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IN A TOROIDAL CAVITY

BY

F. J. PAOLONI

PLASMA PHYSICS
LABORATORY

MASTER

PRINCETON UNIVERSITY
PRINCETON, NEW JERSEY

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COUPLING TO FAST MHD EIGENMODES IN A TOROIDAL CAVITY

F. J. Paoloni
Princeton University, Plasma Physics Laboratory
Princeton, New Jersey

ABSTRACT

The coupling to fast MHD waves in toroidal-like geometry is calculated when eigenmodes exist in the plasma. The torus is considered to be a resonant cavity into which energy is coupled by a half turn loop. The cavity Q is calculated for the minority heating process, for cyclotron harmonic damping, electron transit-time magnetic pumping, wall loading, and Coulomb collisional damping.

The problem of sustaining the eigenmode as the plasma conditions change with time is also discussed. One method that seems to be practical is a feedback scheme that varies the plasma major radius by a small amount as the conditions change.

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INTRODUCTION

One RF process that can be used to heat a plasma is the damping of fast MHD waves by hot plasma effects. Chung and Rothman (1967) obtained appreciable loading of the fast wave in an experiment on the model C stellarator and observed heating of the plasma with an efficiency of about 9%. In toroidal geometry, Adam and Samain (1971) pointed out that because of the variation of the confining magnetic field across the plasma diameter, advantage can be taken of particle resonances in localized regions of the plasma; effects such as second harmonic cyclotron damping of the fast wave, or the cyclotron heating of a minority component of the plasma. Experiments by Hosea and Hooke (1972), (1973) and Adam, et al. (1974) report observation of fast wave generation efficiencies of 35-50% and heating efficiencies of about 20% when the wave frequency was twice the ion cyclotron frequency in the center of the plasma.

In large toroidal geometry it is possible that fast waves which are not heavily damped may make many transits around the circumference. In other words, the toroid can act like a resonant cavity, the loading resistance of the coupling antenna becoming much larger than the radiations resistance, and very efficient coupling can be expected. Toroidal eigenmodes were observed for short periods of time by Adam, et al (1974) who obtained loading resistances of up to 200Ω and wave generation efficiencies of 90%.

The presence of toroidal eigenmodes raises the possibility (Perkins (1972), Stix (1974)) that loading can be maintained over long periods of the plasma's life by using feedback control to sustain the eigenmode. In large volume machines, many modes can exist and mode tracking equipment need not require large bandwidths, since it may be possible to jump to new modes as the plasma conditions change.

In this report a method is described which can be used to calculate the coupling impedance to the fast MHD wave when toroidal eigenmodes exist. The toroid is considered to be a resonant cavity into which energy is coupled by a half-turn coil similar in construction to that used by Hosea and Hooke (1972) on the ST Tokamak. Approximations have been made to keep the equations as simple as possible to preserve useful physical insight into the coupling process. Some energy damping mechanisms are also considered and estimates are obtained for the cavity Q 's for each process.

TOROIDAL CAVITY MODEL

We consider the toroid to be a resonant cavity into which energy is coupled by a loop. The loop construction is assumed small so that the eigenmodes of the system are the resonant modes of the cavity alone. The equivalent circuit is shown in Fig. 1, where the quantities L_c and C_c represent the equivalent inductance and capacitance of the cavity, and the resistor R represents some cavity loss; L_l is the loop inductance in the presence of the cavity.

Assuming sinusoidal time variation of frequency ω and lossless coupling, the input impedance into the terminals AB is

$$Z_{in} = j\omega L_{\ell} + \frac{R_o (\omega/\omega_o)^2}{1+jQ(\omega^2/\omega_o^2-1)}$$

where

$$R_o = \frac{\omega_o^2 \phi^2}{2P} = \frac{Q\omega_o \phi^2}{2W}$$

$$Q = \frac{\omega_o W}{P}$$

ϕ = flux linkage between primary and secondary

P = average power into secondary

W = average energy stored in secondary

ω_o = resonant frequency of secondary.

The equations indicate that, at resonance, the input impedance consists of the inductance of the primary in series with a resistance R_o reflected from the secondary into the primary. No reactive energy is supplied to the cavity, only real power being required to replace energy losses. The real part of the input impedance at frequencies in the neighborhood of resonance is sketched in Fig. 2. The full width at half amplitude is ω_o/Q .

The quantities ϕ^2 , P , and W can be expressed in terms of the square of the wave amplitude in the cavity, and their ratio is independent of wave amplitude. R_o is most conveniently expressed in the form

$$R_o = \frac{Q\omega_o \phi^2}{2W}$$

where the Q of the cavity is determined by the equation

$$Q = \frac{\omega_o W}{P}$$

In the second part of this report we will calculate the Q 's caused by various physical damping processes in the plasma.

APPROXIMATE DISPERSION RELATION AND FIELDS OF THE FAST WAVE

The dispersion relation and wavefields are required when calculating the flux linkage and the total wave energy in the fully ionized plasma. Neglecting magnetic field gradient and curvature, electron mass, and assuming uniform plasma density, the dispersion relation for fast waves is

$$k_{\parallel}^2 = \frac{k_{\perp}^2}{2(1-\Omega^2)} \left[- (1-\Omega^2) + 2 \left(\frac{k_A}{k_{\perp}} \right)^2 - \sqrt{(1-\Omega^2)^2 + 4 \Omega^2 \left(\frac{k_A}{k_{\perp}} \right)^4} \right]$$

where

$$k_A = \omega/V_A$$

$$\Omega = \omega/\omega_{ci}$$

Woods (1962) has solved the wave equations in cylindrical geometry and obtains, for waves with solutions of the form $f(r)e^{i(k_{\parallel}z - \omega t + m\theta)}$,

$$b_r + ib_{\theta} = i (C - k_{\parallel}A) J_{m+1}(k_{\perp}r)$$

$$b_r - ib_\theta = i (C+k_{\parallel}A) J_{m-1}(k_{\perp}r)$$

$$|b_{\perp}|^2 = |b_r|^2 + |b_\theta|^2 = \frac{1}{2} \left[(C-k_{\parallel}A)^2 J_{m+1}^2(k_{\perp}r) + (C+k_{\parallel}A)^2 J_{m-1}^2(k_{\perp}r) \right]$$

$$b_z = k_{\perp}A J_m(k_{\perp}r) = b_{z0} J_m(k_{\perp}r)$$

$$E_r = (\omega/k_{\parallel}) b_\theta$$

$$E_\theta = -(\omega/k_{\parallel}) b_r$$

$$E_z = 0$$

where $C/(k_{\parallel}A) = (k_{\parallel}^2 + k_{\perp}^2) \Omega / (k_A^2 - k_{\parallel}^2)$

and C & A are related to the wave amplitude.

For fast waves it can be shown that

$$C/(k_{\parallel}A) \approx 1$$

so that the wave is mostly right hand circularly polarized.

The boundary conditions for perfectly conducting cylindrical walls are

$$[E_\theta, E_z]_{r=a} = 0$$

where a is the radius of the waveguide. For a plasma filled

waveguide, under conditions where the left hand circularly polarized component of the fast wave is negligible, the boundary conditions become

$$J_{m-1}(k_{\perp}a) = 0.$$

It has been shown by Hosea and Sinclair (1970) and Paoloni (1975), among others, that this simplified boundary condition and the dispersion relation describe fairly well the observed characteristics of $m = 0$ and $m = +1$ fast waves provided $k_A > k_{\perp}$. To be strictly correct over a broad frequency range, the electron mass terms must be included, and much more complicated equations are obtained which require solution by numerical methods. One effect of the inclusion of the electron term is to produce a new component of the wave fields which is appreciable only near the plasma surface. These new fields may provide some surface heating but should not contribute greatly to heating of the total plasma volume.

TOTAL AVERAGE WAVE ENERGY

The average energy density of the wave, ρ , is divided between magnetic, electric, and particle energy densities. Using the result that $E_z = 0$ and assuming a single ion species and neglecting electron mass, the individual time averaged energy densities are:

$$\rho_B = \frac{1}{4\mu_0} \left[|b_r|^2 + |b_{\theta}|^2 + |b_z|^2 \right]$$

$$\rho_E = \frac{1}{4\mu_0} \left(\frac{v_p}{c} \right)^2 \left[|b_r|^2 + |b_{\theta}|^2 \right]$$

$$\rho_p = \frac{1}{4\mu_0} \left(\frac{k_A}{k_{\parallel}} \right)^2 \left[(|b_r|^2 + |b_{\theta}|^2) (1 + \Omega^2) - 2 i \Omega (b_r b_{\theta}^* - b_{\theta} b_r^*) \right] / (1 - \Omega^2)^2$$

where c' is the velocity of light in vacuum

v_p is the phase velocity of the fast wave.

Ignoring the electric field component since $v_p \ll c'$, and substituting for the wave solutions in cylindrical geometry, we find that the total average energy density becomes

$$\rho = \frac{|b_{z0}|^2}{4\mu_0} \left[\frac{1}{2k_{\perp}^2} \left[\frac{k_A^2}{(1-\Omega)^2} + k_{\parallel}^2 \right] \left[\frac{C}{k_{\parallel}A} - 1 \right]^2 J_{m+1}^2(k_{\perp}r) + \frac{1}{2k_{\perp}^2} \left[\frac{k_A^2}{(1+\Omega)^2} + k_{\parallel}^2 \right] \left[\frac{C}{k_{\parallel}A} + 1 \right]^2 J_{m-1}^2(k_{\perp}r) + J_m^2(k_{\perp}r) \right].$$

The first term on the right hand side is related to the left hand circularly polarized component, while the second and last terms are produced by the right hand and the axial magnetic field components, respectively.

For the fast wave we ignore the left hand circularly polarized component and apply the approximate boundary condition. Assuming that the wave amplitude is uniform along the z direction, the total averaged wave energy in the length ℓ_T of plasma filled waveguide is

$$W = \frac{|b_{z0}|^2}{4\mu_0} J_m^2(k_{\perp}a) \left[\frac{1}{2k_{\perp}^2} \left(\frac{k_A^2}{(1+\Omega)^2} + k_{\parallel}^2 \right) \left(\frac{C}{k_{\parallel}A} + 1 \right)^2 + 1 \right] (\pi a^2 \ell_T).$$

In the above expression we have used the result that

$$\frac{-J_m(\chi)}{J_{m-2}(\chi)} \approx 1$$

when $J_{m-1}(\chi) = 0$.

For fast waves with $\frac{C}{k_0 A} \approx 1$ note that

$$\frac{W_{\text{particles}}}{W_{\text{magnetic}}} \approx \frac{\left(\frac{k_A}{k_1}\right)^2}{\left[\left(\frac{k_0}{k_1}\right)^2 + \frac{1}{2}\right] [1+\Omega]^2}$$

FLUX LINKAGE

The flux linkage is the flux of the wave enclosed by the coupling loop and is determined by the axial magnetic field near the plasma edge where the coil is located. Whereas the total wave energy is moderately insensitive to transverse density gradients, the flux linkage will be greatly influenced by density nonuniformity. We adopt as our model a waveguide of radius a , surrounding a uniform plasma column of radius p . The coupling system is a half turn coil of radius c , the return path for the driving current being the waveguide wall nearest the loop. A schematic diagram of the coupling structure is shown in Fig. 3.

If $k_1 \delta \ll 1$ we may write approximate expressions for the wave fields in the vacuum layer:-

$$b_r \propto J_{m-1}(k_{\perp} r)$$

$$b_z \approx b_{z0} J_m(k_{\perp} p) e^{k_{\parallel}(p-r)}.$$

We have assumed that the transverse wave fields in the vacuum layer have approximately the same form as those inside the plasma and thus satisfy the boundary condition $J_{m-1}(k_{\perp} a) = 0$, whereas the axial magnetic field decays exponentially with distance away from the plasma surface. A sketch of the fields for the first radial mode ($n=1$) of the $m=1$ wave is shown in Fig. 4.

The flux enclosed by the loop (which for the moment is considered to be infinitesimally thin) is

$$\begin{aligned} \phi &= \int_0^{\pi} \int_c^a r b_z dr d\theta \\ &\approx \left\{ \frac{\pi}{2} \right\} b_{z0} J_m(k_{\perp} p) a \Delta \exp(-k_{\parallel}(\delta - \frac{\Delta}{2})) \text{ if } m = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix}. \end{aligned}$$

SERIES LOADING RESISTANCE OF LOOP

On resonance, the equivalent input resistance of the loop is

$$R_O = \frac{Q\omega_0 \phi^2}{2W}.$$

Substituting the calculated values for the enclosed flux and the total wave energy, we obtain

$$R_O = \left\{ \frac{2\pi}{8/\pi} \right\} \frac{Q\omega_o \mu_o}{\ell_T} \frac{\Delta^2 e^{-k_{||}(2\delta-\Delta)}}{\left[\frac{1}{2k_{\perp}^2} \left(\frac{k_A^2}{(1+\Omega)^2} + k_{||}^2 \right) \left(\frac{C}{k_{||}A} + 1 \right)^2 + 1 \right]} \text{ for } m = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix}$$

In toroidal geometry we assume that the dispersion relation and field solutions are essentially the same as in cylindrical geometry and take $\ell_T = 2\pi R$ where R is the major radius.

EFFECT OF COIL WIDTH

To calculate the flux enclosed by the loop shown in Fig. 5, we consider only the case when the width $b \ll \frac{\lambda_u}{2}$ and replace the value of b_z under the loop by the average value of b_z calculated along the loop width.

The flux ϕ is therefore

$$\phi = \phi_o \left[\frac{2}{k_{||}b} \sin \left(\frac{k_{||}b}{2} \right) \right] \quad k_{||}b \ll \frac{\pi}{2}$$

where ϕ_o is the flux enclosed by an infinitesimally thin loop which has previously been calculated.

The coil loading resistance R' is given by

$$R' = R_o \left[\frac{2}{k_{||}b} \sin \left(\frac{k_{||}b}{2} \right) \right]^2$$

and therefore the equivalent loading resistance reduces as the width is increased.

Consider now the loop geometry to be a transmission line, such as a strip line. If the length ℓ_c is less than a quarter of the free space wavelength (λ_0), then the loop reactance will be inductive and, in principle, can be removed by adding a series capacitor. The total impedance at resonance will then appear resistive, the resistance divided between the plasma wave loading and the resistive losses in the loop material. The ratio of the power coupled to the plasma to the power lost in the coil structure is

$$\frac{P_{\text{coupled}}}{P_{\text{lost}}} = \frac{R'}{R_{\text{loss}}} \propto \frac{\left[\frac{1}{b} \sin\left(\frac{k_{\parallel} b}{2}\right) \right]^2}{\frac{1}{b}}$$

since the resistive loss is inversely proportional to the coil width. Thus,

$$\frac{P_{\text{coupled}}}{P_{\text{lost}}} \propto b \quad \text{if} \quad b \ll \frac{\lambda_{\parallel}}{2}$$

and therefore the efficiency of energy transfer increases linearly with coil width when coil losses are significant.

We note also that as the width increases, the loop reactance is reduced and thus less loop voltage is required for a given input power. Voltage limitations become less stringent as the loop width is increased.

ESTIMATES FOR THE FAST WAVE TOROIDAL CAVITY Q

In this section we will calculate approximate expressions for the energy losses in the toroidal cavity due to various physical mechanisms. We will consider the cyclotron heating of a minority species in a two component plasma, the damping of the fast wave at the second harmonic of the ion cyclotron frequency, electron transit time magnetic pumping, wall loading, and Coulomb collisional damping.

A. Minority Heating in a Two Component Plasma

Stix (1962) has shown that the average power absorbed per unit volume by ions from a transverse electric field E_{LT} that is left hand circularly polarized, and which has a frequency $\omega \sim 0 (\omega_{ci})$ is

$$P / \text{unit volume} = \frac{\sqrt{\pi}}{2} \frac{\epsilon_0 \omega_{pi}^2}{k_{\parallel} v_T} e^{-\left(\frac{\omega - \omega_{ci}}{k_{\parallel} v_T}\right)^2} |E_{LT}|^2$$

where v_T is the average ion thermal speed $= \sqrt{\frac{2kT_i}{m_i}}$ and ω_{pi} is the ion plasma frequency.

To calculate the power absorbed by a minority ion from the fast wave in toroidal-like geometry, we adopt the method of Adam and Samain (1971). We assume that the minority resonance layer is in the center of the plasma, as shown in Fig. 6, and that the properties of the fast wave are determined by the majority component alone. Toroidal effects on the fast wave are ignored.

The exponential function is appreciable only over distances Δx where

$$\Delta x \approx \left| \frac{k_{\parallel} v_T}{\frac{\partial \omega_{ci}(x)}{\partial x}} \right|$$

and for a toroidal field without shear

$$\Delta x \approx \frac{k_{\parallel} v_T}{\omega_{ci}} R$$

where ω_{ci} is the minority component ion cyclotron frequency in the center of the resonance layer. Assuming $|E_{LT}|^2$ is constant over the band Δx , the total average power absorbed over the whole circumference of the torus (ℓ_T) is

$$P_{MH} \approx \frac{\sqrt{\pi}}{2} \epsilon_0 \frac{\omega_{pi}^2}{\omega_{ci}} R \ell_T \int_{-a}^a |E_{LT}|^2 dx$$

We may write the left hand circularly polarized component of the electric field in terms of the right hand component, and then express the electric field in terms of the magnetic field (since $E_z = 0$). This gives the following expression:-

$$P_{MH} \approx \frac{\sqrt{\pi}}{2} \epsilon_0 R a \ell_T \left(\frac{\omega_{pi}^2}{\omega_{ci}} \right)_{\min} \left[\frac{C}{k_{\parallel} A} - 1 \right]^2 \\ \times \left(\frac{\omega}{k_{\perp}} \right)^2 b_{zo}^2 \left\{ \frac{1}{2a} \int_{-a}^a J_{m+1}^2(k_{\perp} r) dr \right\}$$

where ω_{pi} and ω_{ci} are evaluated for the minority resonant species. As a first approximation to the integral we write:-

$$P_{MH} \approx \frac{\sqrt{\pi}}{2} \epsilon_0 R a \ell_T \left(\frac{\omega_{pi}^2}{\omega_{ci}} \right)_{\min} \left[\frac{C}{k_{\parallel} A} - 1 \right]^2 \\ \times \left(\frac{\omega}{k_{\perp}} \right)^2 b_{zo}^2 \left\{ J_m^2(k_{\perp} a) \right\}$$

and from the definition of Q we obtain

$$Q_{MH} \approx \frac{\sqrt{\pi}}{2} \left(\frac{c'}{v_p} \right)^2 \left(\frac{\omega_{ci}}{\omega_{pi}} \right)_{\min} \left(\frac{a}{R} \right) \left(\frac{k_{\perp}}{k_{\parallel}} \right)^2 \\ \frac{\left[\frac{1}{2k_{\perp}^2} \left(\frac{k_A^2}{(1+\Omega)^2} + k_{\parallel}^2 \right) \left(\frac{C}{k_{\parallel} A} + 1 \right)^2 + 1 \right]}{\left[\frac{C}{k_{\parallel} A} - 1 \right]^2}$$

B. Cyclotron Harmonic Damping in a Single Species Plasma

From hot plasma theory it can be shown that the average power absorbed per unit volume by ions from a left hand circularly polarized electric field of frequency $\omega \sim 0$ ($2\omega_{ci}$) is

$$P / \text{unit volume} = \frac{\sqrt{\pi}}{2} \epsilon_0 \frac{\omega_{pi}^2}{k_{\parallel} v_T} \lambda_i e^{-\left(\frac{\omega - 2\omega_{ci}}{k_{\parallel} v_T} \right)^2} |E_{LT}|^2$$

where the hot plasma expansion parameter λ_i is given by

$$\lambda_i = \frac{1}{2} (k_{\perp} r_L)^2 = \frac{k_{\perp}^2 v_T^2}{2 \omega_{ci}^2}$$

and the quantity $(\omega_{pi}^2/\omega_{ci})$ is evaluated for the single plasma species. Using the method described in the previous section we have, for cyclotron harmonic resonance at the plasma center,

$$P_{CH} \approx \frac{\sqrt{\pi}}{4} \epsilon_0 \left(\frac{\omega_{pi}^2}{\omega_{ci}} \right) \lambda_i R a l_T \left[\frac{C}{k_{\parallel} A} - 1 \right]^2 \left(\frac{\omega}{k_{\perp}} \right)^2 \left[b_{z0}^2 J_m^2(k_{\perp} a) \right]$$

This gives

$$Q_{CH} \approx \frac{2\sqrt{\pi}}{\lambda_i} \left(\frac{c'}{v_p} \right)^2 \left(\frac{\omega_{ci}}{\omega_{pi}} \right)^2 \left(\frac{a}{R} \right) \left(\frac{k_{\perp}}{k_{\parallel}} \right)^2 \frac{\left[\frac{1}{2k_{\perp}^2} \left(\frac{k_A^2}{(1+\Omega)^2} + k_{\parallel}^2 \right) \left(\frac{C}{k_{\parallel} A} + 1 \right)^2 + 1 \right]}{\left[\frac{C}{k_{\parallel} A} - 1 \right]^2}$$

Note that the magnitudes of Q_{MH} and Q_{CH} depend upon the chosen widths of the resonant layer Δx . Their relative magnitude, however, remains independent of Δx .

C. Electron Transit-Time Magnetic Pumping

When the phase velocity v_p of an electromagnetic wave is of the order of the electron thermal speed, the average power absorbed per unit volume by electrons is (Stix, 1962).

$$P / \text{unit volume} = 2\sqrt{\pi} \epsilon_0 \frac{v_{Te}}{k_{\parallel}} \omega^2 \left(\frac{\omega_{pe}}{\omega_{ce}} \right)^2 e^{-(v_p/v_{Te})^2} |b_z|^2.$$

Integrating over the plasma volume and using the total wave energy, we obtain

$$Q_{ETT} \approx \frac{1}{8\sqrt{\pi}} \left(\frac{c'}{v_p} \right)^2 \left(\frac{v_p}{v_{Te}} \right) \left(\frac{\omega_{ce}}{\omega_{pe}} \right)^2 e^{(v_p/v_{Te})^2} \\ \times \left[\frac{1}{2k_1^2} \left(\frac{k_A^2}{(1+\Omega)^2} + k_{\parallel}^2 \right) \left(\frac{C}{k_{\parallel}A} + 1 \right)^2 + 1 \right].$$

Recently, Stix (1974) has shown that in a hot plasma, coherence effects between transit-time damping and Landau damping for electrons reduce the total absorption by the mechanisms to one half that for transit-time damping alone. In this case, the appropriate Q value for the two effects combined would be $2 Q_{ETT}$.

D. Wall Loading

In practice, waveguide walls have a finite conductivity and some power will be lost in the walls. With our approximations, the boundary conditions for perfectly conducting walls are

$$[E_{\theta}, E_z]_{r=a} = 0$$

and thus the only nonzero field quantity at the surface of the waveguide is the axial magnetic field. Suppose the walls have non-infinite conductivity, σ , so that there exists an electric field on the surface of the metal boundary. To calculate this field we take an approximate approach which uses $b_z(r=a)$ from the infinite conductivity results.

Neglecting displacement currents we have

$$\underline{E}_{wall} = \frac{1}{\mu_0 \sigma} (\nabla \times \underline{B})$$

and the average power loss per unit surface area of waveguide is

$$P / \text{unit area} = \frac{1}{2} \operatorname{Re} \left[(\underline{E} \times \underline{H}^*) \cdot \underline{\hat{r}} \right]_{r=a}$$

$$= \frac{\omega \delta_s}{4 \mu_0} |b_z|_{r=a}^2$$

where δ_s is the skin depth $= \frac{1}{\sqrt{\pi f \mu_0 \sigma}}$.

If we define ℓ_s to be the surface length of the torus walls ($\ell_s \geq 2\pi R$), the cavity Q for wall loading is

$$Q_W \approx \frac{1}{2} \left(\frac{a}{\delta_s} \right) \left(\frac{\ell_T}{\ell_s} \right) \left[\frac{1}{2k_\perp^2} \left(\frac{k_A^2}{(1+\Omega)^2} + k_\parallel^2 \right) \left(\frac{C}{k_\parallel A} + 1 \right)^2 + 1 \right] e^{2k_\parallel \delta}$$

E. Ohmic Collisional Damping

The average power lost per unit volume through Ohmic collisions can be estimated from the plasma conductivity $\underline{\sigma}$ by the equation

$$P / \text{unit volume} = \frac{1}{2} \operatorname{Re} (\underline{j} \cdot \underline{E}) = \frac{1}{2} \operatorname{Re} \left[(\underline{\sigma} \cdot \underline{E}) \cdot \underline{E} \right]$$

For fast waves

$$P / \text{unit volume} \approx \frac{1}{4\eta_\parallel} \left[\frac{\eta_\parallel}{\mu_0} C k_\perp J_m(k_\perp r) \right]^2$$

where $\eta_\parallel = \frac{1}{\sigma_\parallel}$ is the Spitzer resistivity

$$= 0.5 \times 10^{-4} \ln \Lambda / T_e^{3/2} \text{ ohm m } (T_e \text{ in eV})$$

Thus

$$Q_{\text{coll}} \approx \frac{\omega \mu_0}{\eta_\parallel} \frac{1}{k_\parallel^2} \left[\frac{1}{2k_\perp^2} \left(\frac{k_A^2}{(1+\Omega)^2} + k_\parallel^2 \right) \left(\frac{C}{k_\parallel A} + 1 \right)^2 + 1 \right]$$

For temperatures above 10eV, collisional effects are unimportant.

EXAMPLES

As examples we consider the geometry and plasma conditions of the ST Tokamak and those projected for PLT.

A. ST Tokamak

The ST Tokamak operated with two half coils in series at a frequency of 25 MHz in a deuterium plasma of density 8.5×10^{18} ions / m³ and a magnetic field corresponding to $\Omega = 2$ ($k_A = 18\text{m}^{-1}$) at the plasma center. The minor and major radii are 0.16 and 1.09 meters respectively, but near the coil the minor radius was increased to 0.19 meters. The parameters used are:

$$R=1.09\text{m}, \quad a=0.16\text{m}, \quad p=0.14\text{m}, \quad \Delta=0.04\text{m}, \quad \delta=0.05\text{m}.$$

To calculate the damping caused by minority component heating we assume a 1% hydrogen content. For wall loading the conductivity is taken to be that of stainless steel. We obtain:-

$k_{\perp} (\text{m}^{-1})$	$k_{\parallel} (\text{m}^{-1})$	$\lambda_{\parallel} (\text{m})$	$C/k_{\parallel} A$		
16	3.9	1.6	1.7		
$Q_{\text{mH}} (1\%)$	$Q_{\text{CH}} (150\text{eV})$	$Q_{\text{ETT}} (1\text{keV})$	Q_{W}	Q_{coll}	$R_{\text{O}} (\text{Ohm})$
84	6440	1180	3500	$>10^7$	35Ω

The Q's for cyclotron harmonic and electron transit-time damping for different ion and electron temperatures, respectively, are given in Fig. 7.

B. PLT

We consider a single half turn loop operating at 60 MHz in a deuterium plasma of density 5×10^{19} ions/m³ and a magnetic field such that $\Omega=2$ ($k_A = 43.2 \text{ m}^{-1}$). The dispersion relation of the fast wave for these conditions is plotted in Fig. 8.

The machine parameters used are:

$$R=1.3\text{m}, \quad a=0.5\text{m}, \quad p=0.45\text{m}, \quad \Delta=0.025\text{m}, \quad \delta=0.05\text{m}.$$

The results are given in Table I. To calculate the minority component heating we assume at 5% resonant hydrogen content. The Q 's for cyclotron harmonic and electron transit-time damping for different radial orders and varying ion and electron temperatures are given in Figs. 9-11.

TABLE I.

PLT

m=0 fast waves (MKS units)

n	$k_{\perp}$	$k_{\parallel}$	$\lambda_{\parallel}$	$\frac{C}{k_{\parallel}A}$	Q_{mH} (5%)	Q_{CH} (1 keV)	Q_{ETT} (1 keV)	Q_W	R_O (ohm)
1	7.66	24.7	0.26	1.04	2×10^4	$>10^7$	3865	-	3.8
2	14	23.4	0.28	1.08	2000	8×10^5	1250	-	3.0
3	20.4	21.2	0.31	1.16	460	8×10^4	673	-	2.7
4	26.6	18.4	0.37	1.27	158	2×10^4	508	-	2.3
5	33	14.4	0.48	1.48	66	4500	605	5×10^4	2.0
6	39.2	9.2	0.74	1.80	33	1570	4840	2×10^4	2.1

m=1 fast waves

1	4.8	25.1	0.25	1.01	10^5	$>10^8$	9600	-	1.75
2	11	24.1	0.27	1.05	5500	3×10^6	1936	-	1.4
3	17.2	22.4	0.29	1.12	911	2×10^5	874	-	1.2
4	23.6	19.8	0.33	1.26	256	3×10^4	560	-	1.0
5	30	16.4	0.40	1.43	97	8000	511	7×10^4	0.86
6	36	12.2	0.56	1.65	46	2640	1000	3.6×10^4	0.82

CONCLUSION

We find that the loading impedance for eigenmodes varies for different radial modes and can become quite high for lightly damped waves. The greatest damping process is minority heating, once the minority content exceeds a few percent. Cyclotron harmonic damping and electron transit-time pumping rates vary for each radial mode. Generally, the cyclotron harmonic damping increases for higher order modes since the left-hand circularly polarized component of the wave is larger for higher orders. For TTMP, damping is highest for the mode whose phase velocity is closest to the electron thermal speed. Wall loading and collisional damping are mostly insignificant for large machines and hot plasmas.

For conditions in the ST Tokamak, the observed loading in the presence of an eigenmode can mainly be attributed to a very small percentage of resonant hydrogen impurity. The process would produce hot hydrogen ions but energy equipartition would be established with deuterium in a time of about 1msec. Surface heating may also occur because of the additional wave fields related to the electron mass term. These new fields have a large value of $k_{\perp}$ and consequently the second harmonic absorption of wave energy by deuterons or by other impurity atoms[†] may become important on the plasma surface.

Loading of a single half-turn loop by eigenmodes in PLT is calculated to be in the range 1-3ohms for a minority concentration of 5%. The loading can be improved by using more coupling loops that are appropriately positioned and phased with respect to

the standing wave in the toroid, or by varying the resonant species concentration. Simple calculations show that, for a given wave amplitude, absorption of energy by the minority heating process can be maximized; e.g., in PLT the optimum percentage concentration is between 5 and 10%.

The major problem in any eigenmode system would be sustaining the mode. One technique that could be used would be a feedback scheme that shifts the frequency as the plasma conditions change. Typical fixed frequency generation schemes, however, have the loops in a resonant circuit to remove their inductive impedance. A frequency varying network cannot include this technique and there will be difficulty in matching the load to the RF supply. A method which avoids this problem is a feedback scheme that varies the plasma major radius as the conditions change. If the plasma is assumed to obey infinite conductivity MHD theory, the Alfvén velocities in the initial and final stages of compression (or expansion) are the same when the change is carried out in a static toroidal field. Thus, the wavelength of the fast wave remains approximately unchanged whilst the plasma length is linearly proportional to the major radius. In large machines, like PLT, the mode spectrum of $m=0$ and $m=+1$ fast waves are moderately dense and a variation of $\pm 5\%$ in the major radius will allow the conditions for different eigenmodes to overlap. Variation of the plasma major radius in times shorter than the containment time can be achieved using the programmable character of the vertical field coils. Such a facility was available on ST where the vertical field could be changed by $\pm 10\%$ in a time of 0.2 msec [H. Hill (1973)], and the present

system on ATC can achieve a 100% change in less than 2 msec. It is believed that a $\pm 5\%$ change in the vertical field of PLT can be obtained in about 4 msec.* Unfortunately, the coupling of the loops to the eigenmode will change as the major radius varies, but it should be possible to compensate for this change by placing loops on both the inside and outside circumferences of the toroidal geometry.

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+ H. TAKAHASHI, (personal communication) has pointed out
that O^{8+} and D^{+} have the same cyclotron frequency and thus both
undergo second harmonic resonance. Collisions of O^{8+} with the
walls may drive excessive impurities into the plasma.

* H. HILL, (personal communication).

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Fig. 1. Equivalent circuit of coupling loop and toroidal cavity.

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Fig. 2. A sketch of the real part of the input impedance at frequencies around resonance.

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Fig. 3. A schematic diagram of the coupling structure.

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Fig. 4. Approximate fields of the $m=1$ fast wave.

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Fig. 5. Schematic diagram of the coupling structure.

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Fig. 6. A diagram illustrating the resonance layer in the plasma where the wave frequency is in the vicinity of the minority component ion cyclotron frequency. The radial variation of the confining field is also included.

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Fig. 7. The quality factors Q for cyclotron harmonic heating and electron transit-time magnetic pumping in the ST Tokamak for different ion and electron temperatures, respectively.

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Fig. 8. Dispersion relation for fast waves in a deuterium plasma of density $5 \times 10^{19} / m^3$ and $\Omega=2$.

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Fig. 9. The quality factors for cyclotron harmonic heating in PLT for various modes of the fast wave at different ion temperatures.

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Fig. 10. The quality factors for electron transit-time damping of the $m=0$ fast wave in PLT for varying electron temperatures.

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Fig. 11. The quality factors for electron transit-time damping of the $m=1$ fast wave in PLT for varying electron temperatures.

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