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REACTOR PHYSICS OF TEAPOT - PART V,
PROPOSED STABILITY EXPERIMENT

SUMMARY

An investigation has been made concerning two methods of determining the stability of boiling reactors. It was found that the one involving a rapid reactivity change (such as rapid removal of a rod) is the better method. In the experiment involved, the reactor flux and rate of reactivity addition over periods of time are required. It is also desirable that the average fluid density be measured.

Means of providing rapid reactivity changes are available, and methods involving rod movement and gas pressure release appear quite reasonable.

INTRODUCTION

In performing a boiling reactor experiment, it is important that an answer be obtained as to the stability of such reactor types. This involves determining the time relationship between input nuclear power and the vapor generated as a consequence. Two methods for doing this are presented below. Both involve adding reactivity in a known manner and observing the resultant neutron flux variation, the difference in the two methods lying in the time variation of reactivity additions and in the manner of interpreting the data.

Method I.

This method is primarily a batch-type experiment. Reactivity is added quickly to the reactor by means such as quickly removing an absorbing rod from the core region, and the resultant neutron flux surge is recorded. The applicable equations of motion are:

$$\frac{d\sigma}{dt} = \lambda_0 \left[k(1 - \beta) - k_c \right] \sigma + \sum_i \lambda_i c_i \quad (128)$$

$$\frac{dc_i}{dt} + \lambda_i c_i = \lambda_0 \beta_i k \sigma \quad (129)$$

$$\sum_i \beta_i = \beta \quad (130)$$

$$k_c = 1 + \frac{k_{co} - 1}{1 + A_2} \left[a + A_2 \left(\frac{\rho_0}{\rho} \right)^2 \right] \quad (131)$$

$$A_2 = \left[\left(\frac{4.81}{R + 2x} + \xi \right) \left(\frac{h_n(0) + 2x}{\pi} \right) \right]^2 \quad (132)$$

$$\xi = \frac{\Delta k}{4.81 M_0^2} \left(\frac{\rho}{\rho_0} \right)^2 \quad (133)$$

$$\frac{dH_t}{dt} + \mu H_t \left(t - \frac{1}{\mu} \right) = P_0 \sigma \quad (134)$$

$$\rho - \rho_0 = \left(\frac{\partial \rho}{\partial H_t} \right)_{fv} \left[H_t - H_{t0} + \xi \int_0^\infty G(s) \left[H_t(t-s) - H_{t0} \right] ds \right] \quad (135)$$

$$\xi = \frac{\left(\frac{\partial \rho}{\partial H_t} \right)_T - \left(\frac{\partial \rho}{\partial H_t} \right)_{fv}}{\left(\frac{\partial \rho}{\partial H_t} \right)_{fv}}$$

where

σ = nuclear power relative to steady state value

$$\lambda_0 = \frac{1}{\tau k_c} \approx \frac{1}{\tau_0 k_{c0}}$$

τ = average lifetime of neutron from birth to capture

k = material multiplication constant

k_c = k required to maintain criticality

k_{c0} = k_c evaluated under initial conditions

c_i = a measure of the nuclear power contributed by the i th group of delayed neutron emitters

λ_i = decay constant of the i th group of delayed neutron emitters

β_i = fraction of fission neutrons which are delayed in i th group of delays

β = fraction of fission neutrons which are delayed

A_2 evaluated under initial conditions is given in Figure 10, (Part II).

a is given in Figure 11, (Part II) as a function of fluid density

x is given in Table IV, (Part II)

R = radius of reactor core, cm

$h_n(0)$ = height of reactor core under initial conditions, cm.

ξ is given in Table IV, (Part II)

Δk = change in k_c due to rod movement alone, and is given in Table IV, (Part II) and used in conjunction with Equations 14 or 15, and 16, (Part II).

H_t = total energy content of reactor core

H_{t0} = H_t evaluated under steady-state conditions

μ = reciprocal average time energy remains in core region

P_0 = steady-state power of the reactor

ρ = average density of core region

ρ_0 = ρ evaluated under steady-state conditions

$\left(\frac{\partial \rho}{\partial H_t}\right)_{fv}$ = change in core fluid density per unit energy injected as superheat alone, (evaluated in Part III for Teapot)

$\left(\frac{\partial \rho}{\partial H_t}\right)_T$ = change in core density per unit energy injected as vapor (evaluated in Part III for Teapot)

G = normalized time distribution rate of vapor formation

The above quantities have already been discussed in previous memos (Part I - CF 52-6-78; Part II - CF 52-6-118; Part III - CF 52-7-38; Part IV - CF 52-8-128). In the above system the important function to evaluate insofar as reactor stability is the form of G . To that end, consider that the reactor is operating in a steady-state boiling condition with the central thimble rod all the way in. As the rod is quickly removed, rod position as a function of time will be known, or

$$z = f(t) \quad (137)$$

where z = length of rod inserted into reactor core proper

$f(t)$ = known function of time

Employing the relation between Δk and z as given in Equation 14,

$$\frac{(\Delta k)_{\text{rod}}}{(\Delta k)_{\text{rt}}} = \frac{z}{h} - \frac{1}{2\pi} \sin \frac{2\pi z}{h} \quad (0 < z < h) \quad (138)$$

where $(\Delta k)_{\text{rod}}$ = deviation of k_c from steady-state value as determined by rod movement alone

$(\Delta k)_{\text{rt}}$ = maximum $(\Delta k)_{\text{rod}}$ available = difference in k_c between rod all the way in and all the way out (preliminary values can be taken from Table IV)

h = effective height of reactor core

Combining Equations (137) and (138)

$$(\Delta k)_{\text{rod}} = (\Delta k)_{\text{rt}} \left[\frac{f(t)}{h} - \frac{1}{2\pi} \sin \frac{2\pi f(t)}{h} \right] = F(t) \quad (139)$$

where $F(t)$ is defined in the above equation. Thus, $(\Delta k)_{\text{rod}}$, which is the Δk in Equation 133, is a known function of time. The accompanying change in reactor reactivity will cause a change in the neutron flux, which will be continuously recorded. With λ_0 , β_i , β , $k = k_{c0}$, λ_i and c_i known, Equations 128-130 together with the recorded flux variations will give

$$k_c = g(t) \quad (140)$$

where $g(t)$ is a known function of time for a given experimental run. Combining the above with Equations 131-133 will give ρ as a known function of time. Since Equation 134 gives the relation between flux and energy within the core, the only unknown function in Equation 135 is the time distribution

rate of vapor formation given by G . If G is of such a form that $G(S)$ is small except when S is small, $H_t(t - S)$ can be approximated by the first two terms of a Taylors' expansion, or

$$H_t(t - S) \approx H_t(t) - S \frac{dH_t(t)}{dt} \quad (141)$$

so

$$\rho - \rho_0 = \left(\frac{\partial \rho}{\partial H_t} \right)_{fv} \left[(H_t - H_{t0})(1 + \zeta) - \bar{S} \frac{dH_t}{dt} \right] \quad (142)$$

where

$$\bar{S} = \int_0^{\infty} SG(S) dS = \text{first moment of } G$$

= average time for energy to be transferred
from liquid to vapor

In the above expression, the only unknown is S . Thus, from measurement of the Δk in the rod, the rate of rod movement, and the neutron flux level, values for $\bar{S}$ can be obtained. However, it is more desirable to know the form of G itself rather than its first moment. To do this, G must be arbitrarily chosen until it satisfies the experimental data.

In specifying G , it is desirable that G be based upon plausible mechanisms for vapor formation. Thus, if bulk liquid superheat is the controlling driving force for vapor formation

$$\alpha \frac{dT}{dt} + K(T - T_0) = \rho_0 \sigma \quad (143)$$

$$\frac{dH}{dt} + \epsilon H(t - \frac{1}{\epsilon}) = K(T - T_0) \quad (144)$$

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where α = volume heat capacity of core
 T = bulk liquid temperature
 T_0 = saturation liquid temperature
 K = energy transfer from liquid to vapor per degree superheat
 H = energy content of vapor within reactor core
 ϵ = reciprocal average time a vapor bubble remains within core

Since the right hand side of Equation 144 represents the rate at which energy is being injected into the core as vapor at time t , and thus dependent upon the past history of the reactor, Equation 144 can be written as

$$\frac{dH}{dt} + \epsilon H(t - \frac{1}{\epsilon}) = P_0 \int_0^{\infty} G(s) \sigma(t - s) ds \quad (145)$$

Combining Equations 143 and 144 yields

$$\frac{dH}{dt} + \epsilon H(t - \frac{1}{\epsilon}) = P_0 \frac{K}{\alpha} \int_0^{\infty} e^{-\frac{K}{\alpha} s} \sigma(t - s) ds \quad (146)$$

and comparison of this result with Equation 145 shows that for this case

$$G(s) = \nu e^{-\nu s} \quad (147)$$

where $\nu = \frac{K}{\alpha}$

Consider now a mechanism which is dependent upon the formation of gas within the reactor fluid, the gas bubbles then being available as vapor nuclei.

Let G = volume of dissolved gas in the reactor core

n = volume of dissolved gas formed per unit input of nuclear energy

θ = average reciprocal time dissolved gas stays in that phase

G = volume of gas in gaseous phase within core

$\oplus$ = average reciprocal time gas remains as such within core

Then

$$\frac{dg}{dt} + \theta g = n P_0 \sigma \quad (148)$$

$$\frac{dG}{dt} + \oplus G = \theta g \quad (149)$$

Postulating that the rate of vapor formation is proportional to G (specifically, that $\frac{1}{\oplus}$ is the average time it takes for a gas bubble to become in effect a vapor bubble), Equation 144 becomes

$$\begin{aligned} \frac{dH}{dt} + \epsilon H(t - \frac{1}{G}) &= \oplus G \\ &= n P_0 \theta \oplus \int_0^{\infty} \frac{e^{-\theta S} - e^{-\oplus S}}{\oplus - \theta} \sigma(t - S) dS \end{aligned} \quad (150)$$

or

$$G(S) = \frac{\theta \oplus}{(\oplus - \theta)} \left(e^{-\theta S} - e^{-\oplus S} \right) \quad (151)$$

Other mechanisms can be postulated which will lead to other forms for G ; however, the purpose of the above is to illustrate how a plausible G may be obtained.

Method II.

In this method the reactivity is changed continuously by means of an oscillating poison within the central thimble. In the small oscillation approximation, Equation 128 becomes

$$\frac{d\phi}{dt} = \lambda_0 (\Delta k) \phi_0 - \lambda_0 \beta k \phi + \sum_1 \lambda_1 c_1 \quad (152)$$

where $\Delta k = k_{c0} - k_c$

ϕ = thermal neutron flux

k_c is influenced by the control rod position and also by the effect of nuclear power upon reactivity, or a power coefficient of reactivity, so

$$\Delta k = \Delta k(\text{rod}) + \Delta k(\phi) \quad (153)$$

where $\Delta k(\text{rod})$ = variation in k_c as determined by rod position

$\Delta k(\phi)$ = variation in k_c due to power coefficient

The reactivity variations due to the control rod motion can be approximated by

$$\Delta k(\text{rod}) = \Delta k_0 e^{i\omega t} \quad (154)$$

where Δk_0 = maximum Δk of rod

ω = angular frequency of rod motion

and the resultant flux level change can be represented by $\eta e^{i\omega t}$ where η is the maximum flux variation and so

$$\Delta k(\phi) = r(i\omega) \eta e^{i\omega t} \quad (155)$$

where r = the power coefficient of reactivity at the angular frequency ω .

γ thus determines the phase lags between flux variations and reactivity changes dependent upon flux variations. Combining the above with Equations 152 and 129 results in

$$i\omega \eta e^{i\omega t} = \lambda_0 \left[\Delta k_0 e^{i\omega t} + \gamma(i\omega) \eta e^{i\omega t} \right] \Phi_0 - \lambda_0 \beta k \eta e^{i\omega t} + \sum_i \frac{\lambda_i \lambda_0 \beta_i k \eta e^{i\omega t}}{\lambda_i + i\omega}$$

or

$$\eta e^{i\omega t} \left[i\omega + \lambda_0 \beta k - \sum_i \frac{\lambda_i \lambda_0 \beta_i k}{\lambda_i + i\omega} - \Phi_0 \gamma(i\omega) \right] = \lambda_0 \Delta k_0 e^{i\omega t} \Phi_0 \quad (156)$$

The above can be written as

$$\frac{\Delta \Phi}{\Phi_0} = \frac{\Delta k_0 e^{i\omega t}}{\Delta k(i\omega) - \Phi_0 \gamma(i\omega)} \quad (157)$$

where $\Delta \Phi = \eta e^{i\omega t}$ = flux deviation from initial level

Φ_0 = initial flux level

$$\Delta k(i\omega) = \frac{i\omega}{\lambda_0} + \beta k - \sum_i \frac{\lambda_i \beta_i k}{\lambda_i + i\omega}$$

It can be noted that $\Delta k(i\omega)$ is defined in terms of $i\omega$ by the same equation which relates a reactivity change Δk to the inverse stable pile period which it produces. Equation 157 can be written as

$$\frac{\Delta \Phi}{\Phi_0} / \Delta k_0 e^{i\omega t} = \Delta k(i\omega) - \Phi_0 \gamma(i\omega) \quad (158)$$

If the quantity given on the left hand side of the above equation is measured at high (boiling) and low (just below boiling) flux levels, then

$$\left[\frac{\Delta \Phi}{\Phi_0} / \Delta k_0 e^{i\omega t} \right]_{\text{high}} - \left[\frac{\Delta \Phi}{\Phi_0} / \Delta k_0 e^{i\omega t} \right]_{\text{low}} = (\Phi_0 \beta)_{\text{low}} - (\Phi_0 \beta)_{\text{high}} \quad (159)$$

$(\beta)_{\text{high}}$ will contain the effects of liquid superheat and vapor formation, while $(\beta)_{\text{low}}$ will contain liquid heating effects only, and so with experimental results inserted into Equations 158 and 159 the two effects can presumably be separated. Thus, from Equation 155,

$$\Delta k(\Phi) = \beta \Delta \Phi = \frac{\Delta k}{\Delta \Phi} \Delta \Phi \quad (160)$$

and so if a phase angle is to be associated with the power coefficient due to vapor formation lagging reactor flux, $\frac{dk}{d\Phi}$ can be calculated as if there were no phase lag, and comparison of this result with the experimental data will yield the actual phase lags. Also, G can be determined in a manner similar to that given in Method I by utilizing Equations 131 (A_2 is now a constant since the effect of the control rod on reactivity has been separated from the density effect), 134, 135 and 160.

Discussion:

The particular method to use depends upon the accuracy of the results, and to a lesser degree upon the mechanical ease of installation and operation of the appropriate apparatus. Both methods are somewhat complicated by the fact that the "steady-state" in a boiling reactor is rather erratic, the flux level varying perhaps ± 10 or 20 per cent from the average value. In addition, the "steady state" can only be applied to the reactor as an

integral unit, any particular portion of it being in an unsteady-state, since vapor bubbles will always rise from the regions in which they are formed. Because of this natural flux variation, it appears that Method I will be better than Method II, for in the latter the flux variations which would have occurred without the oscillating rod would have to be known. Also, Method II implies constant reactor cooling and since vapor bubbles within the reactor will not collapse and in this respect are independent of the flux behavior, an additional complication is added to the analysis. It also appears that the apparatus necessary in performing Method II will be more complex than that required for Method I. Method I is therefore the better one to use.

There are several ways reactivity can be added quickly to the reactor. Three will be mentioned: (1) removing an absorbing rod quickly, (2) removing a poison gas quickly, (3) removing a poison liquid quickly. The one to employ is dependent upon the effectiveness of the method, and the ease of its application.

Pertaining to rod withdrawal, J. H. Payne at KAPL has developed a mechanism which will drive a control rod into a thimble a distance of about 25 inches with an acceleration of $3g$. This apparatus could also be used to withdraw a rod, but more acceleration is afforded in downward motion. By extending the thimble through the core bottom, the apparatus could be used in its present design, and reactivity changes could be accomplished by moving the rod from a core region to a position beneath the core. The size of the apparatus excluding control and electrical equipment is quite small, being

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approximately $3\frac{1}{2}' \times 1' \times 1\frac{1}{2}'$. One unit has already been built, and about 25 are to be built in the shops at KAPL in the near future, and if we should desire a unit, Mr. Payne felt it could easily be built at KAPL. The present unit has fulfilled all expectations as to performance, pertaining to exact rod position and speed at which a $\frac{5}{8}"$ rod can be inserted into the reactor core.

KAPL has also done work on injecting a poison gas into an evacuated thimble inside the reactor to cut down reactivity very quickly. They have found that if two cylindrical containers are separated by a diaphragm, one cylinder containing gas at 200 psi and the other cylinder evacuated, the time for the pressures to be equalized following rupturing of the diaphragm is nearly that associated with the speed of sound. This method could be used in reverse for Teapot reactivity changes, although changes in design would be necessary so that the gas pressure would be inside the thimble, and various pressures could be employed several times. It appears that BF_3 would be suitable as the gas (B^{10}).

Another method of rapid reactivity changes would be to quickly dump by means of compressed air, a thimble filled with poison liquid such as boric acid or borax dissolved in water. It is my understanding that some work with poison liquid has been done at KAPL much along the same line as with gas. However, nothing definite has been heard concerning this as yet.

From the above, it is clear that means of providing rapid reactivity changes are available, although the particular method still remains to be chosen.

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Of the above three methods, the first two are the most appealing: the absorbing rod mechanism because it has already been successfully developed and apparently can be obtained as an integral unit; the compressed gas because the energy required for changing the core reactivity is available in a convenient form.

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