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THE ELECTRON COMPTON EFFECT OF 250
MEV (thesis)

By
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November 1953

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ABSTRACT

The electron Compton effect has been investigated using photons of the bremsstrahlung beam from the Berkeley synchrotron. Photons in a 30-Mev energy band about a mean energy of 253 Mev were selected by selecting recoil electrons in a given momentum interval and measuring the angle through which the photons were scattered. It is estimated that the primary energy band is known within three percent. A Compton event was identified as a coincidence between a pulse from a photon detector and a pulse from an electron detector. The accidental coincidence rate was taken to be the coincidence rate observed when the pulses from the electron detector were delayed 8×10^{-8} seconds. Cesium fluoride crystals were used to detect the photons, and phenylcyclohexane activated with 4 g/liter of terphenyl was used in all other detectors. The amplitude and length of the pulses from the photomultipliers were limited in length and amplitude but not amplified before reaching the germanium diode coincidence circuit. The resolution used was 2×10^{-8} seconds. Counting rates were measured to approximately 10 percent for photon scatter angles of 4° , 8° , 16° , 20° and 25° . When the observed angular distribution and that predicted by the differential Klein-Nishina equation were adjusted to agree at 8° , the other experimental points were found to lie on the theoretical curve within the accuracy of the experiment. The absolute differential cross section at 8° was found to be

$$\frac{d\sigma}{d\Omega} = 8.45 \times 10^{-27} \frac{\text{photons}}{\text{cm}^2}$$

which is approximately 17 percent larger than that predicted by the Klein-Nishina equation. This discrepancy is attributed, at present, to probable errors in the energy calibration of the synchrotron.

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I. INTRODUCTION

In a spectrometric analysis of scattered radiation from a target bombarded by monochromatic x-rays, A. H. Compton¹ showed that the scattered beam is composed of two lines, one of the same wave length as that of the primary beam and the other one of longer wave length. To explain the occurrence of the shifted component, Compton applied the extreme quantum picture, in which a beam of radiation consists of a stream of localized quanta of energy, or photons. It was assumed that the scattering process could be considered as an elastic collision between the photon and a free electron, and by applying the relativistic relations for conservation of energy and momentum, it is easy to show that the change of wave length should be

$$\Delta\lambda = \frac{h}{m_0 c} (1 - \cos \phi)$$
 where ϕ is the angle through which the photon is scattered. Measurements made by Compton², Ross³, and others confirmed this prediction. Also, as the increase in wave length corresponds to a loss of energy by the photon, the electrons must gain energy. These "recoil" electrons were discovered by C. T. R. Wilson⁴ and W. Bothe⁵, and quantitatively studied by Bless⁶, Compton and Simons⁷, and others.

Theoretical angular distributions for the scattered radiation were advanced by Compton⁸, G. Breit⁹, Dirac¹⁰, and Klein and Nishina¹¹. At x-ray energies the differences between the predicted distributions are small; however, the early experimental evidence definitely disagreed with the Thompson scattering equation and followed the Klein-Nishina equation within experimental errors.^{41, 42} R. Hofstadter and J. A. McIntyre¹² studied the angular distribution of the scattered photons when a stilbene

target was bombarded with 1.2-Mev γ -rays. Their measurements were sufficiently accurate to establish the Klein-Nishina formula in preference to the Dirac-Gordon or Compton equations. These same authors¹³ made some measurements using 100-Mev photons from the Berkeley synchrotron, but these measurements were not very accurate, and the maximum scattering angle investigated was 16° .

Total cross sections for Compton scattering have been obtained by many investigators, at low energies by Meitner and Hupfeld,¹⁴ Tarrant,¹⁵ and others; at high energies by Emigh¹⁶, Lawson,¹⁷ and Dewire, Ashkin and Beach.¹⁸ Recently A. I. Berman,¹⁹ using 19.5-Mev photons, has found agreement within 2 percent between the observed total Compton cross section and that predicted by the integrated Klein-Nishina formula, and at 300 Mev R. W. Kenney²⁰ finds agreement within 5 percent. However, the methods by which the total cross sections have been studied are rather indirect; either the number of electrons and positrons from a target are counted and the excess number of electrons attributed to the Compton process, or the total absorption coefficients are measured as a function of Z and the Compton cross section separated from the pair-production cross section by the different dependence upon atomic number for the two processes.

Because the Compton effect is one of the most fundamental interactions between radiation and matter, it was proposed that a direct method be employed to study the effect at high energies. Since no detailed measurements had been made in this energy region, it was also of interest to do more than a total cross section measurement.

Calculation of expected counting rates led to values that would make feasible an accurate comparison of the Compton scattering at various angles. However, as the photon spectrum from the synchrotron is essentially that of the bremsstrahlung from high-energy electrons, it was recognized that serious background problems exist, not only near the primary beam, but also for larger angles. This is true because the relative Compton cross section for large-angle scattering is greater for low-energy photons, of which there is a plentiful supply.

II. SURVEY OF EXPERIMENT

The Compton scattering process occurs when a photon of energy k , strikes a free electron, which then recoils, and the incident photon scatters at an angle ϕ with its original direction, and with a reduced energy of k' . The error introduced by treating all electrons of the scatterer as free is small if the photon energy is large compared with the average binding energy of the atomic electrons, which is the case in the present experiment.

From the conservation of energy and momentum, it follows that, when a Compton scatter occurs, the new energy k' , of the photon is given as a function of the old energy k and of the angle of scatter ϕ by

$$k' = \frac{k}{1 + \gamma (1 - \cos \phi)} \quad (1)$$

where $\gamma = \frac{k}{m_0 c^2}$, and $m_0 c^2$ is the rest energy of the electron.

See the sketch in Section III A.

For the cross section of the Compton scattering process the Klein-Nishina formula gives

$$\frac{d\sigma}{d\Omega} \frac{1}{r_0^2} = H(\gamma, \phi) = \frac{(1 + \cos^2 \phi)}{2[1 + \gamma(1 - \cos \phi)]^2} \left\{ 1 + \frac{\gamma^2(1 - \cos \phi)^2}{(1 + \cos \phi)[1 + \gamma(1 - \cos \phi)]} \right\} \quad (2)$$

where $r_0 = \frac{e^2}{m_0 c^2} = 2.82 \times 10^{-13}$ cm, the classical electron radius.

Here $d\sigma$ is the differential cross section for Compton scattering, wherein the photon is scattered into the differential solid angle $d\Omega$ at ϕ .

My experiment is an attempt (1) to measure the angular dependence of the differential cross section for the Compton process, and (2) to obtain an absolute measure of this cross section at a particular angle. The experimental values are to be compared with those predicted by use of the Klein-Nishina cross section. The primary photon source was the bremsstrahlung beam from the Berkeley synchrotron. This radiation was first collimated to 1/4 inch diameter and cleared of electrons, then scattered by a 0.200-inch beryllium target. A Compton scattering was identified with a coincidence between a signal from a counter which detects the recoil electron, and a counter which detects the scattered photon. These elements are shown as X and P, respectively, in Fig. 4. The magnetic field shown is used to select electrons of a given momentum, while the photon channels allow only photons scattered at a particular angle θ to enter the photon counter. Because a Compton scatter is a two-body process, the determinations of electron momentum and of the angle of the scattered photon allow the energy of the incident photon to be calculated. This result is important to the experiment, for bremsstrahlung radiation is not homogeneous and the primary photon energy can not be selected or determined before the Compton scatter.

To discriminate against pair production, a charged-particle detector, Y in Fig. 4, is placed in front of the photon detector. Triple coincidences of pulses from counters X, Y, and P (triples) occurring in coincidence with coincidences between counters X and P (doubles), are recorded, and later the triples rate is subtracted from the doubles rate. Positive particles that come from the target are deflected by the magnetic field away from the electron counters and are not counted by them; therefore, a decay photon from a neutral meson associated with a proton could not be confused with a Compton scatter.

The combination of the rather long, narrow photon channels and the magnetic field greatly reduces the number of positrons entering counters Y and P. Although the use of counter Y discriminates against pair production, the reduction of the number of positrons entering these counters lowers their singles counting rate, which in turn reduces the

accidental coincidence rate, or allows a more intense primary beam to be used. Actually, on the basis of the observed triples rate and singles rate in counter Y, very few positrons enter the photon counter except at small values of ϕ .

In addition to determining the doubles counting rate, it is necessary to determine the fraction of this rate that is due to accidental coincidences. The accidentals rate was measured by determining the doubles counting rate with an additional length of cable in the line from the electron counter. This cable delayed the pulses from the electron counter 8×10^{-8} sec. Also it is necessary to repeat all observations with the target removed to determine the contribution from the air column that these counters can "see".

Thus, a complete set of observations includes four counting rates: (a) target in and delay out, (b) target in, delay in, (c) target out, delay out, (d) target out, delay in. The net counting rate due to Compton scattering is then taken as $R_c = (a - b) - (c - d)$.

III. DESIGN CONSIDERATIONS

A. Characteristics of the Scattered Photons and Recoil Electrons for Incident Photons of High Energy

The following sketch clearly illustrates the meaning of the quantities used throughout this paper to describe a Compton scatter.

Figures 1, 2, 3 are plotted from relations given in Appendix I, and show the dependence of (1) the recoil angle of the electron, θ ; (2) the energy of the scattered photon, k ; and (3) the momentum of the recoil electron, pc , as a function of the angle of the scattered photon, ϕ . The energy of the incident photon, k_0 , is the parameter for the various curves shown.

From Fig. 1 it is seen that the values of θ are all less than 3.5° for values of ϕ greater than 4° , and values of k_0 greater than 225 Mev. Thus all the recoil electrons leave the target almost in the direction of the beam, and any attempt to detect these electrons without first deflecting them involves serious background problems. Furthermore, as is seen in Fig. 1, the value of θ for a particular value of ϕ is almost independent of k . Thus, detection of the undeflected recoil electrons would not greatly aid in a determination of the energy, k , of the incident photon.

From an examination of Fig. 2, it is seen that the energy, k , of the scattered photon (1) at any angle ϕ is almost independent of k_0 for values of k_0 greater than 200 Mev, and (2) rapidly decreases as ϕ increases. The first property makes a determination of the energy of the scattered photon useless for the determination of the energy k_0 of the incident photon. As the differential cross section is to be measured as a function of ϕ , the value of ϕ must be measured. The fact that the scattered photons are nearly monoenergetic at a given angular position means that a single evaluation for the efficiency of the photon counter for each angle ϕ is sufficient. However, as the energy of the scattered photons is a function of ϕ , this efficiency must be evaluated for each value of ϕ . In fact, the energy range of the scattered photons is large, so the uncertainties in evaluation of the photon counter efficiencies are important and limit the accuracy of the relative angular distribution independently of the accuracy of any measurements or of the "counting statistics" of the observations.

As illustrated in Fig. 3, the energy of the recoil electron increases as the photon is scattered to larger angles. For high values of k_0 the recoil electron receives most of the photon's initial energy even though ϕ may be relatively small, and consequently, a measurement of energy or of the momentum of the recoil energy is very helpful in determining the initial photon energy.

Consequently, the electron momentum was chosen as the second quantity to be measured. Also as all the recoil electrons are highly relativistic, the detection problem is the same for all electron energies.

B. Energy of Incident Photons

A band of energies about a mean value of 250 Mev was chosen for the incident photons, rather than a higher energy band, primarily because (1) the high-energy end of the photon spectrum is not stable, (2) the number of photons drops off rapidly at higher energies, and (3) the computation of the number of photons is uncertain at the high-energy end of the spectrum.

C. Design Consideration for Electron Counters, Photon Channels and Target.

1. Resolution

It is obvious that the best definition of k is obtained for point geometry and that the effect of increasing the dimensions of any of the three elements -- target, electron counter or photon apertures -- is to increase the uncertainty in the value of k .

The effect of the finite size of the counters can be determined as follows. From the relations given in Appendix I it can be shown that k is given in terms of the energy, mc^2 , of the recoil electron and the angle θ by the expression:

$$k = \frac{mc^2 - m_0c^2}{2} + \frac{1}{2} \left[(mc^2 - m_0c^2)^2 + \frac{4m_0c^2(mc^2 - m_0c^2)}{1 - \cos \theta} \right]^{1/2} \quad (3a)$$

Then:

$$dk = \frac{\partial k}{\partial (mc^2)} d(mc^2) + \frac{\partial k}{\partial \theta} d\theta \quad (3b)$$

The expressions for the partial derivatives are:

$$\frac{\partial k}{\partial (mc^2)} = \frac{1}{2} + \frac{1}{2} \frac{\left[(mc^2 - m_0 c^2) + \frac{2 m_0 c^2}{1 - \cos \phi} \right]}{D}$$

and

$$\frac{\partial k}{\partial \phi} = \frac{\left[\frac{m_0 c^2 (mc^2 - m_0 c^2) \sin \phi}{(1 - \cos \phi)^2} \right]}{D}$$

$$\text{where } D = \left[(mc^2 - m_0 c^2)^2 + \frac{4 m_0 c^2 (mc^2 - m_0 c^2)}{1 - \cos \phi} \right]^{1/2}$$

The electron "momentum" pc and the electron energy mc^2 are nearly equal, as seen from the relation

$$mc^2 = \left[(m_0 c^2)^2 + (pc)^2 \right]^{1/2}$$

In Table I are listed values of momentum accepted by the electron counters, and the angular intervals accepted by the photon channels, for electrons and photons that arise at the center of the target. The values of

$$\frac{\partial k_0}{\partial (mc^2)} \quad \text{and} \quad \frac{\partial k_0}{\partial \phi}$$

are computed using average values for mc^2 and ϕ .

θ	Min. pc Mev	Max. pc Mev	Δpc Mev	$\frac{\partial k}{\partial (mc^2)}$	$\frac{\partial k}{\partial (\theta)}$	$\Delta \theta$ Radians	$\frac{\partial k}{\partial mc^2} \Delta mc^2$ Mev	$\frac{\partial k}{\partial \theta} \Delta \theta$ Mev	ΔK Mev
4°	127.8	153.5	25.7	1.255	2270	0.00126	32.25	2.86	35.11
8°	196.7	224.3	27.6	1.030	531	0.00334	28.4	1.77	30.17
16°	228.3	255.8	27.3	1.027	85.4	0.00548	28.25	0.468	28.72
20°	237.6	259.3	26.7	1.001	45.1	0.00919	26.75	0.414	27.16
25°	235.9	262.2	26.3	1.0005	23.6	0.0150	26.7	0.354	26.61

From the values for the partial derivatives, it is seen that the variation in ϕ is much more serious than variation in momentum. As the recoil momentum for any electron is essentially along the beam line, the motion of the electrons is essentially in a horizontal plane, so electron motion can be treated as a two-dimensional problem. Thus increasing the vertical dimension of the electron counters does not increase Δpc . But an increase in the height of the electron counter does decrease the probability the electrons miss it owing to scattering or defocusing by the magnetic field, so this dimension is made as large as the gap in the magnet pole tips will allow. As yet only the effect of finite counters has been considered. The finite extension of the target also contributes to the uncertainty in k , and indeed the combination of the finite target and finite photon counter even introduces uncertainties in Δk_0 because $\Delta\phi$ is no longer single valued.

That $\Delta(pc)$ remains reasonably well defined is due to two conditions:

- (1) that each electron moves in a horizontal plane, and
- (2) that the use of a magnetic field allows electrons of the same momentum but from different parts of the target to be focused along a vertical line. This focusing is simply illustrated in the accompanying sketch for the case of a uniform field where the electron paths are arcs of circles.

It is seen that the points of focus lie along a line which passes through the target and makes an angle of 45° with the beam. Therefore the electron counters were placed as near to such a line as possible without requiring the electrons to traverse the divergent magnetic fields near the edges of the pole tips. The momentum values of electrons that leave the target from points labeled 1, 2, 3, 4, 5 in Fig. 5 and pass through points a, and b of the counter are given in Table II for two values of ϕ .

15 TABLE II.

Photon Angle →	4°			16°		
	PC		Δ PC	PC		Δ PC
Counter Target Point	a	b		a	b	
1	126.6	152.6	26.0	227.6	255.2	27.6
2	127.0	152.5	25.5	227.1	254.1	27.0
3	129.1	154.7	25.6	229.9	257.5	27.6
4	128.8	155.3	26.5	230.2	258.2	28.0
5	127.8	153.5	25.7	228.3	255.8	27.5
Ave			25.9	Ave		27.4

From these values it is seen that electrons are focused sufficiently well that the uncertainty in Δpc is unimportant and that the value of Δpc may be taken as that value obtained for electrons leaving the center point of the target.

A different electron counter was used for each value of ϕ in an effort to select events at every scattering angle from the same energy region of the beam. To determine the size, shape and placement of these counters, electron orbits were traced through the magnetic field by graphical methods from target points 1 and 2. A more detailed discussion of electron orbits is given in Appendix II.

As the electron counters could not be placed exactly at the positions of electron focusing, the target was made rather narrow. Such a target also allows the use of smaller channels in the shielding around both the electron and photon counters, and thus helps eliminate radiation from points other than the target.

D. Choice of Electronics

Counting equipment, to be useful for experiments with beams from pulsed accelerators that have short duty cycles, should have the following properties: (1) fast resolution time for coincidences, (2) absence of dead time, and (3) high discrimination ratio of n -fold coincidences to $(n-1)$ -fold coincidences where n is the order of coincidence desired. It is also desirable to avoid using amplifiers before the coincidence is formed, for these amplifiers must pass a wide band of frequencies and are usually distributed amplifiers such as the Hewlett-Packard model 460A.

The usual expression for the accidental coincidence rate between two counters is $2 \tau N_1 N_2$, where τ is the resolution of the coincidence circuit, and N_1 and N_2 are the individual counting rates. On the basis of this expression, it appears that the shortest resolving time possible is the best, and this is true even for pulsed machines if the radiation during the pulse is continuous. However, Madey, Bandtel and Frank²¹ have shown that beam pulses from the synchrotron have a fine structure, such that the spread-out bremsstrahlung beam is emitted in sharp bursts with the 47.7-megacycle frequency of the electrons in the doughnut.

The individual bursts have half-width less than 1.5×10^{-9} seconds. Thus if the resolving time of the coincidence circuit is less than 2×10^{-8} seconds (the time between bursts) and greater than 1×10^{-9} seconds the probability of a coincidence in any 2×10^{-8} -second interval is the same as the probability that an electron of the right momentum to be detected and a photon in the right angular interval to be detected occur in the same burst. For a single burst there is a probability $P(e)$ for an electron and a probability $P(\gamma)$ for a photon other than those associated with the Compton process, as well as the probability $P(c)$ of a Compton event. Therefore the probability that an electron and photon appear in any one burst is $P(e, \gamma) = P(e)P\gamma + P(C)$. Now it is seen that if the pulse in either the electron channel or photon channel is delayed by any multiple of 2×10^{-8} seconds, the probability for a coincidence in any burst is $P'(e, \gamma) = P(e) P(\gamma)$, which is justification for the delay method used to measure accidental rates. Also it is evident that unless the resolving time can be made short with respect to 10^{-9} seconds, the accidental rate is not improved for resolving times less than 2×10^{-8} seconds. However, there is an advantage in using a resolution time of 2×10^{-8} seconds in that one may still use RG65/U cable for the pulse clipping line, which results in a grid signal at least four times the amplitude obtainable with other clipping lines. As the best resolving times reported are greater than 10^{-9} seconds, and as this experiment is not based on time of flight, a resolving time of 2×10^{-8} was considered optimum.

IV. EXPERIMENTAL PROCEDURE

A. Alignment

The target-electron counters, electron channel and photon channels at 16° , 20° , 25° , and 30° were carefully located in the proper relative positions on a dural template of quarter-inch stock. The four photon channels were constructed as one unit, which was located on the template by stainless steel positioning pins. Similarly the electron counters and target holder were positioned by pins and were removable. Before the pole tips were put in the magnet, the template with the electron channel mounted on it was put in position in the pole tips and secured in place by means of brass jack screws. The pole tips were then put in the magnet, and the magnet was positioned before the counters and

channels were mounted. To insure that the pole tips were always in the same position in the magnet, three stainless steel pins, ($1/2$ inch in diameter) were welded to the edge of the upper pole tip so as to project above it. These pins were always brought in firm contact with the upper magnet pole. Two pointed stainless steel pins were inserted in holes on the center line of the template and their position with respect to the beam line was determined by means of a transit, which has previously been aligned with the beam by photographic methods. These pins were then positioned on the beam line by moving the magnet along a track at right angles to the beam line and rotating the magnet about a vertical axis. The elevation was adjusted by shimming under the track.

A single collimator one inch by $1/4$ inch, mounted on a dural plate of $1/2$ -inch stock, was used in the 4° and 8° photon counter positions. "Line-up" pins were inserted in holes in the plate. These holes were so located that if one of them was a predetermined distance from the target and the pins were aligned by means of the transit, the collimator would be in the desired angular position.

The final step in aligning the experiment was to photograph the beam immediately behind the target. From the photograph it is possible to determine whether or not the target intercepts the central portion of the beam.

It is estimated that errors in positioning are not greater than $1/16$ inch in any one dimension. The following table lists the change in angle such errors would cause. Small changes in electron-counter position are negligible, as they cause only small changes in the energy of the primary photons, and neither the number of photons or the Compton cross section varies rapidly with the photon energy, k .

ϕ	1/16" displacement perpendicular to beam			1/16" displacement parallel to beam		
	$\delta\phi$ Rad.	$\delta k = \frac{\partial k}{\partial \phi} \delta\phi$ Mev	% Error in ΔK	$\delta\phi$ Rad.	$\delta k = \frac{\partial k}{\partial \phi} \delta\phi$ Mev	% Error in ΔK
4°	2.91×10^{-4}	0.661	1.9	4.4×10^{-4}	.99	2.8
8°	8.73×10^{-4}	0.463	1.5	17.5×10^{-4}	.93	3.0
16°	14.5×10^{-4}	0.123	0.4	.0067	.06	0.2
20°	.015	0.735	2.7	.013	.59	2.2
25°	.020	0.474	1.8	.019	.45	1.2

B. Adjustment of Photomultiplier Voltages

1. Sources

The 1.3-Mev gamma rays from Co^{60} and the 4.47-Mev gamma rays from a polonium-beryllium source were used.

The reaction in the polonium-beryllium source is $\text{Be}^9(\alpha, n)\text{C}^{12}$. Sometimes the carbon nucleus is left in the 4.47-Mev excited state, which decays by gamma emission.²² Higher energy states are not excited as the polonium α -particles do not have sufficient energy. Thus the radiation from this source contains only neutrons and gamma rays of a single energy. The α -particles are absorbed in the brass walls of the container.

2. Charged-Particle Counters

The PoBe source was used in adjusting voltages of the photomultipliers in the charged-particle detectors. Pulses from the limiter were displayed on a Tektronix 511AD synchroscope and the photomultiplier voltage raised until the pulse heights no longer increased as the voltage was increased. At this potential the pulses from the photomultiplier have the minimum height necessary to cut off the current in the 6B Q7 limiter tube. The photomultipliers were operated at a potential 75 volts higher than that necessary to cause the largest pulses to "limit." As the adjustment was made visually, the pulses that were limited were probably due to energy dissipation in the phosphor of 4 Mev or less. It is estimated that electrons that had path lengths in the sensitive volume of 0.50 inches or more were detected.

Therefore it was assumed that any electron that crossed the center plane of an electron counter was detected. In Fig. 5 this plane is represented by a line drawn between counter points a and b. It is probable that the estimated path length in the sensitive volume necessary for electron detection is too long. However, the maximum electron momentum range is intercepted by the side 3, 4 and is not appreciably different from that intercepted by the plane a, b.

3. Photon Counters

In calculating the efficiency of the photon counter it is necessary to know the minimum energy loss in the detector that will cause a pulse large enough to be detected in coincidence with a pulse from the electron counter. Hence the photon counters were set to detect energies greater than a certain minimum, as follows. First, the

photomultiplier voltage was adjusted so pulses caused by gamma rays from one of the sources were just limited. Pulses from the limiter were applied to both inputs of the double-coincidence circuit. The pulses from the coincidence circuit are first amplified, then passed through a discriminator circuit in the variable-gate chassis. The gain of the amplifier is adjusted so the coincidence pulses can not pass the discriminator for some intermediate value of its range. The exact value is determined by measuring the counting rate as a function of discriminator setting. Thus the over all gain of the system is such as just to discriminate against pulses caused by the maximum energy of the gamma rays used. A small adjustment of the discriminator setting must be made because the pulses from the limiter appear smaller if applied to two input diodes rather than to a single input diode of the coincidence circuit. One of the photon counters was adjusted with the Co^{60} gammas and used at 16° , 20° , and 25° . The other photon counter was adjusted with PoBe gamma rays and used at 8° and 4° . The effect of errors in adjustment of these counters is discussed in a later section.

The over all gain could be checked for each photon counter by putting the source in a known position and checking the counting rate at a standard discriminator setting.

C. Beam Monitors and Synchrotron Operation

A thin ionization chamber located ahead of the primary collimator was used to measure the total radiation. The current from this ionization chamber was integrated by an electrometer. Initially a thick copper-walled ionization chamber, built to specifications furnished by R. R. Wilson at Cornell University, was placed behind the experiment, but owing to leakage across the internal insulation its use was discontinued. However, this "Cornell" chamber was later compared with the pre-collimator ionization chamber and the results were used in the evaluation of the absolute cross section.

Recently it has become apparent that extreme care must be taken to stabilize the operating conditions of the synchrotron to maintain a single-photon energy spectrum. That seriously large changes in the high-energy end of the photon spectrum occur and are not detected by the usual beam monitors was first observed by Charles McDonald.²³ He was using a pair spectrometer directly in the synchrotron beam, and found variations of 30 percent in the ratio of 300-Mev photons detected by it to the integrated current from the precollimator ionization chamber. Later this variation was found to depend upon the operating parameters of the synchrotron. Several workers using the pair spectrometer and various ionization chambers have shown that all monitors exhibit ratios to the pair-spectrometer counting rate that are constant within 5 percent as the beam intensity is varied, provided the accelerator operating conditions do not change. In particular, variations of the photon energy distribution are minimized if one stabilizes the capacitor high voltage and, hence, the peak magnetic field in the synchrotron and the time in the synchrotron cycle at which the circulating electron beam begins to strike the internal target. During this experiment the capacitor high voltage was 14.9 kilovolts and the peak intensity of the "spreadout" beam was kept at 1500 microseconds before peak magnetic field.

For these conditions McDonald and Anderson²⁴ have found the maximum photon energy to be 335 Mev, which is somewhat higher than a value obtained earlier by Powell, et al²⁵ using a cloud chamber to observe electron-positron pairs.

D. Bombardments

Rather than use beam intensities low enough to insure a negligible accidental coincidence rate, it was found more satisfactory to measure this rate by delaying either the signals from the photon counter or those from the electron counter. As explained previously, the delay time must be a multiple of 2×10^{-8} seconds and should be at least 4×10^{-8} seconds. Values of 4×10^{-8} , 8×10^{-8} , and 16×10^{-8} seconds were used and no detectable differences in the accidental rates were observed. It is found desirable, however, to keep the accidental rate below 30 percent of the counting rate. The runs were about one hour long and were made alternately on the counting rate and the accidental rate. Counting rates were also taken without the target, and were found negligible except at small values of θ . This result is not surprising, as the photon channel accepts radiation from a much longer bombarded air column for positions close to the beam.

Magnet current and discriminator-crystal bias were checked frequently.

V. EXPERIMENTAL DETAILS

A. Beam Collimation and Magnetic Sweeping Field

At a point 55 inches from the internal platinum target of the synchrotron, the beam was collimated by a tapered hole in a lead block nine inches thick. The collimating hole was 0.25 inches in diameter at the entrance end and was part of a cone whose apex lay at the platinum target. Directly behind the primary collimator was a 0.75-inch secondary collimator of lead six inches thick. Following this was a small magnet used to clear the beam of electrons. The pole tips of this magnet were separated 2 inches and are 3 inches by 13.5 inches. The center field value was 5500 gauss. Early tests showed that this field reduced the singles background rate of all counters, particularly in that of the electron counters.

B. Target

The target was made from beryllium, which is a material of low Z and is relatively easy to handle. As pair production increases as Z^2 and the Compton effect only as Z , a material of low atomic number is essential to reduce the singles counting rate in the electron detector. The dimensions of the target were: height, 1.50 inches;

depth in the direction of the beam, 0.200 inches; and width transverse to beam, 0.250 inches. From photographs of the beam taken directly behind the target, the area irradiated was found to be 0.250 inch x 0.500 inch.

The target was supported by cellulose scotch tape from its ends to a dural frame with a 3-inch aperture. No tape covered the center section of the target.

C. Detectors

All detectors were scintillation counters. Electron detectors, X, and the charged-particle detectors, Y, were of a lucite construction similar to that described by Garwin.²⁶ The scintillator fluid consisted of a solution of 4 grams of terphenyl plus 20 milligrams of diphenylhexatriene in one kilogram of phenylcyclohexane. The photon detectors were cesium fluoride crystals.

1. Fluid Counter Construction and Performance

The sensitive volumes of the electron detectors were all 2.75 inches high and 1 inch in the direction parallel to the electron paths through the counter. Electrons of the energies used should lose at least 5 Mev in traversing the sensitive volume. The width of the sensitive volume was chosen so the counters approximately intercepted electron momentum ranges that correspond to the same energy range in the incident photon beam. The sensitive volume was viewed from one edge through a 22-inch light pipe by two RCA 1P21 photomultipliers. Except for one edge of the sensitive volume, which was the polished end of the lucite light pipe, all sides were constructed of 1/16-inch lucite stock and glued together with lucite shavings dissolved in ethylene dichloride.

The sensitive volume of the counter Y was 2-1/2 in. x 2-1/2 in. x 1 in. It was viewed from opposite edges by two RCA 1P21 photomultipliers. The 2-1/2 in. x 2-1/2 in. sides were of 1/16-inch lucite, while the edges were made from 1/4-inch lucite stock.

All liquid counter surfaces, except those through which the sensitive volumes were viewed, were heavily coated with magnesium oxide to improve the light collection. Counter volumes coated with magnesium oxide showed approximately a 30 percent increase over those covered with shiny aluminum foil in light transmitted to the photomultiplier, independent of the use of light pipes. With a collimated

pencil of gamma rays from Co^{60} , the detection efficiency as a function of position in the sensitive volume was found to vary no more than 20 percent, even though the sensitive volume was viewed through a 22-inch light pipe.

All liquid counters and light pipes were covered with 1 mil of aluminum foil and wrapped with two thicknesses of black paper scotch photographic tape.

The use of two photomultipliers increased the detection efficiency and lowered the noise rate. However, it was necessary to connect the anodes to separate grids of a double triode to avoid coupling of the two photomultipliers. The plates of the double triode were connected so the photomultiplier pulses were added.

2. Photon Counters

Two cesium fluoride crystals in the form of truncated cones, 1.25 inches high, base diameter 2.25 inches and top diameter 2.00 inches, were used as the sensitive elements in the photon detectors. The use of this material for photon detection was first investigated by W. Van Sciver and R. Hofstadter.²⁷ These investigators showed that the decay constant for cesium fluoride is $5 \times 10^{-9} \pm 1 \times 10^{-9}$ seconds, that the light output is approximately proportional to the photon energy (at least for photons up to a few Mev), and that the pulse height is about 1/20 that of the anthracene pulse when viewing the crystals with 5819 photomultipliers. Cesium fluoride crystals are useful as photon detectors, because their short decay constant reduces pile-up of pulse, and because their high density (3.586 g/cm^3) and high atomic number contribute to greater detection efficiency. On the other hand, the pulse height is small, and the material is very deliquescent. Each crystal is mounted in a lucite cylinder of 2-1/4 inches internal diameter of 1/8-inch-thick walls.

Photons entered one end of the cylinder through a 1/16-inch lucite window, and light was transmitted through a 1/4-inch-thick lucite window at the other end to a 5819 photomultiplier. The space between the cesium fluoride crystal and the lucite cylinder was filled with clear mineral oil. Also the space between the window and the photomultiplier was filled with mineral oil. The lucite container was coated with magnesium oxide.

No deterioration of the crystals has been noticed six months after packaging. Small crystals have shown no deterioration in such packages in one year.

D. Electronics

1. Pulse Shaping and Limiting

Inasmuch as the pulse height from the photomultiplier depends upon the energy that is lost by the particle in passing through the phosphor, there are large variations in the amplitude of single pulses, which in turn give rise to a variation in the time that is required for the pulse amplitude to decay below a certain value. The time required for a pulse to decay enough to allow the circuit to accept another pulse is the "dead time" of the circuit. Furthermore the resolution of the coincidence circuit can be no shorter than the length of the input pulses to it. For these reasons the photomultiplier pulses are "clipped" in time by a shorted line. They are also limited in amplitude to improve the discrimination ratio of the coincidence circuit.

As mentioned above and as shown in the circuit diagram in Fig. 6, the clipping line is a piece of RG63/U delay cable, 1-1/2 inches long. The characteristic impedance of this cable is 1000 ohms. From the circuit diagram, Fig. 6, it is apparent that the characteristic impedance of the clipping line determines the value of the resistance between the grid of the limiter tube and ground. Because the photomultiplier is a constant-current source of high impedance, the grid signal is directly proportional to the value of grid resistor. Of course, a grid resistance of several thousand ohms is not permissible, either, as the grid resistor and the input capacitance of the limiter tube plus photomultiplier anode capacitance would form an RC circuit whose time constant would then determine the dead time. For clipping times shorter than 10^{-8} second, RG 63/U line can no longer be used, as it ceases to act like a distributed line.

6BQ7A vacuum tubes, which are double triodes, were used as limiter tubes. In the cases where two photomultipliers viewed the same phosphor, separate grids were used for the outputs of the two photomultipliers. Electrostatic shielding as suggested by Morton²⁸ was found effective in reducing the noise counting rates in the 1P21 photomultipliers.

2. Germanium Diode Coincidence Circuits

Triple-coincidence circuits are used in pairs with two inputs of each circuit common to the other circuit of the pair. Such an arrangement permits one to feed a single signal to two circuits without an appreciable loss in amplitude, because the lines from the point of division to the two input germanium diodes are much less than one wave length. Fig. 7 shows a schematic diagram of a single triple-coincidence circuit. The 1N56 germanium diode inputs were used instead of tubes to increase the circuit sensitivity, which further aids one to eliminate input signal amplifiers. The three input diodes are arranged in the conventional manner of a Rossi coincidence circuit. The two parallel 1N56 diodes and the associated capacitor, which increase the discrimination ratio by clamping the output voltage in all cases in which less than three of the input diodes are cut off, are due to Garwin.²⁹ The 1N54 diode in the grid of the output tube further increases the discrimination of the circuit. It was introduced by Madey, who is to publish a more detailed account of this circuit.

It may be noted here that the 1N54 diode acts as an extremely sensitive discriminator, which makes it imperative that the voltage across it be maintained constant. The value of this voltage was kept at 0.60 volts and was checked at frequent intervals.

3. Other Electronic Equipment

Figure 8 shows a block diagram of the equipment in the magnet room and is essentially those elements described above. There was in addition one chassis containing ten inverter channels, which were simply plate followers used to invert the singles signals to the correct polarity to be amplified by standard linear amplifiers.

Figure 9 shows a block diagram of equipment used in the counting area. All circuits are standard in this laboratory and circuit diagrams are available.

The slow coincidence between triples and doubles was used, because it is difficult to adjust the two fast-coincidence circuits used to respond identically. Therefore the discrimination in the triples circuit was intentionally lowered and the slow coincidence made to assure that only triples that occurred with doubles were counted.

E. Magnetic Field Measurements

The magnetic field was mapped by a method based upon the fact that the resistance of a bismuth wire is a function of magnetic induction.³⁰ The resistance is also a function of temperature. In the equipment used, the temperature of the bismuth wire probe was kept approximately constant at -28° C. by evaporating freon. Rather than measuring the resistance directly, a constant current is passed through the wire, and the potential drop across the wire is measured. With care this method is accurate to a tenth of one percent; however, the accuracy of the measurement made of the 350-Mev pair-spectrometer field is about 0.5 percent. The field was monitored by measuring the potential drop across a 50-millivolt, 200-ampere shunt carrying the magnet current. The same shunt was used for the runs as was used for the magnetic measurements. The potential drop across the shunt was measured by a Leeds and Northrup potentiometer, which could be read to 0.001 millivolt.

The bismuth probe is motor-driven in one direction, and the potential drop across the probe continuously recorded on a Speed-O-Max recorder whose range is expanded 20 times.³¹ The field was measured along lines spaced $1/4$ inch apart in the region near the target position. This spacing was increased first to $1/2$ inch, then to 1 inch, as measurements progressed to the uniform field at the center of the magnet, and was decreased again at the other end of the pole tips.

The magnetic field values were used to obtain the electron orbits as described in Section IX B.

F. Magnetic Shielding

All photomultipliers were magnetically shielded by two soft iron cylinders separated by brass spacers. The wall thickness of the cylinders used to shield the 1P21 photomultipliers was $1/16$ inch. The wall thickness of the 5819 shields was $1/4$ inch. Also the 5819 photomultipliers were wrapped with three layers of permalloy one mil thick. The permalloy and the iron shields extended $1-1/2$ inches in front of the photocathode.

Each counter was tested by turning on the magnetic field when it was counting gamma rays from a radioactive source. The counting rates were found to be independent of the magnetic field.

G. Photomultiplier Cooling

If cooling is not provided for the photomultipliers, they reach temperatures above the maximum recommended operating temperatures, and the noise pulses become excessive in number and size. There are two sources of heat. First: a limiter tube is mounted on each photomultiplier base, so that the lead from the photomultiplier anode to the grid of the tube is as short as possible; the power dissipation of this limiter tube is usually enough to cause the photomultiplier to become noticeably "noisier" due to heating. The second heat source is the pair spectrometer. Approximately 17 kilowatts are dissipated in the field coils of this magnet, and to avoid excessive heating of the coils, two fans force air through each of the two coil cases. The exhausts are so located that turbulent currents of hot air pass along the sides of the coil cases where the photomultipliers are located. To provide cooling for the photomultipliers, the space between the two magnetic shields was sealed by soldering brass spacers in position. Adequate cooling was obtained by circulating city water through the shields.

H. Radiation Shielding

Figure 4 shows the location of lead shields used to protect the detectors from the stray radiation in the magnet room. A minimum of 4 inches of lead was used in any position and 8 inches of lead was used wherever space permitted.

VI. COMPUTATIONS

A. Absolute Number of Incident Photons

To evaluate the absolute cross section, it is necessary to determine the number of photons in a given energy interval per unit of integrated signal from the beam monitor. This determination can be made if the following information is available: (1) the relative energy spectrum, (2) the total energy in the beam per unit of integrated monitor signal, and (3) the maximum energy of the electrons accelerated in the synchrotron.

Basically the photon spectrum produced by a beam of monoenergetic electrons striking an infinitely thin target is given by the Bethe-Heitler formula.³² The thin-target spectrum is modified by the thickness of the target by the following process. First, the original electrons lose energy by radiation so that their energy is

lowered, and the photons produced by them farther along in the target show a lower average energy. McMillan²⁵ first pointed out that this effect is reduced considerably by the fact that the electrons that have been multiply scattered are no longer pointing in their original directions. By use of the multiple-scattering formulas of E. J. Williams²⁵, it can be shown that after traversing about the first four mils of the target the electrons are sufficiently out of line so as to make most of the photons produced by them miss the collimator. The result is to give a spectrum more like that of an infinitely thin target. Also Schiff³³ has shown that for targets of finite width, multiple electron scattering in the target has the effect of averaging over-all angles so that the integration over gamma-ray angle is valid even though only the beam in the forward direction is considered. The Bethe-Heitler formula has been checked at high energies experimentally by De Wire and Beach³⁴, Stokes³⁵, and Powell et al.²⁵ However, as the photon spectrum comes from the first quarter of the target, the remaining three quarters act as an absorber for these quanta. About 10 percent of the quanta are absorbed.

The method of producing the "spread-out" photon beam causes the electrons that strike the platinum target to have a distribution of energies. The theoretical bremsstrahlung energy distribution is corrected for the variation in electron energies by a method described in detail by Gilbert.³⁶ In brief, the electron beam intensity is assumed proportional to the photon beam intensity. The electron energy distribution is then determined from a photograph of the time variation of the photon beam intensity. A set of time marks and a fiducial mark at peak magnetic field are included on the photograph. The energy of the electrons in terms of their maximum possible energy is given by

$$\bar{E} = \bar{E}_{\max} \sin \left(\frac{7790 - t}{7790} \right)$$

where t is the time (in microseconds) before peak field. Integration of the bremsstrahlung spectrum over this electron energy distribution gives a resultant spectrum that is correct for the spread-out photon beam. The three curves on Figure 10 illustrate (A) thin-target bremsstrahlung computed from Bethe-Heitler formula (B) bremsstrahlung

curve corrected for thick target, (C) bremsstrahlung curve corrected for thick target and energy distribution of electrons.

The total energy in the beam was measured by use of the "Cornell" ionization chamber. The energy per coulomb of charge collected by the Cornell ionization chamber is

$$Q = 3.74 \times 10^{18} \left(\frac{T}{273} - \frac{760}{P} \right)$$

where T is the temperature in degrees Kelvin and P the pressure in millimeters of mercury. The ordinate, $W(k)$, of the bremsstrahlung curve (Fig. 11) represents the total energy carried by photons of energy k ; i.e., the number of photons of energy E equals $W(k)/k$. The total energy Q corresponds to the total area under the bremsstrahlung curve. Therefore the number of photons in any energy interval is

$$N_o = B \int_{k_1}^{k_2} \frac{W(k)}{k} dk \quad (6)$$

where B is given by $Q = B \int_0^{k_{\max}} W(k) dk$.

These integrations can be carried out numerically. The maximum and minimum values of k , k_2 and k_1 were determined as follows: (1) the energy limits of the electrons from the center of the target that were counted by the electron counter were computed as outlined in Appendix B, (2) the maximum and minimum values for the angle at which photons from the center of the target could enter the photon counter were determined from the geometry, (3) as a photon retains a larger fraction of its energy for smaller scattering angles, the maximum incident photon energy, k_2 , is computed by using the maximum electron recoil energy and the minimum value of ϕ in Eq. 3, and (4) k_1 is similarly computed using the minimum electron recoil energy and maximum value of ϕ .

B. Efficiencies

The efficiency of the electron detector is taken as 100 percent for electrons with path lengths in the sensitive volume greater than 1/2 inch (see section IV B).

The efficiency of the photon detector depends primarily on two factors: 1) the conversion of photons to charged particles, and 2) the minimum detectable energy loss E_d . Photons are converted either by pair production or by the Compton process, as the photoelectric effect is negligible for photon energies considered here. Therefore the absorption coefficient of the cesium fluoride crystal for photons may be written as

$$\mu = \mu_{pp} + \mu_c = \sigma_{pp} \rho_a + \sigma_c \rho_e$$

where the subscript pp denotes pair production and the subscript c denotes Compton effect. The fraction of photons that stop in the crystal is $(1 - e^{-\mu t})$, where t is the length of the crystal. However, some of the charged particles can escape from the back of the crystal without being detected, so μt is replaced by

$$(\mu t)_d = \mu_{pp} (t - t_{pp}) + \mu_{cd} (t - t_c)$$

To determine t_{pp} and t_c , $\frac{dE}{dx}$ was evaluated as a function of E for electrons in cesium fluoride, using the relations given by Aron et al.³⁷ As explained in section IV B, the photon counters are adjusted to detect energy losses greater than an energy E_d , the minimum energy loss for detection. For an electron to lose energy ΔE it must travel a distance

$$t = \int_{E-\Delta E}^E \left(\frac{dE}{dx} \right) dx \quad . \text{ For electron-positron pairs}$$

$\frac{dE}{dx}$ was assumed to be the same for each particle, and ΔE was taken as $1/2 E_d$. For Compton electrons ΔE was assumed to be E_d .

Many of the Compton electrons do not have enough energy to be detected. However, as the energy of the electron is determined by the energy of the incident photon and the angle through which it is scattered, it is possible to compute limits for integration of the differential Klein-Nishina cross section, so only electrons that can be detected are included. Thus,

$$\mu_{cd} = \sigma_{cd} \rho_e = \rho_e \int_{\phi_{\min}}^{180} \left(\frac{d\sigma}{d\Omega} \right) d\Omega$$

where $\frac{d\sigma}{d\Omega}$ is given by Eq. (2) and $\phi_{\min}$ is computed as explained, by use of the relations given in Appendix A. Thus the over-all efficiency R in Eq. (8) is given by

$$R = 1 - e^{-(\mu t)_d} \quad (7)$$

The contribution by the Compton process to the detection efficiency has been calculated using the Klein-Nishina formula. The validity of this procedure, even though Compton cross sections are being investigated, is justified as follows: 1) for high-energy photons, the Compton contribution is small enough that even large uncertainties in it result in only small uncertainties in the efficiency, and 2) at low photon energies where the Compton contribution is appreciable, the Klein-Nishina formula has been experimentally verified.¹²

Pair-production cross sections can be computed using the Bethe-Heitler relations, in which the effect of screening of the nuclear potential by the atomic electrons is based on the assumption of a Thomas-Fermi atomic model. However, the conditions for the validity of the Born approximation, which is used to obtain the theoretical relations, are not fulfilled for these photon energies. In fact, the cross section for pair production in lead has been found about 10 percent low for photon energies from 17.5 Mev to 300 Mev.^(16, 17, 18, 19) Two investigators have recently given experimentally determined corrections to the Bethe-Heitler cross sections. For photons of 19.5 Mev, A. I. Berman¹⁹ finds the data fit the following

relation:

$$\frac{\sigma_{BH} - \sigma_{pp}}{\sigma_{BH}} = (1.55 \pm 0.1) \times 10^{-5} Z^2$$

Similarly, for 300-Mev photons, Emigh¹⁶ finds

$$\frac{\sigma_{BH} - \sigma_{pp}}{\sigma_{BH}} = (1.4 \pm 0.1) \times 10^{-5} Z^2$$

However, measurements made by Chadwick, Blackett and Occhialini,³⁸ using the ThC'' γ -rays, yield a pair-production cross section 10 percent higher than that predicted by the Bethe-Heitler theory. Values for σ_{pp} at $5.2 m_0 c^2$ and $3 m_0 c^2$ have been calculated by Hulme and Jaeger³⁹ using exact wave functions, and are higher than those calculated using the Bethe-Heitler relations.

Consequently, the following values of the pair production cross sections were used: 1) for photon energies above 17 Mev,

$$\sigma_{pp} = \sigma_{BH} \left[1 - 1.47 \times 10^{-5} Z^2 \right];$$

2) for photon energies between 6 Mev and 17 Mev, $\sigma_{pp} = \sigma_{BH}$;

3) for photon energies less than 6 Mev, σ_{pp} (taken from a curve of pair production vs. photon energy, which was passed through the Hulme and Jaeger values and "faired" in to meet the Bethe-Heitler curve at 6 Mev).

C. Cross Sections

The number of Compton events per unit of integrated beam is related to the cross section by

$$N_c = N_i \rho_e \sigma t R, \quad (8)$$

where ρ_e is the electron density, t is the extent of the target in the direction of the beam and R is the over-all efficiency for detection of Compton events (which is taken to equal the photon counter efficiency, as the electron counter has been assumed 100 percent efficient).

$N_i = r N_0$ where r is the fraction of the total beam intercepted by the target. Therefore, the observed differential cross section is

$$\frac{d\sigma}{d\Omega} = \frac{1}{R r N_0 \rho_e t} \frac{dN_c}{d\Omega}, \quad (9)$$

The term dN_c is replaced by the observed counts per unit of beam ΔN and $d\Omega$ by $\Delta\Omega$ evaluated from the geometry of the experiment.

The observed angular dependence is compared with the theoretical angular dependence, by equating the cross sections given by Eqs. (2) and (8), and taking $\phi = 8^\circ$. Though $H(\gamma, \phi)$ differs from $\frac{d\sigma}{d\Omega}$ only by the constant factor r_0^2 , H is more convenient, in that its theoretical values lie between zero and one. Thus, the values of $H(\phi)$ shown in Fig. 12 were computed from the relation

$$H(\phi) = \frac{D}{R \int_{K_1}^{K_2} \frac{W(k)}{k} dk} \frac{\Delta N}{\Delta\Omega} \quad (10)$$

where the numerical value of D is 1.43×10^{-7} . The solid curve was computed using Eq. (2), with γ taken as 495. The use of a single value of γ is justified, as all values of the effective incident photon energies are essentially the same (see Table IV). These effective values were computed from the relation

$$K_e = \frac{\int_{K_1}^{K_2} \frac{W(k)}{k} dk}{\int_{K_1}^{K_2} \frac{W(k) dk}{k}}$$

D. Discussion of Errors

Most of the important factors that influence the accuracy of the quantities appearing in Eq. 9 have already been discussed. Here numerical estimates of the uncertainties are given and the effect of these uncertainties upon the calculated values of $H(\phi)$ is evaluated. The dependence of $H(\phi)$ upon the various quantities is more apparent if the integral in the denominator of Eq. 10 is approximated by

$\frac{W(k)}{k} \Delta k$, where $\Delta k = k_2 - k_1$, and D is replaced by

$$D = D' H(\phi)_8 \frac{W(k)_8}{k_8} (\Delta k)_8 \left(\frac{\Delta\Omega}{\Delta N} \right)_8$$

Thus equation (10) becomes

$$H(\phi) = D' H(\phi)_8 \frac{W(k)_8}{W(k)_\phi} \frac{k_\phi}{k_8} \frac{(\Delta k)_8}{(\Delta k)_\phi} \frac{\Delta(\Omega)_8}{\Delta(\Omega)_\phi} \frac{\Delta N_\phi}{\Delta N_8} \frac{R_8}{R_\phi} \quad (11)$$

As seen from Table IV, $\frac{k_\phi}{k_8} = 1$. Also from the values of k_1 and k_2 listed in table IV it is seen that the same part of the bremsstrahlung curve is used at each angle ϕ , therefore

$$\frac{W(k)_8}{W(k)_\phi} = 1; \text{ i.e., } H(\phi), \text{ as calculated here, is nearly independent of}$$

photon energy spectrum.

Though Δk was not computed by the method discussed in III C, the uncertainties in this quantity due to uncertainties in the measured quantities Δmc^2 and $\Delta\phi$ can be conveniently estimated from Eq. (3b). On the basis of the discussion in section III C and Appendix B the uncertainty in Δmc^2 is taken to be one percent or less. The uncertainty in $\Delta\phi$ if taken as half of the maximum possible variation is no larger than 30 percent. By use of these values the maximum uncertainty in Δk is found to be 3.6 percent for $\phi = 4^\circ$. As $\Delta\phi$ is computed in the same manner for each value of ϕ , the error in the ratios of values of Δk should not exceed three or four percent. As discussed in section IV A, errors introduced by misalignment should not exceed 3 percent (See Table III).

Several assumptions were made in the computation of the efficiencies, R . However, as only ratios of the efficiencies appear in Eq. 10, the errors introduced by these assumptions tend to cancel. But the efficiency assigned to the photon counter also depends on the determination of the minimum energy loss in the counter that can be detected. Their value, E_d , was determined as outlined in Section IV, 3. It is apparent that the value of E_d , so determined, can be in error if pulses "pile up" in the crystal, and because of uncertainties in the end point of the counting-rate curve. To minimize these effects the photon-counter efficiencies were assigned on the basis of the known energies of the γ -rays from the radioactive sources and by comparison of the counting rates of the two photon counters at the same position. Fortunately the efficiency does not

vary rapidly with E_d for small values of E_d (see Fig. 12). Also the efficiency curves are nearly parallel for photon energies greater than 8 Mev, which corresponds to scattering angles θ of 20° and less. The ratio of efficiencies due to errors in assigning values of E_d are probably less than 2 percent; however, the absolute error in the efficiency may be as high as 6 percent. The latter value is based on two comparisons of a single counting rate made with photon counters that were independently adjusted.

The uncertainties shown in Fig. 12 include only those due to the counting statistics. Several 10-percent determinations of the counting rate for $\theta = 8^\circ$ were made using various beam rates. All counting rates were within 5 percent of the mean value.

So far, only the factors that influence the relative cross sections have been discussed. Several additional factors must be determined if the absolute cross section is to be computed by Eq. (8). First, the uncertainties in the photon spectrum do not cancel, and introduce an estimated uncertainty of 5 percent in the value of $\frac{d\sigma}{d\Omega}$. Then errors in the determination of the maximum energy of the electrons accelerated in the synchrotron may cause an error in N_0 , and hence in $\frac{d\sigma}{d\Omega}$, as high as 3 percent. By far the most uncertain quantity in the calculation is the measured total energy in the synchrotron beam. Further, P. D. Edwards⁴³, of the University of Illinois, estimates that the Cornell ionization chamber may give total energy values as much as 15 percent low because of recombination in the chamber.

Other factors that have been investigated and which contribute uncertainties of 2 percent or less are target dimensions, density of target, impurities in target, fraction of beam intercepted by target, electron scattering, impurities in cesium fluoride crystal, variation of magnetic field, and the contribution of the photoelectric effect to photon-counter efficiencies. Corrections up to 10 percent were necessary for the absorption of photons by the air column, target, and detector Y.

VII. CONCLUSIONS

The observed values of $\frac{d\sigma}{d\Omega}$ are given in Table IV, and the points plotted in Fig. 12 are these values divided by r_e^2 . The solid curve in Fig. 12 represents the angular distribution given by the Klein-Nishina formula. It is apparent that the experimental points lie close enough to the curve that the theoretical distribution is verified to 10 percent for photons scattered through angles as large as 25° . Though there seems to be a consistent deviation between the experimental and theoretical distributions, in that points for angles greater than 8° lie below the curve and the point at 4° lies above the curve, no significance can be attributed to such differences. Deviations of the order of magnitude of those present can be attributed to systematic errors such as could be introduced by the pair-production cross section or by the relative adjustment of the two photon detectors.

Also the absolute differential cross section for Compton scattering through 8° has been calculated from the data for photons in a 30-Mev energy interval about an effective energy of 253 Mev. The value obtained is

$$\frac{d\sigma}{d\Omega} = 8.45 \times 10^{-27} \frac{\text{cm}^2}{\text{photon}},$$

which is to be compared with

$$\frac{d\sigma}{d\Omega} = 7.08 \times 10^{-27} \frac{\text{cm}^2}{\text{photon}}$$

computed from the Klein-Nishina equation. Although the difference between the two values corresponds to 17 percent of the observed value, the two values are considered to be in agreement. If future measurements of the synchrotron beam intensity are more accurate than the present calibrations, it will be possible to determine from the present data a more accurate value of $\frac{d\sigma}{d\Omega}$, and a 10-percent deviation from theory should be detectable. However, on the basis of total cross-section measurements quoted before^(19, 20) and the confirmation of the angular distribution obtained here, such a deviation seems unlikely for scatter angles as large as 25° .

No attempt has been made to correct the theoretical values for radiative effects discussed by L. M. Brown and R. P. Feynman⁴⁰, as these corrections are approximately only 3 percent of the Klein-Nishina values.

The techniques employed in this experiment can probably be used to obtain relative cross sections reliable to 5 percent. However, to extend the measurements to larger angles, an experimental method should be used to evaluate the counter efficiencies for low-energy photons.

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APPENDIX

A. Kinematics

The relativistic equations for conservation of energy and momentum written in terms of the laboratory coordinates are

$$k + m_0 c^2 = k' + mc^2 \quad \dots\dots\dots (12)$$

$$\frac{k}{c} = \frac{k'}{c} \cos \phi + p \cos \theta \quad \dots\dots\dots (13)$$

$$0 = \frac{k'}{c} \sin \phi - p \sin \theta \quad \dots\dots\dots (14)$$

$$mc^2 = \left[(m_0 c^2)^2 + p^2 c^2 \right]^{1/2} \quad \dots\dots\dots (15)$$

Equation (14) is just the relativistic relation between energy and momentum. Symbols are defined and illustrated in paragraph III A.

From these four equations the following useful relations were derived:

$$k' = \frac{k}{1 + \gamma (1 - \cos \phi)} \quad (16)$$

$$\gamma = \frac{k}{m_0 c^2}$$

$$k = \frac{mc^2 - m_0 c^2}{2} + \frac{1}{2} \left[(mc^2 - m_0 c^2)^2 + \frac{4 m_0^2 c^2 (mc^2 - m_0 c^2)}{1 - \cos \phi} \right]^{1/2} \quad (17)$$

$$\cot \theta = (1 + \gamma) \tan \frac{\phi}{2} \quad (18)$$

B. Electron Orbits

For each electron-counter position two orbits were graphically traced, using a special beam compass made from a 1/2-in. x 1-in. dural bar. The orbits plotted were, first, the path of an electron of minimum energy from the target edge nearest to the center of curvature (i.e. target point 1, Fig. 5) and second, the path of an electron of maximum energy from target point 2. The minimum and maximum values for the electron energies were computed for primary photon energies of 240 Mev and 265 Mev,

assuming a target 1/4 in. wide and 1-1/2 in. high, and various photon channel dimensions. During the runs, however, the target area irradiated was 1/4 in. x 1/2 in., and several of the channels were reduced.

It was noticed that the field variation along the orbits plotted was small enough that it might be possible to approximate the electron paths by circular arcs. Therefore, for each orbit that had been plotted, an effective field value was determined as follows. First, a circular arc was passed through the target and counter points that lay on a traced orbit, with the condition that the center of curvature lie on a line perpendicular to the initial velocity vector for the electron. That is, the following elementary set of equations was solved for R.

$$(x_t - f)^2 + (y_t - b)^2 = R^2$$

$$(x_c - f)^2 + (y_c - b)^2 = R^2$$

$$y_t = m x_t + y_o$$

where $m = \tan (25^\circ + \theta)$

Here the subscript t refers to target point and the subscript c to counter point. θ is the recoil angle of the electron, and the value 25° must be added because the synchrotron beam is 25° to the coordinate system used to measure the magnetic field and locate counters, target, etc. Then the effective value of B is computed from the relation

$pc = 3 \times 10^{-4} BR$. The effective values of B are listed in Table V together with the extreme values of m for each counter position. It is seen that the electron orbits can be approximated by circular arcs without incurring errors greater than 1 percent. Thus, for an electron orbit between any target point and any counter point, it was possible to compute R from the above set of equations, taking the appropriate average value for m. Then the electron energy could be computed from $pc = 3 \times 10^{-4} BR$, using the appropriate average value for B.

TABLE IV.

ϕ	Limits of primary photon energies from central target point		K effective	Efficiency $\times$ Attenuation by counter Y	$K_e \int_{K_1} \frac{W(k)}{k} dk$	Adjusted to fit Klein-Nishina valve at 8°		Valves from Klein-Nishina equation
	$\frac{K_1}{\text{Mev}}$	$\frac{K_2}{\text{Mev}}$				$H(\phi)$	$\frac{d\sigma}{d\Omega} \times 10^{+27}$	$H(\phi)_{KN}$
4	237.5	272.9	254	0.474	1.58	0.289	22.9	0.272
8	238.4	268.5	252.5	0.407	1.34	0.0882	7.08	0.0882
16	240.1	268.1	253	0.328	1.25	0.0236	1.875	0.0247
20	240.1	267.2	253	0.292	1.21	0.0145	1.152	0.0161
25	240.7	267.2	253	0.245	1.18	0.0094	0.747	0.0106
25	240.7	267.2	253	0.287	1.18	0.0104	0.827	0.0106

43 TABLE V.

θ	m = $\tan (25^\circ + \theta)$ for electron leaving target point		Effective field along electron path (Fig. 5)		Average value of magnetic field
	1	2	1 - 1	2 - 2	
			gauss	gauss	
4°	0.54409	0.53322	13,679	13,716	13,706
8°	0.50466	0.49934	13,750	13,786	13,733
16°	0.4852	0.4827	13,861	13,830	13,845
20°	0.48049	0.47928	13,887	13,876	13,882
25°	0.47829	0.47660	13,866	13,886	13,876

TABLE VI

Summary of observed counting rates and solid angles

θ	Bombardment μ -coulombs from precollimator ionization chamber	Counts	Delayed counts	Counting rate $\Delta N = \frac{\text{Counts}}{\mu\text{-coulomb}}$	Solid angle Steradians
4°	61	89	39	0.913 ± 0.19	0.604×10^{-5}
8°	566.7	1033	345	1.21 ± 0.07	0.362×10^{-4}
16°	107.5	156	27	1.21 ± 0.13	1.79×10^{-4}
20°	138.9	210	78	1.01 ± 0.12	2.83×10^{-4}
25°	280.0	381	192	0.68 ± 0.09	3.40×10^{-4}
25°	137.4	155	35	0.873 ± 0.10	3.40×10^{-4}

Fig. 1—Plot of electron recoil angle versus scattered-photon angle. The initial-photon energy, k , is the parameter for the various curves.

Fig. 2—Plot of photon energy as a function of the angle through which the photon has scattered. The initial-photon, k , is the parameter for different curves.

Fig. 3—Plot of the electron "momentum" (pc) versus the angle through which the photon scattered. Again the initial photon energy k is the parameter. The boxes show the angular resolution and the range of electrons accepted.

Fig. 4—Cross section of experiment.

Fig. 5—Electron paths in the magnetic field.

Fig. 6—Photomultiplier circuit and circuit used to shape, limit, and add photomultiplier pulses.

ALL RESISTORS 1/2 WATT, UNLESS SPECIFIED

ALL CAPACITOR VALUES IN μf

Fig. 7—Germanium diode triple-coincidence circuit, inverter and cathode follower.

Fig. 8—Block diagram of electronic equipment in magnet room, exclusive of power supplies.

Fig. 9—Block diagram of electronic equipment in counting area, exclusive of power supplies.

Fig. 10—Relative distribution of photons in the synchrotron beam.
A. Theoretical bremsstrahlung curve computed from the Bethe-Heitler theory.
B. Bremsstrahlung curve corrected for finite thickness of the internal synchrotron target.
C. Bremsstrahlung curve corrected for finite thickness of the internal target and for the energy distribution of the electrons striking the target.

Fig. 11—Comparison of the observed angular distribution of the scattered photons with that predicted by the Klein-Nishina theory. The solid curve was computed from the theoretical equation and matched to the experimental distribution at 8°. The errors shown are just those due to counting statistics.

Fig. 12

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