was used here, all numbers must be carried as fractions of the largest allowed value of that variable. As a consequence, variables that have a large range cannot be carried to many places at the small end of the range and every reasonable device must be used to restrict the range of variables or of any terms in the computations. Also, where a number either grows or decreases with time during the calculation, it is wise to stop and re-scale some of the variables and some parts of the computations.

Ideally, each problem that is conceived should be accompanied by a set of scaling rules for the variables and computations which are specially tailored to give maximum significance.

For scaling purposes it was found convenient to write the equation of state in terms of $z^{-1} = Y$, for which the various functions in Equation 31 become

$$B_1 = \frac{4}{5} \left[\frac{485 Y}{1 + 1000Y^2} + \frac{(3860Y + 1)}{Y(16.5 + 7500Y)} \right], \quad (57)$$

$$B_3 = \frac{485(1000Y^2 - 1)Y^2}{(1 + 1000Y^2)^2} + \frac{7500 Y(3860 Y + 1) + (7500 Y + 165)}{(16.5 + 7500Y)^2}, \quad (58)$$

$$C_1 = - \frac{2}{5} \left[\frac{2(485)Y^2}{(1 + 1000Y^2)^2} + \frac{(16.5 \times 3860 - 7500)Y}{(16.5 + 7500Y)^2} \right], \quad (59)$$

$$B_2 = - 2 Y^3 \left[\frac{485(3000Y^2 - 1)}{(1000Y^2 + 1)^3} + \frac{7500(16.5 \times 3860 - 7500)}{(16.5 + 7500Y)^3} \right], \quad (60)$$

$$A = \frac{51Y^2 + 18Y + 3}{(3Y + 1)^2} + \frac{800 Y^2 [1 + 2000(2 - Y)Y]}{(2000Y^2 + 1)^2} + \frac{8.72 \times 10^8 Y^3}{(16 \times 10^4 Y^2 + 1)^2} + \frac{1.44 \times 10^{12} Y^3}{(16 \times 10^6 Y^2 + 1)^2} = \mu_0^{-1} + z\mu_0' \quad (61)$$

$$\mu_0^{-1} = \frac{17Y + 3}{3Y + 1} + \frac{800Y(1 - Y)}{2000Y^2 + 1} + \frac{2725 Y}{16 \times 10^4 Y^2 + 1} + \frac{45,000 Y}{16 \times 10^6 Y^2 + 1}, \quad (62)$$

$$B = \frac{B_1 B_2}{B_3} = \mu_1 + z\mu_1', \quad (63)$$

$$C = 1 - \frac{C_1}{B_3}, \quad (64)$$

where A, B and C are as used in the energy equation (Equation 31).

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IV. POINT SOURCE IN AIR

INITIAL CONDITIONS

The analytical solutions to the point source problem^(5,7) are restricted to an ideal gas and a strong shock, and so do not lend themselves immediately to use as initial conditions for an air problem. In a sea level atmosphere the equation of state is fairly well characterized along the Hugoniot by a constant gamma of 1.25 between 4000 atmos and 40,000 atmos shock pressure, so that an approximate initial state is attained by using the strong shock solution for this gamma at a shock pressure of 4500 atmos. Von Neumann's solution was used in this case⁽⁵⁾ with a modified front to adjust to the artificial viscosity requirement of a smeared shock front.

SHOCK FRONT BEHAVIOR

The shock overpressure ($\Delta p_s = \pi_s - 1$) as a function of the shock radius (λ_s) is shown in Figure 1 on a log-log scale. For comparison the peak overpressure for the ideal gas case is also indicated (broken line). The two curves can be made to coincide over most of their range by reducing the radius at each pressure on the ideal gas curve by a factor of 0.843. This reduction corresponds to an effective energy reduction to 60 percent of the total, i.e., the real air case appears to be only 60 percent as efficient as the ideal gas blast insofar as peak overpressures are concerned. The discrepancy lies in the much larger amount of work that is done on the high temperature air in dissociation and ionization. The temperatures are still high in the regions near the origin long after the shock is gone, and so that energy is not fed back into the blast very quickly and adds

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little to the subsequent history. It is interesting to compare the peak overpressures from a bare spherical charge of TNT (dashed curve in Figure 1) with these two point source curves. The early decay rate of the point source curves in the strong shock region is like the inverse cube of the radius, but the early part of the HE curve is more like the inverse first power of the radius since the source in that case is a fairly low temperature, high density gas which acts more like a piston. At later stages the decay increases as the piston energy is exhausted, but at the larger distances the TNT curve looks more like the ideal gas curve since there is not the concentration of energy left near the origin in a high temperature sphere.

Beyond a lambda of 4 or below a peak overpressure of 0.05 atmos the point source real air curve is not reliable, since beyond this point the lack of significant figures and computing accuracy* allowed the total energy to deviate from unity. At this late stage all the air except that in the hot interior can be treated as ideal gas so the last part of the problem was rerun using just an ideal gas equation of state, but accounting for the energy held in the hot inner region by arbitrarily reducing the total energy to 0.6, thus giving the same overpressures at the point of start of the ideal gas extension. This extension was not entirely satisfactory, however, both because of inaccuracies in the then small overpressures and velocities and

* Perhaps the most serious inaccuracy in the computation in the late part of this problem is in the use of the approximation that zone volumes are proportional to $\lambda^2 \Delta \lambda$ rather than to $(\lambda + \Delta \lambda)^3 - \lambda^3$. Equation 29 indicates the latter form since it was used in subsequent problems and is the more correct. In this problem, however, the approximate form was used since it was not originally planned to allow zones near the origin to increase to a point where this approximation would not be valid, i.e., to where $\Delta \lambda$ is no longer small relative to λ over most of the energy-containing volume.

because all the work energy was not yet out of the hot interior where the air was still far from ideal gas.

The ideal gas extension calculation is shown in Figure 1 as a dotted line. Since the time when this calculation was completed, some more accurate calculations have been carried out, giving results that are more reliable in the lower pressure region. These are reported on in RM-1825. (3)

The peak dynamic pressure ($\gamma \eta \beta^2 / 2$) is shown as a function of shock radius in Figure 2. Here again both the original air calculation and the ideal gas extension calculation are of little value beyond a radius of about four. Pressures are shown in units of P_0 , i.e., in atmospheres.

The peak particle velocity (u/c_0), actually the peak particle Mach number, is shown in Figure 3 as a function of the shock radius. The shock compression (the shock overdensity) ($\eta_s - 1$) is shown as a function of shock radius in Figure 4. Where the shock is strong, the density ratio at the shock is given by $(\gamma + 1)/(\gamma - 1)$, and it is evident from the early compression values (equal to $2/(\gamma - 1)$ for strong shocks) that the gamma is fairly small. As the shock decreases in strength the compression begins to fall about like the inverse three halves power of the shock radius, much as the peak particle velocity does.

Figure 5 shows the shock temperature ($T/T_0 - 1$ or temperature above freezing in units of the absolute freezing temperature of water) as a function of shock radius. The shock temperature does not ride as high as the pressure for two reasons, one is the increased density in the shock, and the other is the fact that the non-ideal gas features of air further depress the temperature relative to the pressure.

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The relation between shock radius (λ_g) and shock arrival time (t_g) is shown in Figure 6, where the time at the point where the calculation was begun was computed from the strong shock analytical solution that was used for initial conditions (used a gamma of 1.25). The ideal gas extension shows a sudden jump to a larger radius, indicating that the arbitrary reduction of the energy to six-tenths was not correct for the time of arrival, although it was satisfactory for the peak overpressure-shock radius relation.

VARIATION OF BLAST PARAMETERS WITH SHOCK OVERPRESSURE

A number of variables are shown in Figure 7 as functions of the shock overpressure. The peak particle velocity can be seen to go from about a square-root dependence on the peak overpressure at high pressures to a linear dependence at low peak overpressures. The shock density ratio and the shock temperature (better shown in Figures 4 and 5) approach unity at low pressures, but at no point as rapidly as the pressure does. The duration of the positive pressure phase ($D_{\Delta p}^+$) has a distinct minimum around four atmos pressure and rises slowly toward a constant at lower overpressures. The positive velocity duration (D_Q^+) which must be greater than the positive pressure duration for inertial reasons, has less of a minimum and approaches the same limit as the positive pressure duration for very low overpressure. The negative phase durations ($D_{\Delta p}^-$ and D_Q^-) are very similar to each other and nearly constant at around unity over the whole range of shock overpressure.

The positive impulse of overpressure ($I_{\Delta p}^+$) is defined as the integral over the positive phase of the overpressure.

$$I_{\Delta p}^+ = \int_{\tau_s}^{D_{\Delta p}^+ + \tau_s} \Delta p \, d\tau \quad (65)$$

Figure 7 indicates that the overpressure impulse at high pressures varies about like the cube root of the overpressure, while at the low pressure end it varies more like the first power of the overpressure. Although the strong shock analytical solution would predict a sixth root dependence, such a dependence is not valid since a considerable part of the impulse comes at late times when the shock is no longer a strong shock, and the after-flow which is determining the tail of the impulse at close-in points is then characteristic of a weaker shock.

The negative overpressure impulse ($I_{\Delta p}^-$), which is the absolute value of the integral of the overpressure in the negative phase (i.e. the "underpressure"), is seen to be nearly constant at the high overpressure end and to be dropping off about like the first power of the overpressure at the low pressure end.

$$I_{\Delta p}^- = \int_{\tau_s + D_{\Delta p}^+}^{\tau_s + D_{\Delta p}^+ + D_{\Delta p}^-} \Delta p \, d\tau. \quad (66)$$

Figure 8 has the peak dynamic pressure (Q_s) and the dynamic impulse (I_Q^+) indicated as functions of the shock overpressure. The usual relation from the Hugoniot conditions specifies that the peak dynamic pressure be related to the peak overpressure by the form

$$Q_s = \frac{\Delta p_s^2 + \Delta p_s (7 - 5\gamma)/2}{(\gamma - 1)(7 + \Delta p_s)} \quad (67)$$

where gamma is defined by the relation between pressure, density and internal energy for air ($\gamma = p/\rho E - 1$), and ranges in value between 1.4 and 1.18 for shocks in sea level air. Disregarding the variation of $\gamma - 1$, this relation

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indicates a dependence at high overpressures on the first power of the shock overpressures. The variations of the curve, then, at the high end are due to variations in γ . At the lower pressures the γ is very close to constant at 7/5ths, and the formula reduces to

$$Q_s = \frac{5 \Delta p_s^2}{2 (7 + \Delta p_s)} , \quad \text{for } \Delta p_s < 10. \quad (68)$$

For small Δp_s this expression is nearly quadratic in Δp_s , as the Q_s curve in Figure 8 indicates.

The positive dynamic impulse (I_Q^+) is defined as

$$I_Q^+ = \int_{\tau_s}^{\tau_s + D_Q^+} Q \, d\tau . \quad (69)$$

Figure 8 indicates that the dynamic impulse increases exceedingly little as the shock pressure increases above about 10 atmos, and in fact the dynamic impulse appears to be proportional to the sixth root of the peak overpressure at high overpressures. At low peak overpressure, on the other hand, the dynamic impulse seems to be nearly proportional to the square of the peak overpressure. An approximate expression then is

$$I_Q^+ \simeq \frac{6.78 \Delta p^2}{(1.4 + \Delta p)^{11/6}} . \quad (70)$$

Also shown on Figure 8 is the time of arrival. The nearly straight-line nature of this curve suggests a simple relation:

$$(\tau_s)^{1.14} \Delta p_s = 0.200, \quad \text{or} \quad \tau_s^8 \Delta p_s^7 = 1.28 \times 10^{-5} \quad (71)$$

which is approximately valid over the entire range of shock overpressures.

VARIATION OF IMPULSES WITH DISTANCE

Positive impulses as functions of the distance from the source are shown in Figure 9. Here again some simple approximations are obvious from the straight-line character of parts of these curves. The positive overpressure impulse is given well over its entire range by the expression

$$\lambda \frac{I^+}{\Delta P} = 0.0322 \quad (72)$$

The positive dynamic pressure impulse decreases like the inverse square root of the radial distance in the close-in region, but fairly sharply at around a lambda of 0.3 it changes to a rate of decrease proportional to the inverse five halves power of the radius. This latter decay rate is true out to two orders of magnitude on lambda or down five more orders of magnitude in impulse. An approximate expression which exhibits these features is the following:

$$I_Q^+ = \frac{A}{\sqrt{\lambda} (B + \lambda^2)} = \frac{0.004}{\sqrt{\lambda} (0.089 + \lambda^2)} \quad (73)$$

This expression is low by as much as thirty five percent near the inflection point ($\lambda \sim 0.36$) but is much better than that on either side and goes quite exactly to the curve of Figure 9 at either extreme. For a better fit (at all λ 's) a correction term can be added

$$I_Q^+ = \frac{0.004}{\sqrt{\lambda} (0.089 + \lambda^2)} + \frac{0.000314\lambda}{0.00231 + \lambda^5} \quad (74)$$

KINETIC AND INTERNAL ENERGIES AS FUNCTIONS OF TIME

At any instant of time a certain fraction of the energy appears as ordered kinetic motion and the balance is in the form of internal energy

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of the gas. It is interesting to note how these quantities (defined in Equations 50 and 51) vary with time. As Equation 50 indicates, the internal energy considered is just the total internal energy in the blast wave minus the internal energy possessed initially by the unshocked air now engulfed, i.e., the net internal energy above that of normal air. Figure 10 is a time plot of these quantities K (kinetic energy) and E (internal energy) as a function of time. In the strong shock for an ideal gas of gamma of 1.25 about one quarter of the energy is in kinetic energy, while in a weak shock the kinetic energy tends to zero.

PRESSURES AS FUNCTIONS OF TIME

The negative overpressure phase has a very constant duration, and in the close-in regions it has also a constant minimum value. Figure 11 shows the negative overpressure duration, the minimum overpressure, and the negative overpressure impulse as functions of the distance from the source. Beyond the region of the residual hot gasses, i.e. beyond a radius of about 0.3 the minimum overpressure and the negative impulse begin to decrease about proportional to the inverse of the distance. The apparent proportionality between the negative overpressure impulse and the minimum negative overpressure may be expressed by the following approximate relation:

$$I_{\Delta p}^- \cong -0.55 \Delta p_{\min} \approx \frac{1}{2} \Delta p_{\min} D_{\Delta p}^- \quad (75)$$

The factor of a half may be considered a kind of form factor, since the negative phases do in fact have very similar shapes and nearly the same durations. The shape of the negative phase for overpressure is given approximately by the form

$$\Delta p = 14 \Delta p_{\min} t(1 - t)e^{-4t} \quad (76)$$

where t is measured from the onset of the negative phase.

The minimum negative dynamic pressure, i.e., the maximum inward directed dynamic pressure in the negative phase of the velocities rises very sharply in the close-in region (where the air is left considerably heated) ($\lambda < 0.3$) to a maximum of about 2×10^{-3} at a radius of about a half. Beyond that point it decreases in absolute value at a rate asymptotically proportional to the radius squared. Figure 12 illustrates this rapid rise and fall of the maximum inward dynamic pressure and also the corresponding inward dynamic impulse. Much like the negative overpressure phase, the negative velocity phases follow similar time histories at all points, and having also a nearly constant duration the maximum inward dynamic pressure and the resulting impulse are nearly proportional.

$$I_Q^- = 0.38 Q_{\min} \quad (77)$$

Such blast parameters as the overpressure, the dynamic pressure, the air density, the temperature and the particle velocity are given as functions of the time after shock arrival in Figures 13 - 46. The first ten (Figures 13 - 23) are plotted on semi-log paper to illustrate the exponential nature of their decay. A simple exponential would be represented as a straight line, but none of these are that simple. Actually, the overpressure is very easily fit with the sum of two exponentials, a sharp decaying one for the earliest times and a more leisurely one for the remainder. If the overpressure fit is to be made to pass through zero at the end of the positive phase, then a linear factor $(1 - t/D_{\Delta p}^+)$ may

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be included. At any of the time plots, a reasonable fit may be made with the form

$$\Delta p = \Delta p_s (a e^{-\alpha t} + b e^{-\beta t}) (1 - t/D_{\Delta p}^+), \text{ with } a + b = 1. \quad (78)$$

Less precisely but more generally the same form can be applied over the entire range of distances or of peak overpressures by taking the arbitrary constants in equation 78 to be the following:

$$\begin{aligned} \alpha &= \Delta p_s + 1/2 \\ \beta &= 70 + 10\Delta p_s \\ a &= (1 + 0.16\Delta p_s)/(1 + \Delta p_s) \\ b &= (0.84\Delta p_s)/(1 + \Delta p_s) \end{aligned} \quad (79)$$

This form with these constants expressed in terms of the peak overpressure (as above) gives a fairly good fit to the overpressure at all levels, however, its integral over time, which is the positive overpressure impulse, is lower than the actual numerical integral for the highest pressure levels, but is good to ten percent for peak overpressures less than thirty atmospheres.

A similar exponential fit can be made for the dynamic pressure, using the sum of two exponential decays, but the dynamic pressure drops so rapidly with time that a single exponential fit to the initial decay rate together with a linear term to force the dynamic pressure to zero at the end of the positive phase works quite well.

$$Q = Q_s (1 - t/D_Q^+) e^{-\sigma t}, \quad (80)$$

where again t is the time after time of arrival ($t = \tau - \tau_s$), and D_Q^+ is the

duration of the positive velocity phase. This form can be made to agree fairly well over the whole range of pressure levels with the following approximating analytic form for the exponential coefficient:

$$\sigma = 14.5(Ap_s)^{1.126} + 7 \quad . \quad (81)$$

This form not only approximates the time dependence of the dynamic pressure but also gives the dynamic impulse to better than an order of magnitude.

Table 1 is a list of the shock values at the distances chosen for the time plots of Figures 13 - 46.

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Table 1

Shock Values for Figures 13 thru 46.

Fig. No.	λ_s	Δp_s	η_s	β_s	θ_s	Q_s	τ_s
13	.048644	900.1	8.402	27.33	50.06	4393.	.0 ³ 66004
14	.052857	698.6	9.112	23.459	39.88	3510.	.0 ³ 81207
15	.061134	452.1	10.079	18.70	28.52	2467.	.0 ² 117497
16	.069561	297.2	9.669	15.12	22.65	1547.	.0 ² 163764
17	.078376	202.7	8.941	12.07	18.51	911.8	.0 ² 223367
18.	.097630	99.86	7.360	8.645	12.47	385.0	.0 ² 391663
19	.113408	69.88	6.864	6.872	9.736	226.9	.0 ² 572371
20	.124683	52.66	6.319	5.899	8.141	153.9	.0 ² 724744
21	.151996	30.61	5.421	4.380	5.703	72.80	.01182123
22	.175526	20.70	4.912	3.470	4.356	41.40	.0167477
23	.197034	14.85	4.433	2.922	3.539	26.49	.0221001
24	.223110	10.56	3.987	2.409	2.878	16.20	.0297042
25	.254437	7.343	3.517	1.960	2.359	9.458	.0403989
26	.296806	4.910	3.023	1.558	1.947	5.137	.0574644
27	.370999	3.034	2.489	1.186	1.616	2.451	.0934767
28	.433672	2.021	2.107	.8784	1.430	1.138	.1293144
29	.494858	1.5012	1.8745	.72961	1.3309	.69850	.1686889
30	.609083	.9873	1.614	.5275	1.229	.3144	.2493437
31	.721279	.7212	1.4650	.40438	1.1724	.16769	.3353310
32	.880953	.4970	1.331	.3105	1.123	.08983	.46486489
33	1.121718	.3523	1.2391	.22795	1.0891	.045070	.6793629
34	1.328503	.2679	1.1841	.17939	1.0687	.026674	.8599936
35	1.56402	.20832	1.14459	.13742	1.05364	.015130	1.08110
36	2.13397	.13395	1.09387	.091473	1.03663	.0 ² 64069	1.54872
37	2.58061	.10304	1.072531	.072656	1.02845	.0 ² 39632	1.96674
38	3.50446	.068127	1.048193	.047938	1.01902	.0 ² 16862	2.86984
39	4.32976	.051419	1.036459	.036285	1.01443	.0 ³ 95522	3.66868
40	5.23119	.041958	1.029791	.029598	1.01181	.0 ³ 63150	4.59842
41	6.19689	.034537	1.024548	.024490	1.00975	.0 ³ 43014	5.54919
42	7.54070	.027467	1.019543	.019538	1.00777	.0 ³ 27244	6.88027
43	9.78855	.020813	1.014822	.014790	1.00590	.0 ³ 15539	9.12267
44	11.28682	.017374	1.012379	.012410	1.00493	.0 ³ 10914	10.61940
45	13.45330	.013513	1.0096333	.0 ² 97801	1.00384	.0 ⁴ 67600	12.78051
46	14.12602	.012659	1.0090260	.0 ² 90091	1.00360	.0 ⁴ 57328	13.47936

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FIG. 10 - Internal energy (E) and kinetic energy (directed motion) (K) as functions of the time (τ)

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Fig. 13 — Blast parameters as a function of time after shock arrival at a radial distance of $\lambda = 0.04864$

D_0^+ = Duration of positive dynamic pressure = 0.314

$D_{\Delta p}^+$ = Duration of positive overpressure = 0.160

τ_s = Shock arrival time = 0.0006600

Δp = Overpressure $[(p - p_0)/p_0]$

Q = Dynamic pressure $\gamma \eta \beta^2 / 2$

η = Density ratio (ρ/ρ_0)

θ = Temperature ratio (T/T_0)

β = Particle velocity (u/c_0)

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Fig. 14 — Blast parameters as a function of time after shock arrival at a radial distance of $\lambda = 0.05286$

- D_Q^+ = Duration of positive dynamic pressure = 0.314
- $D_{\Delta p}^+$ = Duration of positive overpressure = 0.160
- τ_s = Shock arrival time = 0.0008121
- Δp = Overpressure $[(p - p_0)/p_0]$
- Q = Dynamic pressure $\gamma \eta \beta^2 / 2$
- η = Density ratio (ρ/ρ_0)
- θ = Temperature ratio (T/T_0)
- β = Particle velocity (u/c_0)

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Fig. 16 — Blast parameters as a function of time after shock arrival at a radial distance of $\lambda = 0.06956$

$D_Q^+ =$ Duration of positive dynamic pressure = 0.314

$D_{\Delta p}^+ =$ Duration of positive overpressure = 0.160

$\tau_s =$ Shock arrival time = 0.001638

$\Delta p =$ Overpressure $[(p - p_0)/p_0]$

$Q =$ Dynamic pressure $\gamma \eta \beta / 2$

$\eta =$ Density ratio (ρ/ρ_0)

$\theta =$ Temperature ratio (T/T_0)

$\beta =$ Particle velocity (u/c_0)

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Fig. 19—Blast parameters as a function of time after shock arrival at a radial distance of $\lambda = 0.1134$

D_a^+ = Duration of positive dynamic pressure = 0.314

$D_{\Delta p}^+$ = Duration of positive overpressure = 0.156

τ_s = Shock arrival time = 0.005724

Δp = Overpressure $[(p - p_0)/p_0]$

Q = Dynamic pressure $\gamma \eta \beta^2 / 2$

η = Density ratio (ρ/ρ_0)

θ = Temperature ratio (T/T_0)

β = Particle velocity (u/c_0)

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Fig. 37 — Blast parameters as a function of time after shock arrival at a radial distance of $\lambda = 2.581$

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