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ADDENDUM I

AEC RESEARCH AND
DEVELOPMENT REPORT

MASTER

**SEAL SHELL-2-A COMPUTER PROGRAM
FOR THE STRESS ANALYSIS OF A
THICK SHELL OF REVOLUTION WITH
AXISYMMETRIC PRESSURES,
TEMPERATURES, AND
DISTRIBUTED LOADS**

APRIL 1967

CONTRACT AT-11-1-GEN-14

BETTIS ATOMIC POWER LABORATORY, PITTSBURGH, PA.,
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UC-32 Mathematics and Computers

SEAL-SHELL-2 - A COMPUTER PROGRAM FOR THE STRESS ANALYSIS OF
A THICK SHELL OF REVOLUTION WITH AXISYMMETRIC PRESSURES, TEMPERATURES,
AND DISTRIBUTED LOADS

C. M. Friedrich

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H.C. \$ 3.00; MN 65

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The SEAL SHELL-2 program has been changed to include two new options. The first option gives a plot of the shell geometry in a meridian plane. The second option allows the user to input the influence coefficients of the flexibility matrix for any segment.

SEAL-SHELL-2 Addendum I

C. M. Friedrich

I. INTRODUCTION

The original SEAL-SHELL-2 program¹ determines stresses, strains, deflections, and reactions in a thick shell of revolution having axisymmetric loading. The loading consists of a temperature distribution, inside and outside pressure distributions, and circumferential forces and moments applied to the middle surface. The shell is linear-elastic with tensile, bending, and shear strains.

The SEAL-SHELL-2 computer program has been modified to include an option for plotting the shape of the shell in a meridian plane with radial R and axial Z co-ordinates. The plot is generated on the CDC-280 microfilm plotter by a set of FORTRAN callable subroutines², and shows the actual shape used by the program.

Another improvement is the inclusion of an option to input the influence coefficients of the flexibility matrix of any segment.

The coefficients may be for any axisymmetric structure, such as a tube sheet in a heat exchanger. Another application would be to condense a group of segments into a single element which may be used in a series of problems and so cut down running time on the computer. A third application is to use the condensed group of segments as a single element in order to allow more elements in the analysis of a critical region of the shell.

¹ C. M. Friedrich, "SEAL-SHELL-2, A Computer Program for the Stress Analysis of a Thick Shell of Revolution with Axisymmetric Pressures, Temperatures, and Distributed Loads," WAPD-TM-308, December, 1963.

² G. J. Pfeifer, "CDC-6600 Fortran Programming Bettis Environmental Report," WAPD-TM-668, January, 1967.

II. PLOTTING OPTION

The signal for printing out the shape of the shell geometry is -110.0 in place of +110.0 in columns 61 to 70 of the title card. Each segment is then plotted as shown for segment K in Figure 1.

The boundary surfaces at positions K-1 and K on segment K are printed correctly, but the outside, middle, and inside surfaces are approximated by chords instead of actual shapes. In other words, points 1 to 6 are correctly placed and straight lines are used to connect the points. If there is a kink of the middle surface at the joint of two adjacent segments, the result printed out is as shown in Figure 2.

Fig. 2. Plot of 2 Kinked Segments

III. INPUT FLEXIBILITY OPTION

A. Theory

1. Deflections of Shell Middle Surface

In the output of each SEAL-SHELL-2 problem, a flexibility matrix of 5 rows and 8 columns is printed. The equations represented by this matrix are:

$$\begin{aligned} D_{zn} = & A_{41}^F t + A_{42}^F p + A_{43}^F w + A_{44}^F z_n \\ & + A_{45}^F r_o + A_{46}^F m_o + A_{47}^F r_n + A_{48}^F m_n \end{aligned} \quad 1a$$

$$\begin{aligned} D_{ro} = & A_{51}^F t + A_{52}^F p + A_{53}^F w + A_{54}^F z_n \\ & + A_{55}^F r_o + A_{56}^F m_o + A_{57}^F r_n + A_{58}^F m_n \end{aligned} \quad 1b$$

$$\begin{aligned} D_{mo} = & A_{61}^F t + A_{62}^F p + A_{63}^F w + A_{64}^F z_n \\ & + A_{65}^F r_o + A_{66}^F m_o + A_{67}^F r_n + A_{68}^F m_n \end{aligned} \quad 1c$$

$$\begin{aligned} D_{rn} = & A_{71}^F t + A_{72}^F p + A_{73}^F w + A_{74}^F z_n \\ & + A_{75}^F r_o + A_{76}^F m_o + A_{77}^F r_n + A_{78}^F m_n \end{aligned} \quad 1d$$

$$\begin{aligned} D_{mn} = & A_{81}^F t + A_{82}^F p + A_{83}^F w + A_{84}^F z_n \\ & + A_{85}^F r_o + A_{86}^F m_o + A_{87}^F r_n + A_{88}^F m_n \end{aligned} \quad 1e$$

where D_{zn} = axial deflection at position n , + $\downarrow$
 D_{ro} = radial deflection at position 0, + $\leftarrow$
 D_{mo} = bending rotation at position 0, + $\curvearrowright$
 D_{rn} = radial deflection at position n , + $\rightarrow$
 D_{mn} = bending rotation at position n , + $\curvearrowright$

as shown in the following figure. It is to be noted that the axial deflection is measured with respect to position 0; that is, $D_{z0} \equiv 0$.

Fig. 4 External Forces, Other than Pressure

2. Forces of Shell

The shell forces corresponding to the deflections of Eqs 1a to 1e are shown in Figure 4. In Eq 1a, A_{41} is the influence coefficient for the thermal deflection of D_{zn} , so that F_t is a unit force representing thermal effects. Similarly, A_{42} is the influence coefficient for the axial deflection D_{zn} produced by pressure loading on the shell; F_p is a unit load representing pressure effects. In the same fashion, A_{43} is the influence coefficient for the axial deflection D_{zn} produced by the other external loads $W_{r1}, W_{m1}, W_{z1}, W_{r2}, \dots, W_{rn}, W_{mn}$ and W_{zn} . Just like F_t and F_p , F_w is a unit load representing the effects of the applied loads $W_{r1}, \dots, W_{zn}$.

$F_{zn}, F_{ro}, F_{mo}, F_{rn}$, and F_{mn} are boundary condition loads applied at positions 0 and n, as shown in Figure 4. By Maxwell's reciprocal theorem, the influence coefficient A_{45} for axial deflection D_{zn} produced by a unit value of radial force F_{ro} equals the influence coefficient A_{54} for radial deflection D_{ro} produced by a unit value of axial force F_{mn} . Similarly, $A_{46} = A_{64}, A_{47} = A_{74}, A_{48} = A_{84}, A_{56} = A_{65}$, etc.

The 5×8 matrix of coefficients A_{41} to A_{88} is really part of an 8×8 symmetrical matrix in which the first 3 rows represent the "working deflections" D_t, D_p , and D_w of the forces F_t, F_p , and F_w . In matrix form,

$$\begin{array}{|c|} \hline D_t \\ \hline D_p \\ \hline D_w \\ \hline D_{zn} \\ \hline D_{ro} \\ \hline D_{no} \\ \hline D_{rn} \\ \hline D_{mn} \\ \hline \end{array} = \begin{array}{|c|c|c|c|c|c|c|c|c|} \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline A_{41} & A_{42} & A_{43} & A_{44} & A_{45} & A_{46} & A_{47} & A_{48} & \\ \hline A_{51} & A_{52} & A_{53} & A_{54} & A_{55} & A_{56} & A_{57} & A_{58} & \\ \hline A_{61} & A_{62} & A_{63} & A_{64} & A_{65} & A_{66} & A_{67} & A_{68} & \\ \hline A_{71} & A_{72} & A_{73} & A_{74} & A_{75} & A_{76} & A_{77} & A_{78} & \\ \hline A_{81} & A_{82} & A_{83} & A_{84} & A_{85} & A_{86} & A_{87} & A_{88} & \\ \hline \end{array} \times \begin{array}{|c|} \hline F_t \\ \hline F_p \\ \hline F_w \\ \hline F_{zn} \\ \hline F_{ro} \\ \hline F_{no} \\ \hline F_{rn} \\ \hline F_{mn} \\ \hline \end{array}$$

where the 24 zeroes are used because we are not interested in the values of D_t , D_p , and D_w .

The axial force F_{zo} is not used as an independent force in the shell because the program determines the value of F_{zo} in terms of the 8 loads F_t to F_{mn} by the equation of axial equilibrium for the entire shell.

3. Deflections and Forces of Segment K

A segment K is really a shell between positions K-1 and K just as the entire shell is between positions 0 and n. Segment deflections and forces are therefore shown in Figures 5 and 6 just as shell forces were shown in Figures 3 and 4. In Figures 5 and 6, the subscript a is used for position K-1 and the subscript b is used for position K on segment K; that is,

$$\begin{array}{lll} F_{ra} = F_{rk-1}, & F_{rb} = F_{rk}, & W_{rb} = W_{rk} \\ F_{ma} = F_{mk-1}, & F_{mb} = F_{mk}, & W_{mb} = W_{mk} \\ F_{za} = F_{zk-1}, & F_{zb} = F_{zk}, & W_{zb} = W_{zk} \end{array}$$

Note that deflections at "a" match the forces at "a" in Figures 5 and 6, so that by comparison of Figures 3 and 5,

$$\begin{array}{l} D_{ra} = +D_{ro} \text{ for } K = 1, \text{ or } D_{ra} = -D_{rk-1} \text{ for } K > 1 \\ D_{ma} = +D_{mo} \text{ for } K = 1, \text{ or } D_{ma} = -D_{mk-1} \text{ for } K > 1 \\ D_{za} = 0 \\ D_{rb} = +D_{rk} \\ D_{mb} = +D_{mb} \\ D_{zb} = +D_{zk} - D_{zk-1} \end{array}$$

Then the flexibility matrix equation for segment K may be written in the same form as Equation 2. As for the shell, F_{za} is found in terms of F_t to F_{zb} by the equation of axial equilibrium for the segment.

Fig. 5 Deflections of Segment K

Fig. 6 Forces of Segment K

$$\begin{bmatrix} D_t \\ D_p \\ D_w \\ D_{zb} \\ D_{ra} \\ D_{na} \\ D_{rb} \\ D_{mb} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ A_{41} & A_{42} & A_{43} & A_{44} & A_{45} & A_{46} & A_{47} & A_{48} \\ A_{51} & A_{52} & A_{53} & A_{54} & A_{55} & A_{56} & A_{57} & A_{58} \\ A_{61} & A_{62} & A_{63} & A_{64} & A_{65} & A_{66} & A_{67} & A_{68} \\ A_{71} & A_{72} & A_{73} & A_{74} & A_{75} & A_{76} & A_{77} & A_{78} \\ A_{81} & A_{82} & A_{83} & A_{84} & A_{85} & A_{86} & A_{87} & A_{88} \end{bmatrix} \times \begin{bmatrix} F_t \\ F_p \\ F_w \\ F_{zb} \\ F_{ra} \\ F_{ma} \\ F_{rb} \\ F_{mb} \end{bmatrix}$$

B. Application to Any Shape of Revolution

With the A matrix of Eq 3 as an input option, shapes other than segment shapes may now be used in SEAL-SHELL-2. For example, consider the plate in Fig. 7 as a free body in Fig. 8.

For axial equilibrium of the equivalent segment K, set

$$\begin{aligned} P_1 &= P_0 = 0 \\ W_{rk} &= W_{mk} = 0 \\ \underline{W_{zk} = \pi R_p^2 (P_1 - P_2)} \end{aligned}$$

where

$$\begin{aligned} R_p &= \text{pressurized radius} \\ P_1 &= \text{upper pressure} \\ P_2 &= \text{lower pressure} \end{aligned}$$

Let

$$\begin{aligned} T_1, T_2 &= \text{upper and lower temperatures, } ^\circ\text{F} \\ E, \nu &= \text{modulus of elasticity and Poisson's ratio} \\ R_k, H &= \text{radius and thickness of plate to positions K-1 and K.} \\ \alpha &= \text{coefficient of linear thermal expansion.} \end{aligned}$$

The net pressure, and loads per unit of circumference are, as shown in Fig. 9,

$$\begin{aligned} P &= W_{zb} / (\pi R_p^2) \\ N_r &= (F_{rb} - F_{ra}) / (2\pi R_k) \\ M &= [(F_{mb} - F_{ma}) + (h/2)(F_{ra} + F_{rb})] / (2\pi R_k) \\ N_z &= F_{zb} / (2\pi R_k) \end{aligned}$$

4

or in matrix form,

$$\begin{bmatrix} F_t \\ P \\ M \\ N_r \\ N_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{W_{zk}}{\pi R_p^2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{h}{4\pi R_k} & \frac{-1}{2\pi R_k} & \frac{h}{4\pi R_k} & \frac{+1}{2\pi R_k} \\ 0 & 0 & 0 & 0 & \frac{-1}{2\pi R_k} & 0 & \frac{1}{2\pi R_k} & 0 \\ 0 & 0 & 0 & \frac{1}{2\pi R_k} & 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} F_t \\ F_b \\ F_w \\ F_{zb} \\ F_{rk} \\ F_{ma} \\ F_{rb} \\ F_{mb} \end{bmatrix}$$

5

Fig. 7 Shell with Plate

Fig. 8 Plate Free Body

Fig. 9 Equivalent Loads on Plate

From simple plate formulas, if we assume p acts over the entire plate:

$$\delta_t = \delta_p = 0 \text{ (since we do not use these values)}$$

$$\delta_m = \text{rotation} \cong \alpha \left(\frac{T_2 - T_1}{h} \right) R_k + \left(\frac{3(1-\nu)R_k}{2\pi E_h^3} \right) \pi R_p^2 + \left(\frac{12(1-\nu)R_k}{E_h^3} \right) M$$

$$\delta_r \cong \alpha \left(\frac{T_2 - T_1}{2} \right) R_k + \left(\frac{R_k}{E_h} \right) N_r$$

$$\delta_z \cong \alpha \left(\frac{T_2 + T_1}{2} \right) h$$

or in matrix form

δ_t	=	0	0	0	0	0	x	F_t
δ_p		0	0	0	0	0		F_p
δ_m		$\alpha \left(\frac{T_2 - T_1}{h} \right) R_k$	$\frac{3(1-\nu)R_k R_p^2}{2E_h^3}$	$\frac{12(1-\nu)R_k}{E_h^3}$	0	0		M
δ_r		$\alpha \left(\frac{T_2 + T_1}{2} \right) R_k$	0	0	$\frac{R_k}{E_h}$	0		N_r
δ_z		$\alpha \left(\frac{T_2 + T_1}{2} \right) h$	0	0	0	0		N_z

Substituting the forces from Eq 5 into Eq 6 gives

δ_t	=	0	0	0	0	0	0	0	x	F_t
δ_p		0	0	0	0	0	0	0		F_p
δ_n		$\alpha \left(\frac{T_2 - T_1}{h} \right) R_k$	0	$\frac{3(1-\nu)R_k W_{zk}}{2\pi E_h^3}$	0	$\frac{3(1-\nu)}{\pi E_h^2}$	$\frac{-6(1-\nu)}{\pi E_h^3}$	$\frac{3(1-\nu)}{\pi E_h^3}$	$\frac{6(1-\nu)}{\pi E_h^3}$	F_w F_{zb}
δ_r		$\alpha \left(\frac{T_2 + T_1}{2} \right) R_k$	0	0	0	$\frac{-1}{2\pi E_h}$	0	$\frac{1}{2\pi E_h}$	0	F_{ra} F_{ma}
δ_z		$\alpha \left(\frac{T_2 + T_1}{2} \right) R_k$	0	0	0	0	0	0	0	F_{rb} F_{mb}

Finally, we want to express the plate deflections in terms of equivalent shell deflections:

$$\begin{aligned}\delta_w &= 0_b \text{ arbitrarily} \\ \delta_{zb} &= \delta_z, \text{ axially} \\ \delta_{ra} &= -\delta_r + 0.5h \delta_m, \text{ inward} \\ \delta_{ma} &= -\delta_m, \text{ clockwise} \\ \delta_{rb} &= \delta_r + 0.5h \delta_m, \text{ outward} \\ \delta_{mb} &= +\delta_m, \text{ counterclockwise}\end{aligned}$$

or in matrix form

$$\begin{bmatrix} \delta_t \\ \delta_p \\ \delta_w \\ \delta_{zb} \\ \delta_{ra} \\ \delta_{ma} \\ \delta_{rb} \\ \delta_{mb} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & +.5h & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & .5h & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} \delta_t \\ \delta_p \\ \delta_m \\ \delta_r \\ \delta_z \end{bmatrix}$$

Substituting the deflections from Eq 8 into Eq 10 gives:

$$\begin{bmatrix} \delta_t \\ \delta_p \\ \delta_w \\ \delta_{zb} \\ \delta_{ra} \\ \delta_{ma} \\ \delta_{rb} \\ \delta_{mb} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha \left(\frac{T_2 + T_1}{2} \right) h & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\alpha T_1 R_k & 0 & \frac{+3(1-\nu) R_k W_{zk}}{4\pi E_h^2} & 0 & \frac{+4-3\nu}{2\pi E_h} & \frac{-3(1-\nu)}{\pi E_h^2} & \frac{2-3\nu}{2\pi E_h} & \frac{3(1-\nu)}{\pi E_h^2} \\ \alpha \left(\frac{T_1 - T_2}{h} \right) R_k & 0 & \frac{-3(1-\nu) R_k W_{zk}}{2\pi E_h^3} & 0 & \frac{-3(1-\nu)}{\pi E_h^2} & \frac{6(1-\nu)}{\pi E_h^3} & \frac{-3(1-\nu)}{\pi E_h^2} & \frac{-6(1-\nu)}{\pi E_h^3} \\ \alpha T_2 R_k & 0 & \frac{3(1-\nu) R_k W_{zk}}{4\pi E_h^2} & 0 & \frac{2-3\nu}{2\pi E_h} & \frac{-3(1-\nu)}{\pi E_h^2} & \frac{4-3\nu}{2\pi E_h} & \frac{3(1-\nu)}{\pi E_h^2} \\ \alpha \left(\frac{T_2 - T_1}{h} \right) R_k & 0 & \frac{3(1-\nu) R_k W_{zk}}{2\pi E_h^3} & 0 & \frac{3(1-\nu)}{\pi E_h^2} & \frac{-6(1-\nu)}{\pi E_h^3} & \frac{3(1-\nu)}{\pi E_h^2} & \frac{6(1-\nu)}{\pi E_h^3} \end{bmatrix} \times \begin{bmatrix} F_b \\ F_p \\ F_w \\ F_{zb} \\ F_{ra} \\ F_{ma} \\ F_{rb} \\ F_{mb} \end{bmatrix}$$

Thus Eq 11 gives the required 8 x 8 flexibility matrix for the equivalent segment in Fig. 7.

C. Input Cards

Four things must be done for each special segment whose flexibility matrix is to be on input cards.

1. Set $E_t = 0$ in the 40I series of cards for each special segment.
2. Set $P_1 = P_0 = 0$ in the 60I series of cards for each special segment.
3. Set $W_{rk} = W_{mk} = 0$ and calculate the value of W_{zk} in the 70I series of cards for each special segment. In the plate example,

$$W_{zk} = (\pi R_p^2)(P_1 - P_2).$$

4. Put the flexibility matrix on 8 cards after the 1001 card. Each card is divided into 9 fields of 8 spaces each for a total of 72 spaces. In columns 65 to 72 is the card integer $100K + I$ where $K = \text{segment number, 1 to N}$ and $I = \text{row number, 1 to 8}$.

In the first 8 fields of card $100K + I$ are the 8 input flexibility coefficients for row I of the matrix for segment K .

Segment flexibility cards must be in increasing numerical order after the 1001 card. For example, if segments 3 and 42 have $E_t = 0$, then after card 1001 come cards 301, 302, ---, 308, 4201, 4202, ---, 4208.

ABSTRACT FORMAT

1. Name or Designation of Program.

The program name is SEAL SHELL-2. The program number at BAPL is M0110.

2. Computer for Which Program is Designed and Programming Languages Used.

Except for the PLOT routine, the program is written in FORTRAN IV for a machine with 32000 words. The program has been used on PHILCO-2000, IBM-704, and CDC-6600.

3. Nature of Physical Problem Solved.

The SEAL-SHELL-2 program determines stresses, strains, deflections, and reactions in a thick shell of revolution with axisymmetric loading. The loading consists of a temperature distribution, inside and outside pressure distributions, and circumferential forces and moments applied to the middle surface. The shell is linear-elastic with tensile, bending, and shear strains.

4. Method of Solution.

The shell is divided into 2 to 100 segments. Each segment is subjected to 11 deformation shapes, whose magnitudes are calculated by the principle of virtual work. That is, for small virtual changes in magnitude, the changes in strain energy and external work are determined; then equating the energy to the work gives a set of simultaneous equations which may be solved for the magnitudes.

5. Restrictions on the Complexity of the Problem.

The basic restrictions are the limit of 100 segments, and the limit of 11 deformation shapes per segment.

6. Related and Auxiliary Programs.

The program has been changed to allow a plot of the shape of the shell, as discussed in reference 2. The plot routine is discussed in reference 3.

7. Typical Running Time.

The time is about 5 minutes on the PHILCO-2000.

8. References.

(1) C. M. Friedrich, "SEAL-SHELL-2, A Computer Program for the Stress Analysis of a Thick Shell of Revolution with Axisymmetric Pressures, Temperatures, and Distributed Loads," WAPD-TM- 398, December, 1963.

(2) C. M. Friedrich, "SEAL-SHELL-2...Addendum I," WAPD-TM-398, to be issued.

(3) G. J. Pfeifer, "CDC-6600 Fortran Programming Bettis Environmental Report," WAPD-TM-668, January, 1967.

9. Unusual Features of the Program.

The program will handle thick shells of revolution, as well as the usual thin shell structures. As discussed in reference 2, an option allows the requestor to input influence coefficients of the flexibility matrix of any segment. The coefficients may be for any axisymmetric structure, such as a tube sheet in a heat exchanger. In this way, different branches of shells may be connected together.

Another application is to condense a group of segments into a single element which may be used in a series of problems and so cut down running time on the computer. Also, using the condensed group of segments as a single segment allows the user to specify more elements in the analysis of a critical region in the shell.

10. Status

The program is in production with all options tested.

11. Machine Requirements.

The program is written in FORTRAN IV and requires 32000 words of high speed memory.

12. Operating System on Monitor Under Which Program is Executed.

Any system using standard FORTRAN-IV may be used. At BAPL, the system of reference 3 is used.

13. Other Programming or Operating Information.

The plot routine is described in reference 3.

14. Name and Establishment of Author.

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