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Energy Dependence of Mean Square Charged Multiplicity
in Regge-Mueller Formalism

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Abstract

The energy dependence of the mean square multiplicity $\langle n^2 \rangle$ is obtained on the basis of a Regge-Mueller analysis, when a Pomeron with $\alpha_p(0) = 1$ and subsidiary trajectories with $\alpha_{p'}(0) = 1/2$ are present. The expression for $\langle n(n-1) \rangle$ is shown to be of the form $c_1(\ln s)^2 + c_2(\ln s) + c_3 + c_4 \frac{(\ln s)^2}{\sqrt{s}} + c_5 \frac{\ln s}{\sqrt{s}} + \frac{c_6}{\sqrt{s}}$, with the coefficients given in terms of certain well-defined integrals over physically measurable quantities. It is found that c_1 and c_4 must be positive. We discuss how the sign of c_2 can be negative to account for the present experimental data. From a knowledge of a similar expression for the mean multiplicity $\langle n \rangle$, plausible numerical estimates of the coefficients c_1 and c_2 are obtained. These two coefficients alone reproduce reasonably well the $\langle n_{ch}(n_{ch}-1) \rangle$ experimental data in p-p collisions up to a constant term.

I. Introduction

The recent measurements¹ of the mean multiplicity $\langle n_{ch} \rangle$ and the mean square multiplicity $\langle n_{ch}^2 \rangle$ of charged particles in high energy hadronic collisions provide a great deal of information about high energy collisions. These multiplicities can be expressed as integrals over inclusive cross-sections and hence the study of the energy dependence of these multiplicities is a very important means to check our understanding of inclusive cross-sections. Several models² have been proposed in the past to study hadronic interactions for infinitely large energies. One cannot claim with certainty that the present experimental observations have already reached this asymptotic region. There are already indications that the sub-asymptotic terms are important all the way up to ISR energies³. It is thus very important to know the sub-asymptotic models, so that one can profitably compare the present experimental data with the expectations of a theory. Unfortunately, most of the proposed models are not defined enough in the sub-asymptotic regions because of large freedoms inherent in the models. In this respect the Regge-Mueller⁴ description of the high energy inclusive reactions is unique in the sense that it has a definite prescription of calculating the sub-asymptotics. If the exchanged objects in a Regge-Mueller theory are taken to be simple poles that one is familiar with in the study of two-body reactions, then the asymptotic behavior of an inclusive cross-section is given by the Pomeron

exchange whereas the exchanges of lower Regge poles give the sub-asymptotics. It is known that the main features of two-body reactions can be reasonably well described by a Pomeron with $\alpha_p(0) = 1$ and four subsidiary trajectories f_0 , ω , ρ , and A_2 all with an intercept of $\alpha_{p,i}(0)$ approximately $1/2$. In the description of inclusive reactions, where only forward helicity non-flip protons are involved, we can neglect, as a good first approximation the isovector trajectories. Calculation of $\langle n_{ch} \rangle$ on the basis of such a Regge-Mueller model has already been done⁵ and they seem to accomodate well the present data from lab momentum 4 to 10^4 GeV/c. It has become clear, however, that the mean square multiplicity $\langle n_{ch}^2 \rangle$ rather than $\langle n_{ch} \rangle$ is more of a distinctive characteristic of hadronic models. It is therefore important to know the energy dependence of $\langle n_{ch}^2 \rangle$ in any theoretical model and we intend to discuss this from the Regge-Mueller theory. We consider Pomeron, ω , and P' exchanges, and derive the expression for the leading and the non-leading terms for $\langle n_{ch}(n_{ch}-1) \rangle$.

In section II we define the relevant couplings and write down the cross-sections in different regions of the phase-space. The mean multiplicities $\langle n_{ch} \rangle$ and $\langle n_{ch}^2 \rangle$ are then calculated keeping all the terms in this theory. The coefficients of these expansions are discussed in the next section and it is shown that reasonable fits of the data for large s are possible with our estimates of the leading terms.

II. Behavior of $\langle n \rangle$ and $\langle n^2 \rangle$

We consider for definiteness collisions of two protons each of mass m . We work throughout in the c.m. system so that the two incoming momenta are $p_a \equiv m(\cosh\theta, 0, 0, -\sinh\theta)$ and $p_b \equiv m(\cosh\theta, 0, 0, \sinh\theta)$. Assuming exchanges of Pomeron with $\alpha_p(0) \equiv \alpha_p = 1$ and f_0 and ω with $\alpha_i(0) \equiv \alpha_i = 1/2$, $i = \omega, f_0$ the total cross-section has the expansion

$$\sigma_t = \frac{1}{s} \left[e^{2\theta} \gamma_p^2 + \sum_i e^\theta \gamma_i^2 \right]; \quad (1)$$

γ_i 's are the Regge-particle couplings; in Eq(1) and in equations to follow all summations run over f_0 and ω . Consider now the one-particle inclusive reaction

$$p + p \rightarrow \pi^- + \text{Anything}, \quad (2)$$

for a π^- of momenta $p_1 \equiv \mu(\cosh\theta_1 \cosh\beta_1, \sinh\beta_1, 0, \sinh\theta_1 \cosh\beta_1)$, where μ is the pion mass. β_1 ranges from 0 to ∞ , whereas θ_1 varies between $\pm\theta_m$ where $\theta_m = \theta + \ln(m/\mu_t)$, μ_t being the transverse mass of the pion. For the fragmentation region where $(\theta + \theta_1)$ is less than a certain number θ_0 say, we have the expansion⁶,

$$\frac{d\sigma}{d\theta_1 d(\cosh\beta_1)^2} = \frac{1}{s} \left[e^{2\theta} f_p(\theta_1 + \theta, \beta_1) \gamma_p + \sum_i e^\theta f_i(\theta_1 + \theta, \beta_1) \gamma_i \right]; \quad (3)$$

an exactly similar expression holds at the other end where $(\theta - \theta_1)$ is less than θ_0 . The f_i 's are the usual couplings of the Regge pole i to the $(p\pi)$ system. In the pionization region where θ lies between $(-\theta + \theta_0)$ and $(\theta - \theta_0)$, the inclusive cross-section is given by

$$\begin{aligned} \frac{d\sigma}{d\theta_1 d(\cosh\beta_1^2)} &= \frac{1}{s} \left[e^{2\theta} f_{pp}(\beta_1) \gamma_p^2 \right. \\ &+ 2 \exp\left(\frac{3}{2}\theta\right) \cosh(1/2 \theta_1) \sum_i f_{ip}(\beta_1) \gamma_p \gamma_i \\ &\left. + \exp(\theta) \sum_i f_{ii}(\beta_1) \gamma_i^2 \right]. \end{aligned} \quad (4)$$

The f_{ii} are the four point coupling of Regge-poles i, i' to a pion line. We defer a discussion of the actual value of θ_0 until later in this section.

Consider now the two-particle inclusive cross-section.

$$p + p \rightarrow \pi^-(p_1) + \pi^-(p_2) + \text{Anything}, \quad (5)$$

where the momenta p_1 is defined as before and $p_2 = \mu(\cosh\theta_2 \cosh\beta_2, \sinh\beta_2 \cos\varpi_2, \sinh\beta_2 \sin\varpi_2, \sinh\theta_2 \cosh\beta_2)$. We divide the entire phase space into the following regions and write down the expansions appropriate to each region. We restrict ourselves to $\theta_2 > \theta_1$; an identical set of expansions hold for the complementary region $\theta_2 < \theta_1$.

(i) Simple fragmentation region characterized by $\theta_2 < -\theta + \theta_0$:

In this region we have

$$\begin{aligned} \frac{d\sigma}{d\theta_1 d(\cosh\beta_1^2) d\theta_2 d(\cosh\beta_2^2) d\varpi_2} &= \frac{1}{s} \left[e^{2\theta} D_p(\theta_1 + \theta, \theta_2 - \theta_1, \beta_1, \beta_2, \varpi_2) \right. \\ &\left. + \sum_i e^\theta D_i(\theta_1 + \theta, \theta_2 - \theta_1, \beta_1, \beta_2, \varpi_2) \right]; \end{aligned} \quad (6)$$

D_i 's represent the coupling of the Regge pole i to the $(p\pi\pi)$ system.

(ii) Double fragmentation region characterized by $\theta_1 < -\theta + \theta_0$ and $\theta_2 > \theta - \theta_0$: The inclusive cross-section in this region is given by

$$\frac{d\sigma}{d\theta_1 d(\cosh\beta_1^2) d\theta_2 d(\cosh\beta_2^2) d\omega_2} = \frac{1}{s} \left[e^{2\theta} f_p(\theta_1+\theta, \beta_1) f_p(\theta-\theta_2, \beta_2) \frac{1}{2\pi} \right. \\ \left. + \sum_i e^\theta f_i(\theta_1+\theta, \beta_1) f_i(\theta-\theta_2, \beta_2) \frac{1}{2\pi} \right]; \quad (7)$$

The f_i 's are the same ones occurring in Eq(3) so that we are building in factorization right from the beginning.

(iii) Pionization-Fragmentation region characterized by $-\theta < \theta_1 < -\theta+\theta_0$ and $-\theta+\theta_0 < \theta_2 < \theta-\theta_0$: The cross-section in this domain is given by

$$\frac{d\sigma}{d\theta_1 d(\cosh\beta_1^2) d\theta_2 d(\cosh\beta_2^2) d\omega_2} = \frac{1}{s} \cdot \frac{1}{2\pi} \left[\exp(2\theta) \gamma_p f_p(\theta_1+\theta, \beta_1) f_{pp}(\beta_2) \right. \\ + \exp\left(\frac{3\theta+\theta_2}{2}\right) \sum_i \gamma_i f_p(\theta_1+\theta, \beta_1) f_{pi}(\beta_2) \\ + \exp\left(\frac{3\theta-\theta_2}{2}\right) \sum_i \gamma_p f_i(\theta_1+\theta, \beta_1) f_{pi}(\beta_2) \\ \left. + \exp(\theta) \sum_i \gamma_i f_i(\theta_1+\theta, \beta_1) f_{ii}(\beta_2) \right]. \quad (8)$$

We next come to the region where both the pions are emitted in the central region. We further subdivide this into two domains depending on whether $(\theta_2-\theta_1)$ is bigger than or less than a certain fixed number δ . The cross-sections in these two regions have the following form.

(iv) Simple pionization region characterized by $-\theta+\theta_0 < \theta_1 < \theta_2 < \theta-\theta_0$ and $\theta_2-\theta_1 < \delta$: In this region

$$\frac{d\sigma}{d\theta_1 d(\cosh\beta_1^2) d\theta_2 d(\cosh\beta_2^2) d\omega_2} = \frac{1}{s} \\ \left[\exp(2\theta) \exp(\theta_1-\theta_2) g_{pp}(\theta_2-\theta_1, \beta_1, \beta_2, \omega_2) \gamma_p^2 \right]$$

$$\begin{aligned}
 & + \exp\left(\frac{3}{2}\theta\right) \exp(\theta_1 - \theta_2/2) \sum_i g_{ip}(\theta_2 - \theta_1, \beta_1, \beta_2, \omega_2) \gamma_p \gamma_i \\
 & + \exp\left(\frac{3}{2}\theta\right) \exp(\theta_1/2 - \theta_2) \sum_i g_{ip}(\theta_2 - \theta_1, \beta_1, \beta_2, \omega_2) \gamma_p \gamma_i \\
 & + \exp(\theta) \exp(\theta_1/2 - \theta_2/2) \sum_i g_{ii}(\theta_2 - \theta_1, \beta_1, \beta_2, \omega_2) \gamma_i^2 \Big]. \quad (9)
 \end{aligned}$$

The g_{ii} , represent the six-point coupling of the Regge poles i, i' to the $(\pi\pi)$ system.

(v) Double pionization region characterized by $-\theta + \theta_0 < \theta_1 < \theta_2 < \theta - \theta_0$ and $\theta_2 - \theta_1 > \delta$. The expansion for this region is

$$\begin{aligned}
 \frac{d\sigma}{d\theta_1 d(\cosh\beta_1^2) d\theta_2 d(\cosh\beta_2^2) d\omega_2} &= \frac{1}{s} \cdot \frac{1}{2\pi} \\
 & \Big[\exp(2\theta) \gamma_p^2 f_{pp}(\beta_1) f_{pp}(\beta_2) \\
 & + \exp\left(2\theta + \frac{\theta_1}{2} - \frac{\theta_2}{2}\right) \sum_i \gamma_p^2 f_{ip}(\beta_1) f_{ip}(\beta_2) \\
 & + \exp\left(\frac{3\theta}{2} + \frac{\theta_2}{2}\right) \sum_i \gamma_p \gamma_i f_{pp}(\beta_1) f_{ip}(\beta_2) \\
 & + \exp\left(\frac{3\theta}{2} + \frac{\theta_1}{2}\right) \sum_i \gamma_p \gamma_i f_{pi}(\beta_1) f_{ii}(\beta_2) \\
 & + \exp\left(\frac{3\theta}{2} - \frac{\theta_1}{2}\right) \sum_i \gamma_p \gamma_i f_{ip}(\beta_1) f_{pp}(\beta_2) \\
 & + \exp\left(\frac{3\theta}{2} - \theta_2\right) \sum_i \gamma_p \gamma_i f_{ip}(\beta_1) f_{pp}(\beta_2) \\
 & + \exp\left(\frac{3\theta}{2} - \frac{\theta_2}{2}\right) \sum_i \gamma_p \gamma_i f_{ii}(\beta_1) f_{pi}(\beta_2) \\
 & + \exp(\theta + \theta_2 - \theta_1) \sum_i \gamma_i^2 f_{ip}(\beta_1) f_{ip}(\beta_2)
 \end{aligned}$$

$$+ \exp(\theta + \frac{1}{2}\theta_2 - \frac{1}{2}\theta_1) \sum_i \gamma_i^2 f_{ii}(\beta_1) f_{ii}(\beta_2) \quad (10)$$

The mean and mean square multiplicities are related to inclusive cross-section by the relations:

$$\sigma_t \langle n_- \rangle = \int_1^\infty d(\cosh\beta_1^2) \int_{-\theta_m}^{\theta_m} d\theta_1 \frac{d\sigma}{d\theta_1 d(\cosh\beta_1^2)} \quad (11)$$

and

$$\begin{aligned} \sigma_t \langle n_- (n_- - 1) \rangle &= \int_0^{2\pi} d\varphi_2 \int_1^\infty d(\cosh\beta_1^2) \int_1^\infty d(\cosh\beta_2^2) \\ &\times \int_{-\theta_{m1}}^{\theta_{m1}} d\theta_1 \int_{-\theta_{m2}}^{+\theta_{m2}} d\theta_2 \frac{d\sigma}{d\theta_1 d(\cosh\beta_1^2) d\theta_2 d(\cosh\beta_2^2) d\varphi_2} \end{aligned} \quad (12)$$

where $\theta_{mi} = \theta + \ln \frac{m}{\mu \cosh\beta_i}$, $i = 1, 2$. Using the expressions

in Eqs(6) through (10), we obtain the following expansion for the mean multiplicity⁵:

$$\begin{aligned} \sigma_t \langle n_- \rangle &= F_{pp} \gamma_p^2 \ln S + 2 \left[\gamma_p F_p - \theta_0 F_{pp} \gamma_p^2 + 2 \sum_i F_{pi} \gamma_p \gamma_i \exp(-\frac{1}{2} \theta_0) \right] \\ &+ \sum_i F_{ii} \gamma_i^2 \ln S / \sqrt{s} \\ &+ \frac{2}{\sqrt{s}} \left[\sum_i \gamma_i F_i - \theta_0 \sum_i F_{ii} \gamma_i^2 - 2 \sum_i F_{pi} \gamma_p \gamma_i \exp(1/2 \theta_0) \right]; \end{aligned} \quad (13)$$

where

$$F_i = \int_1^\infty d(\cosh\beta_1^2) \int_{-\theta_{m1}}^{-\theta_{m1} + \theta_0} d\theta_1 F_i(\theta + \theta_1, \beta_1); \quad i = p, f_0, w \quad (14)$$

and

$$F_{ii'} = \int_1^{\infty} d(\cosh \beta_1^2) f_{ii'}(\beta_1); i, i' = p, f_o, w. \quad (15)$$

The functions F_i and $F_{ii'}$ are independent of energy and are functions of θ_o only. Further, since w has negative G-parity $F_{pw} = F_{f_o w} = 0$. For $\langle n_-(n_- - 1) \rangle$ we obtain

$$\sigma_t \langle n_-(n_- - 1) \rangle = \sigma_t \left[c_1 (\ell ns)^2 + c_2 (\ell ns) + c_3 + c_4 \frac{(\ell ns)^2}{\sqrt{s}} + c_5 \frac{\ell ns}{\sqrt{s}} + \frac{c_6}{\sqrt{s}} \right], \quad (16)$$

where c_i 's are given as follows:

$$\sigma_t c_1 = g_{pp}^p, \quad (17)$$

$$\begin{aligned} \sigma_t c_2 = & -4(\theta_o + \delta) g_{pp}^p + \sum_i 4g_{pp}^i \exp(-\delta) + \sum_i 8g_{ip}^p \exp(-\theta_o/2) \\ & + 2\gamma_p^2 h_{pp}^3 + 4\gamma_p F_p F_{pp}, \end{aligned} \quad (18)$$

$$\begin{aligned} \sigma_t c_3 = & 4g_{pp}^p (\theta_o + \delta)^2 - \sum_i 8g_{pp}^i (\theta_o + \delta + 1) \exp(-\delta) \\ & + 8 \sum_i g_{ii}^p \exp(-\theta_o) - \sum_i 16 g_{ip}^p (\theta_o + \delta + 1) \exp(-\theta_o/2) \\ & + 16 \exp(-\theta_o/2 - \delta) \sum_i g_{ip}^i - 4\theta_o \gamma_p^2 h_{pp}^{(3)} \\ & - 2\gamma_p^2 h_{pp}^{(1)} + 8 \sum_i g_{ip}^{(3)} \gamma_p \gamma_{p'} \exp(-\theta_o/2) \\ & - 8\theta_o \gamma_p F_p F_{pp} + 8 \exp(-\theta_o/2) \sum_i \gamma_i F_i F_{pi} \\ & + 8 \exp(-\theta_o/2) \sum_i \gamma_p F_i F_{pi} + 2\gamma_p D_p + 2F_p^2, \end{aligned} \quad (19)$$

$$\sigma_t c_4 = \sum_i g_{ii}^i, \quad (20)$$

$$\begin{aligned} \sigma_t c_5 = & -4(\theta_0 + \delta) \sum_i g_{ii}^i - 4 \exp(\delta) \sum_i g_{ii}^p - 8 \exp(\theta_0/2) \sum_i g_{ip}^i \\ & + 2 \sum_i \gamma_i^2 h_{ii}^{(2)} + 4 \sum_i \gamma_i F_i F_{ii}, \end{aligned} \quad (21)$$

and

$$\begin{aligned} \sigma_t c_6 = & 8 \sum_i g_{pp}^i \exp(\theta_0) + 4(\theta_0 + \delta)^2 \sum_i g_{ii}^i \\ & + 8(\theta_0 + \delta - 1) \exp(\delta) \sum_i g_{ii}^p \\ & + 16(\theta_0 + \delta - 1) \exp(\theta_0/2) \sum_i g_{ip}^i \\ & + 16 \exp(\theta_0/2 + \delta) \sum_i g_{ip}^p - 4\theta_0 \sum_i \gamma_i^2 h_{ii}^{(2)} \\ & - 2 \sum_i \gamma_i^2 h_{ii}^{(4)} - 8 \exp(\theta_0/2) \sum_i \gamma_p \gamma_i h_{ip}^{(2)} \\ & - 8 \theta_0 \sum_i \gamma_i F_i F_{ii} - 8 \exp(\theta_0/2) \sum_i \gamma_i F_p F_{pi} \\ & - 8 \exp(\theta_0/2) \sum_i \gamma_p F_i F_{pi} + 2 \sum_i \gamma_i D_i + 2 \sum_i F_i^2. \end{aligned} \quad (22)$$

We have defined here

$$g_{i,i}^i = \gamma_i \gamma_i F_{ii} F_{ii} \quad (23)$$

The $h_{ii}^{(1)}$, etc. are given by:

$$\begin{aligned} h_{ii}^{(\lambda)} = & \int_0^{2\pi} d\varphi_2 \int_1^\infty d(\cosh \beta_1^2) \int_1^\infty d(\cosh \beta_2^2) \int_0^\delta d\theta_{21} h^{(\lambda)}(\theta_{21}) \\ & \times g_{ii}(\theta_{21}, \beta_1, \beta_2, \varphi_2), \quad (\lambda=1, \dots, 4), \end{aligned} \quad (24)$$

with $h^{(1)}(x) = x \exp(-x)$, $h^{(2)}(x) = \exp(-x/2)$, $h^{(3)}(x) = \exp(-x)$,
and $h^{(4)}(x) = x \exp(-x/2)$; The D's are

$$D_i = \int_0^{2\pi} d\varphi_2 \int_1^{\infty} d(\cosh\beta_1^2) \int_1^{\infty} d(\cosh\beta_2^2) \int_{-\theta_{m1}}^{-\theta_{m1}+\theta_0} d\theta_1 \int_{-\theta_{m2}}^{-\theta_{m2}+\theta_0} d\theta_2$$

$$\times D_i(\theta_1+\theta, \theta_2-\theta_1, \beta_1, \beta_2, \varphi_2). \quad (25)$$

We conclude this section with the following two observations:

(1) To be technically correct, we should use the expansion in Eq(1) for σ_t to obtain the final expansion for $\langle n_- \rangle$ and $\langle n_-(n_-1) \rangle$. However, for all practical purposes σ_t is a remarkably constant quantity from the accelerator energies to ISR energies, so that in studying the energy dependence of $\langle n_- \rangle$ and $\langle n_-(n_-1) \rangle$ via Eqs(13) and (16) it is reasonable to assume σ_t to be constant.

(2) The quantity θ_0 we have introduced divides the phase space into fragmentation and pionization domains. It is trivially true that if the expansion used in various regions matches at the boundaries, the expressions for $\langle n_- \rangle$ and $\langle n_-(n_-1) \rangle$ will not be sensitive to θ_0 as θ_0 is varied in the pionization-fragmentation boundary region. To obtain a reasonable value of θ_0 consider the subenergy in the one-particle inclusive reaction given by

$$(p_0 + p_1)^2 = m^2 + \mu^2 + 2\mu_t \exp(\theta + \theta_1). \quad (26)$$

From our experience in analysis of two-body scattering, we can now expect that if the sub-energy (26) is greater than say

$5(\text{GeV})^2$, it can be considered asymptotic. In other words, the expansions we have used in the pionization region with P , f_0 , w exchanges, would be reasonable if the sub-energy (26) is greater than say about $5(\text{GeV})^2$. The boundary of the pionization region is $\theta_1 = -\theta + \theta_0$, so that we obtain an estimate for $\theta_0 \approx 1.7$. In a similar way, the parameter δ divides the entire pionization into simple and double pionization regions depending on the sub-energy of the two pions. Following an analogous argument as used above for θ_0 we can estimate a value for δ which would not be very different from that of θ_0 .

III. Numerical Evaluation and Discussion

We have seen that the coefficients in the expansion for $\langle n_- \rangle$ and $\langle n_- (n_- - 1) \rangle$ can be evaluated once we know certain Regge-particle couplings defined in the previous section. We shall now discuss possible numerical estimates for these coefficients. We note at the very beginning that precise predictions for these coefficients are not possible at present, due mainly to our lack of exact knowledge of the couplings of the subsidiary trajectories in inclusive cross-sections.

First, the best fit to the total cross-section data tells us that⁷

$$\gamma_p^2 = 37.4. \quad (27)$$

Further, in the approximation of neglecting couplings of ρ and A_2 to the forward helicity non-flip amplitude, we have from Ref. 7

$$\gamma_f^2 = \text{coefficient of } 1/\sqrt{s} \text{ term in } (\sigma_{pp} + \sigma_{\bar{p}p})/2 \approx 42.$$

Next, Tow⁵ has carried out a fit for $\langle n_{ch} \rangle$ of the form

$$\langle n_{ch} \rangle = A + b \ln s + d s^{-1/2}, \quad (29)$$

and the values for the best fit are $a = -4.8$, $b = 2.0$, and $d = 10.0$. This fit is comparable to the fits obtained by other people^{1, 14} and we shall use it only to obtain some rough estimates. Assuming $n_{ch} = 2n_- + 2$, we then immediately obtain

$$F_{pp} \approx 1.0, \quad (30)$$

which tells us that $c_1 = F_{pp}^2 \approx 1.0$.

Now the fit (29) does not distinguish between $1/\sqrt{s}$ and $\ln s/\sqrt{s}$ term, so that in comparing (29) with (13), we will replace $\ln s/\sqrt{s}$ in (13) by an effective average value $(\ln s)_{av}/\sqrt{s}$. Assuming that, we have

$$10.0 \approx \frac{2}{\sigma_t} \left[\sum_i \left\{ (\ln s)_{av} - 2\theta_0 \right\} F_{ii} \gamma_i^2 - 4 \exp(\theta_0/2) F_{pf} \gamma_p \gamma_f + \sum_i 2\gamma_i F_{ii} \right]. \quad (31)$$

For large θ_0 , the R.H.S. of (31) will be dominated by the second term. For our choice of θ_0 , we will retain as a first approximation only the second term; for $\theta_0 = 2$, this gives

$$F_{pf} \approx -0.43. \quad (32)$$

The -ve sign of F_{pf} is evident from the fact that the pionization cross-section rises with energy from the accelerator to ISR energies. The fragmentation integral F_p can be easily estimated, because it is known that fragmentation reaches a limiting behavior rather early and at accelerator energies

reasonably detailed information is available⁸ for us to estimate the integral F_p in Eq(14). For $\theta_0=2$, our estimate is

$$F_p \approx 2.0 \quad (33)$$

At this stage, we are unable to make numerical estimates of the other quantities needed to determine all the c_i 's. However, by using duality arguments, we can expect that in the pionization region, the double pionization formula given in Eq(10) continues to describe the simple pionization region IV in an average sense. This means that a reasonable estimate for c_2 can be made by putting $\delta=0$ and $h_{pp}^3 = 0$ in Eq(18). Numerically this means

$$c_2 \approx -8 + 0.74 - 1.34 + 1.31 = -7.29. \quad (34)$$

Because of our lack of knowledge of the c_i 's beyond c_2 , the best we can do at this stage in comparing experimental data on $\langle n_{ch}(n_{ch}-1) \rangle$ with the Regge-Mueller model, is to attempt a fit to the data retaining the first three terms in Eq(16) (which are the only ones that survive asymptotically), using the determined values of c_1 and c_2 and treating c_3 as a parameter. To obtain $\langle n_{ch}(n_{ch}-1) \rangle$ from our results we use the expression $4\langle n_-(n_- - 1) \rangle + 5\langle n_{ch} \rangle - 8$ for it. Figure 1 shows the fit obtained for $\theta_0 = 2.0$. We also show in the figure the results for $\theta_0 = 1.0$. The two curves show some difference, the reason being that in our approximate estimates for the various couplings, the fragmentation and pionization cross-section have failed to merge into each other smoothly. Because of the uncertainties in determining the Regge parameters, and our inability to make numerical estimates for the coeffi-

icients c_3 to c_6 , we cannot make too strong a conclusion from our fits. All that we can say is that the experimental data seem to be consistent with our model and the negative sign of c_2 which is a marked feature of the data, comes out reasonably well in our model.

Let us now see what more we can learn about the sign of the $\ln s$ coefficient. We note that only three regions of the phase space contribute to this coefficient. Out of these three the contributions coming from the pionization-fragmentation and the simple pionization regions (via Pomeron exchange) give positive contributions to this coefficient. Our rough evaluation of the fragmentation integral tells us that this gives a small contribution, $4F_p/\gamma_p$, of the order of 1. The negative sign can then only arise from the Double-Pionization contributions which depend on three factors θ_0 , δ , and F_{pf} . We have some ideas about the possible ranges of these parameters. We have plotted in Figure 2 the contributions of the double-pionization region to c_2 , denoted by $c_2^{D.P.}$ as a function of F_{pf} for several reasonable values of θ_0 and δ , namely from 0 to 2. A clear picture is indicated in this graph. If F_{pf} has a small negative value, say between 0 and -1.5, then $c_2^{D.P.}$ is negative and reasonably large to produce a negative $\ln s$ coefficient in $\langle n(n-1) \rangle$. This should be true for all reasonable θ_0 and δ . We further find that if $\delta = 0$ is a good approximation, i.e. the simple pionization region can be approximated by the double pionization region, then Figure 2 shows that F_{pf} must be between -2 and 1 to account for the negative c_2 . Thus the sign of c_2 is

quite delicately balanced by θ_0 , onset of the pionization region, & the limit of simple pionization in 2 particle inclusive reactions and the strength of the coupling F_{pf} .

It is worthwhile at this point to mention the results that have been derived for other models. In multiperipheral models¹⁰, one obtains the $(\ln s)^2$ leading term for $\langle n(n-1) \rangle$, but so far one has no prediction for the succeeding terms. In simple fragmentation models¹¹ one finds the cross-section for n charged particle production $\sigma_n \propto 1/n^2$, from which all moments can be calculated. Experimental data¹ disagree with this simple prediction. A modification of this simple fragmentation model has been made by Hwa¹² and by Abarbanel and Kane¹³ by taking into account the effect of the minimum momentum transfer. These latter results can be made to agree with data¹⁴ by certain choices of parameters in the model but then the elegance of the model is somewhat lost. These fragmentation models are again asymptotic models and no estimates are available as to how the asymptotic values of the various observables are approached. As we have stated before, our work shows that once some more information on the inclusive cross-sections is available, we will be able to predict all the sub-asymptotic terms and a clear confrontation with experimental data on both $\langle n \rangle$ and $\langle n(n-1) \rangle$ can be made in this theory.

References and Footnotes

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Figure Captions

Fig. 1. The available experimental data for $\langle n_{ch}(n_{ch}-1) \rangle$ and the predictions for large s with the coefficients estimated in section 3 are plotted as a function of s . The solid curve is for $\theta_0 = .2$ and is given by $55 + 4(\ln s)^2 - 29\ln s + 5\langle n_{ch} \rangle$. The dashed curve is for $\theta_0 = 1$ and is given by $18 + 4(\ln s)^2 - 21\ln s + 5\langle n_{ch} \rangle$. The constant term in both cases is adjusted for the fit.

Fig. 2. Contributions of the Double-Pionization regions to the coefficient of the $\ln s$ term in $\langle n_-(n_- - 1) \rangle$, $c_2^{D.P.}$, are plotted as a function of F_{pf} for different values of θ_0 and δ . We have set $F_{pp} = 1$ and used the value of γ_f/γ_p as given by Eqs(28) and (29).

Fig. 1

Fig. 2