UNCLASSIFIED

OAK RIDGE NATIONAL LABORATORY

Operated By

UNION CARBIDE NUCLEAR COMPANY

POST OFFICE BOX P
OAK RIDGE, TENNESSEE
ORNL
CENTRAL FILES NUMBER
0 CF-55-10-118

DATE: October 27, 1955

COPY NO. 135SUBJECT: Effect of Non-Uniform Neutron Flux Distribution
on the Temperature Distribution in Thick Fuel
Elements.

TO: OAK RIDGE SCHOOL OF REACTOR TECHNOLOGY

FROM: L. G. Alexander

DISTRIBUTION

100	L. G. Alexander
101-120	R. A. Chapple
121	F. C. VanderLage
122	M. W. Rosenthal
123	F. C. Zoole
124	J. Wentland
125	J. Dreske
126	F. Neill
127	A. Estes
128	F. Spievak
129	R. Grevier
130	E. Howell
131	D. Hamilton
132	S. Jennings
133	C. Smith
134-135-141	Lab Records
142-143	Central Research Library
144	Lab Records

NOTICE

This document contains information of a preliminary nature and was prepared primarily for internal use at the Oak Ridge National Laboratory. It is subject to revision or correction and therefore does not represent a final report.

UNCLASSIFIED

834-1

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

1

EFFECT OF NON-UNIFORM NEUTRON FLUX DISTRIBUTION ON THE TEMPERATURE DISTRIBUTION IN THICK FUEL ELEMENTS

Oak Ridge School of Reactor Technology
Oak Ridge National Laboratory

by

L. G. Alexander

In calculating the temperature distribution in fuel elements it is commonly assumed that the heat generation rate is uniform over cross section of fuel zone. This is a good approximation for thin fuel elements such as are used in the Materials Testing Reactor; however, in heterogeneous reactors, such as the Oak Ridge Graphite Reactor, where the fuel is segregated from the moderator in order to increase the resonance escape probability, there is necessarily a non-uniform distribution of the fission-heat liberation rate. It is the purpose of this paper to investigate the effect of the non-uniform heating rate on the temperature distribution in such cases.

Theoretical

The general equation for conduction of heat with internal generation is

$$\nabla k \nabla T + G = 0 \quad 1$$

where T is the temperature, k the thermal conductivity, and G the heat release rate in heat units per unit volume per unit time. The case where k is independent of the temperature will be treated.

Consider an infinite slab of thickness W , uniformly and equally cooled on both sides, where G is symmetrical about the midplane and varies only with x , the coordinate normal to the surface of the slab. The temperature will also be symmetrical about the midplane, and its derivative with respect to x there

$$G(x) = G(-x)$$

$$T(x) = T(-x)$$

$$dT/dx = 0 \text{ when } x = 0$$

For this case, the temperature distribution is readily obtained in the form

$$T(x) - T_0 = - \int_0^x (1/k) \int_0^{x'} G(x'') dx'' dx' \quad 2$$

where T_0 is the temperature of the midplane. In cylinders of diameter D , with comparable boundary conditions,

$$T(r) - T_0 = - \int_0^r (1/r'k) \int_0^{r'} G(r'') r'' dr'' dr' \quad 3$$

where r is the radial coordinate.

If the release rate, G , is uniform, Equations 2 and 3 are integrable into

$$(\Delta T)^* = T_0 - T_s = G W^2/8k \quad 4$$

for slabs, and

$$(\Delta T)^* = T_0 - T_s = G D^2/16k \quad 5$$

for cylinders, where T_s is the temperature at the surface of the slab or cylinder, and $(\Delta T)^*$ denotes the temperature difference between surface and T_0 when the heat release rate is uniform.

The heat release rate is proportional to the thermal neutron flux. The equation governing the distribution of the flux is

$$\nabla^2 \phi - \kappa^2 \phi = 0 \quad 6$$

where χ^2 is the ratio of the thermal neutron absorption cross section Σ_a to the thermal neutron diffusion coefficient D_n . Its value is about 0.49 cm^{-2} for natural uranium. The applicable boundary condition is that the gradient of the flux is zero at the midplane or centerline. Thus, in slabs

$$G(x) = G_0 \cosh (\chi x) \quad 7$$

and in cylinders

$$G(x) = G_0 I_0 (\chi r) \quad 8$$

where G_0 is the heat generation rate at the centerline and I the modified Bessel function of the first kind. These expressions can be set into Equations 2 and 3. However, if a comparison of the resulting temperature differences is to be made with those given by Equations 4 and 5, then the restraint that the average heat release rate, $\bar{G}$, in the non-uniform case should be set equal to G in Equations 4 and 5 should be imposed. $\bar{G}$ is readily evaluated by integrating Equations 7 and 8 over the fuel zones. In slabs

$$\bar{G} = G_0 (2/\chi W) \sinh(\chi W/2) \quad 9$$

while in cylinders,

$$\bar{G} = 2G_0(2/\chi D) I_1 (\chi D/2) \quad 10$$

Use of Equations 9 and 10 to eliminate G_0 from Equations 7 and 8, substitution thence into Equations 2 and 3, followed by integration gives

$$(\Delta T) = T_0 - T_s = (\bar{G}^2/8k) \quad 2 \frac{[\cosh (\chi W/2) - 1]}{(\chi W/2) \sinh (\chi W/2)} \quad 11$$

for slabs, and

$$(\Delta T) = T_o - T_s = (\bar{G}D^2/16k) \frac{2[I_0(\frac{D}{2}) - 1]}{(D/2)I_1(\frac{D}{2})} \quad 12$$

for cylinders. Dividing these by Equations 4 and 5, it is found that

$$\frac{(\Delta T)}{(\Delta T)^*} = \frac{2[\cosh(\lambda W/2) - 1]}{(\lambda W/2) \sinh(\lambda W/2)} \quad 13$$

for slabs, and

$$\frac{(\Delta T)}{(\Delta T)^*} = \frac{2[I_0(\lambda D/2) - 1]}{(\lambda D/2)I_1(\lambda D/2)} \quad 14$$

for cylinders. These temperature ratios were computed for various values of the arguments. The results are listed in Table I and graphed in Figure 1. The radii and thicknesses of natural uranium rods and plates corresponding to the arguments are also given.

TABLE I

Temperature Defect in Fuel Slabs and Rods

<u>Natural Uranium</u> W or D, cm	<u>Dimensionless Argument</u> $\lambda W/2$ or $\lambda D/2$	$\Delta T/\Delta T^*$	
		Slabs	Rods
0.0	0.0	1.000	1.000
0.5	0.175	1.000	1.000
1.0	0.350	0.990	0.994
1.5	0.525	0.978	0.986
2.0	0.700	0.962	0.970
2.5	0.875	0.938	0.952
3.0	1.050	0.916	0.935
3.5	1.225	0.891	0.916
4.0	1.400	0.865	0.891

It is seen that the temperature difference between fuel center and surface based on neutron diffusion theory is about ten per cent less in natural uranium elements about 4 cm thick than is the difference obtained when the neutron flux

is assumed uniform. The effect is stronger, of course, in enriched elements.

Experimental

Woods and Biehl⁽¹⁾ present experimental data on the distribution of thermal neutron densities in natural uranium rods 1.0 and 1.5 inches in diameter in D₂O moderated lattices of various dimensions which may be used in place of Equation 7 in evaluating (ΔT) in Equation 3. The dip in the flux, and therefore also in the heat release rate, is rather greater than would be predicted from diffusion theory. A discrepancy is to be expected, of course, because diffusion theory gives poor results near a boundary--roughly within one mean free path, which is of the order of 1 cm in the uranium.

The results calculated from the experimental data are shown plotted on Figure 1 as circles. It is seen that whereas diffusion theory results in prediction of temperature effects $(\Delta T)/(\Delta T)^*$ of 0.95 and 0.90 for the 1.0 and 1.5 inch rods respectively, the experimental data yield the values 0.90 and 0.86 respectively.

Conclusions

Taking a 10 per cent decrease as the criterion for significance, it is seen that the decrease in the temperature difference ΔT due to "self-shielding" is important in natural uranium rods having diameters in excess of 3 cm. The effect is greater, of course, in enriched fuels. However, enriched fuel elements must have high ratios of heat transfer surface to volume in order to avoid high temperature differences. As a result, the elements take the forms of thin plates, thin-walled tubes, or wires; the corresponding $\kappa W/2$ and $\kappa D/2$ values are quite small, and hence, the $(\Delta T)/(\Delta T)^*$ values are small.

The temperature difference in the fuel is frequently the controlling design factor in production reactors where the coolant temperature rise is small.

(1) Woods, D. and Biehl, A. T., NAA-SR-138, Part I (1951) and Part II (1953).

Such reactors employing natural uranium fuel rods 3 cm in diameter could be operated at power level 10 per cent higher than would be calculated on the assumption that the neutron flux is uniform over a cross section of the fuel elements.

Use of neutron diffusion theory leads to predictions of the temperature change, $(\Delta T)^* - (\Delta T)$, about thirty per cent less than changes calculated from the experimental data.

Fig.1. Temperature Defect in Thick Fuel Elements.

THIS PAGE
WAS INTENTIONALLY
LEFT BLANK