**THE COMPTON COOLING OF ELECTRONS BY PHOTONS:
DEPENDENCE ON THE PHOTON ENERGY SPECTRUM**

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MASTER

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THE COMPTON COOLING OF ELECTRONS BY PHOTONS: DEPENDENCE ON THE PHOTON ENERGY SPECTRUM

Abstract

The rate (per electron) at which a Maxwellian distribution of electrons with a temperature (T_e) loses energy via Compton scattering to a Planckian distribution of photons with an energy density (u) can be written:

$$\dot{E} = 4(\sigma_0/mc)u[kT_e - k(u/a)^{1/4}]$$

where (σ_0) is the classical Thompson scattering cross section and (a) is the Stefan-Boltzmann constant. The corresponding rate is calculated for (non-

Planckian) photon spectra associated with a uniform fully-ionized isothermal gas sphere in local thermodynamic equilibrium.

It is found that the formula quoted above overestimates the cooling rate if scattering is significant and absorption is negligible; underestimates the cooling rate if the converse is true; and is approximately correct if both scattering and absorption are negligible.

The photon spectrum that maximizes the cooling rate is also determined.

Introduction

The rate at which electrons are cooled by photons via Compton scattering is determined by three parameters^{*}:

1. The average electron energy $\langle E_e \rangle$.
2. The energy density u of the radiation field.
3. The (weighted) average photon energy $\langle (1+n)h\nu \rangle$. This average is taken over the energy spectrum of the photons, and the weighting factor $(1+n)$ is the familiar factor that accounts for stimulated emission.

In a three-temperature code such as LASNEX the first two parameters are known, in fact they provide the definition

^{*}We assume the radiation field to be isotropic, and ignore relativistic effects.

of the electron temperature T_e and the radiation temperature T_r according to the relations

$$kT_e = (2/3)\langle E_e \rangle, \quad (1)$$

$$T_r = (u/a)^{1/4}, \quad (2)$$

where $a (= 7.56 \times 10^{-15} \text{ erg/cm}^3 \text{ } ^\circ\text{K}^4)$ is the Stefan-Boltzmann radiation constant. The remaining parameter is assigned the value $4kT_r$ appropriate to a Planckian photon spectrum,

$$\langle (1+n)h\nu \rangle = 4kT_r \text{ (Planck spectrum)}, \quad (3)$$

T_r being defined in terms of the energy density u according to Eq. (2).^{*}

^{*}Eq. (2) will be taken as the defining relation for the radiation temperature T_r throughout this paper.

This procedure is of course valid if the photon spectrum is indeed Planckian. Even if the spectrum is not Planckian, however, there is a temptation to use this procedure because of its simplicity. It is therefore of interest to determine the extent to which it is valid when applied to non-Planckian photon spectra.

We shall see that this procedure is approximately correct so long as the

photon spectrum itself is not significantly influenced by Compton scattering; that is, when we are dealing with an absorbing rather than a scattering atmosphere. An estimate of the correction that is required when the effect of Compton scattering on the photon spectrum is not negligible will be given. The maximum possible Compton cooling rate and the corresponding photon spectrum will also be derived.

The Compton Cooling Rate

The rate (per electron) of Compton cooling of electrons by photons is given by the expression

$$\dot{E} = (\sigma_0/mc) \int_0^\infty [4kT_e - (1+n)h\nu] u_\nu d\nu \quad (4)^*$$

$$= (\sigma_0/mc) u [4kT_e - \langle (1+n)h\nu \rangle] \quad (5)$$

where σ_0 (≈ 0.665 barns) is the classical Thomson scattering cross section, T_e is the electron temperature defined in accordance with Eq. (1), ν is the photon frequency, u_ν is the energy density spectrum of the photons,

$$u = \int_0^\infty u_\nu d\nu = aT_R^4, \quad (6)$$

and n is the expected number of photons per radiation oscillator of frequency ν ,

$$n = (c^3/8\pi h\nu^3) u_\nu. \quad (7)$$

Since n is nonnegative we can immediately draw from Eq. (4) the important

* This convenient form of the expression for the Compton cooling rate is obtained directly from Eq. (5) of R. Weymann, *Phys. Fluids* 11, 2112 (1965), by multiplying by $h\nu$ and performing two integrations by parts over all momentum space.

conclusion that: the effect of stimulated emission is always to reduce the rate of electron cooling. Or: The neglect of stimulated emission will always lead to an overestimate of the Compton cooling rate.

If we assume the photon spectrum to be Planckian, that is

$$n = 1/[\exp(h\nu/kT_R) - 1], \quad (8)$$

and ask for the Compton cooling rate contributed by photons of frequency less than ν_0 , we obtain the result

$$(mc/\sigma_0) \dot{E} = 4(kT_e - kT_R) \int_0^{\nu_0} u_\nu d\nu + kT_R \nu_0 u_\nu(\nu_0). \quad (9)$$

The Compton cooling $\dot{E}_P$ due to a Planck spectrum at a temperature T_R , or $\dot{E}_T$ due to a truncated Planck spectrum at "temperature" T_e , follow directly from Eq. (9):

$$\dot{E}_P = 4u(\sigma_0/mc)(kT_e - kT_R), \quad (\nu_0 = \infty), \quad (10)$$

$$\dot{E}_T = (\sigma_0/mc) kT_e \nu_0 u_\nu(\nu_0), \quad (T_R = T_e). \quad (11)$$

The Maximum Possible Compton Cooling Rate and the Corresponding Photon Energy Spectrum

We wish to determine that photon energy spectrum u_ν which maximizes the Compton cooling rate, the total energy density u of the radiation field being held constant. A glance at Eq. (4) shows that we must find u_ν such that

$$\int_0^\infty (1+n)h\nu u_\nu d\nu = \text{Minimum}, \quad (12)$$

subject to the constraint that

$$\int_0^\infty u_\nu d\nu = u = \text{Constant}. \quad (13)$$

Fig. 1. Normalized photon energy spectra, comparing the Planck and bremsstrahlung distributions with one that maximizes the Compton cooling rate.

This is a standard type of variational problem and is readily solved by the method of undetermined multipliers. One finds that u_ν is given by Eq. (7) where

$$n = \begin{cases} (1/2) [(\nu_{\max}/\nu) - 1], & \nu < \nu_{\max} \\ 0, & \nu > \nu_{\max} \end{cases} \quad (14)$$

and

$$\begin{aligned} h\nu_{\max} &= (2\pi/10^{1/4}) kT_r \\ &= (3.5333) kT_r. \end{aligned} \quad (15)$$

The (normalized) energy spectrum u_ν (curve C) that maximizes the Compton cooling rate is compared with the (normalized) Planck spectrum (curve P) and the (normalized) bremsstrahlung spectrum (curve B) in Fig. 1.

Using this spectrum u_ν we find the maximum Compton cooling rate to be

$$\dot{E}_{\max} = 4 \left(\frac{\sigma_0}{mc} \right) u k T_e \left[1 - \left(\frac{4\pi}{10^{5/4}} \right) \frac{T_r}{T_e} \right]. \quad (16)$$

The ratio of the cooling rate $\dot{E}_P$ by a Planck spectrum (see Eq. 10) to the maximum rate possible is given by:

$$\dot{E}_P / \dot{E}_{\max} = (1 - r) / [1 - (0.707)r], \quad (17)$$

where $r = T_r / T_e$. It is obvious from this result that

$$\dot{E}_P \xrightarrow{r \rightarrow 0} \dot{E}_{\max}, \quad (18)$$

or: A Planckian spectrum of photons provides the maximum possible rate of Compton cooling in the low temperature limit ($T_r/T_e \ll 1$). In fact $\dot{E}_P$ is 77% of the maximum rate possible for T_r/T_e as large as 1/2.

It also follows from Eq. (16) that: If $T_r > (1.4151)T_e$, electrons are always heated by radiation no matter what the photon spectrum may be.

Finally, one can obtain the unrestricted maximum rate of Compton cooling by choosing T_r in Eq. (16) so that $\dot{E}_{\max}$ is maximized. The required value of T_r is

$$T_r = (2/\pi) 10^{1/4} T_e, \quad (19)$$

and the unrestricted maximum cooling rate is given by:

$$\dot{E}_{\max} = \frac{1}{2} \left(\frac{4}{\pi} \right) \left(\frac{\sigma_0}{mc} \right) (a T_e^4) k T_e. \quad (20)$$

This is the maximum rate of Compton cooling achievable by any radiation field, whatever its spectrum or energy density.

Compton Cooling by a Planck-Bremsstrahlung Spectrum

If scattering is negligible, the photon spectrum u_ν at the center of a homogeneous isothermal gas sphere of radius R in local thermodynamic equilibrium is given by the relation

$$u_\nu = u_\nu^P \left\{ 1 - \exp \left[-(R/c) (\epsilon_\nu / u_\nu^P) \right] \right\}. \quad (21)$$

This spectrum has the property that

$$u_\nu = \begin{cases} u_\nu^P, & R \rightarrow \infty \\ (R/c) \epsilon_\nu, & R \rightarrow 0, \end{cases} \quad (22)$$

where u_ν^P is the Planckian energy density spectrum and ϵ_ν is the spontaneous emission rate spectrum per unit volume of the radiating plasma. We assume the plasma

ions to be completely stripped of electrons, so that the only contribution to the emission rate ϵ_ν is due to bremsstrahlung. We have then

$$\epsilon_\nu = (hb/2kT_e) K_0(x/2) \exp(-x/2), \quad (23)$$

$$b = (4Z^2/137\pi^2) N_e N_i \sigma_0 c^2 \sqrt{2\pi m k T_e}, \quad (24)$$

where N_e and N_i are the number of electrons and ions (of charge Z) per unit volume, $x = h\nu/kT_e$, and K_0 is the modified Bessel function of zero order and second kind.

The Compton cooling rate $\dot{E}_{PB}$ resulting from the "Planck-bremsstrahlung" spectra defined by Eqs. (21) and (23) is

conveniently expressed in terms of dimensionless quantities as follows:

$$\dot{E}_{PB} = (\sigma_0/mc) (aT_e^4) (4kT_e)z, \quad (25)$$

where

$$z = \int_0^\infty [1 - (1+L)(x/4)] PS dx, \quad (26)$$

$$n = S/(e^x - 1), \quad (27)$$

$$S = 1 - e^{-\tau}, \quad \tau = \alpha B/P, \quad (28)$$

$$B = (1/2) \exp(-x/2) K_0(x/2), \quad (29)$$

$$P = (15/\pi^4) x^3/(e^x - 1). \quad (30)$$

The functions $B(x)$ and $P(x)$ are the normalized bremsstrahlung and Planck

Table 1. Properties of the Planck-bremsstrahlung spectra.

α	$(T_r/T_e)^4$	z	T_r/T_e	$\frac{\langle(1+n)h\nu\rangle}{kT_e}$	$\langle h\nu \rangle/kT_e$	$\frac{\langle h\nu \rangle}{\langle(1+n)h\nu\rangle}$	z/y	$\dot{E}_{PB}/\dot{E}_P$
0	0	0	0	0.667	0.667	1.000	0.833	0.833
10^{-6}	9.83×10^{-7}	8.14×10^{-7}	0.0315	0.688	0.678	0.985	0.828	0.855
10^{-5}	9.59×10^{-6}	7.87×10^{-6}	0.0557	0.718	0.694	0.967	0.820	0.867
10^{-4}	9.07×10^{-5}	7.30×10^{-5}	0.0976	0.784	0.732	0.934	0.804	0.891
10^{-3}	8.05×10^{-4}	6.21×10^{-4}	0.168	0.915	0.813	0.889	0.771	0.927
0.01	6.31×10^{-3}	4.49×10^{-3}	0.282	1.15	0.977	0.850	0.712	0.992
0.1	0.0400	0.0246	0.447	1.54	1.29	0.838	0.614	1.11
0.3	0.0866	0.0479	0.542	1.79	1.51	0.844	0.553	1.21
1	0.182	0.0863	0.653	2.10	1.82	0.867	0.475	1.37
3	0.321	0.126	0.753	2.43	2.16	0.889	0.392	1.59
10	0.523	0.153	0.850	2.83	2.58	0.912	0.293	1.96
30	0.717	0.142	0.920	3.21	2.99	0.931	0.199	2.49
100	0.882	0.093	0.969	3.58	3.39	0.947	0.105	3.39
∞	1.000	0	1.000	4.000	3.83	0.958	0	∞

$$\alpha = Rb/\sigma T_e^4 = (0.459 (Z^3/A^2) \rho^2 R/T_e^{7/2}), \quad y = (T_r/T_e)^4$$

$$\dot{E}_{PB} = (\sigma_0/mc) (aT_e^4) (4kT_e)z$$

spectra. They are shown in Fig. 1 as the graphs labeled (B) and (P). The normalized spectrum that maximizes the Compton cooling rate is shown in Fig. 1 as the graph labeled (C). It is given analytically in terms of x by the relation

$$C = (30/\pi^4) x^3 [(3.5333/x) - 1]. \quad (31)$$

The radiation temperature T_r is determined by the relation

$$(T_r/T_e)^4 = y = \int_0^\infty PS \, d\kappa, \quad (32)$$

and the dimensionless parameter α of the one-parameter family of Planck-bremsstrahlung spectra is given by

Fig. 2. A family of Planck-bremsstrahlung spectra (PS) for various values of the parameter α . Note how each curve follows the black-body curve ($\alpha = \infty$) at low energies and then peels off when the leakage of higher-energy photons becomes appreciable.

$$\alpha = Rb/\sigma T_e^4, \quad \sigma = ac/4, \quad (33)$$

or in a form more convenient for numerical evaluation,

$$\alpha = (0.459) (Z^3/A^2) \rho^2 R/T_e^{7/2}, \quad (34)$$

where ρ is the density of the plasma (g/cm^3), R its radius (cm), and T_e its electron temperature (keV). Z and A are

the charge and mass numbers, respectively, of the plasma ions.

The one-parameter family of Planck-bremsstrahlung spectra (PS) is shown in Fig. 2 for a sequence of values of the parameter α equal to (0.1), (0.3), (1), (3), (10), (30), (100), and (∞). The corresponding normalized spectra (PS/y) are shown in Fig. 3.

Fig. 3. Normalized Planck-bremsstrahlung spectra (PS/y) for various values of the parameter α . These are the curves of Fig. 2, redrawn so that each covers the same area.

Fig. 4. The Compton cooling rate (z) for various photon spectra as a function of the energy density (y).

The dimensionless Compton cooling rate z due to Planck-bremsstrahlung spectra was computed numerically, and is plotted as a function of the (dimensionless) energy density y of the radiation field in Fig. 4 (curve labeled PB). For comparison, the maximum possible Compton cooling rate (curve labeled C), and the cooling rate due to a Planck spectrum (curve labeled P) are also shown. Table 1 presents computed properties of the Planck-bremsstrahlung spectra for reference.

Figure 4 shows that E_{PB} exceeds E_P over most of the range of energy density y from 0 to 1, reaching a peak value that is nearly twice as large as the peak value of E_P . Although it is not

evident in Fig. 4, the ratio $\dot{E}_{PB}/\dot{E}_P$ becomes less than unity for small values of y , approaching the limit $5/6$ as y tends toward zero.

We conclude that: The traditional Compton cooling rate $\dot{E}_P$ will generally underestimate the true cooling rate $\dot{E}_{PB}$ due to Planck-bremsstrahlung spectra, with the exception that $\dot{E}_P$ slightly overestimates (up to 20%) the cooling rate when the energy density of the radiation is very low ($y < 7 \times 10^{-3}$).

We also mention in passing that the Compton cooling rate $\dot{E}_T$ due to a truncated Planck spectrum, Eq. (11), is roughly the same as $\dot{E}_{PB}$ over the entire range of y between 0 and 1, a not altogether unexpected result.

The Change in the Photon Spectrum Due to Compton Scattering and its Effect on the Compton Cooling Rate

None of the photon spectra dealt with so far lead to a Compton cooling rate that is significantly less than that due to a Planck spectrum. However, it is easy to conceive one that does, namely: A spectrum of relatively hard photons $\langle (1+n)h\nu \rangle \sim 4kT_e$ with a relatively low energy density u ($T_r \ll T_e$). A situation in which such a spectrum arises is the interior of a mass of high temperature plasma that is transparent ($\alpha \ll 1$), but that is not small compared with a Compton mean free path ($\beta \gtrsim 1$),

$$\beta = R/\lambda_c = \sigma_0 N_e R \approx (0.4) (Z/A) \rho R. \quad (35)$$

Such a plasma represents an essentially purely scattering atmosphere.

Let us consider the life of a typical photon in this purely scattering atmosphere.

It is born with an average energy of $(2/3)kT_e$ by the process of bremsstrahlung emission. It is then Compton scattered N times (on the average) before it finally escapes from the scattering atmosphere and is lost. After the first Compton collision it has gained (on the average) an amount of energy $\Delta(h\nu)$, where

$$\Delta(h\nu) = (h\nu/\mu) (4kT_e - h\nu), \quad (\mu = mc^2). \quad (36)$$

Averaging this energy gain over the photon energy spectrum and denoting the average by a bar, one obtains the result

$$\mu \Delta(\bar{h\nu}) = (4kT_e) \bar{h\nu} - \overline{(h\nu)^2}, \quad (37)$$

$$= \bar{h\nu}(4kT_e - \bar{h\nu}) - \overline{(h\nu - \bar{h\nu})^2}. \quad (38)$$

Denoting $\bar{h\nu}$ by E , Eq. (38) leads to the inequality

$$\Delta E < \widetilde{\Delta E} = (E/\mu) (4kT_e - E), \quad (39)$$

where $\widetilde{\Delta E}$ is an approximation to the increase in energy ΔE of the average photon obtained by neglecting the dispersion term on the right side of Eq. (38).

The average photon energy E_1 after the first Compton collision is then

$$E_1 < \widetilde{E}_1 = E_0 + (E_0/\mu) (4kT_e - E_0), \quad (40)$$

or more generally after the $(n+1)$ th collision,

$$E_{n+1} < \widetilde{E}_{n+1} = E_n + (E_n/\mu) (4kT_e - E_n) \quad (41)$$

$$< \widetilde{E}_n + (\widetilde{E}_n/\mu) (4kT_e - \widetilde{E}_n), \quad (42)$$

the last inequality requiring only that

$$\widetilde{E}_n < (\mu + 4kT_e)/2, \quad (43)$$

a condition that is certainly satisfied in all cases of interest. We have then the general result that

$$\widetilde{E}_{n+1} - \widetilde{E}_n < (\widetilde{E}_n/\mu) (4kT_e - \widetilde{E}_n), \quad n = 0, 1, 2, \dots \quad (44)$$

subject to the initial condition that

$$\widetilde{E}_0 = E_0 = (2/3)kT_e. \quad (45)$$

Each member of $\widetilde{E}_n$ of the sequence of values obtained from Eq. (44) represents an overestimate of the average photon energy E_n after n Compton collisions. Although it is not difficult to use Eq. (44) to obtain $\widetilde{E}_n$ for any desired value of n by successive substitution, it is convenient (and sufficiently accurate for present purposes) to obtain an approximate solution of Eq. (44) by replacing the finite difference equation by its corresponding differential equation. We have then,

$$d\widetilde{E}/dn = (\widetilde{E}/\mu) (4kT_e - \widetilde{E}), \quad (46)$$

which, with the initial condition (45), has the solution,

$$\widetilde{E}(n) = \frac{4kT_e}{1 + 5 \exp[-(4kT_e/\mu)n]}. \quad (47)$$

In order to determine the Compton cooling rate we need to ascertain the weighted average photon energy $\langle (1+n)h\nu \rangle$. We will approximate this average by ignoring the stimulated emission factor $(1+n)$. The weighted average photon energy exceeds the average photon energy $\langle h\nu \rangle$ by at most 20% for any Planck-bremsstrahlung spectrum (see Table 1), and by a still smaller amount after such spectra have been modified by Compton scattering, so the neglect of stimulated emission is justified in our present approximate treatment.

We need, then, to evaluate the average photon energy $\langle h\nu \rangle_N$ for a scattering atmosphere in which a photon is emitted as bremsstrahlung and experiences an average of N Compton collisions before escaping. This is obtained by averaging $\widetilde{E}_n$ appropriately over n from zero to N , because at any given instant equal numbers of photons will have undergone 0, 1, ..., N Compton collisions. (We are assuming that each photon collides exactly N times before escaping.) The required average is

$$\langle h\nu \rangle_N = \sum_{n=0}^N \widetilde{E}_n^2 / \sum_{n=0}^N \widetilde{E}_n, \quad (48)$$

or replacing the sums by integrals, we have

$$\langle h\nu \rangle_N = (2/3)kT_e \int_0^N y^2 dn / \int_0^N y dn, \quad (49)$$

where $y = \widetilde{E}_n/(2/3)kT_e$. The reason that $\langle h\nu \rangle_N$ is not a simple average of $\widetilde{E}_n$ is

that we require the average of $h\nu$ over the photon energy spectrum rather than the photon number spectrum.

The Compton cooling rate $\dot{E}_P$ due to a Planck spectrum of photons Eq. (10) can be written in the form

$$\dot{E}_P = \dot{E}_{P_0} [1 - (T_r/T_e)], \quad (50)$$

where

$$\dot{E}_{P_0} = 4(\sigma_0/mc) u kT_e. \quad (51)$$

Comparing Eq. (50) with the general expression for the Compton cooling rate Eq. (5), and disregarding the stimulated emission factor $(1 + n)$, we see that the cooling rate $\dot{E}_N$ in a scattering atmosphere

of such dimensions that the average photon experiences N Compton collisions before escaping is given approximately by

$$\dot{E}_N = \Omega_N \dot{E}_{P_0}, \quad (52)$$

$$\Omega_N = 1 - \{ \langle h\nu \rangle_N / 4kT_e \} \quad (53)$$

$$= 1 - (1/6) \int_0^N y^2 dn / \int_0^N y dn. \quad (54)$$

The integrals in Eq. (54) are easily evaluated, with the result that

$$\Omega_N = (5/6) (f - 1) / f \ln f, \quad (55)$$

$$f = (e^g + 5)/6, \quad (56)$$

$$g = (4kT_e/\mu)N. \quad (57)$$

Fig. 5. The dimensionless Compton scattering factor Ω_N versus the dimensionless parameter g . See Eqs. (52)-(57).

A graph of the dimensionless Compton scattering factor Ω_N versus the dimensionless parameter g is shown in Fig. 5.

If, for example, we consider an isothermal plasma with an electron temperature of 30 keV, and of such dimensions that a photon emitted inside is Compton

scattered eight times on the average before escaping, then $g = (4) (30) (8) / (511) = (1.88)$, and we obtain from Fig. 5 the value (0.61) for the Compton scattering factor Ω_N . That is: The Compton cooling of the plasma electrons is roughly half as great as would be the case if the photon spectrum were Planckian.

Summary and Conclusions

The relation between the true Compton cooling rate $\dot{E}$ and the cooling rate $\dot{E}_P$ that would result from a Planck spectrum of photons having the same energy density is summarized as follows:

i) Scattering negligible ($\beta \ll 1$) and absorption not significant ($\alpha < 10^{-2}$),

$$(5/6) \dot{E}_P \leq \dot{E} < \dot{E}_P. \quad (58)$$

ii) Scattering negligible ($\beta \ll 1$) and absorption significant ($\alpha > 10^{-2}$),

$$\dot{E}_P < \dot{E} < 2(\dot{E}_P)_{\max}. \quad (59)$$

iii) Scattering significant ($\beta \geq 1$) and absorption negligible ($\alpha < 10^{-6}$),

$$\Omega_N \dot{E}_{P0} \sim \dot{E} \leq (5/6) \dot{E}_P. \quad (60)$$

In cases (i) and (ii) the effect of stimulated emission (as measured by the ratio of $\langle (1+n)h\nu \rangle$ to $\langle h\nu \rangle$) is less than 20%; in case (iii) it is less than 2%.

We wish to stress that our calculation of the Compton scattering factor Ω_N provides only a very rough estimate of this quantity.