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Offshell 1S_0 Proton-Proton Scattering
Mediated by a Soft Core Potential

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Offshell 1S_0 Proton-Proton Scattering
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by

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OFFSHELL 1S_0 PROTON-PROTON SCATTERING MEDIATED
BY A SOFT CORE POTENTIAL

by

John E. Brolley and L. Kent Morrison

INTRODUCTION

The corpus⁽¹⁻¹⁵⁾ of calculations and observations on the inelastic scattering of nucleons by nucleons, characterized by photon emission, has increased to the point where definite trends may be observed in the data and some limited inferences may be drawn from the calculations. Most of the calculations fall into a set pattern. A nuclear potential is folded into a standard two-potential type of formalism, the electromagnetic field possessing the weaker potential. Appropriate matrix elements and, thence, cross sections are then calculated to ascertain whether or not the assumed nucleon-nucleon potential provides a reasonable description of the offshell interaction. Many of the experiments measure symmetric proton-proton scattering where the pair half angle is less than 45° in the laboratory and all protons and photons are coplanar. Our calculation is addressed to this sort of experimental configuration.

ELEMENTS OF THE CALCULATION

Nuclear structure calculations suggest the desirability of exploring the offshell scattering predictions associated with soft core potentials and velocity-dependent potentials. In the present paper we will treat the soft core potential of Ulehla, Bystricky, Lehar, Lelek, and Wiezner.⁽¹⁵⁾ The 1S_0 portion of their potential for the proton-proton interaction is given in Eq. (1), where V is in MeV and r is in fermis.

$$V(r) = A \frac{e^{-R}}{R} (A_0 + A_1 e^{-R} + A_2 e^{-2R} + A_3 e^{-3R}) + 1.44/r \quad (1)$$

where

$$\begin{aligned} A &= \frac{(135.01)^2}{938} \\ R &= 0.6842 r \\ A_0 &= -2.30 \\ A_1 &= 40.0 \\ A_2 &= -213.1 \\ A_3 &= 245.9 \end{aligned}$$

The behavior of this potential is depicted in Fig. 1. We shall be concerned with the application of this potential to low energy experiments and we have therefore added the coulomb field. The coulomb field is assumed to be screened by the electrons of the hydrogen atoms in the target and we therefore cut it off at the radius of the hydrogen atom.

We then assume the validity of the two-potential formalism of Gell-Mann and Goldberger.⁽¹⁶⁾ The strong interaction and the coulomb field, now regarded as having a finite range, are consolidated in one potential. The second potential describes the electromagnetic radiation. Within this framework it is then possible to construct the appropriate T matrix. Only elements of the T matrix pertaining to the four diagrams of Fig. 2 are retained. The T matrix can be specialized still further by prescribing a particular experimental configuration. We impose the following boundary conditions: All particles shall be coplanar, final state protons shall be observed at symmetric angles in the laboratory, and the final momentum of one proton shall be specified. Since the photon momentum is small compared with the proton momenta, the second constraint implies that all scattering will be approximately 90° in the center of mass. Therefore we need only deal with singlet spin states. Contributions to $\ell > 0$ scattering will come only from the screened coulomb field.

Within the foregoing framework we may employ the singlet projection operators to generate the relevant T matrix. Apart from constants, the result is given by Eq. (2)

$$T = \left(\underline{p}'_1 \cdot \underline{e} - \frac{i\mu}{2} \underline{g}_1 \cdot \underline{K} \times \underline{e} \right) \frac{M_a}{e_a} \Lambda_s + \frac{M_b}{e_b} \Lambda_s \left(\underline{p}_1 \cdot \underline{e} - \frac{i\mu}{2} \underline{g}_1 \cdot \underline{K} \times \underline{e} \right) + \left(\underline{p}'_2 \cdot \underline{e} - \frac{i\mu}{2} \underline{g}_2 \cdot \underline{K} \times \underline{e} \right) \frac{M_c}{e_c} \Lambda_s + \frac{M_d}{e_d} \Lambda_s \left(\underline{p}_2 \cdot \underline{e} - \frac{i\mu}{2} \underline{g}_2 \cdot \underline{K} \times \underline{e} \right) \quad (2)$$

where p = proton momentum, prime denotes final momentum

e = photon polarization

μ = proton magnetic moment = 2.79

σ = proton spin matrix

K = photon momentum

$M_{a,b,c,d}$ = offshell coulomb and strong interaction amplitudes

$\Lambda_s = \frac{1}{4}(1 - \underline{g}_1 \cdot \underline{g}_2)$ = singlet projection operator

$e_{a,b,c,d}$ = energy factors.

All quantities relate to the center-of-mass coordinate system. The $M_{a,b,c,d}$ are the quantities of interest, and we discuss them briefly. Formally we have

$$M \sim \int e^{ik'r} V(r) \psi_k(r) dr, \quad (3)$$

where k' , k are the final and initial relative momenta of the two protons and ψ_k is the wavefunction of the Schrödinger equation for relative momentum k . This integral can be decomposed according to angular momentum contributions. The strong interaction and the coulomb field will contribute to the $\ell = 0$ component. The contributions to all $\ell > 0$ terms will come from the coulomb field. Thus M may be resolved into two types of terms

$$M_{\text{coul}} = M_{\text{coul}}, \ell = 0 \quad (4)$$

and

$$\int j_0(k'r) V(r) \psi_{k,\ell=0}(r) dr. \quad (5)$$

j_0 is a spherical bessel function. Ford⁽¹⁷⁾ has computed the amplitudes involved in Eq. (4). They have the form

$$M_{\text{coul}} = \frac{-4\pi^2 m V_0}{\hbar^2} C_0(\gamma) e^{i[\sigma_0 - \gamma \log(2kR)]} B(k'-k) \times \lim_{\lambda \rightarrow 0} \left[k'^2 - (k + i\lambda)^2 \right]^{i\gamma} / \left[(k' - k)^2 + \lambda^2 \right]^{1+i\gamma} \quad (6)$$

$$M_{\text{coul}, \ell=0} = \frac{C_0(\gamma) e^{i[\sigma_0 - \gamma \log(2kR)]}}{2ik'} B(k' - k) \times \left[\left(\frac{k' - k - i\lambda}{k' + k - i\lambda} \right)^{i\gamma} - \left(\frac{k' + k + i\lambda}{k' - k + i\lambda} \right)^{i\gamma} \right], \quad (7)$$

$$\text{where } \gamma = \frac{mV_0}{\hbar^2 k}$$

$$C_0(\gamma) = e^{-\frac{\pi\gamma}{2}} |\Gamma(1 + i\gamma)|$$

$$\sigma_0 = \arg \Gamma(1 + i\gamma)$$

$$B(k' - k) = \left\{ \frac{1}{\pi\gamma} \right\} \text{ if } \begin{cases} k' > k \\ k' < k \end{cases} \quad (8)$$

and m is the proton mass, V_0 is the coulomb potential constant, R is the radius of the hydrogen atom, and k' and k are final and initial momenta.

Integral (5) is, in principle, integrated from the origin out to the radius of the hydrogen atom. In practice we start the integration on the computer at 0.01 fermi. At some point, $\psi_{k,\ell=0}$ the wavefunction for relative proton momentum k and zero angular momentum becomes a mixture of regular and irregular coulomb functions. Numerical integration can then be discontinued and the remainder calculated analytically. We actually compute the remainder by a few-term approximation for which we are indebted to J. L. Gammel.⁽¹⁸⁾

With these components in hand we compute the cross section in the center of mass and transform to the laboratory frame. Apart from a number of factors we find the cross section to be given by

$$\begin{aligned}
\sigma_{c.m.} \sim \frac{1}{4} \text{tr } T^\dagger T = \frac{1}{4} \left| \left(\frac{M_a}{e_a} - \frac{M_c}{e_c} \right) \underline{P}_1 \cdot \hat{u} \right. \\
\left. + \left(\frac{M_b}{e_b} - \frac{M_d}{e_d} \right) \underline{P}_1 \cdot \hat{u} \right|^2 + \frac{\mu^2 K^2}{16} \times \\
\left\{ \left| \left(\frac{M_a}{e_a} - \frac{M_c}{e_c} \right) + \left(\frac{M_b}{e_b} - \frac{M_d}{e_d} \right) \right|^2 + \left| \left(\frac{M_a}{e_a} - \frac{M_c}{e_c} \right) \right. \right. \\
\left. \left. - \left(\frac{M_b}{e_b} - \frac{M_d}{e_d} \right) \right|^2 \right\} \quad (9)
\end{aligned}$$

where $\hat{u} = \hat{K} \times \hat{n}$, and $\hat{n}$ is the unit vector normal to the plane of scattering. The various differences displayed in Eq. (9) are relatively small because the photon momentum is small compared with the proton momentum. Therefore we have performed Taylor expansions of one member in terms of the other and executed the principal subtractions prior to numerical computation. Antisymmetrization, averaging over initial and summing over final proton spins and summing over photon polarizations, has been performed. For the energy region we are interested in, the contribution of the magnetic moment terms appears minor. The total cross section, for a given pair angle, can be obtained by integrating over the kinematically permitted energies. For a given E_1 , E_2 is double valued. As an example, we note in Fig. 3 some characteristics of the experiment of Thompson, Naqvi, and Warner.⁽¹³⁾ The inner solid ring is the locus of allowed solutions. We integrate from very near $E_{1,MIN}$ to very near $E_{1,MAX}$.

RESULTS

The lowest energy experiment that we may compare with is that of the Manitoba group.⁽¹³⁾ We do not consider participation by P waves in the strong interaction. However at 30 MeV it is likely that the main contribution will still be from the S wave. At this energy and a pair half-angle of 35° (LAB), Thompson, Naqvi, and Warner find $\sigma = 1.85 \pm 0.25$ $\mu\text{b}/(\text{sr})^2$ with an upper limit of 4. We compute $\sigma = 3.4$ $\mu\text{b}/(\text{sr})^2$. Signell and Marker⁽¹³⁾ compute 15, and Sobel and Cromer⁽¹²⁾ compute 12.7. At 33.5 MeV and 30° pair half-angle, the UCLA group⁽⁷⁾ have measured $\sigma = 3.0 \pm 0.8$ $\mu\text{b}/(\text{sr})^2$. We compute $\sigma = 4.3$ $\mu\text{b}/(\text{sr})^2$.

These and other results of our calculations are

presented in Table I. In Table II we present some elastic phases we have calculated for comparison with the experimental results of Knecht, Messelt, and Dahl.⁽¹⁹⁾ In Fig. 4 we present a plot of some of the 30° data with calculated points of Duck and Pearce,⁽¹⁾ Sobel and Cromer,⁽¹²⁾ and the present authors. In Fig. 5 we present a plot of our calculation in the region of manifest interference between the offshell electric and strong interaction amplitudes.

Table I. Cross sections calculated on the basis of the soft core 1S_0 potential. (All quantities refer to the laboratory frame and are for a pair half-angle of 30° unless noted.)

Energy (MeV)	σ $\mu\text{b}/(\text{sr})^2$
30.0	3.4 (35°)
33.5	4.3
5.0	0.94
4.0	0.77
3.0	0.70
0.600	0.090
0.500	0.074
0.450	0.059
0.440	0.055
0.430	0.052
0.420	0.050
0.410	0.050
0.400	0.053
0.390	0.058
0.380	0.067
0.370	0.087
0.360	0.11

Table II. Comparison of 1S_0 proton-proton phase shifts computed with the soft core potential, and the experimental values of Knecht, Dahl, and Messelt.

Energy (MeV)	Computed	Experimental
1.855	44.9°	44.3°
2.425	48.7°	48.3°
3.037	51.5°	51.0°

DISCUSSION

It would appear that the introduction of the soft core potential and the coulomb field has finally provided this type of calculation with a

reasonable confrontation with experiment. More data of higher precision would enable us to attempt simultaneous fitting of elastic and offshell scattering by searching over the potential parameters. Moreover we note that the interference minimum is close to that of the elastic case as we might expect.⁽²⁰⁾ The reason for large disagreement with Signell and Marker, and Sobel and Cromer is obscure. It is unlikely that the absence of the coulomb field from their calculation could account for the disparity. Duck and Pearce do not incorporate the coulomb field either. They utilize the Tabakin potential which does not fit the elastic data as well as some others and also is severely nonlocal.

Nyman⁽¹¹⁾ has raised the question as to whether or not experiments of the type alluded to here can provide very much information on the offshell interaction. From utilization of the Low expansion⁽¹⁴⁾ he notes that the first two terms of the offshell cross section are model independent. They may be computed without any knowledge of offshell behavior. Whilst he has not yet finished calculation of the second term, it is clear that high energy data may not provide as direct a connection with offshell processes as we would desire. However at low energies the model-independent terms vanish. It is therefore a matter of some interest, as we pointed out at the Gainesville Conference, to study the offshell interference minimum. Referring to Fig. 5 we note the very considerable difference in the predictions⁽¹⁵⁾ of the boundary-condition model Henley-Morrison potential and the present soft core potential. Some physicists have speculated that at low energies all potentials should give nearly the same results. If this speculation turns out to have a firm foundation, we may infer that the contribution from inside the boundary may be nonnegligible. In our boundary-condition model calculation we have developed an approximate treatment of the interior contribution that suggested it was small. Alternatively, we might infer that the value of the logarithmic derivative at the boundary changes as the scattering becomes increasingly inelastic.

These questions will be investigated in due course.

Experiments in the 5-MeV region are presently being implemented. Experiments in the interference region are in a preliminary design phase. When more

accurate data become available, it may be desirable to improve some of the approximations in the present calculation and to attempt the simultaneous fitting of elastic and inelastic data. In this connection, the family of velocity-dependent potentials will also be used.

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Fig. 1. Behavior of the soft core potential. Also included for comparison is a plot of the boundary-condition model potential of Henley and Morrison.

Fig. 2. Lowest order diagrams considered in the present calculation.

Fig. 3. Data plot of Thompson, Naqvi, and Warner. In an ideal experiment all data points would lie on the inner solid curve representing the kinematically permitted locus of solutions.

Fig. 4. Comparison of calculations and experiments.

Fig. 5. Calculated behavior of the cross section in the region of the interference minimum.