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HEAT RELEASE IN NUCLEAR REACTORS

By

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ENERGY SOURCES AND MODES OF LOCAL DEPOSITION OF HEAT

Heat is liberated in reactors by the dissipation of the kinetic energy of fission fragments, beta particles, gamma photons, and neutrons. These particles, in penetrating matter, collide with electrons and atomic nuclei and impart energy to them.

Fission fragments, primary gamma photons, and neutrons result directly from fission. Fission fragments emit beta particles and gamma photons by radioactive decay. Neutrons diffuse through the reactor and generate gamma photons by inelastic collisions with and absorptions in various nuclei. Photons arising from all these processes may experience many scattering collisions, losing part of their energy at each collision.

FISSION FRAGMENT AND BETA DECAY ENERGY

The most probable value for the average kinetic energy of the fission fragments, E_{ff} , lies in the range 168 ± 5 mev per fission. This energy is dissipated at the point of fission.

The energy released by the beta decay of fission fragments, $E_{ff\beta}$, is about 7 mev per fission. ⁽¹⁾ Effective ranges are of the order of 1 mm. or less in reactor materials; hence, the beta energy is also liberated at the point of fission.

The sum E_f of the fission fragment and beta ray energies E_{ff} is thus about 175 mev per fission. The heat release rate is proportional to the number of fissions per unit volume per unit time and may be formulated as follows:

$$(G_{ff} + G_{ff\beta}) = \int_{E_n = 0}^{E_n = \infty} \Sigma_f(E_n) \phi_n(E_n) E_n dE_n \quad (1)$$

Here, $\phi_n(E_n)$ represents the energy dependent neutron flux density; i.e., total path length (cm) traversed by neutrons having energy E_n (mev) per unit volume (cm^3) per unit time (sec) per unit energy range (mev) and $\Sigma_f(E_n)$ is macroscopic fission cross section. The integration is carried over all energies; however, in "thermal" reactors, the great majority of the flux is concentrated into a narrow band around the temperature of the medium. Furthermore, the fission cross section, $\Sigma_f(E_n)$ decreases rapidly with increasing neutron energy. Accordingly, in thermal reactors, the integral is approximated very well by the product of the "thermal flux (say, the flux lying between 0.0 and 0.1 ev) and the fission cross section evaluated at the temperature of the medium. Let the thermal flux be denoted by $\phi_n(\text{th})$, whence using Equation 2, and converting to BTU/hr.ft.³, Equation 1 takes the form,

$$G_f = (G_{ff} + G_{ff\beta}) \frac{\text{BTU}}{\text{hr.ft.}^3} = 2.70 \times 10^{-6} \Sigma_f(\text{th}) \phi_n(\text{th}) \quad (2)$$

NEUTRON ENERGY

The two most important processes by which neutrons impart kinetic energy to atomic nuclei are capture and elastic scattering.

Defining δ_{nc} as the average fraction of the kinetic energy of the incident neutron appearing as kinetic energy of the target nucleus after a capture collision, one has

$$\delta_{nc} = 1/(M + 1) \quad (3)$$

where 1 denotes the atomic mass of the neutron, M the atomic mass of the target nucleus.

The local heating effect due to the capture of fast neutrons is thus

$$G_{nc} = \int_{E_n = 0.01 \text{ mev}}^{\infty} \delta_{nc} E_n \Sigma_{nc}(E_n) \phi_n(E_n) dE_n \quad (4)$$

where E_n denotes the energy of the neutrons, and where Σ_{nc} is the macroscopic neutron-capture cross section. G_{nc} must be summed with respect to all kinds of atoms present. In the case of elastic scattering the average fraction of neutron energy E_n that is transferred to the target nucleus is given by

$$\delta_{ne} = (1 - \alpha) / 2 \quad (5)$$

where

$$\alpha = \left[(M - 1) / (M + 1) \right]^2 \quad (6)$$

The elastic collision heating rate is then

$$G_{ne} = \int_{0.01 \text{ mev}}^{\infty} \delta_{ne} E_n \Sigma(E_n) \phi_n(E_n) dE_n \quad (7)$$

where Σ_{ne} is the macroscopic cross section for elastic scattering. Note, thermal neutrons make no contribution. G_{ne} must be summed with respect to all kinds of atoms present in the medium.

ENERGY RELEASE BY BETA DECAY OF ACTIVATED NUCLEI

Nuclei rendered unstable by the capture of a neutron may decay by beta emission. For a given transition, the beta particles are emitted with energies ranging continuously from zero up to some characteristic maximum, E_β (max). (12). Marinelli lists E_β for several nuclides (13). Since the contribution to the local heating from beta decay of activated nuclei is usually small compared to other contributions, it is generally sufficiently precise to take the average electron energy to be one-third E_β (max). Thus, the local heating rate due to "capture" betas becomes

$$G_{nc\beta} = \int_0^{\infty} \Sigma_{nc}(E_n) \phi_n(E_n) dE_n \cdot \sum_j N_j E_{\beta j}(\max)/3 \quad (8)$$

where N_j is the number of betas of characteristic energy $E_{\beta j}(\max)$ emitted in the decay chain per neutron capture. The decay process may proceed through several isomeric states and atomic numbers. Note that the contribution from "thermal" neutrons in Equation 8 may be large and override the contribution from fast neutrons.

In moderators, the contribution of beta decay to the local heating rate is negligible in all except possibly light water. It may become appreciable in structural materials, fuel diluents and clad, coolants and other materials exposed to high thermal-neutron fluxes when these have appreciable capture cross sections.

NEUTRON FLUX DISTRIBUTION

The neutron diffusion and transport theories yield descriptions of the spectral and spatial distributions of the neutron fluxes, $\phi_n(E_n)$. These theories have been discussed elsewhere and will not be considered here.

A one or two-group diffusion calculation will suffice to describe the "gross" variation of the thermal flux in many thermal reactors with sufficient precision for calculations of G_f . Let $\phi_n(th,0)$ be the flux at some reference point in a homogeneous reactor (say the center), and let $f(\vec{r})$ denote a function, obtained from diffusion theory, describing the spatial variation of $\phi_n(th,\vec{r})$, where $\vec{r}$ denotes the radius vector from the reference point, such that

$$\phi_n(th,\vec{r}) = \phi_n(th,0) f(\vec{r}) \quad (9)$$

The heating rate is then given by the equation

$$G_f(\vec{r}) = \frac{q_f f(\vec{r}) \Sigma(th,\vec{r})}{\int_{V_f} f(\vec{r}) \Sigma(th,\vec{r}) dV} = \frac{q_f}{V_f} \left(\frac{G_o}{G_{av}} \right) f(\vec{r}) \quad (10)$$

where Q_f is the total heat liberated by fission fragments and their β decay.

The energy available from fission fragments (kinetic and beta decay) is about 175 mev per fission. The total energy per fission is about 200 mev. Thus Q_f is about 87 per cent of the total reactor heat output, Q_t . The balance of Q_t is generated by fission gammas (8 mev), fission fragment decay gammas (6 mev), fast neutrons (5 mev), capture gammas and betas (7 mev). The particles bearing this energy (about 25 mev) tend to leak out of the fuel-bearing zones, and liberate heat in the coolant, reflector, thermal shield, pressure shell, and biological shield. However, in designing the fuel-bearing core, it is sufficient and conservative to assume that all of the reactor heat, Q_t , of a reactor is released in the fuel, and that it has the same spatial distribution as Q_f .

The function $f(\beta)$ has been evaluated by Glasstone and Edlund⁽⁶⁾ for certain bare, homogeneous, thermal reactors of simple shape, with $\Sigma_f(th, \beta)$ uniform.

TABLE 1

Geometry	$f(\beta)$	G_o/G_{av}
Sphere of radius R	$\sin(\pi\rho/R)/(\pi\rho/R)$	3.29
Cube of side A	$\cos(\pi x/A)\cos(\pi y/A)\cos(\pi z/A)$	3.64
Right cylinder of radius R	$J_0(2.405 r/R)\cos(\pi z/2R)$	3.87

By integrating the given functions over the corresponding geometric forms, the ratio of the maximum heat release rate, G_o to the average rate G_{av} , is readily obtained. Values are listed in Table 1. The presence of moderating reflectors, control elements, structural materials, coolants, ducts, and non-uniform loading (variable Σ_f) modify these simple distributions. Space does not permit treatment of these effects, but it is import-

ant to reduce G_o/G_{av} so that a larger fraction of the core may operate at or near the limiting conditions and so that burn-up of nuclear fuel and accompanying effects will be more uniform.

In order to increase resonance escape in low enrichment uranium reactors, the fuel is sometimes segregated from the moderator. Fast neutrons, released by fission in the fuel, have a high probability of escaping into the moderator where they are quickly slowed down without exposure to resonance absorption in U^{238} . The heat release ⁽¹⁾ in polar coordinates (corresponding to the case of an infinitely long fuel rod immersed in a uniform external flux), is given by

$$G(r) = \frac{\lambda q I_0(\lambda r)}{I_1(\lambda R)} \quad (11)$$

where r is the distance from the axis of the fuel rod, $\lambda = \sqrt{3\Sigma_a\Sigma_{tr}}$, R = rod radius, I denotes the modified Bessel function of the first kind, and q is the local heat flux at the surface of the rod.

In the design of fuel elements, it is customary and conservative practice to assume that all of the fission energy (200 mev per fission) is released in the fuel. In estimating heat release in moderator, coolant pressure shell, and shield, however, it is necessary to estimate the leakage out of the fuel zone of energy associated with fast neutrons and gamma rays. A fair estimate of the fast neutron spectral distribution as a function of position in the moderator of heterogeneous reactors employing long fuel rods may be had from the "Fermi age" treatment ⁽⁶⁾ of simple models e.g., an infinite matrix of line sources with negligible capture of fast neutrons in U^{235} and U^{238} and only weak capture in the moderator. The flux at radius r from a single infinite line source of strength S_n nascent neutrons per unit length per unit time immersed in infinite moderator is given by

$$\phi_n^*(r, E_n) = \frac{1}{\xi \Sigma_t} \int_{E_s=E_n}^{\infty} \frac{S P(E_s) p(E_s, E_n) dE_s}{4\pi \gamma(E_s, E_n)} \exp \left[-r^2/4 \gamma(E_s, E_n) \right] \quad (12)$$

where E_s is the energy of the nascent neutrons, $P(E_s)$ their spectral distribution, $p(E_s, E_n)$ is the capture-escape probability, and $\gamma(E_s, E_n)$ is the Fermi age. D is the diffusion coefficient, Σ_a and Σ_t are the absorption and total cross sections respectively.

The flux at any point in the moderator is constructed by summing the contributions given by Equation 12 for all line sources.

$$P_E(E_s) = 0.484 (\sinh \sqrt{2E_s}) \exp(-E_s) \quad (13)$$

$$p(E_s, E_n) = \exp \left[- \int_{E_n}^{E_s} \Sigma_a dE / \xi \Sigma_t E \right] \quad (14)$$

$$\gamma(E_s, E_n) = \int_{E_n}^{E_s} D dE / \xi \Sigma_t E \quad (15)$$

$$\alpha \approx \frac{2}{M + \frac{2}{3}} \quad (16)$$

Gamma Heating Calculations

The straight-ahead-scattering, integral spectrum method of Hurwitz⁽⁷⁾ is here presented. While more sophisticated and more accurate methods are available, these are laborious, must be applied carefully, and do not lend themselves to recipe type of presentation. The integral spectrum method has

the merit that it accounts for all the gamma energy, that it is reasonably accurate if the number of attenuation lengths is not too great, and that estimates of the heating rate are quickly and easily obtained in many cases.

Suppose it is desired to compute the rate of heat release at some point P (Figure 1) in a medium M_m separated by several other regions of various compositions from a region M_1 containing a distributed gamma source. Let an element of volume in the source at the point having vector coordinates $\vec{\rho}_s$ be denoted by δV , and let other geometric relations be as indicated in the figure. The heating rate at P, in the straight-ahead-scattering, integral-spectrum method, is given by

$$G(\vec{\rho}_P) = \int_{V_1} S_\gamma(\vec{\rho}_s) dV_1 \bar{E}_\gamma(\vec{\rho}_s) P_R(\alpha, \beta, \gamma, \vec{\rho}_s) \bar{\mu}_{e,m} (1/\rho)^2 \exp \left[- \sum_{n=1}^{n=\infty} \bar{\mu}_{e,n} \Delta \rho_n \right] \quad (\text{C.F.}) \quad (17)$$

where

$G(\vec{\rho}_P)$ = heating rate at P in some convenient set of units,
e.g., BTU/hr.ft.³ or mev./cm.³ sec.

V_1 = volume of the source medium.

$S_\gamma(\vec{\rho}_s)$ = Magnitude of the gamma source strength at $\vec{\rho}_s$ in
convenient units, e.g., gammas/cc.sec.

$\bar{E}_\gamma(\vec{\rho}_s)$ = Mean energy of the gammas emitted at $\vec{\rho}_s$, mev/gamma.

$P_R(\alpha, \beta, \gamma, \vec{\rho}_s)$ = angular distribution of photons emitted at
 $\vec{\rho}_s$, i.e., the fraction of the photons $S_\gamma(\vec{\rho}_s)$
that are emitted parallel to a line whose directions
cosines are α, β, γ , per steradian of solid angle
about that line.

$\bar{\mu}_{e,n}$ = Mean photon energy absorption coefficient in medium n for photons having the spectral distribution of the source photons.

$\Delta \rho_n$ = Length of path in medium n of line-of-sight path between P and volume element δV .

ρ = Length of line-of-sight path between P and volume element δV .

There are four important primary gamma sources: (a) prompt fission (b) fission product decay (c) neutron "radiative" capture in fuel and other components, and (d) inelastic scattering of fast neutrons. The energy available from the first three sources is now known reasonably well, but only fragmentary data are available for the last. The number of events of a given type is found by multiplying the neutron flux by the appropriate cross section. The gamma source is obtained by multiplying the number of events by the number of gammas emitted per event. Thus

$$S_\gamma(\vec{r}_s) E_\gamma(\vec{r}_s) = \phi_n(\vec{r}_s) \Sigma_A(\vec{r}_s) N_\gamma(A) E_\gamma(\vec{r}_s) \quad (18)$$

where ϕ_n denotes the neutron flux at $\vec{r}_s$, Σ_A the appropriate cross section, and N_γ the number of gammas emitted per event of type A .

There are about 7.5 prompt fission gammas emitted per fission having an average energy of about 1.0 mev (5). Thus, for prompt fission gammas $N_\gamma E_\gamma = 7.5$ mev/fission. Recent data (8) indicate that $N_\gamma E_\gamma$ for decay of fission products is about 4.7 mev. of gamma energy per fission. The gamma energy $N_\gamma E_\gamma$ radiated by nuclei following capture of a neutron is equal to the binding energy of the incident neutron plus its kinetic energy. Since most captures involve neutrons having energies in the thermal range, it is sufficient to set the energy release equal to the binding energy. This should be computed for each individual atomic species, but is approximately 7 mev.

per capture for heavy and intermediate elements. The data on the energy available from inelastic scattering are fragmentary at best. For heavy elements, one may apply the Weisskopf formula; however, the calculation is complicated and requires precise knowledge of the fast flux distribution in energy and space. The heating done by gammas resulting from inelastic scattering is usually compared to heating by gammas from other sources, and the error introduced by neglecting inelastic scattering is probably of the same order of magnitude as that introduced by use of the integral spectrum method itself.

For isotropic sources, such as thin fuel plates, fuel rods of small diameter, or an element of source volume, $P_{\Omega}(\alpha, \beta, \gamma, \vec{\rho}_s) = 1/4 \pi$. For slab sources exhibiting appreciable self-absorption and having a uniform distribution of source strength (Fig. 2)

$$S_{\gamma}(\vec{\rho}_s) dV_1 E_{\gamma}(\vec{\rho}_s) P_{\Omega}(\alpha, \beta, \gamma, \vec{\rho}_s) = \frac{\phi_n \sum N_i E_{\gamma} \alpha}{4\pi \mu_{e,1}} \left[1 - \exp(-\bar{\mu}_{e,1} b/\alpha) \right] dA \quad (19)$$

where dA is an element of surface area of the slab, b is the thickness, and α is the cosine of the angle made by the line-of-sight between dA and P with the normal to the surface of the slab. For thick slabs, the exponential term is negligible for all angles; for thin slabs, $\mu_{e,1} b/2 \ll 1$, and the expression reduces to

$$S_{\gamma}(\vec{\rho}_s) dV_1 E_{\gamma}(\vec{\rho}_s) P_{\Omega}(\alpha, \beta, \gamma, \vec{\rho}_s) = \frac{\phi_n \sum N_i E_{\gamma} b}{4\pi} dA \quad (20)$$

For thick slab sources in which the source strength increases linearly away from the surface (2) one has

$$S_{\gamma}(\vec{\rho}_s) dV_1 E_{\gamma}(\vec{\rho}_s) P_{\Omega}(\alpha, \beta, \gamma, \vec{\rho}_s) = \frac{\phi_n \sum N_i E_{\gamma} a}{4\pi \mu_{e,1}} \left[1 + \frac{C a}{\mu_{e,1}} \right] dA \quad (21)$$

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where ϕ_0 is the flux level at the surface of the slab, and C is the slope of the flux distribution.

For cylindrical sources (fuel rods) the following expression may be used provided the point P is more than one rod diameter from the rod.

$$P_{\Omega}(a, \beta, \gamma, \vec{\rho}_s) dV_1 = \left[\frac{I_1(2\bar{\mu}_{e,1} R/a) - H_1(2\bar{\mu}_{e,1} Ri/a)}{4\pi R \bar{\mu}_{e,1}} \right] \pi R^2 dz \quad (22)$$

where I_1 is the modified Bessel function of the first kind of order one, H_1 is Struve's function of order one, i is $\sqrt{-1}$ (the argument of H_1 is imaginary), R is the radius of the rod, dx is an element of rod length, and a is the cosine of the angle made by the line joining the element dx and P with a normal to the axis of the rod. Struve's function for imaginary argument is tabulated below.

TABLE II
Struve's Function

Y	H (i Y)
0.0	0.0000
0.1	0.0033
0.2	0.0135
0.4	0.0538
0.6	0.149
1.0	0.356
2.0	1.727
3.0	5.33
4.0	14.38

The mean energy absorption coefficient $\bar{\mu}_e$ is obtained by integrating the energy dependent absorption coefficient over the spectrum of the gamma source.

$$\bar{\mu}_{e,n} = \frac{1}{E_s} \int_0^{\infty} \mu_{e,n}(E_r) P_E(E_r) E_r dE_r \quad (23)$$

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where E_γ is the gamma energy, P_E denotes the spectral distribution, and

$\mu_{e,n}(E_\gamma)$ is the gamma-energy absorption coefficient for photons of energy E_γ in the nth medium. For prompt fission gammas

$$P_E(E_\gamma) = 7.5 \exp(-E_\gamma) \quad (24)$$

where E_γ is in mev., and $\bar{E}_\gamma$ is 1.0 mev (5). For fission product decay gammas,

$$P_E(E_\gamma) = 9.0 \exp(-1.33 E_\gamma) \quad (25)$$

and $\bar{E}_\gamma$ is about 1.1 mev (8). The data on the spectrum of gammas resulting from neutron capture and from inelastic scattering is fragmentary. The spectra vary with the nuclide and the energy of the incident neutron. Data for certain nuclides will be found in references 12 and 13.

The energy absorption coefficient $\mu_a(E_\gamma)$ is defined by

$$\mu_a = \mu_{pe}(E_\gamma) + \mu_{pp}(E_\gamma) + \left[f_c(E_\gamma) + f_o(E_\gamma, E_o) \right] \mu_c(E_\gamma) \quad (26)$$

where μ_{pe} is the photoelectric absorption coefficient for the nth medium,

μ_{pp} is the coefficient for pair production, μ_c is the coefficient for Compton

scattering, f_c is the average fraction of its energy E_γ that a photon loses

in a Compton collision, and f_o is the fraction of E_γ carried away by photons

scattered to energies below some cut-off energy E_o . Since pair production

results in the formation of a positron which shortly combines with an electron

to produce two 0.51 mev photons, and since it is convenient to assume these are

absorbed in the vicinity of the point where the pair production took place, it

is consistent to take E_o as 0.51 mev. The coefficients f_c and f_o (for E_o

of 0.51) are graphed in Figure 3. Values of μ_e for various materials calculated

by Snyder and Powell (9) are graphed in Figure 4. These, however, are

uncorrected for low-energy cut-off (i.e., f_o is zero).

Using these relations, the heat release due to gamma rays may be estimated for many cases of interest without excessive labor. A straight-forward and powerful approach used by Claiborne and Fowler (3) consists in dividing up the entire reactor into a convenient number of zones, computing the gamma source strength of each, applying the isotropic emission probability $1/4\pi$, and summing over all the sources. It will turn out in many cases that only zones adjacent to the point P will make significant contributions to the heating at P, and that more remote zones may be neglected. This will become obvious in the course of the calculation. The more complex probabilities and related products of Equations 19 through 22 will simplify the treatment of many cases.

In applying the straight-ahead-scattering approximation, it should be kept in mind that the method overestimates the penetration of the rays. Since the total energy is correctly accounted for, it is, by this method, spread out more than is physically the case. Hence, the heat release rate near the source will be underestimated, and will be overestimated at points farther out, the error becoming large percentage-wise at several relaxation lengths from the source.

Integration of the Gamma Heating Equation for Simple Cases

in Eq. 17

For several of the most frequently occurring cases the integral has been evaluated. Most of these cases involve the limitation that the gamma source shall be uniformly distributed.

Cylindrical Sources

For the case where the self-absorption by the gamma source is negligible, and where the source strength is uniform along the cylinder, the heat release rate at a point P (Figure 5) is given by

$$G = \frac{S_\gamma \bar{E}_\gamma \bar{\mu}_{e,2} R^2}{4a} \left[F(\theta_2, b) - F(\theta_1, b) \right] \quad (27)$$

where S_γ = source strength, gammas per unit volume per unit time.

R = rod radius

$\theta_1 = \arctan(z_1/a)$

$\theta_2 = \arctan(z_2/a)$

$b = \bar{\mu}_{e,2} a$

The function $F(\theta, b)$ is graphed in Figure 6 taken from the work of Foderaro and Obenshain (4). The cylindrical source is surrounded by a single medium characterized by an energy absorption coefficient $\bar{\mu}_{e,2}$. The presence of other source cylinders in the matrix is necessarily ignored. Equation 27 fails for points lying near the surface of the cylinder; for this case Equation 28 below is recommended.

If the absorption of gamma rays in the source itself is appreciable, the heating at points near the cylinder may be estimated by the method developed by Taylor and Obenshain (10)

$$G = \frac{S_\gamma \bar{E}_\gamma \bar{\mu}_{e,2} R^2}{\bar{\mu}_{e,1}} F(b_1, b_2, R/a) \quad (28)$$

where

$$b_1 = \bar{\mu}_{e,1} a \quad (29)$$

$$b_2 = \bar{\mu}_{e,2} a \quad (30)$$

The function $F(b_1, b_2, R/a)$ is graphed in Figure 7, taken from the cited reference. The range of these charts is rather limited, and for points lying not too close to the source cylinder, the following approximation may be used.

$$G = \frac{S \gamma \bar{\mu}_{e,2} R^2}{4(a+Z)} \left[F(\theta_2, b) - F(\theta_1, b) \right] \quad (31)$$

where $F(\theta, b)$ is the function graphed in Figure 6, and

$$b = \bar{\mu}_{e,1} R + \bar{\mu}_{e,2} a \quad (32)$$

and where, if $a/R \geq 10$, $\mu_{e,1} Z = f(\bar{\mu}_{e,2} R)$ graphed in Figure 8, and if

$a/R < 10$, $\mu_{e,1} Z/m = f(a/R, \bar{\mu}_{e,1} R)$ as graphed in Figure 9 and $m = f[\bar{\mu}_{e,1}(a+R), a/R]$

as graphed in Figure 10. Z is found by solving these relations simultaneously.

The effect of concentric heterogeneities such as cladding, coolant annuli, etc., may be handled by replacing $\bar{\mu}_{e,2} a$ in the preceding equations by $\sum_{n=2}^{n=m} \bar{\mu}_{e,n} \Delta a_n$

where Δa_n is the thickness of the n th concentric layer of materials.

Slab Sources

Slab sources of interest include fuel plates, fuel solutions in spherical or cylindrical reactors where the curvature of the wall is small (-i.e., where the radius of the reactor is at least several times larger than $\bar{\mu}_{e,1}$ for the fuel), core tanks, reflectors, blankets, thermal shields, and reactor vessels.

The gamma heating in a reactor core consisting of an assemblage of fuel plates can often be estimated by judicious application of the equation developed for an infinite slab containing a uniformly distributed gamma source. Figure shows a section through a reactor core containing clad fuel plates cooled by a liquid flowing between the plates. If now the various sections are of infinite extent in the y and z directions, one has, for the contribution of one infinite slab

$$G = \frac{S_r \bar{E}_r \bar{\mu}_{e,m}}{2 \bar{\mu}_{e,1}} \left[b_1 E_1(b_1) - b_2 E_1(b_2) + \exp(-b_2) - \exp(-b_1) \right] \quad (33)$$

where

$$b_1 = \sum_{n=1}^{n=m} \bar{\mu}_{e,n} \Delta a_n \quad (34)$$

$$b_2 = \sum_{n=2}^{n=m} \bar{\mu}_{e,n} \Delta a_n \quad (35)$$

and the function E_1 (exponential integral of order one) is graphed in Figure 11 taken from Foderaro and Obenshain (4).

An important case of non-uniform distribution of the gamma source is encountered in the large aqueous homogeneous reactors. If, near the wall of the core of such a reactor the neutron flux varies linearly, as shown in Figure 12, then the heating is given by

$$G = \frac{\beta_o \Sigma_r N_r \bar{E}_r \bar{\mu}_{e,m}}{2 \bar{\mu}_{e,1}} \left[\left(\frac{C_b}{2 \bar{\mu}_{e,1}} \right) E_2(b) + \frac{C}{2 \bar{\mu}_{e,1}} \exp(-b) \right] \quad (36)$$

where

$$b = \sum_{n=2}^{n=m} \bar{\mu}_{e,n} \Delta a_n$$

and C is the slope of the neutron flux level.

The heating in regions adjacent to reflectors and blankets may frequently be treated similarly.

Another important case of non-uniform distribution is that of the heating of pressure shells and thermal shields by gamma rays resulting from neutron capture in the shell or shield. Suppose the net current J of thermal neutrons into a shield, such as shown in Fig. 13, is known. The heating at P within the

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shield due to neutron captures in the shield may be estimated by the relation

$$G = \frac{\chi J N_{\gamma} \bar{E}_{\gamma}}{2} \exp(-\chi a) \left[F(b_1, c) + F(b_2, c) \right] \quad (38)$$

where

$$\chi = \sqrt{3\Sigma_a \Sigma_{tr}} \quad (39)$$

$$b_1 = \bar{\mu}_e a \quad (40)$$

$$b_2 = \bar{\mu}_e (b-a) \quad (41)$$

$$c = \chi / \bar{\mu}_e \quad (42)$$

and $F(b, c)$ is graphed in Fig. 14 (4).

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NOMENCLATURE

- a Distance from gamma source to point P ($\vec{p}$)
- b Thickness of slab source of radiation, also parameter in Eq. 27 et seq.
- c Slope of flux distribution near boundary of gamma source region;
 $\phi = \phi_0(1+Cx)$, where ϕ_0 is the flux level at the boundary, and x is the distance from the boundary
- d Density of medium, g/cc
- E Energy of particle, mev.
- $E_{1,2,\dots}$ Exponential integrals graphed in Fig. 11.
- E_f Average kinetic energy of fission fragments plus average energy of betas resulting from decay of fission fragments, 187 mev/fission.
- E_s Energy of nascent neutrons, mev.
- f_c Average fraction of its energy that is released by a photon during a Compton collision.
- $f(\vec{p})$ Geometric form of neutron flux density distribution.
- f_0 Fraction of the energy E_γ of a photon that is carried away by photons having energies less than E_0 following a Compton collision.
- F Attenuation functions graphed in Figs. 6, 7, and 14.
- G Specific rate of heat release due to dissipation of the energy of nuclear fission and accompanying radiations, energy per unit volume per unit time.
- H Struve's function.
- I Modified Bessel function of the first kind.
- i $\sqrt{-1}$
- J Net current of thermal neutrons into a thermal shield.
- M Mass of a nucleus in atomic mass units.
- (mc^2) Rest mass of electron--0.51 mev.
- n Number of neutrons per unit volume.
- N_j Number of particles of characteristic energy $E_{\beta j}(\text{max})$ emitted in decay chain following a neutron capture.
- N_γ Number of photons emitted per neutron interaction.
- $p(E_s, E_n)$ Capture escape probability for neutron slowed down from E_s to E_n .
- P Point at which it is desired to compute gamma heating rate.
- P_Ω Angular distribution of radiation source, $P_\Omega d\Omega$ is the fraction of particles radiated into solid angle $d\Omega$ about the direction (α, β, γ) .
- P_E Spectral distribution of neutrons or photons, $P_E dE$ is the fraction of particles having energies in range dE about the energy E.

q	Heat flux, heat units per unit time.
Q	Rate of heat release in nuclear reactor, energy per unit time.
R	Radius of fuel rod or cylindrical gamma source.
r	Radial coordinate in cylindrical geometry.
S	Source function, particles generated per unit volume per unit time.
th	Thermal energy, 0.025 ev.
V	Volume throughout which radiation sources are distributed.
V_f	Fuel volume.
x,y,z	Cartesian coordinates.
Z	Parameter in graphs 8,9,10.
α	Physical property of nucleus defined by Eq. 6; also direction cosine of a ray with respect to the x-axis.
β	Direction cosine with respect to the y-axis.
γ	Direction cosine with respect to z-axis.
δ	Average fraction of energy of incident particle that is released locally as heat during a collision.
χ	Physical property of diffusing medium, $\chi = \frac{1}{3} \frac{\Sigma_s}{\Sigma_a + \Sigma_{tr}}$.
θ	Angle of deflection in a scattering collision, also parameter in Eq. 27 et seq.
$\vec{\rho}$	Radius vector from origin of coordinates.
ρ	Radial coordinate in spherical geometry.
Σ	Macroscopic cross section, probability of a collision of specified type occurring per cm of travel, events per neutron-cm.
τ	Fermi age, cm^2 .
κ	Photon interaction coefficient, probability of a collision of specified type occurring per cm of travel, events per photon-cm.
μ_0	Mean effective gamma absorption coefficient, defined by Equation 23.
μ_0	Gamma photon energy-absorption coefficient defined by Equation 26.
μ_0	Logarithmic energy decrement for neutron scattering, $\xi \approx 2/(M + 2/3)$
Φ	Particle flux density, number of cm traversed by neutrons or photons per cm^2 per second.
∇^2	Laplacian operator.

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Subscripts

a	Absorption.
c	Compton scattering of photons or capture of neutrons.
e	Elastic scattering of neutrons, see also μ_e .
f	Fission, see also E_f .
ff	Fission fragments.
ff β	Beta particles resulting from beta decay of fission fragments.
i	Inelastic scattering.
k	Neutron interaction process of kth type--absorption, inelastic scattering, etc.
m	Medium m in which point P is located.
n	Neutrons, also denotes nth medium along path of gamma ray attenuation.
o	Cut-off energy for gamma heating calculations.
pe	Photo-electric.
pp	Pair-production.
S	Source.
β	Beta particle.
γ	Gamma particle.

Fig. 1.

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Fig. 2

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ORNL-LR-DWG 15000 Fig. 3. Gamma Energy Transfer Coefficients; $E_0 = 0.51$ Mev.

Fig 4 Energy Absorption Coefficients (Macroscopic Cross Section for Energy Absorption divided by Density)

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Fig. 5.

Fig. 6.

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Fig. 8.

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Fig. 9

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Fig. 10.

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Fig. 11.

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Fig. 12

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Fig.13

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Fig. 14.